## Multivariate Time Series Models for Monthly $PM_{2.5}$

Model	Loop Details	Train	Validation	Test	Parameters	MAPE Train + Test
ARIMAX Loop for p, d, q and feature_set	p range(6) d [2,3] q range(6) set 1 to set 7	2014-2018	2019-2020	2021	order (5,2,2) set2	.206
ARIMAX TimeSeriesSplit Forecast one month 24 splits	set 1 to set 7	Begins with 2014-2018 Increase one month	1 month begins from 2019 Jan.	2021	order (1,3,2) set7	.051
ARIMAX TimeSeriesSplit Forecast 6 months 4 splits	p range(1,6) d [2,3] q range(1,6) set 1 to set 7	Begins with 2014-2018 Increase 6 months	6 months begins from 2019 Jan	2021	order (1,2,2) set7	0.240
	feature_set_1	2014-2020	-	2021	(1,2,2)	0.64
<b>ARIMAX</b> auto_ARIMA	feature_set_2				(0,2,0)	.106
	feature_set_3				(2,2,2)	.197
	feature_set_4				(5,2,0)	.1966
	feature_set_5				(3,2,0)	.264
	feature_set_6				(0,2,2)	.189
	feature_set_7				(0,2,2)	.2501
SARIMAX Loop for p, d, q, ps, ds, qs and feature_set	p range(6) d [2,3] q range(6) set 1 to set 7	2014-2018	2019-2020	2021	order (2,1,2) s_order (1,0,0,12) set_2	.211
SARIMAX TimeSeriesSplit Forecast 6 months 4 splits	p range(1,6) d [2,3] q range(1,6) set7	Begins with 2014-2018 Increase 6 months	6 months begins from 2019 Jan	2021	order (0,3,1) s_order (0,0,0,12) set_2	.256
SARIMAX TimeSeriesSplit	p range(1,6) d [2,3]	Begins with 2014-2018	6 months begins	2021	order (0,2,2)	.201

Forecast 6 months 4 splits	q range(1,6) set7	Increase 6 months	from 2019 Jan		s_order (1,0,1,12) set_2	
SARIMAX auto_ARIMA	feature_set2	2014-2020	-	2021	order (2,2,3) s_order (0,0,0,12)	0.205
	feature_set7				order (2,2,2) s_order (0,0,0,12)	0.225
<b>FbProphet</b> Default Setting	feature_set_1	2014-2020	-	2021	default	0.23
	feature_set_2					0.149
	feature_set_3					0.139
	feature_set_4					0.233
	feature_set_5					0.51
	feature_set_6					0.161
	feature_set_7					0.150
<b>FbProphet</b> HP Tuning just one validation set	feature_set_3	2014-2018	2019-2020	2021	changepoint_p rior_scale': 0.4,	
					'n_changepoin ts': 150,	0.185
					'seasonality_m ode': 'multiplicative'	
PyCaret	feature_set_2	2014-2020	-	2021	0.17	.30