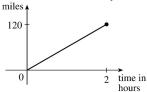
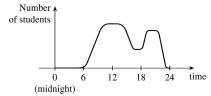
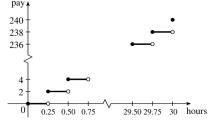
- 1. (a) The point (-1,-2) is on the graph of f, so f(-1)=-2.
- **(b)** When x=2, y is about 2.8, so $f(2) \approx 2.8$.
- (c) f(x)=2 is equivalent to y=2. When y=2, we have x=-3 and x=1.
- (d) Reasonable estimates for x when y=0 are x=-2.5 and x=0.3.
- (e) The domain of f consists of all x -values on the graph of f. For this function, the domain is $-3 \le x \le 3$, or [-3,3]. The range of f consists of all g -values on the graph of g. For this function, the range is $-2 \le y \le 3$, or [-2,3].
- (f) As x increases from -1 to 3, y increases from -2 to 3. Thus, f is increasing on the interval [-1,3].
- 2. (a) The point (-4,-2) is on the graph of f, so f(-4)=-2. The point (3,4) is on the graph of g, so g(3)=4.
- (b) We are looking for the values of x for which the y -values are equal. The y -values for f and g are equal at the points (-2,1) and (2,2), so the desired values of x are -2 and 2.
- (c) f(x)=-1 is equivalent to y=-1. When y=-1, we have x=-3 and x=4.
- (d) As x increases from 0 to 4, y decreases from 3 to -1. Thus, f is decreasing on the interval [0,4]
- (e) The domain of f consists of all x -values on the graph of f. For this function, the domain is $-4 \le x \le 4$, or [-4,4]. The range of f consists of all g -values on the graph of f. For this function, the range is $-2 \le g \le 3$, or [-2,3].
- (f) The domain of g is [-4,3] and the range is [0.5,4].
- 3. From Figure 1 in the text, the lowest point occurs at about (t,a)=(12,-85). The highest point occurs at about (17,115). Thus, the range of the vertical ground acceleration is $-85 \le a \le 115$. In Figure 11, the range of the north-south acceleration is approximately $-325 \le a \le 485$. In Figure 12, the range of the east-west acceleration is approximately $-210 \le a \le 200$.
- 4. *Example 1:* A car is driven at 60 mi / h for 2 hours. The distance d traveled by the car is a function of the time t. The domain of the function is $\{t \mid 0 \le t \le 2\}$, where t is measured in hours. The range of the function is $\{d \mid 0 \le d \le 120\}$, where d is measured in miles.



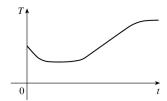
Example 2: At a certain university, the number of students N on campus at any time on a particular day is a function of the time t after midnight. The domain of the function is $\{t \mid 0 \le t \le 24\}$, where t is measured in hours. The range of the function is $\{N \mid 0 \le N \le k\}$, where N is an integer and k is the largest number of students on campus at once.



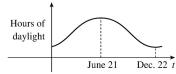
Example 3: A certain employee is paid \$8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter of an hour. This employee's gross weekly pay P is a function of the number of hours worked h. The domain of the function is [0,30] and the range of the function is $\{0,2.00,4.00,\ldots,238.00,240.00\}$.



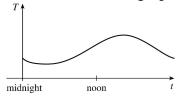
- 5. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.
- 6. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is [-2,2] and the range is [-1,2].
- 7. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is [-3,2] and the range is $[-3,-2) \cup [-1,3]$.
- 8. No, the curve is not the graph of a function since for x=0, ± 1 , and ± 2 , there are infinitely many points on the curve.
- 9. The person's weight increased to about 160 pounds at age 20 and stayed fairly steady for 10 years. The person's weight dropped to about 120 pounds for the next 5 years, then increased rapidly to about 170 pounds. The next 30 years saw a gradual increase to 190 pounds. Possible reasons for the drop in weight at 30 years of age: diet, exercise, health problems.
- 10. The salesman travels away from home from 8 to 9 A.M. and is then stationary until 10:00. The salesman travels farther away from 10 until noon. There is no change in his distance from home until 1:00, at which time the distance from home decreases until 3:00. Then the distance starts increasing again, reaching the maximum distance away from home at 5:00. There is no change from 5 until 6, and then the distance decreases rapidly until 7:00 P.M., at which time the salesman reaches home.
- 11. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



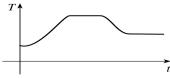
12. The summer solstice (the longest day of the year) is around June 21, and the winter solstice (the shortest day) is around December 22.

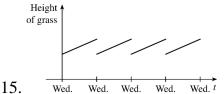


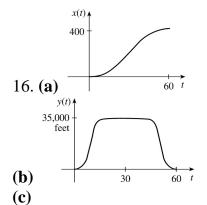
13. Of course, this graph depends strongly on the geographical location!

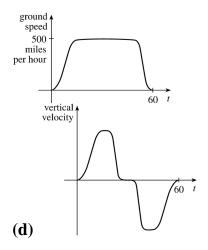


14. The temperature of the pie would increase rapidly, level off to oven temperature, decrease rapidly, and then level off to room temperature.



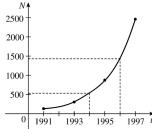




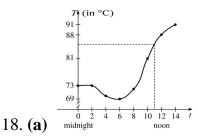


17. (a)

(b) From the graph, we estimate the number of cell-phone subscribers in Malaysia to be about 540 in



1994 and 1450 in 1996.



(b) From the graph in part (a), we estimate the temperature at 11:00 A.M. to be about 84.5 $^{\circ}$ C.

19.
$$f(x)=3x^2-x+2$$
.
 $f(2)=3(2)^2-2+2=12-2+2=12$.
 $f(-2)=3(-2)^2-(-2)+2=12+2+2=16$

$$f(a)=3a^{2}-a+2.$$

$$f(-a)=3(-a)^{2}-(-a)+2=3a^{2}+a+2.$$

$$f(a+1)=3(a+1)^{2}-(a+1)+2=3(a^{2}+2a+1)-a-1+2=3a^{2}+6a+3-a+1=3a^{2}+5a+4.$$

$$2f(a)=2\cdot f(a)=2(3a^{2}-a+2)=6a^{2}-2a+4.$$

$$f(2a)=3(2a)^{2}-(2a)+2=3(4a^{2})-2a+2=12a^{2}-2a+2.$$

$$f(a^{2})=3(a^{2})^{2}-(a^{2})+2=3(a^{4})-a^{2}+2=3a^{4}-a^{2}+2.$$

$$[f(a)]^{2}=\left[3a^{2}-a+2\right]^{2}=\left(3a^{2}-a+2\right)\left(3a^{2}-a+2\right)$$

$$=9a^{4}-3a^{3}+6a^{2}-3a^{3}+a^{2}-2a+6a^{2}-2a+4=9a^{4}-6a^{3}+13a^{2}-4a+4.$$

$$f(a+h)=3(a+h)^{2}-(a+h)+2=3(a^{2}+2ah+h^{2})-a-h+2=3a^{2}+6ah+3h^{2}-a-h+2.$$

20. A spherical balloon with radius r+1 has volume $V(r+1) = \frac{4}{3}\pi(r+1)^3 = \frac{4}{3}\pi(r^3+3r^2+3r+1)$. We wish to find the amount of air needed to inflate the balloon from a radius of r to r+1. Hence, we need to find the difference $V(r+1)-V(r)=\frac{4}{3}\pi(r^3+3r^2+3r+1)-\frac{4}{3}\pi r^3=\frac{4}{3}\pi(3r^2+3r+1)$.

21.
$$f(x)=x-x^2$$
, so $f(2+h)=2+h-(2+h)^2=2+h-(4+4h+h^2)=2+h-4-4h-h^2=-(h^2+3h+2)$, $f(x+h)=x+h-(x+h)^2=x+h-(x^2+2xh+h^2)=x+h-x^2-2xh-h^2$, and
$$\frac{f(x+h)-f(x)}{h}=\frac{x+h-x^2-2xh-h^2-x+x^2}{h}=\frac{h-2xh-h^2}{h}=\frac{h(1-2x-h)}{h}=1-2x-h$$
.

22.
$$f(x) = \frac{x}{x+1}$$
, so $f(2+h) = \frac{2+h}{2+h+1} = \frac{2+h}{3+h}$, $f(x+h) = \frac{x+h}{x+h+1}$, and
$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} = \frac{(x+h)(x+1)-x(x+h+1)}{h(x+h+1)(x+1)} = \frac{1}{(x+h+1)(x+1)}$$
.

23. f(x)=x/(3x-1) is defined for all x except when $0=3x-1 \Leftrightarrow x=\frac{1}{3}$, so the domain is $\left\{x \in R \mid x \neq \frac{1}{3}\right\} = \left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right).$

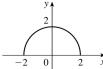
24. $f(x)=(5x+4)/(x^2+3x+2)$ is defined for all x except when $0=x^2+3x+2\Leftrightarrow 0=(x+2)(x+1)\Leftrightarrow x=-2$ or -1, so the domain is $\{x\in R|x\neq -2, -1\}=(-\infty, -2)\cup (-2, -1)\cup (-1, \infty)$.

 $f(t) = \sqrt{t} + \sqrt[3]{t}$ is defined when $t \ge 0$. These values of t give real number results for \sqrt{t} , whereas any value of t gives a real number result for $\sqrt[3]{t}$. The domain is $[0,\infty)$.

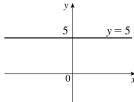
26. $g(u) = \sqrt{u} + \sqrt{4-u}$ is defined when $u \ge 0$ and $4-u \ge 0 \Leftrightarrow u \le 4$. Thus, the domain is $0 \le u \le 4 = [0,4]$.

27. $h(x)=1/\sqrt[4]{x^2-5x}$ is defined when $x^2-5x>0 \Leftrightarrow x(x-5)>0$. Note that $x^2-5x\neq 0$ since that would result in division by zero. The expression x(x-5) is positive if x<0 or x>5. (See Appendix A for methods for solving inequalities.) Thus, the domain is $(-\infty,0)\cup(5,\infty)$.

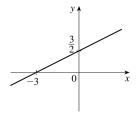
28. $h(x) = \sqrt{4-x^2}$. Now $y = \sqrt{4-x^2} \Rightarrow y^2 = 4-x^2 \Leftrightarrow x^2+y^2 = 4$, so the graph is the top half of a circle of radius 2 with center at the origin. The domain is $\left\{x \mid 4-x^2 \geq 0\right\} = \left\{x \mid 4 \geq x^2\right\} = \left\{x \mid 2 \geq |x|\right\} = [-2,2]$. From the graph, the range is $0 \leq y \leq 2$, or [0,2].



29. f(x)=5 is defined for all real numbers, so the domain is R, or $(-\infty,\infty)$. The graph of f is a horizontal line with y –intercept 5.

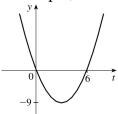


30. $F(x) = \frac{1}{2}(x+3)$ is defined for all real numbers, so the domain is R, or $(-\infty, \infty)$. The graph of F is a line with x –intercept –3 and y –intercept $\frac{3}{2}$.

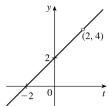


31. $f(t)=t^2-6t$ is defined for all real numbers, so the domain is R, or $(-\infty,\infty)$. The graph of f is a parabola opening upward since the coefficient of t^2 is positive. To find the t –intercepts, let y=0 and solve for t. $0=t^2-6t=t(t-6) \Rightarrow t=0$ and t=6. The t –coordinate of the vertex is halfway between the t –

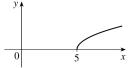
intercepts, that is, at t=3. Since $f(3)=3^2-6\cdot 3=-9$, the vertex is (3,-9).



32. $H(t) = \frac{4-t^2}{2-t} = \frac{(2+t)(2-t)}{2-t}$, so for $t \neq 2$, H(t) = 2+t. The domain is $\{t \mid t \neq 2\}$. So the graph of H is the same as the graph of the function f(t) = t+2 (a line) except for the hole at (2,4).



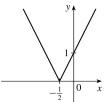
33. $g(x) = \sqrt{x-5}$ is defined when $x-5 \ge 0$ or $x \ge 5$, so the domain is $[5,\infty)$. Since $y = \sqrt{x-5} \Rightarrow y = x-5 \Rightarrow x=y^2+5$, we see that g is the top half of a parabola.



34.

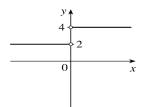
$$F(x)=|2x+1| = \begin{cases} 2x+1 & \text{if } 2x+1 \ge 0\\ -(2x+1) & \text{if } 2x+1 < 1 \end{cases}$$
$$= \begin{cases} 2x+1 & \text{if } x \ge -\frac{1}{2}\\ -2x-1 & \text{if } x < -\frac{1}{2} \end{cases}$$

The domain is R, or $(-\infty, \infty)$.



$$G(x) = \frac{3x + |x|}{x} \text{ . Since } |x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}, \text{ we have}$$

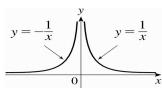
$$G(x) = \begin{cases} \frac{3x+x}{x} & \text{if } x > 0 \\ \frac{3x-x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} \frac{4x}{x} & \text{if } x > 0 \\ \frac{2x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} 4 & \text{if } x > 0 \\ 2 & \text{if } x < 0 \end{cases}$$



Note that *G* is not defined for x=0. The domain is $(-\infty,0) \cup (0,\infty)$.

36.
$$g(x) = \frac{|x|}{x^2}$$
. Since $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$, we have

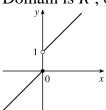
$$g(x) = \begin{cases} \frac{x}{2} & \text{if } x > 0 \\ \frac{-x}{2} & \text{if } x < 0 \end{cases} = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ \frac{1}{-x} & \text{if } x < 0 \end{cases}$$



Note that g is not defined for x=0. The domain is $(-\infty,0) \cup (0,\infty)$.

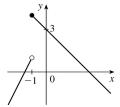
37.
$$f(x) = \begin{cases} x & \text{if } x \le 0 \\ x+1 & \text{if } x > 0 \end{cases}$$

Domain is R, or $(-\infty, \infty)$.



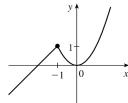
38.
$$f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x \ge -1 \end{cases}$$

Domain is R, or $(-\infty, \infty)$.



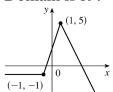
39.
$$f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ x & \text{if } x > -1 \end{cases}$$

Note that for x=-1, both x+2 and x^2 are equal to 1. Domain is R.



$$40. \ f(x) = \begin{cases} -1 & \text{if } x \le -1 \\ 3x + 2 & \text{if } -1 < x < 1 \\ 7 - 2x & \text{if } x \ge 1 \end{cases}$$

Domain is \hat{R} .



- 41. Recall that the slope m of a line between the two points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ is $m = \frac{y_2 y_1}{x_2 x_1}$ and an equation of the line connecting those two points is $y y_1 = m(x x_1)$. The slope of this line segment is $\frac{-6 1}{4 (-2)} = -\frac{7}{6}$, so an equation is $y 1 = -\frac{7}{6}(x + 2)$. The function is $f(x) = -\frac{7}{6}x \frac{4}{3}$, $-2 \le x \le 4$.
- 42. The slope of this line segment is $\frac{3-(-2)}{6-(-3)} = \frac{5}{9}$, so an equation is $y+2=\frac{5}{9}(x+3)$. The function is $f(x) = \frac{5}{9}x \frac{1}{3}$, $-3 \le x \le 6$.
- 43. We need to solve the given equation for y.

 $x+(y-1)^2=0\Leftrightarrow (y-1)^2=-x\Leftrightarrow y-1=\pm\sqrt{-x}\Leftrightarrow y=1\pm\sqrt{-x}$. The expression with the positive radical represents the top half of the parabola, and the one with the negative radical represents the bottom half. Hence, we want $f(x)=1-\sqrt{-x}$. Note that the domain is $x\leq 0$.

- 44. $(x-1)^2 + y^2 = 1 \Leftrightarrow y = \pm \sqrt{1 (x-1)^2} = \pm \sqrt{2x x^2}$. The top half is given by the function $f(x) = \sqrt{2x x^2}$, $0 \le x \le 2$.
- 45. For $-1 \le x \le 2$, the graph is the line with slope 1 and y- intercept 1, that is, the line y=x+1. For $2 < x \le 4$, the graph is the line with slope $-\frac{3}{2}$ and x- intercept 4, so $y-0=-\frac{3}{2}(x-4)=-\frac{3}{2}x+6$. So the

function is $f(x) = \begin{cases} x+1 & \text{if } -1 \le x \le 2 \\ -\frac{3}{2}x+6 & \text{if } 2 < x \le 4 \end{cases}$

46. For $x \le 0$, the graph is the line y=2. For $0 < x \le 1$, the graph is the line with slope -2 and y- intercept 2, that is, the line y=-2x+2. For x>1, the graph is the line with slope 1 and x- intercept 1,

that is, the line y=1(x-1)=x-1 . So the function is f(x)= $\begin{cases} 2 & \text{if } x \leq 0 \\ -2x+2 & \text{if } 0 < x \leq 1 \\ x-1 & \text{if } 1 < x \end{cases}.$

47. Let the length and width of the rectangle be L and W. Then the perimeter is 2L+2W=20 and the area is A=LW. Solving the first equation for W in terms of L gives $W=\frac{20-2L}{2}=10-L$. Thus,

 $A(L)=L(10-L)=10L-L^2$. Since lengths are positive, the domain of A is 0 < L < 10. If we further restrict L to be larger than W, then 5 < L < 10 would be the domain.

- 48. Let the length and width of the rectangle be L and W. Then the area is LW=16, so that W=16/L. The perimeter is P=2L+2W, so P(L)=2L+2(16/L)=2L+32/L, and the domain of P is L>0, since lengths must be positive quantities. If we further restrict L to be larger than W, then L>4 would be the domain.
- 49. Let the length of a side of the equilateral triangle be x. Then by the Pythagorean Theorem, the height y of the triangle satisfies $y^2 + \left(\frac{1}{2}x\right)^2 = x^2$, so that $y^2 = x^2 \frac{1}{4}x^2 = \frac{3}{4}x^2$ and $y = \frac{\sqrt{3}}{2}x$. Using the formula for the area A of a triangle, $A = \frac{1}{2}$ (base) (height), we obtain $A(x) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$, with domain x > 0.
- 50. Let the volume of the cube be V and the length of an edge be L. Then $V = L^3$ so $L = \sqrt[3]{V}$, and the

surface area is $S(V)=6\left(\frac{3}{\sqrt{V}}\right)^2=6V^2/3$, with domain V>0.

- 51. Let each side of the base of the box have length x, and let the height of the box be h. Since the volume is 2, we know that $2=hx^2$, so that $h=2/x^2$, and the surface area is $S=x^2+4xh$. Thus, $S(x)=x^2+4x(2/x^2)=x^2+(8/x)$, with domain x>0.
- 52. The area of the window is $A=xh+\frac{1}{2}\pi\left(\frac{1}{2}x\right)^2=xh+\frac{\pi x^2}{8}$, where h is the height of the rectangular portion of the window. The perimeter is $P=2h+x+\frac{1}{2}\pi x=30 \Leftrightarrow 2h=30-x-\frac{1}{2}\pi x \Leftrightarrow h=\frac{1}{4}\left(60-2x-\pi x\right)$. Thus,

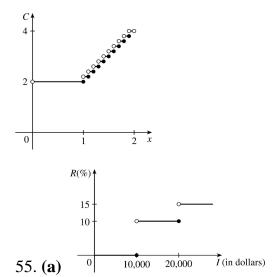
$$A(x) = x \frac{60 - 2x - \pi x}{4} + \frac{\pi x^{2}}{8} = 15x - \frac{1}{2}x^{2} - \frac{\pi}{4}x^{2} + \frac{\pi}{8}x^{2} = 15x - \frac{4}{8}x^{2} - \frac{\pi}{8}x^{2} = 15x - x^{2}\left(\frac{\pi + 4}{8}\right)$$

Since the lengths x and h must be positive quantities, we have x>0 and h>0. For h>0, we have 2h>0 $\Leftrightarrow 30-x-\frac{1}{2}\pi x>0 \Leftrightarrow 60>2x+\pi x\Leftrightarrow x<\frac{60}{2+\pi}$. Hence, the domain of A is $0< x<\frac{60}{2+\pi}$.

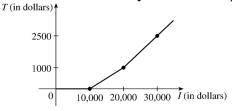
53. The height of the box is x and the length and width are L=20-2x, W=12-2x. Then V=LWx and so

 $V(x)=(20-2x) (12-2x)(x)=4(10-x)(6-x)(x)=4x (60-16x+x^2)=4x^3-64x^2+240x$. The sides L, W, and X must be positive. Thus, $L>0\Leftrightarrow 20-2x>0\Leftrightarrow x<10$; $W>0\Leftrightarrow 12-2x>0\Leftrightarrow x<6$; and x>0. Combining these restrictions gives us the domain 0< x<6.

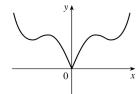
$$C(x) = \begin{cases} \$2.00 & \text{if } 0.0 < x \le 1.0 \\ 2.20 & \text{if } 1.0 < x \le 1.1 \\ 2.40 & \text{if } 1.1 < x \le 1.2 \\ 2.60 & \text{if } 1.2 < x \le 1.3 \\ 2.80 & \text{if } 1.3 < x \le 1.4 \\ 3.00 & \text{if } 1.4 < x \le 1.5 \\ 3.20 & \text{if } 1.5 < x \le 1.6 \\ 3.40 & \text{if } 1.6 < x \le 1.7 \\ 3.60 & \text{if } 1.7 < x \le 1.8 \\ 3.80 & \text{if } 1.8 < x \le 1.9 \\ 4.00 & \text{if } 1.9 < x < 2.0 \end{cases}$$



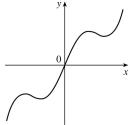
- **(b)** On \$14,000, tax is assessed on \$4000, and 10%(\$4000)=\$400.
- On \$26, 000, tax is assessed on \$16, 000, and 10%(\$10, 000)+15%(\$6000)=\$1000+\$900=\$1900.
- (c) As in part (b), there is \$1000 tax assessed on \$20,000 of income, so the graph of T is a line segment from (10,000,0) to (20,000,1000). The tax on \$30,000 is \$2500, so the graph of T for x>20, 000 is the ray with initial point (20,000,1000) that passes through (30,000,2500).



- 56. One example is the amount paid for cable or telephone system repair in the home, usually measured to the nearest quarter hour. Another example is the amount paid by a student in tuition fees, if the fees vary according to the number of credits for which the student has registered.
- 57. f is an odd function because its graph is symmetric about the origin. g is an even function because its graph is symmetric with respect to the y –axis.
- 58. f is not an even function since it is not symmetric with respect to the y-axis. f is not an odd function since it is not symmetric about the origin. Hence, f is *neither* even nor odd. g is an even function because its graph is symmetric with respect to the y-axis.
- 59. (a) Because an even function is symmetric with respect to the y –axis, and the point (5,3) is on the graph of this even function, the point (-5,3) must also be on its graph.
- (b) Because an odd function is symmetric with respect to the origin, and the point (5,3) is on the graph of this odd function, the point (-5,-3) must also be on its graph.
- 60. (a) If f is even, we get the rest of the graph by reflecting about the y -axis.



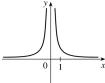
(b) If f is odd, we get the rest of the graph by rotating 180° about the origin.



61.
$$f(x)=x^{-2}$$
.

$$f(-x) = (-x)^{-2} = \frac{1}{(-x)^{2}} = \frac{1}{x^{2}}$$
$$= x^{-2} = f(x)$$

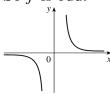
So f is an even function.



62.
$$f(x)=x^{-3}$$
.

$$f(-x) = (-x)^{-3} = \frac{1}{(-x)^3} = \frac{1}{-x^3}$$
$$= -\frac{1}{x^3} = -(x^{-3}) = -f(x)$$

So f is odd.



63. $f(x)=x^2+x$, so $f(-x)=(-x)^2+(-x)=x^2-x$. Since this is neither f(x) nor -f(x), the function f is

neither even nor odd.

64.
$$f(x)=x^4-4x^2$$
.
 $f(-x) = (-x)^4-4(-x)^2$
 $=x^4-4x^2=f(x)$

So f is even.



65.
$$f(x)=x^3-x$$
.

$$f(-x) = (-x)^{3} - (-x) = -x^{3} + x$$
$$= -(x^{3} - x) = -f(x)$$

So f is odd.



66. $f(x)=3x^3+2x^2+1$, so $f(-x)=3(-x)^3+2(-x)^2+1=-3x^3+2x^2+1$. Since this is neither f(x) nor -f(x), the function f is neither even nor odd.