Applied Stochastic Modeling
Case study 1: renewal theory and simulation

Introduction

This case study concerns the application of renewal theory and simulation to the capacity of the Operating Theatre (OT). The goal of this assignment is two-fold: (i) it gives an impression of how renewal theory can be applied in practical situations, and (ii) it provides experience and insights in using renewal theory and simulation.

This case should be carried out in groups of 2–3 students. Each group carries out the case using both renewal theory and simulation. Of course, both methods should (roughly) provide the same results and are a means for verification. Each group submits a brief report containing:

- The (verified) results of the case study,
- An *outline* of both approaches (i.e. the overall steps/formula's being used, *not* the exact implementation).

The target length of the report is 1-2 pages, but it should certainly be no longer than 3 pages. Please submit the report via canvas at or before *Friday October 4, 2024*.

Background and problem statement

We consider the Operating Theatre (OT) that is available for a specific medical discipline. Procedures at the OT are carried out during office hours; that is between 8:00 and 16:00. The length of a procedure is random and unknown in advance. In the file 'procedures.xlsx' the length of procedures can be found for two primary diagnosis codes. Each type of procedure is carried out in a dedicated OT, as the type of procedure depends on the working schedule of the doctors. Moreover, it is typically considered to be efficient for doctors to do similar procedures behind each other. For the model, we assume that the first procedure always starts exactly at 8:00. A new procedure is started as long as it is no later than time x, with $x \in$ [8:00, 16:00]. A procedure that is started is always completed. The procedure time after 16:00 hours, is called overtime.

For the OT, for various values of x, we are interested in two main *performance measures*:

- The expected number of procedures carried out
- The expected overtime

These performance measures can be determined for both diagnosis codes.

Assignment

For this case study the two performance measures above can be computed using Excel or almost any (conventional) programming language[1]. More specifically, for some value of x (start with e.g. $x = 16{:}00$), you should address the following:

Q1 Compute the expected number of procedures, using renewal theory and simulation.

Q2 Compute the expected overtime, using (i) renewal theory and (ii) simulation. For (i) it is best to start with $x = 16{:}00$ and to assume that the OT is running long enough such that overtime behaves as the stationary excess. For (ii) the expected overtime should be extended to the situation where $x < 16{:}00$; for (i) this is optional.

Q3 Visualize the impact of choosing different values for x and for both procedure types.

Determining the expected overtime using renewal theory is (slightly) more involved. Make sure that you are able to determine this at least for the case that $x = 16{:}00$. (As mentioned above, determing the expected overtime using renewal theory for $x < 16{:}00$ is optional.)

Hints

Some useful hints and remarks are given below.

- For the length of a procedure, you may fit a Gamma distribution. Recall that this is *not* a course on statistics, so you may keep estimating parameters simple. We advocate to use the methods of moments, i.e., choose the parameters such that the first two moments (or the mean and the variance) of the Gamma distribution and the data are the same.
- For the renewal approach, realize that the convolution of Gamma random variables with the same scale parameter follows again a Gamma distribution.
- For the simulation, include confidence intervals.
- If you use Excel for implementation, make use of convenient Excel functions, such as GAMMADIST, RAND(), GAMMAINV. Please note the confusing names for the parameters of the Gamma distribution in Excel (i.e. be careful with the scale/rate parameter)!
- Review the material at the slides corresponding to the practical session.

[1] Of course, we cannot guarantee support for any programming language.