TTD Design

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API

- start from one `ad_group` and use statistical test to determine if the improvement in KPI is significant compared to control `ad_groups`. (`t-test`)
- each bid list must contain a distinct combination type (e.g., site and geo, device type and ad environment, etc.).
- API example response


```
{
    "IsGlobal": false,
    "BidListDimensions": [
        "HasDomainFragmentId",
        "HasSupplyVendorId",
        "HasImpressionPlacementId",
        "HasPublisherId"
    ],
    "BidListId": "2131712",
    "Name": "Bidlist 1",
    "BidListSource": "User",
    "BidListAdjustmentType": "Optimized",
    "ResolutionType": "ApplyMultiplyAdjustment",
    "BidLines": [
        {
            "BidLineId": "1216340",
            "BidAdjustment": 2.030000,
            "DomainFragment": "com.cardgame.solitaire.flat",
            "SupplyVendorId": 1,
            "PublisherId": "f77344fc701f4b1f981238390e0b8ffb",
            "ImpressionPlacementValue": "f37ac1804f8040a4b49295e5d13167ff"
        },
        {
            "BidLineId": "1216341",
            "BidAdjustment": 1.020000,
            "DomainFragment": "com.merriamwebster",
            "SupplyVendorId": 118,
            "PublisherId": "11653",
            "ImpressionPlacementValue": "177976"
        },
        {
            "BidLineId": "1216342",
            "BidAdjustment": 1.020000,
            "DomainFragment": "com.pandora.android",
            "SupplyVendorId": 50,
            "PublisherId": "71711271",
            "ImpressionPlacementValue": "cKClcPNTu1Te"
        },
        {
            "BidLineId": "1216343",
            "BidAdjustment": 1.020000,
            "DomainFragment": "com.strawdogstudios.simonscatcrunchtime",
            "SupplyVendorId": 99,
```

```
            "PublisherId": "FYBER-139222",
            "ImpressionPlacementValue": "530c4d2894e444a1976f41d527c03a07"
        }
    ],
    "BidListOwner": "AdGroup",
    "BidListOwnerId": "sberg7l",
    "IsAvailableForLibraryUse": false
}
```

Associate the new bid list to an ad group

```
{
    "AdGroupId": "sberg7l",
    "AssociatedBidLists": [
        {
            "BidListId": "2131711",
            "IsEnabled": true,
            "IsDefaultForDimension": true,
            "BidListAdjustmentType": "Optimized",
        },
        {
            "BidListId": "2131712",
            "IsEnabled": true,
            "IsDefaultForDimension": true,
            "BidListAdjustmentType": "Optimized",
        }
    ]
}
```

UI

- PACING_TYPE_AHEAD Ahead pacing attempts to spend faster than evenly, to make sure the entire budget is spent by the end of the flight.
- PACING_TYPE_ASAP Spend all of pacing budget amount as quick as possible.
- PACING_TYPE_EVEN Spend a consistent budget amount every period of time.
- Ad group: assembling targeting specifics and strategies

Constraints

- `impression` table refreshing frequecy is **daily**.
  - Start of **day 1**, we provide a sequence of bid factors between TTD allowed **MIN** and **MAX** bid prices for each bid dimension
  - During **day 1**, we bid using each bid factor k_t for a **period of time** h_t.
  - After **day 1**, collect
    * number of impressions n_t won with k_t during h_t.
    * compute the average impression cost for each k_t.
    * estiamte α and β for each dimension.
  - Starting from **day 2**, bid with the estimated α and β.
  - Retrain after a few days.
- *curse of dimensionality* if considering all combinations of bid dimensions.
  - assume stable marginal distribution / percentage of impressions p_1, p_2, p_3,
  - *cost per impression* (CPI) is tricky

Design 1: disjoint dimensions

- **Objective**: maximize ROAS under budget constraint for each **adgroup**.
- **Assumptions**:
  - Cost per impression: b_i
  - Number of impressions won: $n_i = g(b_i)$, e.g., $n_i = \beta_i \cdot b_i$.
  - Revenue per impression (RPI_i)
    * independent from b_i.
    * can be estimated for each dimension i.
    * property of dimension i and stay constant during the day.
- **Notations**
  - $CPI_i = b_i = b_0 \cdot f_i$. (assume $b_0 = 1$ for simplicity)
  - $n_i = \beta_i \cdot b_i$.
  - Total spending in i: S_i.
  - Total revenue in i: R_i.
  - Daily budget constraint: B.
- **Optimization**

$$\max_{f_i} \sum_{i=1}^{m} RPI_i \cdot g(f_i)$$

$$\text{subject to } \sum_{i=1}^{m} S_i = \sum_{i=1}^{m} f_i \cdot g(f_i) = B.$$

From the assumptions we know

$$\begin{aligned} S_i &= n_i \cdot CPI_i = \beta_i \cdot f_i^2 \\ R_i &= RPI_i \cdot n_i = RPI_i \cdot \beta_i \cdot f_i = RPI_i \cdot \sqrt{\beta_i \cdot S_i}. \end{aligned}$$

$$\begin{aligned} &\max_{f_i} \sum_{i=1}^{m} RPI_i \cdot \beta_j \cdot f_i \\ &\text{subject to } \sum_{i=1}^{m} f_i^2 \cdot \beta_j = B \end{aligned}$$

The Lagrangian can be written as:

$$\mathcal{L}(\lambda, \{_\infty, \ldots, \{_\updownarrow) = \sum_{i=1}^{m} PRI_i \cdot \beta_i \cdot f_i - \lambda \left(\sum_{j=1}^{m} f_j^2 \cdot \beta_j - B \right)$$

$$\begin{cases} \nabla_\lambda \mathcal{L}(\lambda, f_1, \ldots, f_m) & = \sum_{j=1}^{m} \beta_j f_j^2 - B = 0 \\ \nabla_{f_i} \mathcal{L}(\lambda, f_1, \ldots, f_m) & \propto RPI_i - 2\lambda f_i = 0 \end{cases}$$

Solve these two equations gives the optimal soluation:

$$\hat{f}_i^2 = \frac{RPI_i^2}{\sum_{j=1}^{m} RPI_i^2 \cdot \beta_j} \cdot B$$

The spending on the ith dimension at optimum is

$$S_i = \frac{RPI_i^2 \cdot \beta_i}{\sum_{j=1}^{m} RPI_i^2 \cdot \beta_j} \cdot B$$

Design 2: overlapping bid dimensions

- **Example:**

	State1 (f_1^2)	State2 (f_2^2)	State3 (f_3^2)	
Site 1 (f_1^1)	1	2	3	**6**
Site 2 (f_2^1)	4	5	6	**15**
Site 3 (f_3^1)	7	8	9	**24**
	12	**15**	**18**	

- **Notations**
  - Let j index the marginal dimensions. i.e., $j = 1, \ldots, m$, where $m = K \cdot D$ if each dimension (bid list) has D categories.
  - If the example above, $K = 2, D = 3$.
- **Assumptions**
  - RPI_j^d is defined for each marginal dimension
    * constant
    * estimable
  - Number of impressions won at marginal dimension (d, j):

$$\begin{aligned} n_j^d &= g(f_1^1, \ldots, f_D^K) \\ &= \left(\beta_j^d \cdot f_j^d\right) \cdot \left(\prod_{k \neq d}^{K} \sum_{i=1}^{D_k} \beta_i^k \cdot f_i^k \right), \quad k, d \in [K] \end{aligned}$$

  - Cost for marginal dimension (d, j):
    * Average cost per impression

$$CPI_j^d = \left(\alpha_j^d \cdot f_j^d\right) \cdot \left(\prod_{k \neq d}^{K} \sum_{i=1}^{D_k} \alpha_i^k \cdot f_i^k \right), \quad k, d \in [K]$$

    * Total spending for marginal (d, j)

$$S_j^d = n_j^d \cdot CPI_j^d$$

- **Remark**
  - Compared to Model 1, in Model 2 we make the following additional assumptions:
    * $\beta_{j,k} = \beta_j^{d_1} \cdot \beta_k^{d_2}$ (decomposable impression number coefficients).
    * $f_{j,k} = f_j^{d_1} \cdot f_k^{d_2}$ (decomposable bid feactor).
  - The number of parameters reduces from $O(D^K)$ to $O(DK)$.
- **Optimization**

– Total revenue R:

$$R = \frac{1}{K}\sum_{k=1}^{K}\sum_{j=1}^{D_k} n_j^k \cdot RPI_j^k.$$

– Total spending S:

$$S = B = \frac{1}{K}\sum_{k=1}^{K}\sum_{j=1}^{D_k} n_j^k \cdot CPI_j^k.$$

– Objective function

$$\max_{\{f_i^d\}} \sum_{d=1}^{K}\sum_{j=1}^{D_d} \left(\beta_j^d \cdot f_j^d\right) \cdot \left(\prod_{k\neq d}^{K}\sum_{i=1}^{D_k}\beta_i^k \cdot f_i^k\right) \cdot RPI_j^d$$

$$\text{subject to} \quad \sum_{d=1}^{K}\sum_{j=1}^{D_d}\left[\beta_j^d \cdot \alpha_j^d \cdot (f_j^d)^2 \cdot \prod_{k\neq d}^{K}\left(\sum_{i=1}^{D_k}\beta_i^k \cdot f_i^k\right) \cdot \left(\sum_{i=1}^{D_k}\alpha_i^k \cdot f_i^k\right)\right] = K \cdot B$$

– Lagrangian

$$\begin{aligned}\mathcal{L}(f_j^d, \lambda) = &\sum_{d=1}^{K}\sum_{j=1}^{D_d} \left(\beta_j^d \cdot f_j^d\right) \cdot \left(\prod_{k\neq d}^{K}\sum_{i=1}^{D_k}\beta_i^k \cdot f_i^k\right) \cdot RPI_j^d \\ &- \lambda\left(\sum_{d=1}^{K}\sum_{j=1}^{D_k}\left[\beta_j^d \cdot \alpha_j^d \cdot (f_j^d)^2 \cdot \prod_{k\neq d}^{K}\left(\sum_{i=1}^{D_k}\beta_i^k \cdot f_i^k\right) \cdot \left(\sum_{i=1}^{D_k}\alpha_i^k \cdot f_i^k\right)\right] - K \cdot B\right)\end{aligned}$$

$$\begin{aligned}0 = \frac{\partial}{\partial f_j^d}\mathcal{L}(f_j^d, \lambda) = &\beta_j^d \cdot \left(\prod_{m\neq d}^{K}\sum_{i=1}^{D_m}\beta_i^m \cdot f_i^m\right) \cdot RPI_j^d + \sum_{k\neq d}\sum_{j=1}^{D_k}\left(\beta_j^k \cdot f_j^k\right) \cdot \left(\beta_j^d \prod_{m\neq\{d,k\}}\sum_{i=1}^{D_m}\beta_i^m \cdot f_i^m\right) \\ &- \lambda\left[2\beta_j^d \cdot \alpha_j^d \cdot f_j^d \cdot \prod_{k\neq d}^{K}\left(\sum_{i=1}^{D_m}\beta_i^m \cdot f_i^m\right) \cdot \left(\sum_{i=1}^{D_k}\alpha_i^k \cdot f_i^k\right)\right] \\ &- \lambda\left[\sum_{m\neq d}^{K}\sum_{n=1}^{D_m}\beta_n^m \cdot \alpha_n^m \cdot (f_n^m)^2 \cdot \left(\beta_j^d \prod_{k\neq\{m,d\}}^{K}\left(\sum_{i=1}^{D_k}\alpha_i^k \cdot f_i^k\right) + \alpha_j^d \prod_{k\neq\{m,d\}}^{K}\left(\sum_{i=1}^{D_k}\beta_i^k \cdot f_i^k\right)\right)\right]\end{aligned}$$

$$0 = \frac{\partial}{\partial\lambda} = \left(\sum_{d=1}^{K}\sum_{j=1}^{D_k}\left[\beta_j^d \cdot \alpha_j^d \cdot (f_j^d)^2 \cdot \prod_{k\neq d}^{K}\left(\sum_{i=1}^{D_k}\beta_i^k \cdot f_i^k\right) \cdot \left(\sum_{i=1}^{D_k}\alpha_i^k \cdot f_i^k\right)\right] - K \cdot B\right)$$

– Equality-constrained Newton's method.

PID controller

- $e(t)$: difference in target spending and current spending of the dimension with **MAX** roas

$$u(t) = K_p e(t) + K_d \frac{d}{dt}e(t) + K_i \int_0^t e(\tau)d\tau$$

Q&A

1. how is budget allocated across **ad groups**? We need fixed budget for each ad group.
    1. Ans:
2. Checked ROAS for campaign `n3b53qu` for a given date
    1. The number of impressions **matched**
    2. Total spending **matched**
    3. Total revenue does ***not match*** because I don't use match rate to inflat **store sales**
3. Are changes of bid cap uniform across all dimensions?