The transshipment problem

OPTIMIZATION 2

PRESENTED BY
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The transshipment model

PRELIMINARIES, DEFINITION

SOLUTION METHOD

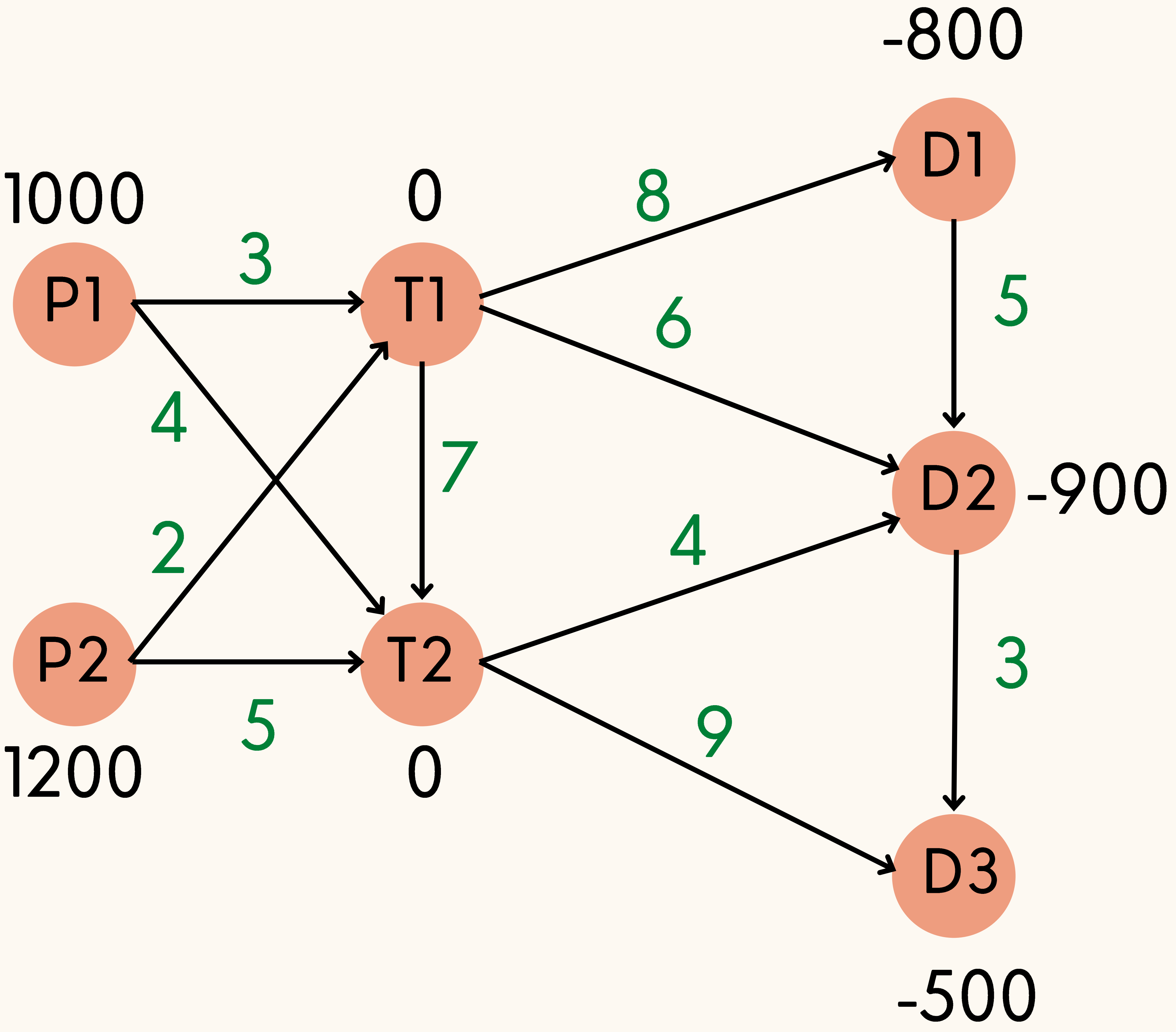
EXAMPLE

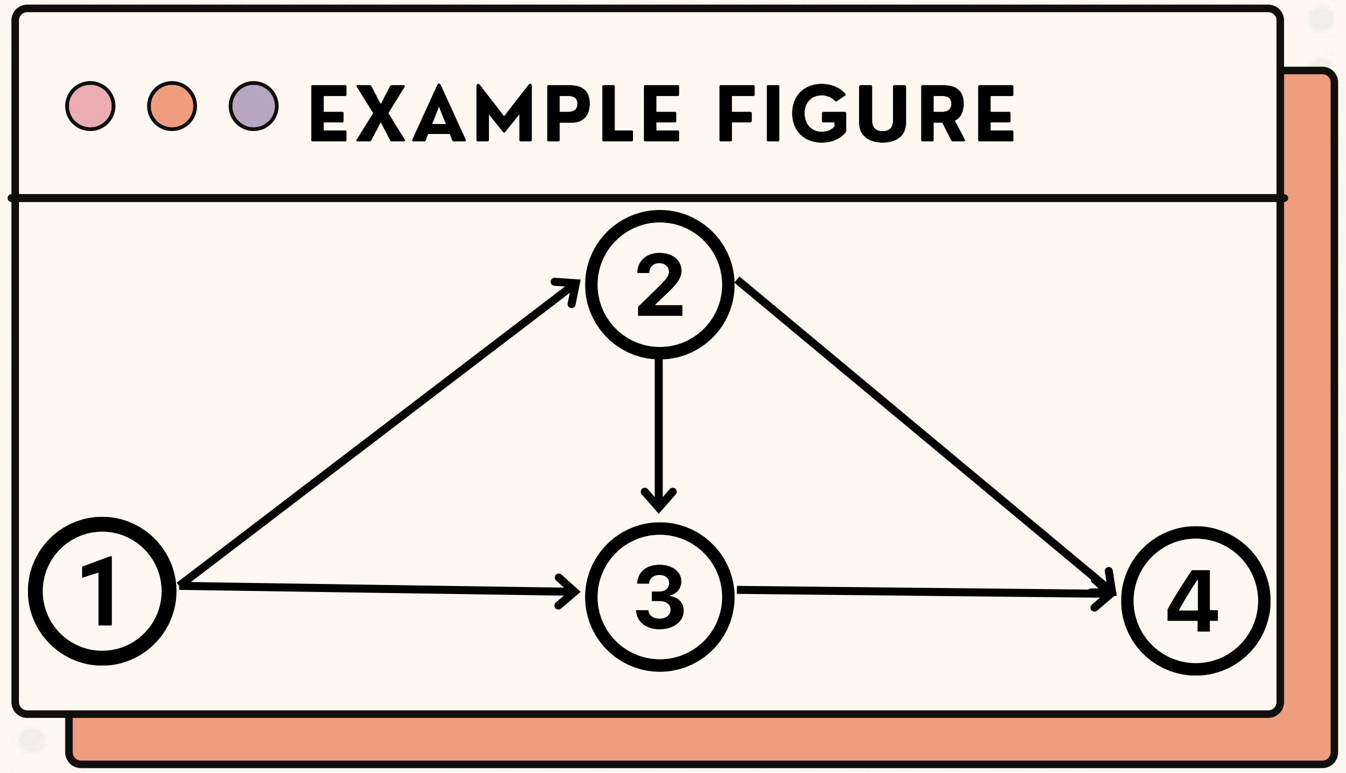
PRELIMINARIES

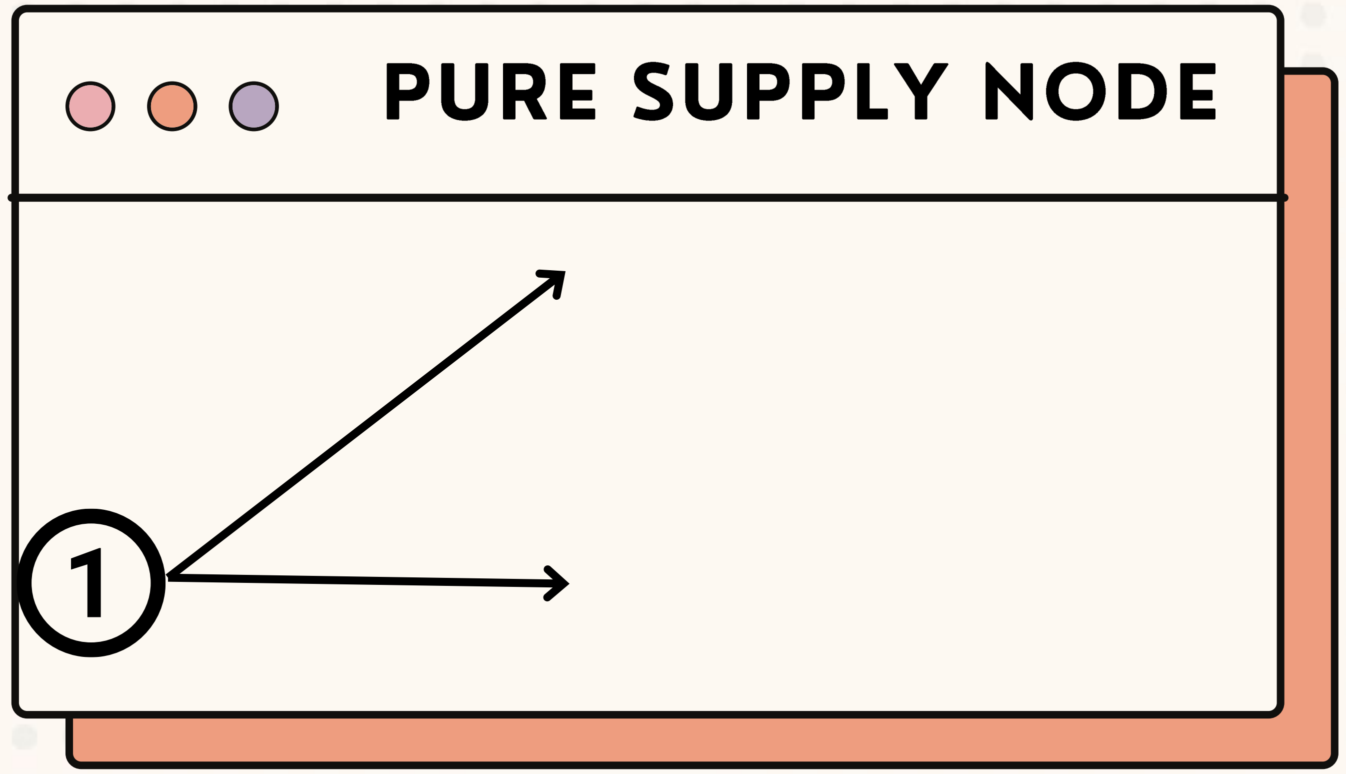
➢ Transshipment node: the node of the network with both output and input arcs (acts as both a source and a destination)

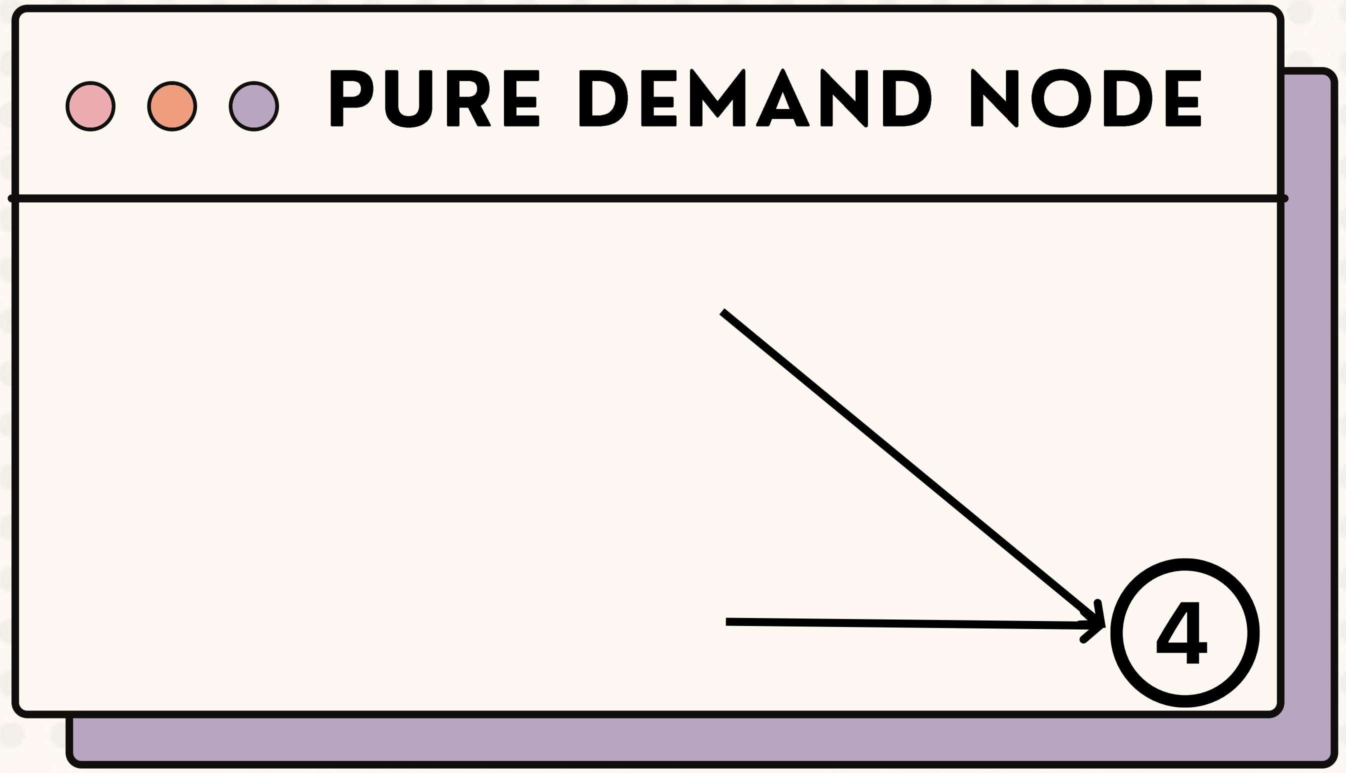
➢ Pure Supply node: the node that only has arcs coming out of it (acts as the source only).

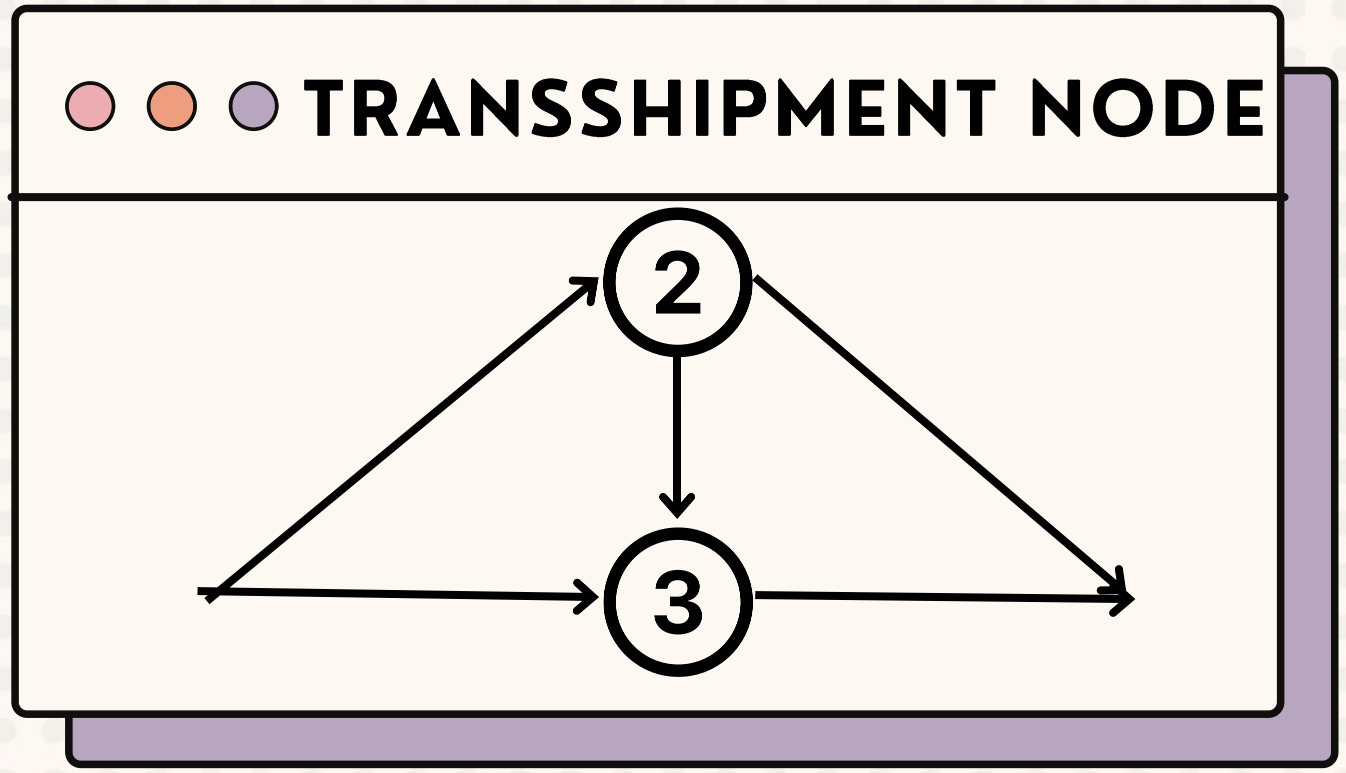
➢ Pure Demand node: the node that only has arcs going in it (acts as the destination only).



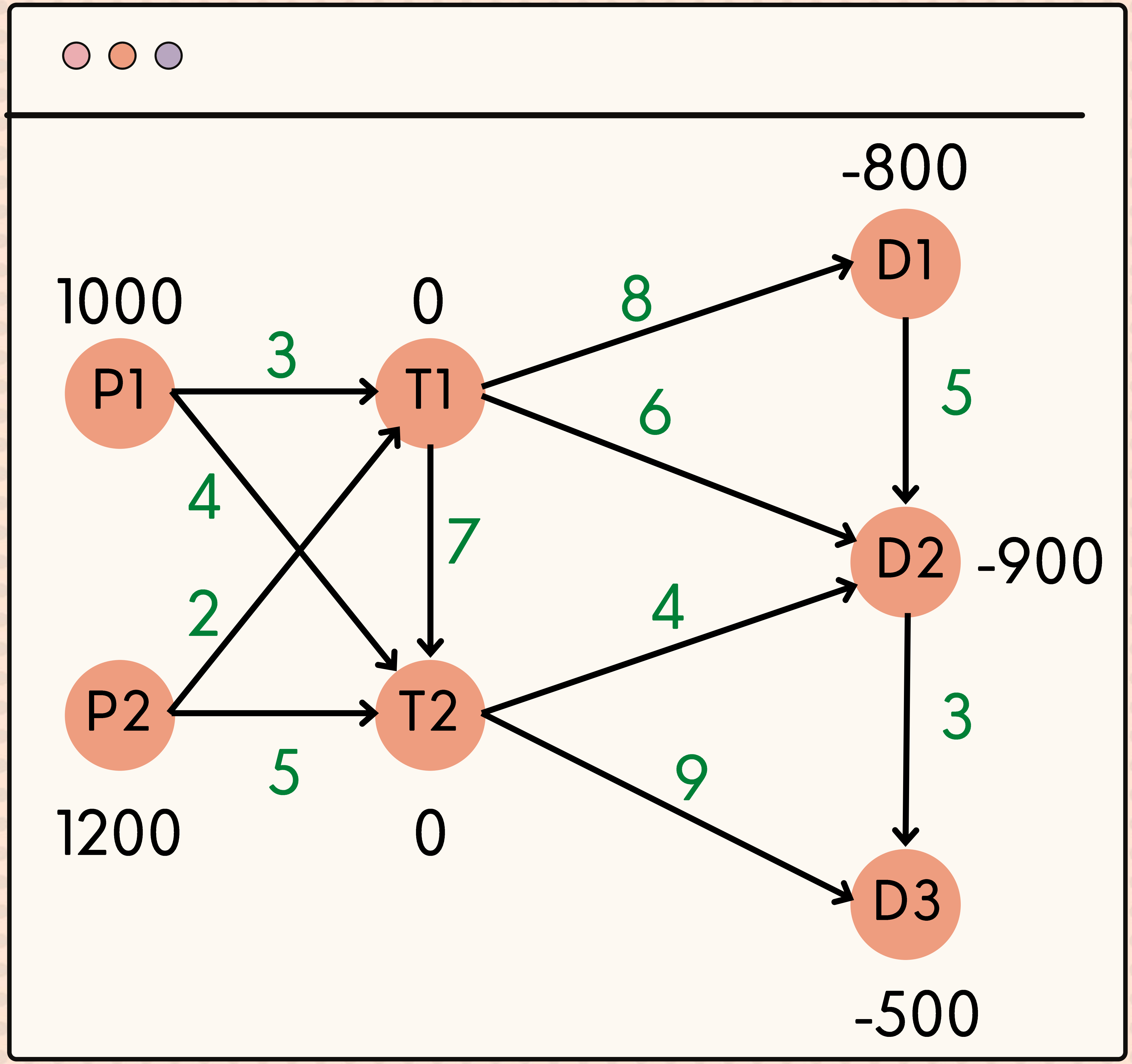

EXAMPLE FIGURE
1
2
3
4


PURE SUPPLY NODE
1


PURE DEMAND NODE
4


TRANSSHIPMENT NODE
2
3

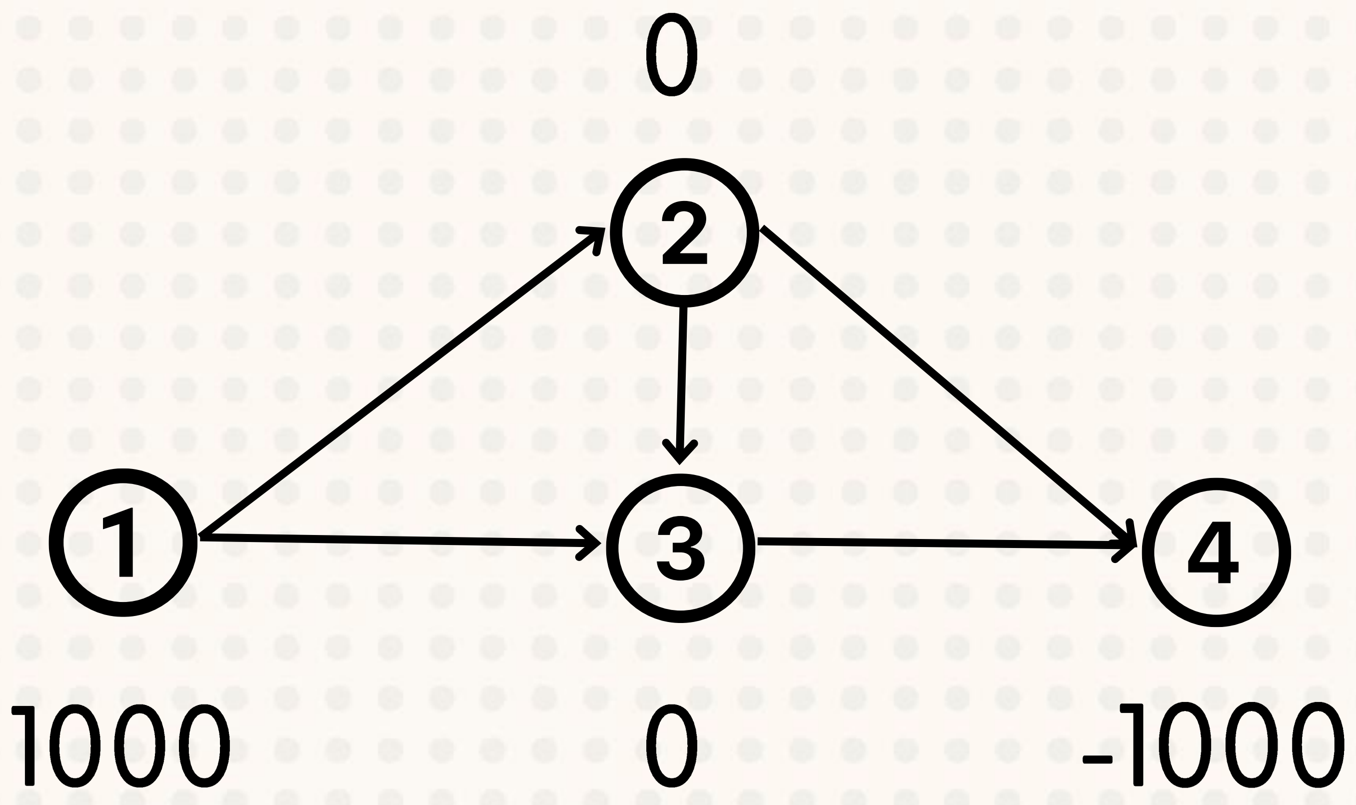
Definition



The transshipment problem is a variation of the well-known transportation problem. It introduces the existence of **transshipment** nodes.

Note that this is a balance problem.

2. SOLUTION METHOD



Consider the example graph, in order to move 1000 units of goods from node 1 to node 4, one can either go through node 2 or node 3 or both of them. So, the capacity of nodes 2 and 3 must be sufficiently large to take the goods. We should add an amount called **buffer amount** to the current supply amount of the transshipment nodes to satisfy that condition.

Buffer amount

Let B represent the buffer amount, then

B = Total supply = |Total demand|

The amounts of supply and demand at each node are computed as:

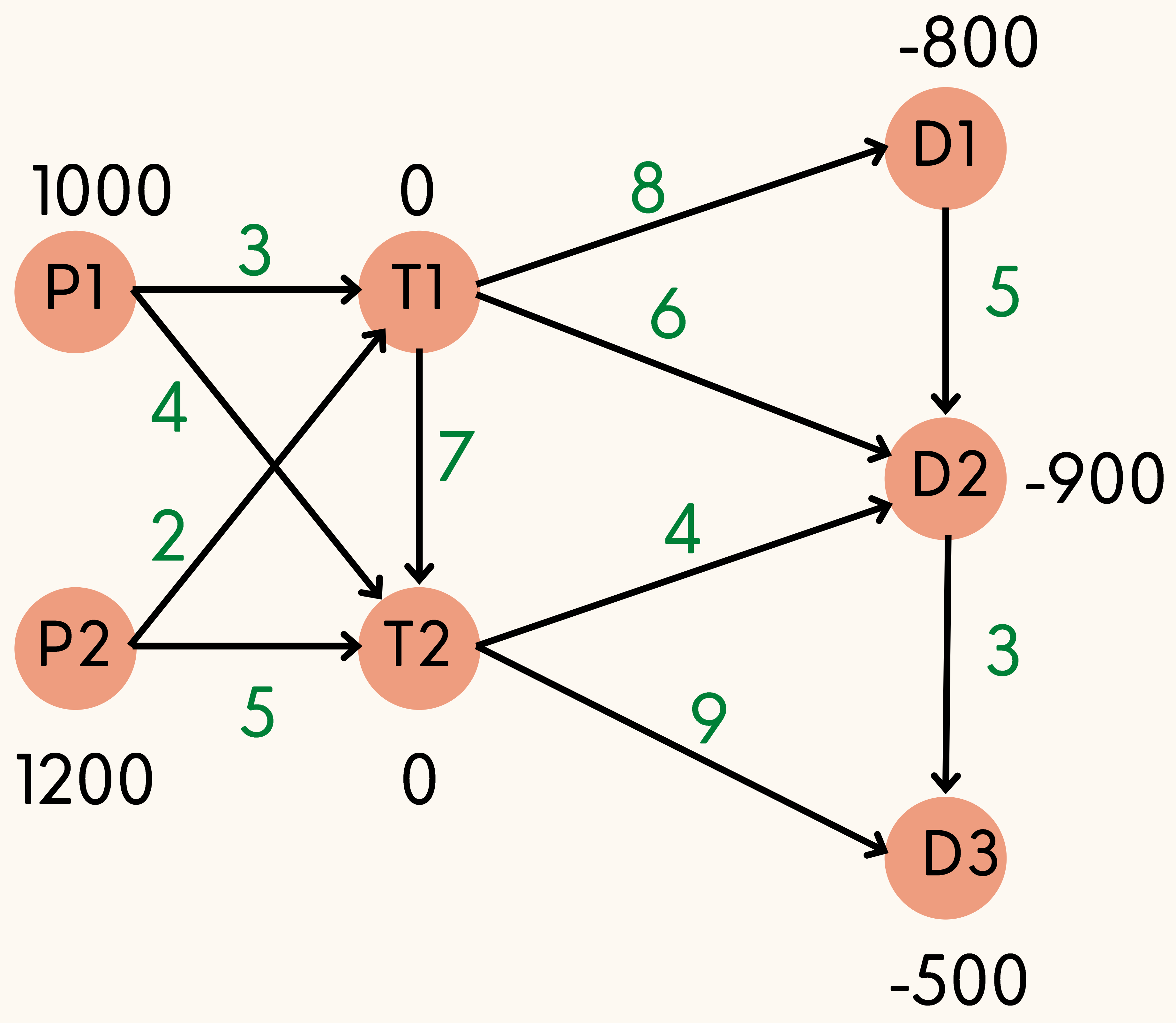
-**Supply** at a pure supply node = Original supply.

-**Demand** at a pure demand node = |Original demand|

-**Supply** at a transshipment node = Original supply + B

-**Demand** at a transshipment node = |Original demand|+ B

S \ D	T1	T2	D1	D2	D3	
P1	3 1000	4	M	M	M	1000
P2	2 1200	5 0	M	M	M	1200
T1	0	7 B	8 0	6	M	B
T2	M	0	M B	4	9	B
D1	M	M	0 800	5 B-800	M	B
D2	M	M	M	0 1700	3 500	B
	B	B	B+800	900+B	500	



S \ D	T1	T2	D1	D2	D3		u
P1	3 1000	4	M	M	M	1000	0
P2	2 1200	5 0	M	M	M	1200	-1
T1	0	7 B	8 0	6	M	B	1
T2	M	0	M B	4	9	B	M-7
D1	M	M	0 800	5 B-800	M	B	-7
D2	M	M	M	0 1700	3 500	B	-12
	B	B	B+800	900+B	500		
v	3	6	7	12	15		

Iteration 1

Step 1: Let G(x) be the set of selected cells of the basic feasible solution

∀(i, j) ∈ G(X), $c_{ij} = u_i + v_j$

Let u_1 = 0, we get:

$$u_2 = -1, u_3 = 1, u_4 = M - 7,$$
$$u_5 = -7, u_6 = 12$$

$$v_1 = 3, v_2 = 6, v_3 = 7,$$
$$v_4 = 12, v_5 = 15$$

S \ D	T1	T2	D1	D2	D3		u
P1	1000 (3)	(4)	(M)	(M)	(M)	1000	0
P2	1200 (2)	0 (5)	(M)	(M)	(M)	1200	-1
T1	(0)	B (7)	0 (8)	(6)	(M)	B	1
T2	(M)	(0)	B (M)	(4)	(9)	B	M-7
D1	(M)	(M)	800 (0)	B-800 (5)	(M)	B	-7
D2	(M)	(M)	(M)	1700 (0)	500 (3)	B	-12
	B	B	B+800	900+B	500		
v	3	6	7	12	15		

Iteration 1

Step 2: $\forall$(i, j) $\notin$ G(x), $\bar{c}_{ij} = c_{ij} - u_i - v_j$

$$\bar{c}_{12} = 4 - 0 - 6 = -2 < 0$$

$$\bar{c}_{31} = 0 - 1 - 3 = -4 < 0$$

$$\bar{c}_{34} = 6 - 1 - 12 = -7 < 0$$

$$\bar{c}_{42} = 0 - (M - 7) - 6 = -M + 1 < 0$$

$$\bar{c}_{44} = 4 - (M - 7) - 12 = -M + 1 < 0$$

$$\bar{c}_{45} = 9 - (M - 7) - 15 = -M + 1 < 0$$

……

Choose x_{44} as entering variable

S \ D	T1	T2	D1	D2	D3		u
P1	3 1000	4	M	M	M	1000	0
P2	2 1200	5 0	M	M	M	1200	-1
T1	0	7 B	8 0	6	M	B	1
T2	M	0	M B-θ	4 θ	9	B	M-7
D1	M	M	0 800+θ	5 B-800-θ	M	B	-7
D2	M	M	M	0 1700	3 500	B	-12
	B	B	B+800	900+B	500		
v	3	6	7	12	15		

Iteration 1:

Step 3:

$$C^+ = \{(3,4),(5,3)\}$$
$$C^- = \{(5,4),(4,3)\}$$

$$\theta^* = \min\{x_{ij}: (i,j) \in C^-\}$$
$$= \min\{B, B-800\} = B-800$$

Leaving variable: x_{54}

S \ D	T1	T2	D1	D2	D3		u
P1	3 1000	4	M	M	M	1000	0
P2	2 1200	5 0	M	M	M	1200	-1
T1	0	7 2200	8 0	6	M	B	1
T2	M	0	M 800	4 1400	9	B	M-7
D1	M	M	0 2200	5	M	B	-7
D2	M	M	M	0 1700	3 500	B	M-11
	B	B	B+800	900+B	500		
v	3	6	7	11-M	14-M		

Iteration 2

Step 1: Let G(x) be the set of selected cells of the basic feasible solution

∀(i, j) ∈ G(X), $c_{ij} = u_i + v_j$

Let u_1 = 0, we get:

$$u_2 = -1, u_3 = 1, u_4 = M - 7,$$
$$u_5 = -7, u_6 = M - 11$$

$$v_1 = 3, v_2 = 6, v_3 = 7,$$
$$v_4 = 11 - M, v_5 = 14 - M$$

S \ D	T1	T2	D1	D2	D3		u
P1	1000 (3)	(4)	(M)	(M)	(M)	1000	0
P2	1200 (2)	0 (5)	(M)	(M)	(M)	1200	-1
T1	(0)	2200 (7)	0 (8)	(6)	(M)	B	1
T2	(M)	(0)	800 (M)	1400 (4)	(9)	B	M-7
D1	(M)	(M)	2200 (0)	(5)	(M)	B	-7
D2	(M)	(M)	(M)	1700 (0)	500 (3)	B	M-11
	B	B	B+800	900+B	500		
v	3	6	7	11-M	14-M		

Iteration 2

Step 2: ∀(i, j) ∉ G(x), $\bar{c}_{ij} = c_{ij} - u_i - v_j$

$$\bar{c}_{12} = 4 - 0 - 6 = -2 < 0$$

$$\bar{c}_{31} = 0 - 1 - 3 = -4 < 0$$

$$\bar{c}_{34} = 6 - 1 - (11 - M) = M + 6 > 0$$

$$\bar{c}_{42} = 0 - (M - 7) - 6 = -M + 1 < 0$$

$$\bar{c}_{54} = 5 - (11 - M) - (-7) = M + 1 > 0$$

$$\bar{c}_{45} = 9 - (M - 7) - (14 - M) = 2 > 0$$

……

Choose x_{42} as entering variable

D S	T1	T2	D1	D2	D3		u
P1	3 1000	4	M	M	M	1000	0
P2	2 1200	5 0	M	M	M	1200	-1
T1	0	7 2200-θ	8 0+θ	6	M	B	1
T2	M	0 θ	M 800-θ	4 1400	9	B	M-7
D1	M	M	0 2200	5	M	B	-7
D2	M	M	M	0 1700	3 500	B	M-11
	B	B	B+800	900+B	500		
v	3	6	7	11-M	14-M		

Iteration 2:

Step 3:

$$C^+ = \{(4,2),(3,3)\}$$

$$C^- = \{(3,2),(4,3)\}$$

$$\theta^* = \min\{x_{ij} : (i,j) \in C^-\}$$

$$= \min\{2000,800\} = 800$$

Leaving variable: x_{43}

S \ D	T1	T2	D1	D2	D3		u
P1	3 1000	4	M	M	M	1000	0
P2	2 1200	5 0	M	M	M	1200	-1
T1	0	7 1400	8 800	6	M	B	1
T2	M	0 800	M	4 1400	9	B	-6
D1	M	M	0 2200	5	M	B	-7
D2	M	M	M	0 1700	3 500	B	-10
	B	B	B+800	900+B	500		
v	3	6	7	10	13		

Iteration 3

Step 1: Let G(x) be the set of selected cells of the basic feasible solution

∀(i, j) ∈ G(X), $c_{ij} = u_i + v_j$

Let u_1 = 0, we get:

$$u_2 = -1, u_3 = 1, u_4 = -6,$$
$$u_5 = -7, u_6 = -10$$

$$v_1 = 3, v_2 = 6, v_3 = 7,$$
$$v_4 = 10, v_5 = 13$$

S \ D	T1	T2	D1	D2	D3		u
P1	1000 ^{3}	^{4}	^{M}	^{M}	^{M}	1000	0
P2	1200 ^{2}	0 ^{5}	^{M}	^{M}	^{M}	1200	-1
T1	^{0}	1400 ^{7}	800 ^{8}	^{6}	^{M}	B	1
T2	^{M}	800 ^{0}	^{M}	1400 ^{4}	^{9}	B	-6
D1	^{M}	^{M}	2200 ^{0}	^{5}	^{M}	B	-7
D2	^{M}	^{M}	^{M}	1700 ^{0}	500 ^{3}	B	-10
	B	B	B+800	900+B	500		
v	3	6	7	10	13		

Iteration 3

Step 2: ∀(i, j) ∉ G(x), $\bar{c}_{ij} = c_{ij} - u_i - v_j$

$$\bar{c}_{12} = 4 - 0 - 6 = -2 < 0$$

$$\bar{c}_{31} = 0 - 1 - 3 = -4 < 0$$

$$\bar{c}_{34} = 6 - 1 - 10 = -5 < 0$$

$$\bar{c}_{54} = 5 - 10 - (-7) = 2 > 0$$

$$\bar{c}_{45} = 9 - (-6) - 13 = 2 > 0$$

……

Choose x_{34} as entering variable

S \ D	T1	T2	D1	D2	D3		u
P1	1000 (3)	(4)	(M)	(M)	(M)	1000	0
P2	1200 (2)	0 (5)	(M)	(M)	(M)	1200	-1
T1	(0)	1400-θ (7)	800 (8)	θ (6)	(M)	B	1
T2	(M)	800+θ (0)	(M)	1400-θ (4)	(9)	B	-6
D1	(M)	(M)	2200 (0)	(5)	(M)	B	-7
D2	(M)	(M)	(M)	1700 (0)	500 (3)	B	-10
	B	B	B+800	900+B	500		
v	3	6	7	10	13		

Iteration 3:

Step 3:

$$C^+ = \{(3,4),(4,2)\}$$
$$C^- = \{(3,2),(4,4)\}$$

$$\theta^* = \min\{x_{ij}:(i,j) \in C^-\}$$
$$= \min\{1400,1400\} = 1400$$

Leaving variable: x_{32}

S \ D	T1	T2	D1	D2	D3		u
P1	1000 (3)	(4)	(M)	(M)	(M)	1000	0
P2	1200 (2)	0 (5)	(M)	(M)	(M)	1200	-1
T1	(0)	(7)	800 (8)	1400 (6)	(M)	B	-4
T2	(M)	2200 (0)	(M)	0 (4)	(9)	B	-6
D1	(M)	(M)	2200 (0)	(5)	(M)	B	-12
D2	(M)	(M)	(M)	1700 (0)	500 (3)	B	-10
	B	B	B+800	900+B	500		
v	3	6	12	10	13		

Iteration 4

Step 1: Let G(x) be the set of selected cells of the basic feasible solution

∀(i, j) ∈ G(X), $c_{ij} = u_i + v_j$

Let u_1 = 0, we get:

$$u_2 = -1, u_3 = -4, u_4 = -6,$$
$$u_5 = -12, u_6 = -10$$

$$v_1 = 3, v_2 = 6, v_3 = 12,$$
$$v_4 = 10, v_5 = 13$$

S \ D	T1	T2	D1	D2	D3		u
P1	3 1000	4	M	M	M	1000	0
P2	2 1200	5 0	M	M	M	1200	-1
T1	0	7	8 800	6 1400	M	B	-4
T2	M	0 2200	M	4 0	9	B	-6
D1	M	M	0 2200	5	M	B	-12
D2	M	M	M	0 1700	3 500	B	-10
	B	B	B+800	900+B	500		
v	3	6	12	10	13		

Iteration 4

Step 2: $\forall$(i, j) $\notin$ G(x), $\bar{c}_{ij} = c_{ij} - u_i - v_j$

$$\bar{c}_{12} = 4 - 0 - 6 = -2 < 0$$

$$\bar{c}_{31} = 0 + 4 - 3 = 1 > 0$$

$$\bar{c}_{32} = 7 + 4 - 6 = 5 > 0$$

$$\bar{c}_{54} = 5 + 12 - 10 = 7 > 0$$

$$\bar{c}_{45} = 9 - (-6) - 13 = 2 > 0$$

……

Choose x_{12} as entering variable

S \ D	T1	T2	D1	D2	D3		u
P1	[3] $1000-\theta$	[4] θ	[M]	[M]	[M]	1000	0
P2	[2] $1200+\theta$	[5] $0-\theta$	[M]	[M]	[M]	1200	-1
T1	[0]	[7]	[8] 800	[6] 1400	[M]	B	-4
T2	[M]	[0] 2200	[M]	[4] 0	[9]	B	-6
D1	[M]	[M]	[0] 2200	[5]	[M]	B	-12
D2	[M]	[M]	[M]	[0] 1700	[3] 500	B	-10
	B	B	B+800	900+B	500		
v	3	6	12	10	13		

Iteration 4:

Step 3:

$$C^{+} = \{(1,2),(2,1)\}$$
$$C^{-} = \{(1,1),(2,2)\}$$

$$\theta^{*} = \min\{x_{ij} : (i,j) \in C^{-}\}$$
$$= \min\{1000, 0\} = 0$$

Leaving variable: x_{22}

S \ D	T1	T2	D1	D2	D3		u
P1	(3) 1000	(4) 0	(M)	(M)	(M)	1000	0
P2	(2) 1200	(5)	(M)	(M)	(M)	1200	-1
T1	(0)	(7)	(8) 800	(6) 1400	(M)	B	-2
T2	(M)	(0) 2200	(M)	(4) 0	(9)	B	-4
D1	(M)	(M)	(0) 2200	(5)	(M)	B	-10
D2	(M)	(M)	(M)	(0) 1700	(3) 500	B	-8
	B	B	B+800	900+B	500		
v	3	4	10	8	11		

Iteration 5

Step 1: Let G(x) be the set of selected cells of the basic feasible solution

∀(i, j) ∈ G(X), $c_{ij} = u_i + v_j$

Let u_1 = 0, we get:

$$u_2 = -1, u_3 = -2, u_4 = -4,$$
$$u_5 = -10, u_6 = -8$$

$$v_1 = 3, v_2 = 4, v_3 = 10,$$
$$v_4 = 8, v_5 = 11$$

S \ D	T1	T2	D1	D2	D3		u
P1	3 1000	4 0	M	M	M	1000	0
P2	2 1200	5	M	M	M	1200	-1
T1	0	7	8 800	6 1400	M	B	-2
T2	M	0 2200	M	4 0	9	B	-4
D1	M	M	0 2200	5	M	B	-10
D2	M	M	M	0 1700	3 500	B	-8
	B	B	B+800	900+B	500		
v	3	4	10	8	11		

Iteration 5

Step 2: ∀(i, j) ∉ G(x), $\bar{c}_{ij} = c_{ij} - u_i - v_j$

$$\bar{c}_{22} = 5 + 1 - 4 = 2 > 0$$

$$\bar{c}_{31} = 0 + 2 - 3 = -1 < 0$$

$$\bar{c}_{32} = 7 + 2 - 4 = 5 > 0$$

$$\bar{c}_{54} = 5 + 10 - 8 = 7 > 0$$

$$\bar{c}_{45} = 9 - (-4) - 11 = 2 > 0$$

……

Choose x_{31} as entering variable

S \ D	T1	T2	D1	D2	D3		u
P1	1000-θ (3)	0+θ (4)	(M)	(M)	(M)	1000	0
P2	1200 (2)	(5)	(M)	(M)	(M)	1200	-1
T1	θ (0)	(7)	800 (8)	1400-θ (6)	(M)	B	-2
T2	(M)	2200-θ (0)	(M)	0+θ (4)	(9)	B	-4
D1	(M)	(M)	2200 (0)	(5)	(M)	B	-10
D2	(M)	(M)	(M)	1700 (0)	500 (3)	B	-8
	B	B	B+800	900+B	500		
v	3	4	10	8	11		

Iteration 5:

Step 3:

$$C^+ = \{(3,1),(1,2),(4,4)\}$$
$$C^- = \{(1,1),(3,4),(4,2)\}$$

$$\theta^* = \min\{x_{ij} : (i,j) \in C^-\}$$
$$= \min\{1000,1400,2200\} = 1000$$

Leaving variable: x_{11}

S \ D	T1	T2	D1	D2	D3		u
P1	3	4 1000	M	M	M	1000	0
P2	2 1200	5	M	M	M	1200	0
T1	0 1000	7	8 800	6 400	M	B	-2
T2	M	0 1200	M	4 1000	9	B	-4
D1	M	M	0 2200	5	M	B	-10
D2	M	M	M	0 1700	3 500	B	-8
	B	B	B+800	900+B	500		
v	2	4	10	8	11		

Iteration 6

Step 1: Let G(x) be the set of selected cells of the basic feasible solution

∀(i, j) ∈ G(X), $c_{ij} = u_i + v_j$

Let u_1 = 0, we get:

$$u_2 = 0, u_3 = -2, u_4 = -4,$$
$$u_5 = -10, u_6 = -8$$

$$v_1 = 2, v_2 = 4, v_3 = 10,$$
$$v_4 = 8, v_5 = 11$$

S \ D	T1	T2	D1	D2	D3		u
P1	3	4 1000	M	M	M	1000	0
P2	2 1200	5	M	M	M	1200	0
T1	0 1000	7	8 800	6 400	M	B	-2
T2	M	0 1200	M	4 1000	9	B	-4
D1	M	M	0 2200	5	M	B	-10
D2	M	M	M	0 1700	3 500	B	-8
	B	B	B+800	900+B	500		
v	2	4	10	8	11		

Iteration 6

Step 2: ∀(i, j) ∉ G(x), $\bar{c}_{ij} = c_{ij} - u_i - v_j$

$$\bar{c}_{11} = 3 - 2 - 0 = 1 > 0$$

$$\bar{c}_{22} = 5 - 0 - 4 = 1 > 0$$

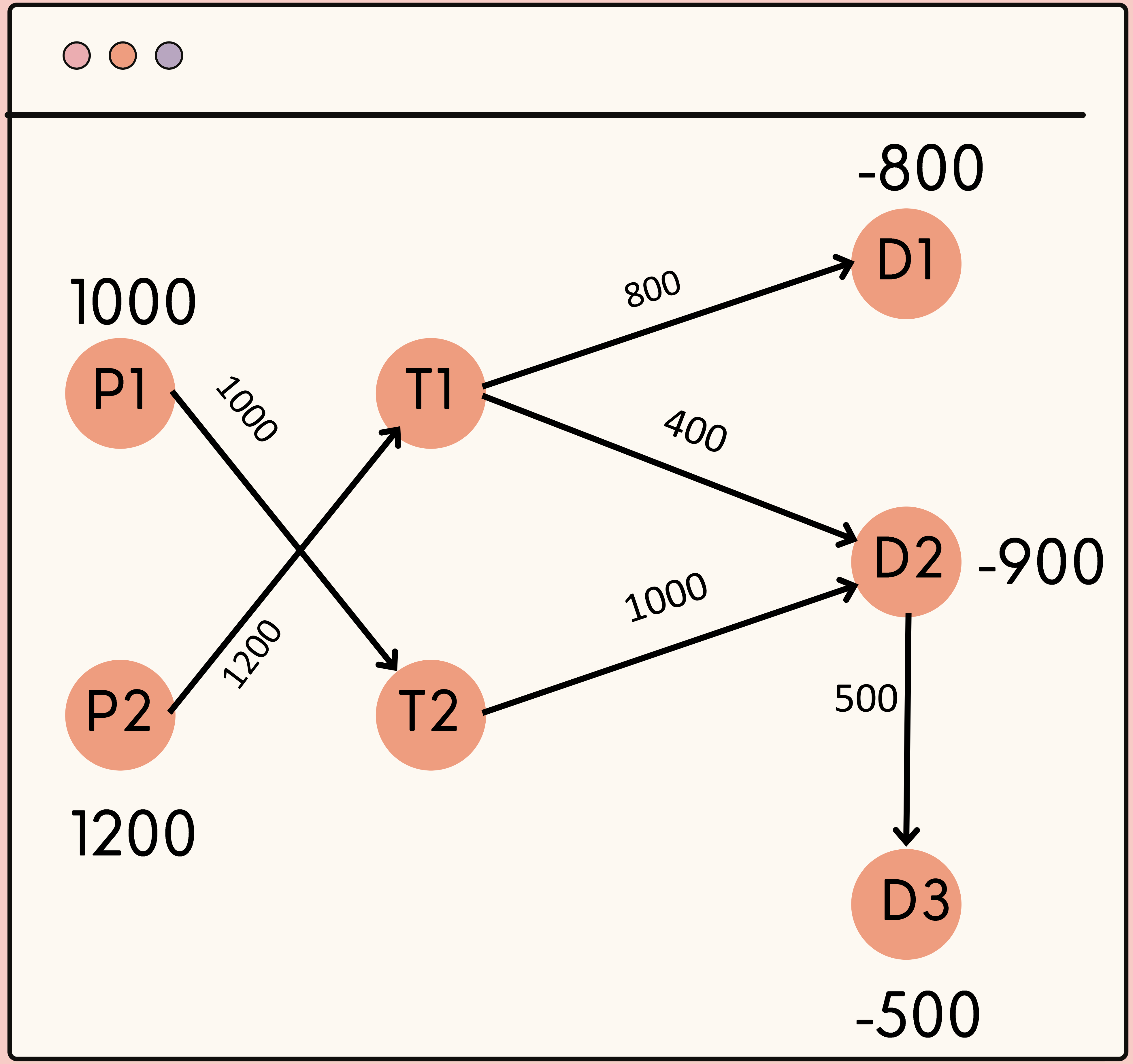
$$\bar{c}_{32} = 7 + 2 - 4 = 5 > 0$$

$$\bar{c}_{54} = 5 + 10 - 8 = 7 > 0$$

$$\bar{c}_{45} = 9 - (-4) - 11 = 2 > 0$$

……

All reduced costs are non-negative, thus the current basic feasible solution is optimal.


-800
D1
1000
P1
1000
T1
800
400
D2
-900
1000
1200
P2
T2
500
1200
D3
-500

D / S	T1	T2	D1	D2	D3		u
P1	3	4 1000	M	M	M	1000	0
P2	2 1200	5	M	M	M	1200	0
T1	0 1000	7	8 800	6 400	M	B	-2
T2	M	0 1200	M	4 1000	9	B	-4
D1	M	M	0 2200	5	M	B	-10
D2	M	M	M	0 1700	3 500	B	-8
	B	B	B+800	900+B	500		
v	2	4	10	8	11		

Minimum total transportation costs = 1000*4 + 1200*2 + 1000*0 + 800*8 + 400*6 + 1200*0 + 1000*4 + 2200*0 + 1700*0 + 500*3 = 20700.