Stellar wind notes

September 28, 2017

1 ATMOSPHERE PRIOR HELIUM IGNITION

We have

$$\mathrm{d}y = -\rho \mathrm{d}r \qquad \text{(Column depth definition.)} \tag{1}$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -g\rho \qquad \text{(Hydrostatic equilibrium.)} \tag{2}$$

$$\frac{\mathrm{d}u_{\mathrm{rad}}}{\mathrm{d}r} = -\frac{3\kappa\rho F_{\mathrm{rad}}}{c} \qquad \text{(Radiation transportation in LTE.)} \tag{3}$$

where $u_{\mathrm{rad}} = aT^4$ is the radiation energy density, $\kappa = \kappa_0 \left[1 + (T/4.5 \times 10^8)^{0.86}\right]^{-1}$ with $\kappa_0 = 0.2$ the opacity.

Integrating from $r = \infty$ down to r and use the fact that prior burning, F_{rad} is roughly a constant, we have

$$y \simeq \frac{P}{g} \simeq \frac{M(>r)}{4\pi r^2}, \tag{4}$$

$$aT^4 \simeq \frac{3\kappa F_{\mathrm{rad}}}{c} y. \tag{5}$$

The atmosphere can be approximated by ideal gas, $P = \rho k_b T/\mu m_{\mathrm{p}}$ with $\mu = 4/3$ for pure He. Thus have

$$\rho = g\frac{\mu m_{\mathrm{p}}}{k_b}\frac{caT^3}{3\kappa F_{\mathrm{rad}}}. \tag{6}$$

The F_{rad} term is due to pycnonuclear and electron capture reactions that escape from the surface, as

$$F_{\mathrm{rad}} = 10^{21}\left(\frac{Q_{\mathrm{crust}}}{0.1\mathrm{MeVnucleon}^{-1}}\right)\left(\frac{\dot{m}}{10^4\mathrm{g\ cm}^{-2}\ \mathrm{s}^{-1}}\right). \tag{7}$$

If accretion rate is 10% of the local Eddington limit, we have a local accretion rate of $\dot{m}_{\mathrm{accr}} = 1.5 \times 10^4 \mathrm{g\ cm}^{-2}\ \mathrm{s}^{-1}$, or a global accretion rate of $\dot{M}_{\mathrm{accr}} = 3.0 \times 10^{-9} M_{\odot}/\mathrm{yr}$.

The entropy evolves as

$$T\frac{\mathrm{d}s}{\mathrm{d}t}=\frac{\mathrm{d}F_{\mathrm{rad}}}{\mathrm{d}y}+\epsilon. \tag{8}$$

Note $T\mathrm{d}s=C_{\mathrm{P}}\mathrm{d}T$, and define

$$-\epsilon_{\mathrm{cool}}=\frac{\mathrm{d}F_{\mathrm{rad}}}{\mathrm{d}y}\simeq-\frac{caT^4}{3\kappa y^2}, \tag{9}$$

we have

$$C_{\mathrm{P}}\frac{\mathrm{d}T}{\mathrm{d}t}=-\epsilon_{\mathrm{cool}}+\epsilon. \tag{10}$$

Run-away burning starts when

$$\epsilon\geq\epsilon_{\mathrm{cool}}. \tag{11}$$

We already have ϵ_{cool}. The ϵ term is given by

$$\epsilon\simeq\epsilon_{3\alpha}=5.3\times10^{21}\frac{\rho_5^2Y^3}{T_8^3}\exp\left(-\frac{44}{T_8}\right)\mathrm{erg\ g^{-1}\ s^{-1}}. \tag{12}$$

We can thus solve for T_{b} and y_{b} for different accretion rates. The result is plotted in the figure below. We tuned $Q_{\mathrm{crust}}=0.05\mathrm{MeV nucleon}^{-1}$ such that at $\dot{M}_{\mathrm{accr}}=0.1\dot{M}_{\mathrm{Edd}}=3\times10^{-9}M_{\odot}/\mathrm{yr}$, we have $y_{\mathrm{b}}=3\times10^{8}\mathrm{g/cm^2}$.

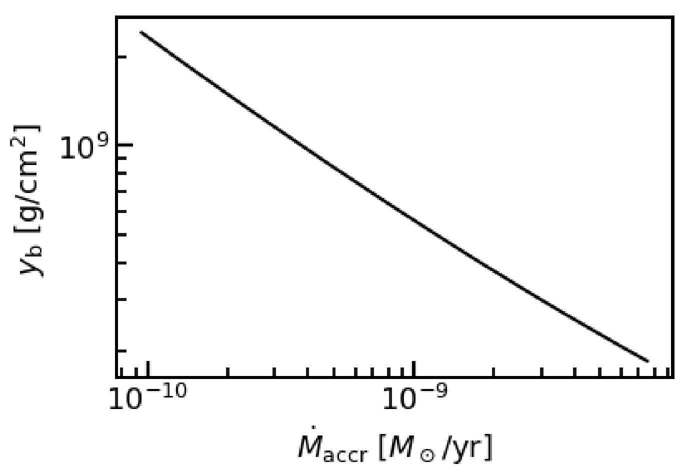

Figure 1: Ignition column depth as a function of global accretion rate.

2 STEADY STATE WIND

When the wind reaches steady state, it is governed by the following equations:

$$4\pi r^2 \rho v = \dot{M}_{\rm wind} = {\rm const}, \qquad \text{(Mass conservation)}, \tag{13}$$

$$\dot{M}_{\rm w}(\frac{v^2}{2} - \frac{GM}{r} + h) + L_{\rm rad} = \dot{E} = {\rm const}, \qquad \text{(Energy conservation)}, \tag{14}$$

$$v\frac{{\rm d}v}{{\rm d}r} = -\frac{1}{\rho}\frac{{\rm d}P}{{\rm d}r} - \frac{GM}{r^2}, \qquad \text{(Momentum conservation)}, \tag{15}$$

$$\frac{{\rm d}T}{{\rm d}r} = -\frac{3\kappa\rho}{4caT^3}F_{\rm rad}, \qquad \text{(Radiation transportation)}, \tag{16}$$

where h is the enthalpy density.

Focusing on the energy equation. Evaluating it at the base of the wind ($r_{\rm w} \simeq R$) and noting that the gravitational binding energy dominates over enthalpy and velocity there leads to

$$-\frac{GM\dot{M}_{\rm w}}{R} + L_{\rm w} \simeq \dot{E}. \tag{17}$$

Can also evaluate it at infinity and take the ansatz that mechanical energy is much smaller than the radiative energy loss

$$L_{\rm rad}(r = \infty) = L_{\rm Edd0} \simeq \dot{E}, \tag{18}$$

where $L_{\rm Edd0} = 4\pi cGM/\kappa_0$ is the Eddington limit at $T = 0$. Thus get mass loss rate due to the mind:

$$\dot{M}_{\rm w} = (L_{\rm w} - L_{\rm Edd0})\frac{R}{GM}. \tag{19}$$

To get $L_{\rm w}$, the luminosity at the wind base, can integrate the radiation transportation equation from $r = \infty$ down to the wind base, and note that the flux there is once again a constant (as the burning region is much deep-down). We have

$$L_{\rm w} = 4\pi R^2 \frac{c}{3\kappa}\frac{aT^4}{y_{\rm w}}. \tag{20}$$

On the other hand, at the base it is almost hydrostatic, $y_w \simeq P_{\rm w}/g$. Further assume radiation pressure dominates, $P_{\rm w} = aT_{\rm w}^4/g$. We thus arrive at

$$L_{\rm w} \sim 4\pi\frac{cGM}{\kappa} \sim L_{\rm Edd}, \tag{21}$$

which is the local Eddington limit. This is consistent with our expectation that the wind should launch where the Luminosity first becomes super-Eddington.

To solve for $L_{\rm w}$, consider two time scales:

$$t_{\rm w} = \frac{Q_{\rm burn}}{\dot{E}}\frac{4\pi R^2 y_{\rm b}}{\mu m_{\rm p}} = \lambda_{\rm w}\frac{4\pi R^2 y_{\rm w}}{\dot{M}_{\rm w}}. \tag{22}$$

The first equality gives the timescale for energy loss, and the second for mass loss. $Q_{\rm burn} = 1.6$Mev/nucleon. We also introduce a scaling factor $\lambda_{\rm w}$, as the two times are related but not necessarily equal. We can then solve for the value of $T_{\rm w}$, as

$$\left[\frac{L_{\rm w}(T_{\rm w})}{L_{\rm Edd0}} - 1\right] \frac{y_b}{R} \frac{Q_{\rm burn}}{\mu m_{\rm p}} = \lambda_{\rm w} \frac{a T_w^4}{3}. \tag{23}$$

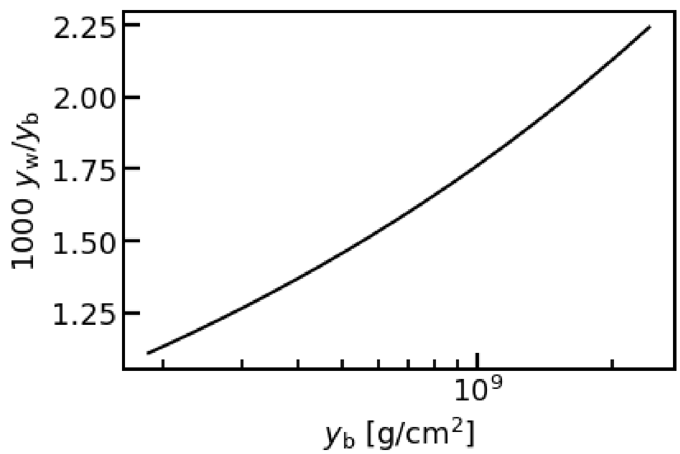

Figure 2: Wind column depth as a function of base column depth, $\lambda_{\rm w} = 3$.

Differentiating the mass conservation equation with respect to r and then combining it with the momentum equation, we have

$$\frac{{\rm d}\rho}{{\rm d}r} = \left(-\frac{GM}{r^2} + \frac{1}{2}\frac{v^2}{r}\right)\left(\gamma c_{\rm s} - v^2\right)^{-1} \tag{24}$$

where we define $c_{\rm s}^2 \equiv P/\rho$ and $\gamma = 4/3$ when radiation pressure dominates.

This leads to the sonic point equations

$$c_{\rm s}^2 = \frac{3}{4} v_{\rm s}^2, \tag{25}$$

$$2 v_{\rm s}^2 = \frac{GM}{r_{\rm s}}, \tag{26}$$

where $r_{\rm s}$ and $v_{\rm s}$ is the radius and velocity at the sonic point, respectively.

Using the mass conservation once again and noting that the radiation pressure dominates at the sonic point we further have

$$\rho_{\rm s} = \frac{\dot{M}_{\rm w}}{4\pi r_{\rm s}^2 v_{\rm s}}, \tag{27}$$

$$P_{\rm s} = c_{\rm s}^2 \rho_{\rm s} = \frac{a}{3} T_{\rm s}^4. \tag{28}$$

Thus once we know one of $r_{\rm s}$ or $v_{\rm s}$, we know all the other parameters at the sonic point.

Evaluate the energy conservation equation both at the sonic point and at infinity, and use the fact that temperature there are sufficiently low so $\kappa \simeq \kappa_0$ so $L_{\rm rad}(r_{\rm s}) \simeq L_{\rm rad}(\infty) \simeq L_{\rm Edd0}$, we then have

$$\frac{v_\infty^2}{2} \simeq \frac{v_{\rm s}^2}{2} - \frac{GM}{r_{\rm s}} + 4c_{\rm s}^2. \tag{29}$$

Thus the sonic point parameters also determines the velocity at infinity.

If we take an ansatz of $v_{\rm s} = 8.5 \times 10^8$cm/s, the above discussion leads to

$$r_{\rm s} = 1.3 \times 10^8 \left(\frac{8.5 \times 10^8 \text{cm/s}}{v_{\rm s}}\right)^2 \text{cm}, \tag{30}$$

$$\rho_{\rm s} = 9.6 \times 10^{-9} \left(\frac{v_{\rm s}}{8.5 \times 10^8 \text{cm/s}}\right)^3 \text{g cm}^{-3}, \tag{31}$$

$$T_{\rm s} = 1.2 \times 10^6 \left(\frac{v_{\rm s}}{8.5 \times 10^8 \text{cm/s}}\right)^{5/4} \text{K}, \tag{32}$$

$$v_\infty = 1.5 \times 10^9 \left(\frac{v_{\rm s}}{8.5 \times 10^8 \text{cm/s}}\right)^2 \text{cm s}^{-1}. \tag{33}$$