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Technical Report

1. Introduction

It is a large-scale glass recycling scheme. Raw glass is collected in collection areas. From there it is transported to processing facilities, where it is sorted, cleaned, and processed to yield cullet (output glass). The cullet is then transported to customer factories, where it is used in glass making. We will create a scheme to determine which processing facilities we use and maximize our profits.

2. Case analysis

There are three main kinds of cost in the whole process:

1) The transportation cost of raw glass from each collection area to each facility.
2) The transportation cost of cullet from each facility to each factory.
3) The processing cost of each facility.

Our company can only gain revenue by selling the cullet glass to different factories. Therefore, the total profits equal to all selling revenue minus all transportation costs (including transportation costs from collection areas to facilities and transportation costs from facilities to factories) and minus all processing costs. And our goal is to maximize the total profits.

All raw glass will be collected from collection areas to processing facilities and assume that there would be no loss during the transportation. In every month the raw glass delivered to each facility will be less than that facility's capacity. In other words, the facility will neither accept more raw glass than its capacity nor keep them till next month to deal with. And assume that given the bids from each facility, we can get a maximal amount among all bids, and the maximal amount represents the total net amount it is willing to accept (net amount means the total raw glass minus mass loss part). So, the facility would not accept the amount of raw glass out of its maximal bid amount.

Assume that except the given mass loss rate during the processing in the facilities, there would be no other loss during the whole processing process. There are five different types of raw glass and it will be converted to three types of cullet based on the given convert rate. After summing up the amount of cullet for 12 months, we can get the amount of cullet for the whole year.

After the raw glass is processed in the facilities, it will be transported to the factory. Each factory has a limit capacity about the amount in the certain period, so they do not accept extra amount of cullet beyond their production capacity. Therefore, in each period, the total amount of cullet transported to the factory should not be greater than the maximum capacity amount. Meanwhile, once the whole year contract for each factory is settled and given the expected demand and allowed variation in demand, we can get an amount range, which we call it demand interval. And then the cullet each factory receives for the whole year must be in this range.

The interval can be calculated by the following formula:

$$[expected\ demand\ -\ allowed\ variation, expected\ demand\ +\ allowed\ variation]$$

3. Simplification

(1) Simplify the rule that small amount cannot be transported

Since this rule may result in the fact that for small collection area, in which the amount of raw glass is so small that these raw glass cannot be transported to get processed. Definitely it is not environmentally friendly at all. Considering that the given transportation cost is the unit transportation cost for raw glass and cullet, it may be unrealistic and not cost-effective to deliver small amount of those raw glass, if the amount of them are less than 25 tons. Thus, in order to simplify the calculation, we assume that the transportation cost will start from the cost of 25 tons. And if the raw glass equal or exceed 25 tons, we calculate this part at price of unit price. To sum up, for the amount less than 25 tons, we calculate it as the price of 25 tons, and then every additional ton will cost one more unit price.

(2) Simplify the rule about processing costs

Given different bid price, to simplify the model, we assume that for a specific amount, we split this amount to corresponding interval, and the bid price varies according to different interval. For example, in Figure 1, for x_3 tons raw glass, the bid price is different in every subinterval. For the first x_0 tons, the bid price is y_0 pound, for the next x_1-x_0 tons, the bid price is y_1 pound, for the next x_2-x_1 tons, the bid price is y_2 pound, and finally for the next x_3-x_2 tons, the bid price is y_3 pound. After analysing the data, we think that this simplified assumption has a small impact on the processing costs, and it improves the running time of our bidding model.

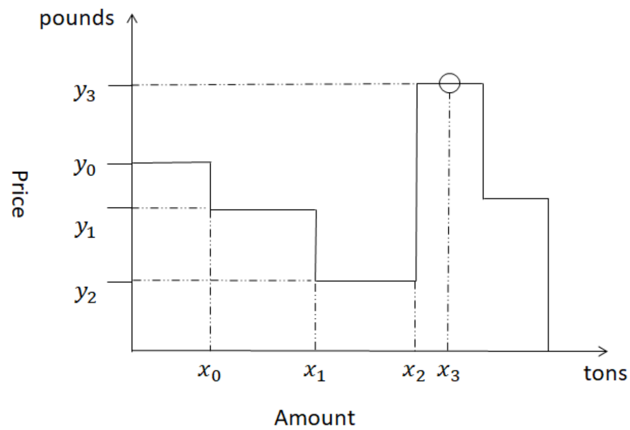

Figure 1 Piecewise bid price

4. Result and discussion

(1) Results

After all these works, we get the following results. Briefly we can get the costs and profits for three scenarios.

Table 1 Result summary Unit: million pound

	Scenario 1	Scenario 2	Scenario 3
Revenues	117.44	130.95	144.56
Transportation cost (areas to facilities)	13.50	15.28	16.76
Transportation cost (facilities to factories)	6.65	7.92	10.37
Processing cost	41.67	46.49	51.62
Profits	55.62	61.26	65.81

As we can see in Table 1, the profits are growing as the amount of raw glass is growing, which means these results are logical because we can allocate more raw glass to process and sell more products.

Besides, although the amount of raw glass in each area varies in 3 scenarios, we can still get a robust allocation plan: the general allocation from assigned collection areas to assigned processing facilities and then from assigned processing facilities to assigned factories will keep the same, except the specific allocation amount will change. This is reasonable because as the total amount collected from area changes, the allocation amount will change as well. To be more specific, if we choose to deliver 22618 tons cullet from processing facilities 1 to factory 1 in scenario 1, we will also deliver from facility 1 to factory 1 in scenario 2 and scenario 3 when the total amount we collect changes. Similarly, if we don't deliver from some processing facilities to factories in one scenario, we will not deliver in other scenarios. For instance, we don't deliver from facility 1 to factory 2 in scenario 1, and we also won't deliver between facility 1 and factory 2 in other scenarios. Thus, in this way, we can firstly make a preliminary allocation scheme between areas and facilities, and between facilities and factories.

Also, after analysing the amount delivered from each processing facility to the factories, we can see that the annual amount of all kinds of cullet each factory gets will keep in the demand interval, which can be calculated from the given demand and allowed variation in demand, and also will not be too close to the boundary value. Consequently, we can conclude that our model is elastic when applied in practice and allows a certain degree of deviation. And normally the amount we supply for each factory will satisfy each factory's expectation rather than deviate from it.

Table 2 Difference in three bid price plan Unit: pound

facility	Bid price		
	Scenario 1	Scenario 2	Scenario 3
1	28.3	28	28
9	25.2	25	25

As shown in the Table 2, the bid plan doesn't fit facility 1 and 9 only, but also the differences are slight enough to ignore, which indicates that to simplify, we can choose the bid plan for scenario 2 and 3 as the final bid plan for all scenarios and it has popularization in a way. The final bid plan is shown in Figure 2 below.

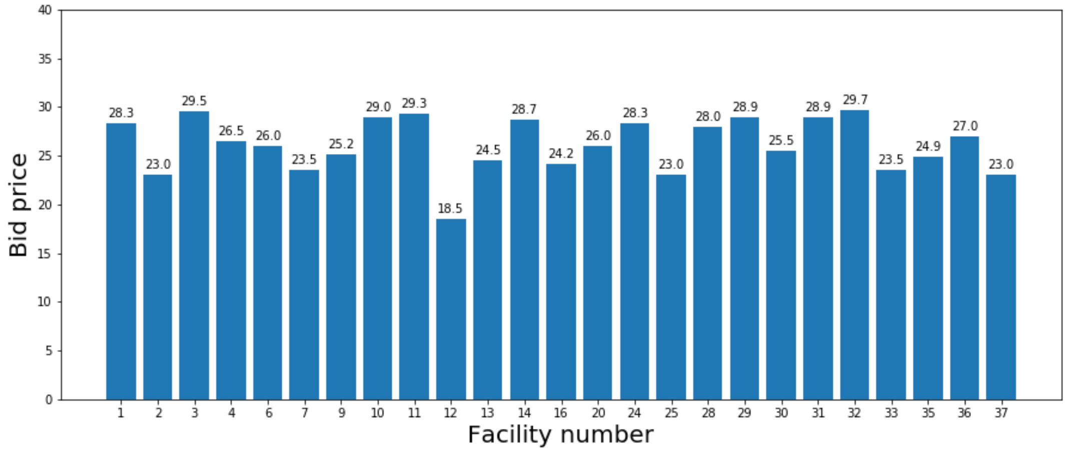

Figure 2 Final bid price plan

(2) Risk

Admittedly, we cannot ignore the risk behind it. It is still possible that we cannot meet the actual demand, because the demand variation is based on past experience. Thus, once something uncontrollable happens, for example, if the amount of raw glass in each area changes rapidly, we cannot collect enough raw glass to produce cullet and then provide to factories, resulting in our default to some factories. To solve this problem, we put forward a suggestion below.

(3) Running time

To sum up, it takes around 40 minutes to run our model for each scenario and get the result. Considering the large amount of data and complexity of the scheme, we consider this time is reasonable and acceptable. Therefore, it is possible to run multiple bidding rounds per day.

5. Suggestion

After analysing the model and the result, we can provide the following suggestions and problems which are worth to discuss further.

(1) Not necessarily collect all areas or reduce the collection frequency (for small amount less than 25 tons）

After observing the given data, we find the total glass in collection area 167 is always less than 25 tons in the whole year in three scenarios. There is no need to collect all the raw glass in such small collection area, because the transportation cost may be larger than their corresponding selling revenue. There are two ways to solve this problem.

First, we don't need to collect all areas. By comparing our schemes, we find if we do not collect all amount of raw glass, the total profit can increase 100 thousand pounds.

Second, we can introduce a proposal that for the small collection area (the total amount of raw glass is less than 25 tons), the frequency of collecting the raw glass in this period can be decreased to every other period. It is practical because the transportation frequency now is once a month. And if we change it to once two months or once three months (it needs to be discussed further with respect to realistic situation), the raw glass would not increase too much burden on the environment but increase our total profit by saving transportation cost.

(2) Allow extra raw glass to be transported to facilities

From the assumption above, the facilities will not accept more raw glass because of the month capacity of the processing facilities. It will be better if we can pay them to keep certain amount of surplus raw glass until next month. Although it may cost some money in certain period, we can still gain profit from it because of less transportation cost.

(3) Negotiate with factories to provide them more cullet in a lower price for the surplus

There is a business rule that the amount of cullet sent to the factories each month cannot exceed the capacity of the factories, which means the factories cannot keep the cullet till next month to deal with. We suggest making some adjustments on this rule. We can negotiate with the factories and then establish a long-term cooperative relationship with the factory: we agreed to sell the cullet to the factory at a relatively lower fixed price, and the factory can accept a slightly higher total amount of glass than the processing capacity, as long as we can save more transportation cost for the surplus amount than selling to other factories. For example, in scenario 3, the capacity of factory 1 for clear cullet is 43548 tons, and total amount delivered to factory 1 from all facilities equals to 43548 tons, which exactly reaches its limit. After analysis, we find that it is due to the relatively low transportation cost from facilities to factory 1. Therefore, if we can negotiate with factory 1 and persuade them to accept more amount of cullet by giving them a lower price, we can save more transportation costs, although we gain less revenue per ton cullet. In consequence, we increase our profit. At the same time, the factory may be willing to accept more cullet if the discount provided by our company could make up for their inventory costs. In the end, our company and factories are able to reach the win-win situation.

Appendix

To help understanding, we list some kernel equations here.

1) $$\boldsymbol{maximise}\ \ profit = revenue - pre\ transportation\ cost - post\ transportation\ cost - processing\ cost$$
2) $facility\ amount = x * (1 - mass\ loss)$
3) $amount\ after\ converted = facility\ amount * convert$
4) $facility\ amount \leq facility\ capacity$
5) If $factory\ amount\ in\ all\ period > 0$

   Then $minimum\ capacity\ of\ factory \leq factory\ amount\ in\ all\ period \leq maximum\ capacity\ of\ factory$

6) For each facility, to get the bid price, we use piecewise to specify all intervals the amount lies in, and then get the corresponding bid price.

$$total\ year\ amount = x = \sum \delta_i$$

$$processing\ cost = \delta_i * bid\ price\ i$$

$$\delta_i \leq (\bar{x}_i - \bar{x}_{i-1}) * z_i, for\ all\ i\ in\ \{1,,,n\}$$

$$\delta_i \geq (\bar{x}_i - \bar{x}_{i-1}) * z_{i+1}, for\ all\ i\ in\ \{1,,,n-1\}$$

$$z_i\ is\ 0\ or\ 1, \delta_i > 0$$

7) To see if the delivered amount is greater than 25, still, we use piecewise method to do it.

$$delivered\ amount = x = \sum \delta_i$$

$$pre\ transportation\ cost = (25 * m_i + \delta_i) * transportation\ cost\ per\ unit(i)$$

$$\delta_i \leq (\bar{x}_i - \bar{x}_{i-1}) * n_i, for\ all\ i\ in\ \{1,,,n\}$$

$$\delta_i \geq (\bar{x}_i - \bar{x}_{i-1}) * n_{i+1}, for\ all\ i\ in\ \{1,,,n-1\}$$

$$\delta_i \leq M * m_i, for\ all\ i\ in\ \{1,,,n\}$$

$$m_i, n_i is\ 0\ or\ 1,\ \delta_i > 0$$