TTD Design (WIP)

Hangjian Li. version: 0.3

What is dimensional bidding provided by TradeDesk?

- **Ad Group**: made up of multiple elements that combine into a cohesive strategy.
  - Audience.
  - Bid Lists.
  - Budgets.
  - Creatives.
  - Base and Max bids, defined by CPM.
- Dimensional bidding.
  - A **dimension** is a targeting item that you can target, block, or apply bid factors to. e.g, "Site", "Ad Format", "Geolocation".
- Example of a **bid list**


```
{
        "BidListId": "2131712",
    "Name": "Bidlist 1",
    "BidListSource": "User",
    "BidListAdjustmentType": "Optimized",
    "ResolutionType": "ApplyMultiplyAdjustment",
    "BidLines": [
        {
            "BidLineId": "1216340",
            "BidAdjustment": 2.030000,
            "DomainFragment": "com.cardgame.solitaire.flat",
            "SupplyVendorId": 1,
            "PublisherId": "f77344fc701f4b1f981238390e0b8ffb",
            "ImpressionPlacementValue": "f37ac1804f8040a4b49295e5d13167ff"
        },
        {
            "BidLineId": "1216341",
            "BidAdjustment": 1.020000,
            "DomainFragment": "com.merriamwebster",
            "SupplyVendorId": 118,
            "PublisherId": "11653",
            "ImpressionPlacementValue": "177976"
        },
    ]
}
```


- Currently, we only use `BlockList` and `TargetList`.

General Requirements

- Objectives:
  - Make use of the `Optimized` bid list type.
  - **Maximize total (attributed) revenue** subject to the budget constraint.
- Our deliverable
  - Bid dimensions.
  - An optimization framework that can produce the bid factors.

Data available

- `attributed sales` (14 or 30 days attribution window)
- `Impression cost` (equal to bid price under first price auction)
- `Impression attributes` (time, geo, publisher, platform, etc.)
- `campaign metadata` (budget, start and end dates, etc.)

Design 1: disjoint dimensions

- **Objective**: maximize ROAS under budget constraint for each **adgroup**.
- **Assumptions**:
  - Cost per impression (**CPI**) (under first price auction): $b_i = b_0 \cdot f_i$
    * i refers to dimension i.
    * assume based bid $b_0 = 1$ for simplicity, and we manipulate the bid factor f_i for each dimension
  - Number of impressions won: $\mathbb{E}[n_i] = g(f_i)$, e.g., $n_i = \beta_i \cdot f_i$.
  - Revenue per impression (RPI_i)
    * independent from b_i.
    * can be estimated for each dimension i.
    * property of dimension i and stay constant during the day.
- **Notations**
  - $CPI_i = f_i$
  - $n_i = \mathcal{N}\left(\beta_i \cdot f_i, \sigma^2\right)$.
  - Total spending in i: S_i.
  - Total revenue in i: R_i.
  - Daily budget constraint: B.

	State1	State2	State3	
Site 1	1 ($= n_1 = \beta_1 f_1$)	2 ($\beta_2 f_2$)	3 ($\beta_3 f_3$)	**6**
Site 2	12 ($\beta_4 f_4$)	1 ($\beta_5 f_5$)	6 ($\beta_6 f_6$)	**19**
Site 3	7 ($\beta_7 f_7$)	5 ($\beta_8 f_8$)	9 ($\beta_9 f_9$)	**21**
	20	**8**	**18**	

- Parameter estimation
  - RPI_i can be estimated based on attributed sales for dimension i.
  - β_i can be estimated through linear regression.
- **Optimization**

$$\max_{f_i} \sum_{i=1}^{m} RPI_i \cdot g(f_i)$$

$$\text{subject to } \sum_{i=1}^{m} S_i = \sum_{i=1}^{m} f_i \cdot g(f_i) = B.$$

From the assumptions we know

$$\begin{aligned} S_i &= n_i \cdot CPI_i = \beta_i \cdot f_i^2 \\ R_i &= RPI_i \cdot n_i = RPI_i \cdot \beta_i \cdot f_i = RPI_i \cdot \sqrt{\beta_i \cdot S_i}. \end{aligned}$$

$$\max_{f_i} \sum_{i=1}^{m} RPI_i \cdot \beta_i \cdot f_i$$
$$\text{subject to} \quad \sum_{i=1}^{m} f_i^2 \cdot \beta_i = B$$

The Lagrangian can be written as:

$$\mathcal{L}(\lambda, \{_\infty, \ldots, \{_\Updownarrow) = \sum_{i=1}^{m} PRI_i \cdot \beta_i \cdot f_i - \lambda \left(\sum_{j=1}^{m} f_j^2 \cdot \beta_j - B \right)$$

$$\begin{cases} \nabla_\lambda \mathcal{L}(\lambda, f_1, \ldots, f_m) & = \sum_{j=1}^{m} \beta_j f_j^2 - B = 0 \\ \nabla_{f_i} \mathcal{L}(\lambda, f_1, \ldots, f_m) & \propto RPI_i - 2\lambda f_i = 0 \end{cases}$$

Solving these two equations gives the **optimal choice of bid factor $\hat{f}_i$**:

$$\hat{f}_i^2 = \frac{RPI_i^2}{\sum_{j=1}^{m} RPI_j^2 \cdot \beta_j} \cdot B$$

The spending on the ith dimension at optimum is

$$S_i = \frac{RPI_i^2 \cdot \beta_i}{\sum_{j=1}^{m} RPI_j^2 \cdot \beta_j} \cdot B$$

Limitation

- Not scalable with more dimensions and groups need to be considered.
  - *curse of dimensionality* if considering all combinations of bid dimensions. Not enough impression at each section.
- Not making full use of the multidimensional bidding feature.

Design 2: overlapping bid dimensions

- Motivation
  - bid price = $b_0 * f_1 * f_2$
- **Example:**

  We now consider two **types** of dimensions (superscript): site and state, each has 3 unique **groups** (subscript).

	State1 (f_1^2)	State2 (f_2^2)	State3 (f_3^2)	
Site 1 (f_1^1)	1	2	3	**6**
Site 2 (f_2^1)	4	5	6	**15**
Site 3 (f_3^1)	7	8	9	**24**
	12	**15**	**18**	

- **Notations**
  - ~~Let j index the marginal dimensions. i.e., $j = 1, \ldots, m$, where $m = K \cdot I$ if each dimension (bid list) has I categories.~~
  - If the example above, $K = 2, I = 3$.

- **Assumptions**
  - RPI_j^d is defined for each marginal dimension
    * constant
    * estimable
  - Number of impressions won at marginal dimension (d, j):

$$\begin{aligned}\mathbb{E}\left[n_j^d\right] &= g(f_1^1, \ldots, f_I^K) \\ &= \left(\beta_j^d \cdot f_j^d\right) \cdot \left(\prod_{k \neq d}^{K} \sum_{i=1}^{I_k} \beta_i^k \cdot f_i^k\right), \quad k, d \in [K]\end{aligned}$$

    * Example:
      · Recall the bidding price is defined as $b_{12} = f^1 \times f^2$ in TTD API. In Design 1 we would have that $\mathbb{E}\left[n_{12}\right] = \beta_{12} \times b_{12}$.

$$\begin{aligned}\mathbb{E}\left[n_1^1\right] &= \beta_1^1 \cdot f_1^1 \cdot (\beta_1^2 f_1^2 + \beta_2^2 f_2^2 + \beta_3^2 f_3^2) \\ \mathbb{E}\left[n_2^1\right] &= \beta_2^1 \cdot f_2^1 \cdot (\beta_1^2 f_1^2 + \beta_2^2 f_2^2 + \beta_3^2 f_3^2) \\ \mathbb{E}\left[n_3^1\right] &= \beta_3^1 \cdot f_3^1 \cdot (\beta_1^2 f_1^2 + \beta_2^2 f_2^2 + \beta_3^2 f_3^2)\end{aligned}$$

$$\begin{aligned}\mathbb{E}\left[n_1^2\right] &= \beta_1^2 \cdot f_1^2 \cdot (\beta_1^1 f_1^1 + \beta_2^1 f_2^1 + \beta_3^1 f_3^1) \\ \mathbb{E}\left[n_2^2\right] &= \beta_2^2 \cdot f_2^2 \cdot (\beta_1^1 f_1^1 + \beta_2^1 f_2^1 + \beta_3^1 f_3^1) \\ \mathbb{E}\left[n_3^2\right] &= \beta_3^2 \cdot f_3^2 \cdot (\beta_1^1 f_1^1 + \beta_2^1 f_2^1 + \beta_3^1 f_3^1)\end{aligned}$$

      · If there were one more type of dimension, e.g., **time**, with 3 unique **groups**, the estimated total impresion from `Site1` becomes

$$\mathbb{E}\left[n_1^1\right] = \beta_1^1 \cdot f_1^1 \cdot (\beta_1^2 f_1^2 + \beta_2^2 f_2^2 + \beta_3^2 f_3^2) \cdot (\beta_1^3 f_1^3 + \beta_2^3 f_2^3 + \beta_3^3 f_3^3)$$

  - Cost for marginal dimension (d, j):
    * Average cost per impression

$$CPI_j^d = \left(\alpha_j^d \cdot f_j^d\right) \cdot \left(\prod_{k \neq d}^{K} \sum_{i=1}^{I_k} \alpha_i^k \cdot f_i^k\right), \quad k, d \in [K]$$

    * Total spending for marginal (d, j)

$$S_j^d = n_j^d \cdot CPI_j^d$$

- **Remark**
  - Compared to Model 1, in Model 2 we make the following additional assumptions:
    * $\beta_{j,k} = \beta_j^{d_1} \cdot \beta_k^{d_2}$ (decomposable impression number coefficients).
    * $f_{j,k} = f_j^{d_1} \cdot f_k^{d_2}$ (decomposable bid feactor).
  - The number of parameters reduces from $O(I^K)$ to $O(IK)$.
- **Parameter Estimation**

– l_2 Loss function

$$l(\beta_1^1, \ldots, \beta_3^2 \mid f_1^1, \ldots, f_3^2, n_1^1, \ldots, n_3^2) = \sum_{k=1}^{K} \sum_{i=1}^{I_k} \| n_i^k - \left(\beta_j^d \cdot f_j^d\right) \cdot \left(\prod_{k \neq d}^{K} \sum_{i=1}^{I_k} \beta_i^k \cdot f_i^k \right) \|_2^2$$

– Matrix form

$$\begin{bmatrix} n_1^1 \\ n_2^1 \\ n_3^1 \\ n_1^2 \\ n_2^2 \\ n_3^2 \end{bmatrix} = \begin{bmatrix} \beta_1^2 & & & & & \\ & \beta_2^2 & & & & \\ & & \beta_3^2 & & & \\ & & & \beta_1^1 & & \\ & & & & \beta_2^1 & \\ & & & & & \beta_3^1 \end{bmatrix} \cdot \begin{bmatrix} f_1^2 f_1^1 & f_1^2 f_2^1 & f_1^2 f_3^1 & & & \\ f_2^2 f_1^1 & f_2^2 f_2^1 & f_2^2 f_3^1 & & & \\ f_3^2 f_1^1 & f_3^2 f_2^1 & f_3^2 f_3^1 & & & \\ & & & f_1^1 f_1^2 & f_1^1 f_2^2 & f_1^1 f_3^2 \\ & & & f_2^1 f_1^2 & f_2^1 f_2^2 & f_2^1 f_3^2 \\ & & & f_3^1 f_1^2 & f_3^1 f_2^2 & f_3^1 f_3^2 \end{bmatrix} \begin{bmatrix} \beta_1^1 \\ \beta_2^1 \\ \beta_3^1 \\ \beta_1^2 \\ \beta_2^2 \\ \beta_3^2 \end{bmatrix}$$

– Data generation

We need to observe $\{n_i^k\}_{ik}$ for a range of values of $\{f_i^k\}_{ik}$ in order to estimate $\{\beta_i^k\}_{ik}$.

- **Optimization**

– Total revenue R:

$$R = \frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{I_k} n_i^k \cdot RPI_i^k.$$

– Total spending S:

$$S = B = \frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{I_k} n_i^k \cdot CPI_i^k.$$

– Objective function

$$\max_{\{f_i^d\}} \sum_{d=1}^{K} \sum_{j=1}^{D_d} \left(\beta_j^d \cdot f_j^d\right) \cdot \left(\prod_{k \neq d}^{K} \sum_{i=1}^{D_k} \beta_i^k \cdot f_i^k \right) \cdot RPI_j^d$$

$$\text{subject to} \quad \sum_{d=1}^{K} \sum_{j=1}^{D_d} \left[\beta_j^d \cdot \alpha_j^d \cdot (f_j^d)^2 \cdot \prod_{k \neq d}^{K} \left(\sum_{i=1}^{D_k} \beta_i^k \cdot f_i^k \right) \cdot \left(\sum_{i=1}^{D_k} \alpha_i^k \cdot f_i^k \right) \right] = K \cdot B$$

– Lagrangian

$$\begin{aligned} \mathcal{L}(f_j^d, \lambda) = & \sum_{d=1}^{K} \sum_{j=1}^{D_d} \left(\beta_j^d \cdot f_j^d\right) \cdot \left(\prod_{k \neq d}^{K} \sum_{i=1}^{D_k} \beta_i^k \cdot f_i^k \right) \cdot RPI_j^d \\ & - \lambda \left(\sum_{d=1}^{K} \sum_{j=1}^{D_k} \left[\beta_j^d \cdot \alpha_j^d \cdot (f_j^d)^2 \cdot \prod_{k \neq d}^{K} \left(\sum_{i=1}^{D_k} \beta_i^k \cdot f_i^k \right) \cdot \left(\sum_{i=1}^{D_k} \alpha_i^k \cdot f_i^k \right) \right] - K \cdot B \right) \end{aligned}$$

– Quasi-Newton's method (`BFGS`, `L-BFGS-B`, `SLSQP`).

API

- start from one `ad_group` and use statistical test to determine if the improvement in KPI is significant compared to control `ad_groups`. (`t-test`)
- each bid list must contain a distinct combination type (e.g., site and geo, device type and ad environment, etc.).
- API example response


```json
{
    "IsGlobal": false,
    "BidListDimensions": [
        "HasDomainFragmentId",
        "HasSupplyVendorId",
        "HasImpressionPlacementId",
        "HasPublisherId"
    ],
    "BidListId": "2131712",
    "Name": "Bidlist 1",
    "BidListSource": "User",
    "BidListAdjustmentType": "Optimized",
    "ResolutionType": "ApplyMultiplyAdjustment",
    "BidLines": [
        {
            "BidLineId": "1216340",
            "BidAdjustment": 2.030000,
            "DomainFragment": "com.cardgame.solitaire.flat",
            "SupplyVendorId": 1,
            "PublisherId": "f77344fc701f4b1f981238390e0b8ffb",
            "ImpressionPlacementValue": "f37ac1804f8040a4b49295e5d13167ff"
        },
        {
            "BidLineId": "1216341",
            "BidAdjustment": 1.020000,
            "DomainFragment": "com.merriamwebster",
            "SupplyVendorId": 118,
            "PublisherId": "11653",
            "ImpressionPlacementValue": "177976"
        },
        {
            "BidLineId": "1216342",
            "BidAdjustment": 1.020000,
            "DomainFragment": "com.pandora.android",
            "SupplyVendorId": 50,
            "PublisherId": "71711271",
            "ImpressionPlacementValue": "cKClcPNTu1Te"
        },
        {
            "BidLineId": "1216343",
            "BidAdjustment": 1.020000,
            "DomainFragment": "com.strawdogstudios.simonscatcrunchtime",
            "SupplyVendorId": 99,
            "PublisherId": "FYBER-139222",
            "ImpressionPlacementValue": "530c4d2894e444a1976f41d527c03a07"
        }
    ],
```

```json
    "BidListOwner": "AdGroup",
    "BidListOwnerId": "sberg7l",
    "IsAvailableForLibraryUse": false
}
```

Associate the new bid list to an ad group

```json
{
    "AdGroupId": "sberg7l",
    "AssociatedBidLists": [
        {
            "BidListId": "2131711",
            "IsEnabled": true,
            "IsDefaultForDimension": true,
            "BidListAdjustmentType": "Optimized",
        },
        {
            "BidListId": "2131712",
            "IsEnabled": true,
            "IsDefaultForDimension": true,
            "BidListAdjustmentType": "Optimized",
        }
    ]
}
```