ETH zürich | D BSSE | Computational Biology Group

Evolutionary Dynamics

Exercises 9

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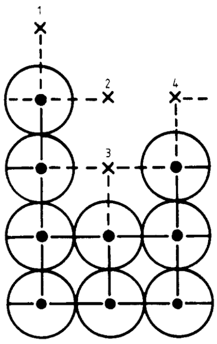
Problem 1: Eden model dynamics
Consider the R code at https://git.io/vF0KH, which defines a two-dimensional Eden growth model.

(a) What is the default update rule for this model? Explain how to modify the code so that the proliferation rate is the same for all cells that have at least one empty neighbour. **(1 point)**

(b) After making the above change, how is the boundary different? Explain this difference. Hint: for each model, consider the probability that an empty site becomes occupied, depending on number of occupied neighbours; for example, you might consider the configuration shown below, where dashed lines represent bonds between occupied and unoccupied sites.

(2 points)



Problem 2: Diffusion approximation of a spatial Moran model
Consider the spatial Moran model for a mutation spreading through an infinite row of demes. Let μ denote the death rate, s the fitness advantage of the mutant, m the dispersion probability, and N the number of individuals per deme. Assume initially that $n_i = N\ \forall i \leq 0$ and $n_i = 0$ otherwise, where n_i is the number of mutants in deme i.

(a) Write down the probability density that a wild type individual will die, per unit of time. Also write down the probability that a dead individual will be replaced by a parent from its own deme, and the probability it will be replaced by a parent from a neighbouring deme. Using these results, show that the transition probability density for the number of mutants in deme i increasing by one individual is

$$W_i^+(\mathbf{n}) = \frac{\mu(1+s)}{N}(N-n_i)\left[n_i + \frac{m}{2}n_i''\right], \tag{1}$$

where $n_i'' = n_{i-1} + n_{i+1} - 2n_i$.

(2 points)

(b) In general,

$$\frac{d\langle n_i \rangle}{dt} = \langle W_i^+(\mathbf{n}) - W_i^-(\mathbf{n})\rangle, \tag{2}$$

where angle brackets denote the expected value. Using the expression for $W_i^-(\mathbf{n})$ given in the lecture, and applying the approximation $\langle n_i n_k \rangle \approx \langle n_i \rangle \langle n_k \rangle$ (called the mean-field approximation), show that

$$\frac{d\langle n_i\rangle}{dt} = \frac{\mu m}{2}\langle n_i''\rangle + \frac{s\mu}{N}(N - \langle n_i\rangle)\left(\langle n_i\rangle + \frac{m}{2}\langle n_i''\rangle\right). \tag{3}$$

(2 points)

(c) We can approximate distance along the row of demes using the continuous variable $x = li$, where l is the deme width. Setting $u(x) = \langle n_i\rangle/N$, show that process (3) can be approximated by the diffusion equation

$$\frac{\partial u}{\partial t} = D[1 + s(1-u)]\frac{\partial^2 u}{\partial x^2} + \mu s u(1-u), \tag{4}$$

where diffusion coefficient $D = \mu m l^2/2$. Hint: Note that $\frac{\partial u}{\partial x}$ is simply the expected difference in u between neighbouring demes, and $\frac{\partial^2 u}{\partial x^2}$ is the expected difference in $\frac{\partial u}{\partial x}$ between neighbouring demes.

(2 points)

(d) Diffusion equation (4) can be a useful approximation of the spatial Moran process when N and s are large but fails to describe the dynamics when $s = 0$ (because then the mean-field approximation breaks down). Explain why this failure is evident from examining the form of equation (4) when $s = 0$.

(1 point)