ETH zürich D BSSE Computational Biology Group

Evolutionary Dynamics

Exercises 8

Prof. Dr. Niko Beerenwinkel
Dr. Katharina Jahn
Dr. Rob Noble
14th November 2019

Problem 1: Weak selection **(5 points)**

Consider the general two-strategy game

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array}$$

in a finite population of size N.

(a) Show that for weak selection, $w \ll 1$, the fixation probability of strategy A is given by

$$\rho_A = \frac{1}{N} \frac{1}{1-(\alpha N - \beta)w/6}$$

with $\alpha = a + 2b - c - 2d$ and $\beta = 2a + b + c - 4d$.

You can use the following formulae:

(i) For small w, one can approximate $\prod_{i=1}^{k}(1-wx_i) \approx 1 - w\sum_{i=1}^{k} x_i$.

(ii) For small w, it holds $\frac{1-wy}{1-wz} \approx 1 - w(y-z)$.

(iii) $\sum_{k=1}^{N}\sum_{i=1}^{k} i = N(N+1)(N+2)/3!$.

Now consider the specific game

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 21 & 2 \\ 17 & 1 \end{pmatrix} \end{array}. \tag{1}$$

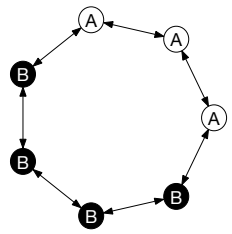
(b) Decide for which N strategies A and B are evolutionarily stable in the limit of weak selection.

(c) Compute for which N strategy A is risk dominant, $\rho_A > \rho_B$, in the limit of weak selection.

(d) Compare your results with the deterministic case.

Problem 2: Evolutionary games on graphs **(5 points)**

Consider the evolution of a population of two types A and B on a regular graph with $k = 2$ (two neighbors per individual, i.e. a circle):



Suppose that fitness is constant and B individuals have a relative fitness advantage r. Write a simulation to verify the *isothermal theorem*, i.e. the absorption probability ρ of a single B individual is the same as for the unstructured Moran process. Use $N = 20$ individuals and run your simulations for a neutral process, $r = 1$, and for a process with fitness advantage of $r = 1.1$.