Image Processing
INT3404 20
Week 8: Morphological Operations

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Schedule

Tuần	Nội dung	Yêu cầu đối với sinh viên (ngoài việc đọc tài liệu tham khảo)
1	Giới thiệu môn học	Cài đặt môi trường: Python 3, OpenCV 3, Numpy, Jupyter Notebook
2	Ảnh số (Digital image) – Phép toán điểm (Point operations) Làm quen với OpenCV + Python Điều chỉnh độ tương phản (Contrast adjust)– Ghép ảnh (Combining images)	
3	Histogram - Histogram equalization	Làm bài tập 1: điều chỉnh gamma tìm contrast hợp lý
4	Phép lọc trong không gian điểm ảnh (linear processing filtering)	Thực hành ở nhà
5	Phép lọc trong không gian điểm ảnh (linear processing filtering) (cont.) Thực hành: Cách tìm filters	Thực hành ở nhà
6	Thực hành: Ứng dụng của histogram; Tìm ảnh mẫu (Template matching)	Bài tập mid-term
7	Trích rút đặc trưng của ảnh Cạnh (Edge) và đường (Line) và texture	Thực hành ở nhà
8	Các phép biến đổi hình thái (Morphological operations)	Làm bài tập 2: tìm barcode
9	Chuyển đổi không gian – Miền tần số – Phép lọc trên miền tần số Thông báo liên quan đồ án môn học	Đăng ký thực hiện đồ án môn học
10	Xử lý ảnh màu (Color digital image)	Làm bài tập 3: Chuyển đổi mô hình màu và thực hiện phân vùng
11	Các phép biến đổi hình học (Geometric transformations)	Thực hành ở nhà
12	Nhiễu – Mô hình nhiễu – Khôi phục ảnh (Noise and restoration)	Thực hành ở nhà
13	Nén ảnh (Compression)	Thực hành ở nhà
14	Hướng dẫn thực hiện đồ án môn học	Trình bày đồ án môn học
15	Hướng dẫn thực hiện đồ án môn học Tổng kết cuối kỳ	Trình bày đồ án môn học

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Recall: Image processing operations

- Point:
  - +, -, *, :
  - Gamma transformation
  - Thresholding
  - Histogram
- Area:
  - Correlation
  - Convolution
- Feature:
  - Edge
  - Line
  - Texture

- global
- local

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Morphological operations

Erosion, Dilation, Opening, Closing

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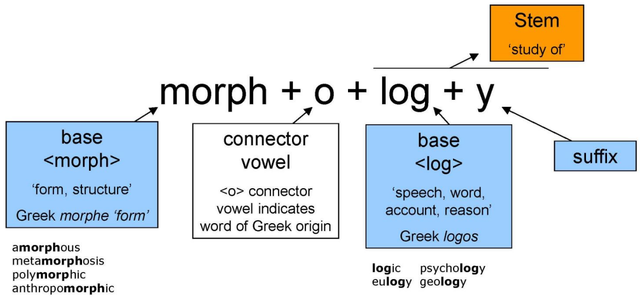
Morphology



→ Study of form and structures

→ In image processing: Study of image components that are useful in the representation and description of region shape
→ Eg: boundaries, skeletons, convex hull
→ Operations: filtering, thinning, pruning

Illustration credit: https://educ.queensu.ca/research/spotlights/morphology

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Set theory for morphology

- Binary image = a set of 2-D integer space
- Grayscale digital image = a set of 3-D elements
- Morphological operations are defined in terms of sets
- 2 sets of pixels:
  - Objects → foreground pixels
  - Structuring elements

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Represent image as a set

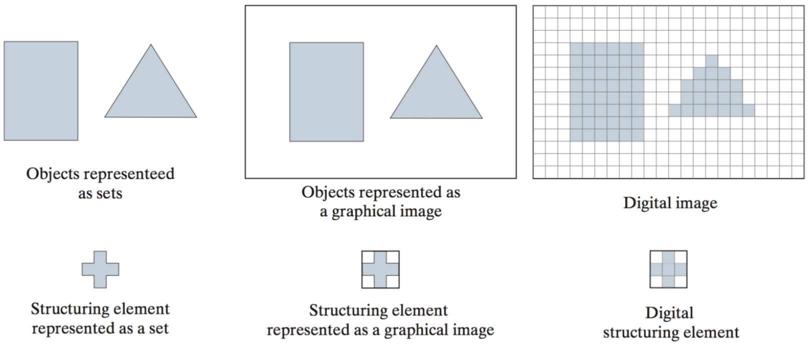

FIGURE 9.1 Top row. *Left:* Objects represented as graphical sets. *Center:* Objects embedded in a background to form a graphical image. *Right:* Object and background are digitized to form a digital image (note the grid). Second row: Example of a structuring element represented as a set, a graphical image, and finally as a digital SE.

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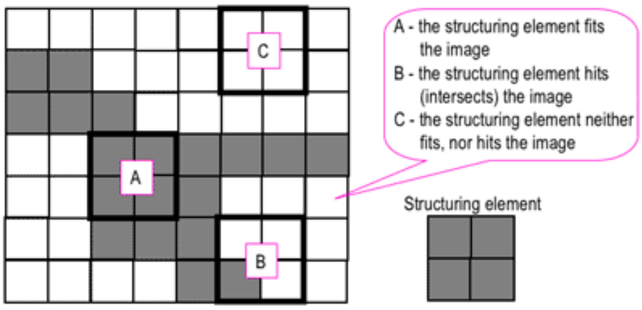
Probing image with structuring element



https://medium.com/@himnickson/morphological-operations-in-image-processing-cb8045b98fcc

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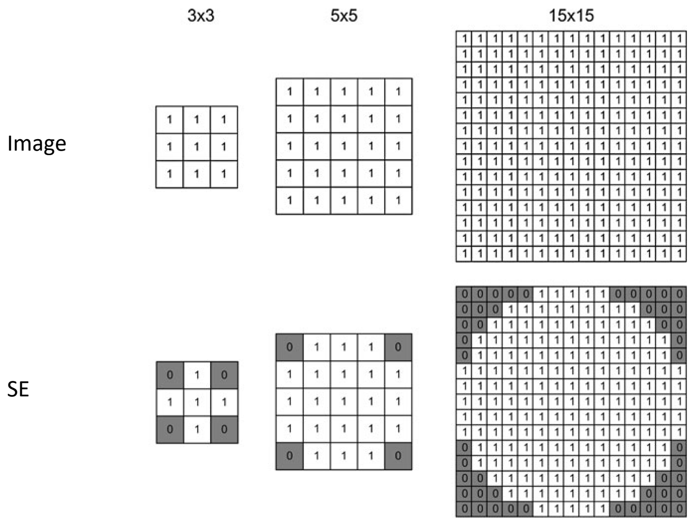
Example of structuring elements



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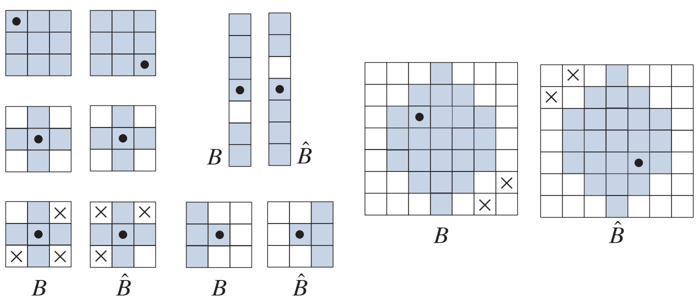
Reflection

The *reflection* of a set (structuring element) B about its origin, denoted by $\hat{B}$, is defined as

$$\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}$$

→ (x, y) is replaced by (-x, -y)

FIGURE 9.2 Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.



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Translation

The *translation* of a set B by point $z = (z_1, z_2)$, denoted $(B)_z$, is defined as

$$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$$

(x, y) is replaced by (x+z1, y+z2)

→ Is used to translate (slide) a structuring element over an image, an each location perform a set operation between the structuring element and the area of the image directly under it

Fig. 9.3

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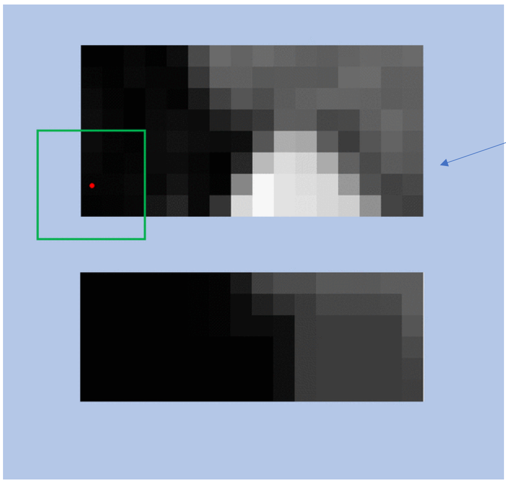
Erosion



Act like a local minimum filter

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

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Erosion – formal definition

With A and B as sets in Z^2, the *erosion* of A by B, denoted $A \ominus B$, is defined as

$$A \ominus B = \{ z \mid (B)_z \subseteq A \} \qquad (9\text{-}3)$$

→ B has to be contained in A
→ Equivalent to B not sharing any common elements with the background (i.e., the set complement of A)

$$A \ominus B = \{ z \mid (B)_z \cap A^c = \varnothing \}$$

For the whole image

$$I \ominus B = \{ z \mid (B)_z \subseteq A \text{ and } A \subseteq I \} \cup \{ A^c \mid A^c \subseteq I \} \qquad (9\text{-}4)$$

where I is a rectangular array of foreground and background pixels.

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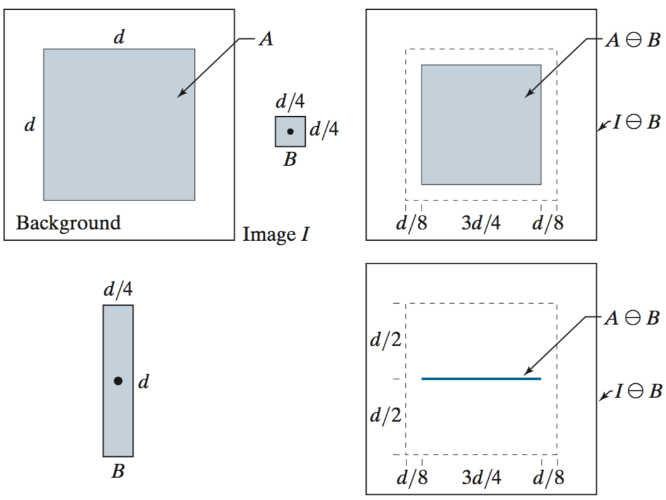
Example of erosion

a b c
d e

FIGURE 9.4
(a) Image I, consisting of a set (object) A, and background.
(b) Square SE, B (the dot is the origin).
(c) Erosion of A by B (shown shaded in the resulting image).
(d) Elongated SE.
(e) Erosion of A by B. (The erosion is a line.) The dotted border in (c) and (e) is the boundary of A, shown for reference.



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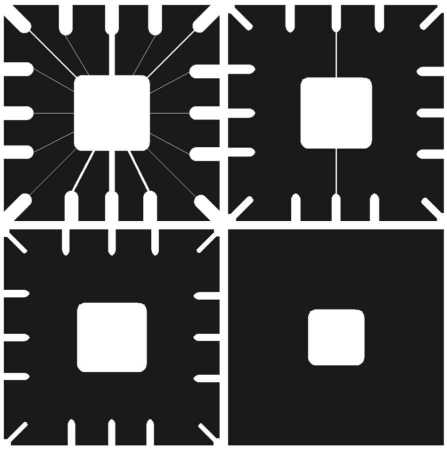
Example: Using erosion to remove image components

Erosion shrinks or thins objects in a binary image
→ Erosion as a morphological filtering operation in which image details smaller than the structuring element are filtered (removed) from the image

a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45 elements, respectively, all valued 1.



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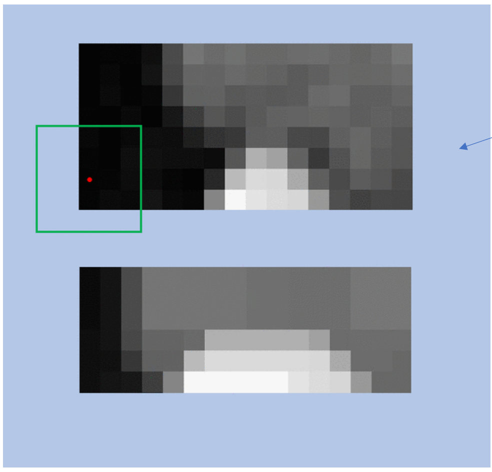
Dilation



Act like a local maximum filter

$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$

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Dilation – formal definition

With A and B as sets in Z^2, the dilation of A by B, denoted as $A \oplus B$, is defined as

$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\} \qquad (9\text{-}6)$$

→ Reflecting B about its origin and translating the reflection by z
→ The set of all displacements, z, such that the foreground elements of $\hat{B}$ overlap at least one element of A

$$A \oplus B = \left\{ z \,\middle|\, [(\hat{B})_z \cap A] \subseteq A \right\}$$

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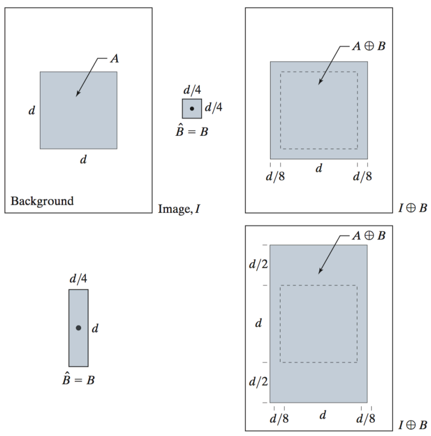
Dilation example

Erosion = shrinking or thinning operation
Dilation "grows" or "thickens" objects

a b c
d e

FIGURE 9.6
(a) Image I, composed of set (object) A and background.
(b) Square SE (the dot is the origin).
(c) Dilation of A by B (shown shaded).
(d) Elongated SE.
(e) Dilation of A by this element. The dotted line in (c) and (e) is the boundary of A, shown for reference.



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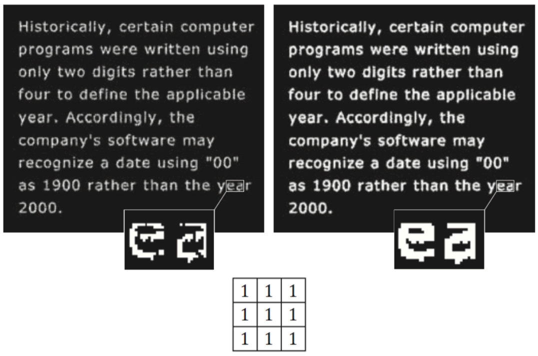
Example: Using dilation to repair broken characters

One of the simplest applications of dilation is for bridging gaps

a c
b

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.



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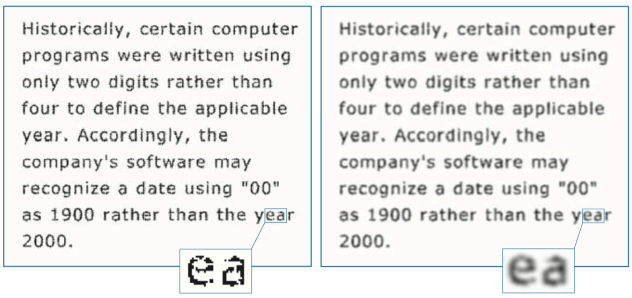
Bridging gaps comparing with spatial filtering

Lowpass filtering: starts with a binary image and produces a grayscale image
Dilation: results directly in a binary image

a b

FIGURE 4.48
(a) Sample text of low resolution (note the broken characters in the magnified view).
(b) Result of filtering with a GLPF, showing that gaps in the broken characters were joined.



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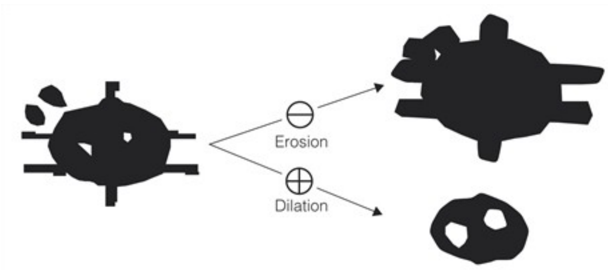
Dilation vs Erosion



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Duality of Erosion and Dilation

Erosion and dilation are *duals* of each other with respect to set complementation and reflection. That is,

$$(A \ominus B)^c = A^c \oplus \hat{B} \tag{9-8}$$

and

$$(A \oplus B)^c = A^c \ominus \hat{B} \tag{9-9}$$

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Opening

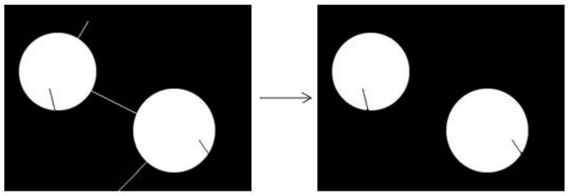
- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions

The *opening* of set A by structuring element B, denoted by $A \circ B$, is defined as

$$A \circ B = (A \ominus B) \oplus B \qquad (9\text{-}10)$$

→ The opening of A by B is the union of all the translations of B so that B fits entirely in A

$$A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \}$$



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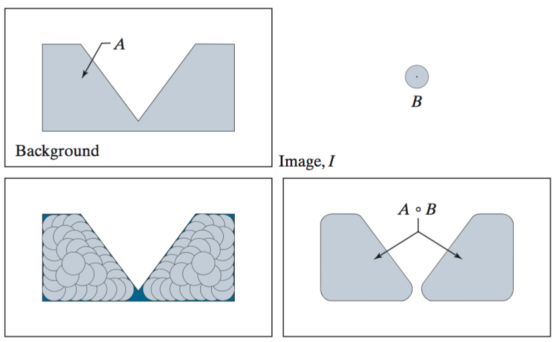
Geometric interpretation of Opening

a b
c d

FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.



Opening → eliminate regions narrower than the structuring element

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Closing

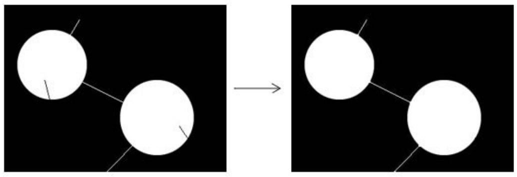
- Closing tends to smooth sections of contours, but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

Similarly, the *closing* of set A by structuring element B, denoted $A \bullet B$, is defined as

$$A \bullet B = (A \oplus B) \ominus B \tag{9-11}$$

→ Complement of the union of all translations of B that do not overlap A

$$A \bullet B = \left[\bigcup \left\{ (B)_z \,\middle|\, (B)_z \cap A = \varnothing \right\} \right]^c$$



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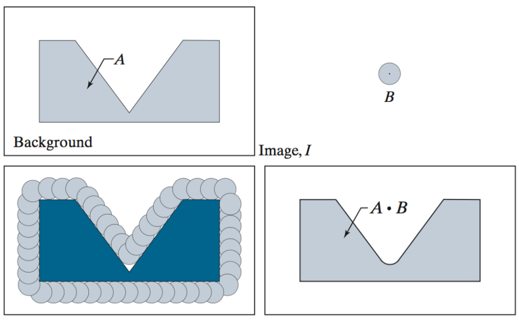
Geometric interpretation of Closing

a b
c d

FIGURE 9.9
(a) Image I, composed of set (object) A, and background.
(b) Structuring element B.
(c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.)
(d) Closing of A by B.



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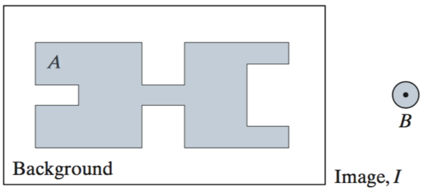
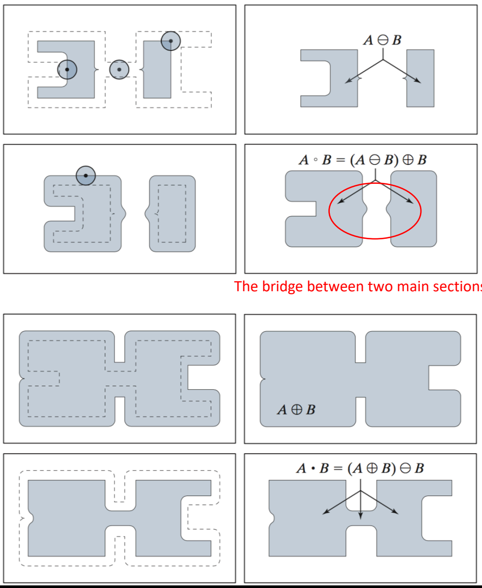
Opening vs closing





The bridge between two main sections was eliminated

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. (a) Image I, composed of a set (object) A and background; a solid, circular structuring element is shown also. (The dot is the origin.) (b) Structuring element in various positions. (c)-(i) The morphological operations used to obtain the opening and closing.

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Opening & Closing duality

As with erosion and dilation, opening and closing are duals of each other with respect to set complementation and reflection:

$$\left(A \circ B\right)^c = \left(A^c \bullet \hat{B}\right) \qquad (9\text{-}14)$$

and

$$\left(A \bullet B\right)^c = \left(A^c \circ \hat{B}\right) \qquad (9\text{-}15)$$

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Properties of Opening and closing

Morphological opening

(a) $A \circ B$ is a subset of A.
(b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
(c) $(A \circ B) \circ B = A \circ B$.

Multiple openings or closings of a set have no effect after the operation has been applied once.

Similarly, closing satisfies the following properties:

(a) A is a subset of $A \bullet B$.
(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
(c) $(A \bullet B) \bullet B = A \bullet B$.

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Opening and closing as morphological filtering

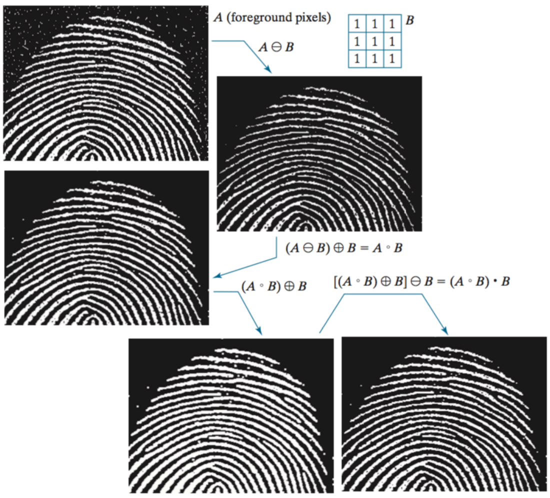
Eroding → remove white specks, but increase size of dark spots

Dilation → new gaps between the fingerprint ridges

Dilation → store breaks but ridges are thickened

Erosion → remedy thickened ridges

Overall: only some white specks



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Hit-or-miss transform (HMT)

- Basic tool for shape detection
- Using two structuring elements

B_1, for detecting shapes in the foreground
B_2, for detecting shapes in the background

$$I \circledast B_{1,2} = \left\{ z \,\middle|\, (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c \right\}$$
$$= (A \ominus B_1) \cap (A^c \ominus B_2)$$

→ *Simultaneously* B_1 find a match in the foreground and B2 find a match in the background

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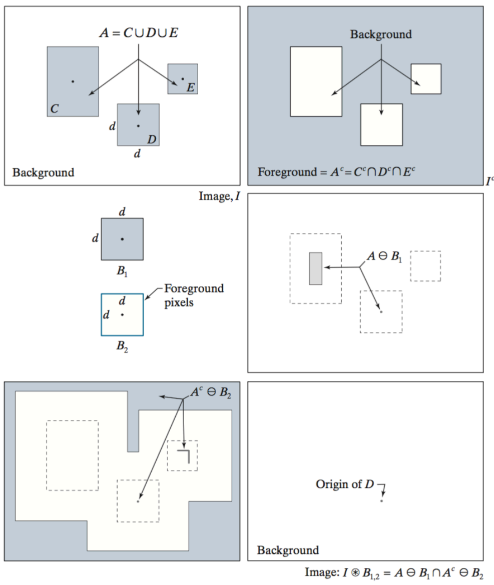
HMT example

$$I \circledast B_{1,2} = \left\{ z \,\middle|\, (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c \right\}$$
$$= (A \ominus B_1) \cap (A^c \ominus B_2)$$

a b
c d
e f

FIGURE 9.12 (a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's. (b) Image with its foreground defined as A^c. (c) Structuring elements designed to detect object D. (d) Erosion of A by B_1. (e) Erosion of A^c by B_2. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.



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Morphological algorithms

- Extract boundaries
- Fill holes
- Find connected components

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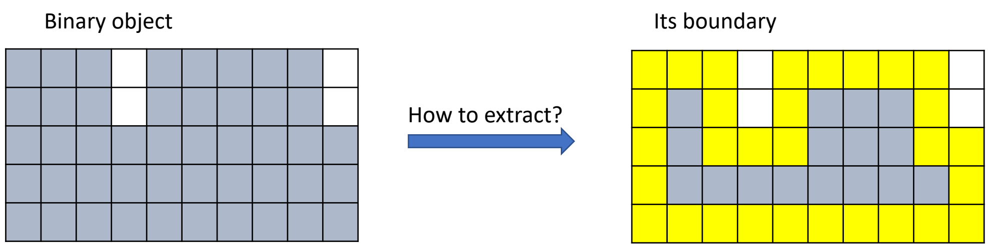
Foreground boundary



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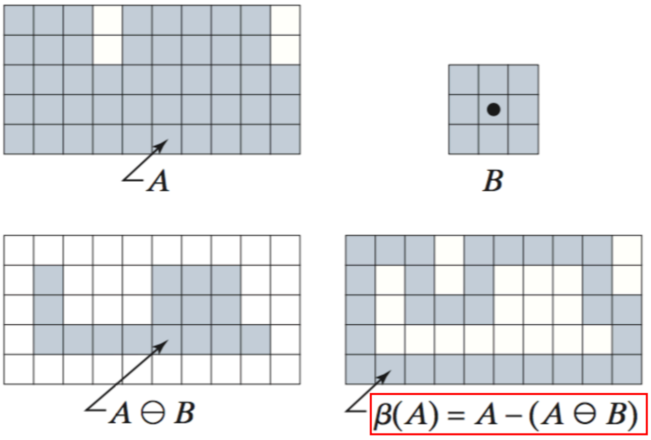
Boundary extraction algorithm



Reference: Gonzalez et. al., Fig. 9.15

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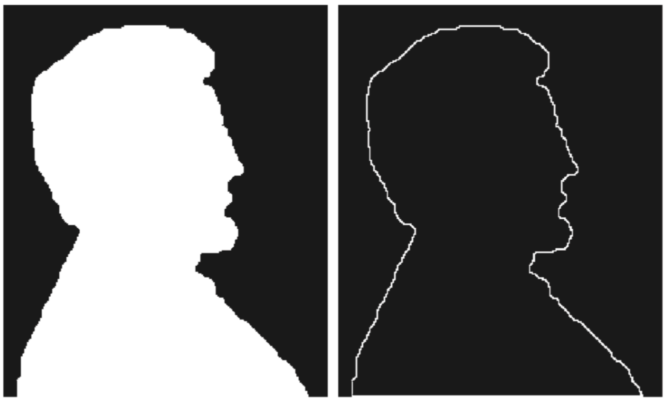
Boundary extraction example

a b

FIGURE 9.16
(a) A binary image.
(b) Result of using Eq. (9-18) with the structuring element in Fig. 9.15(b).



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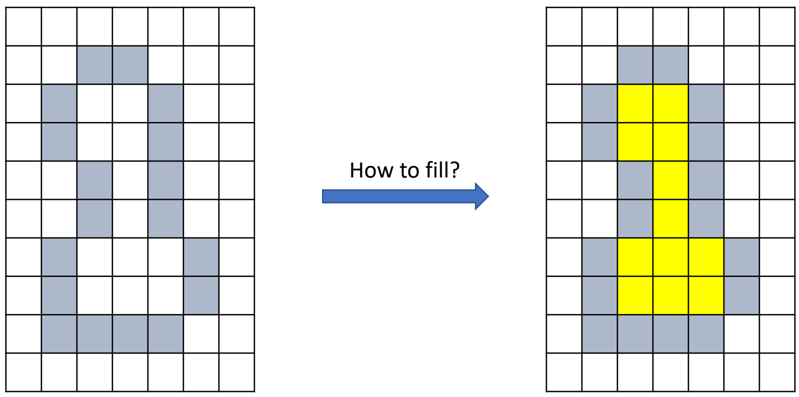
Hole filling



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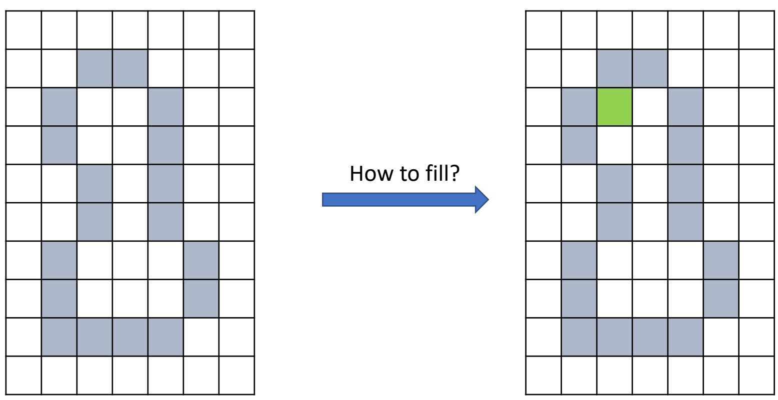
Hole filling



Given that we know a point in the hole

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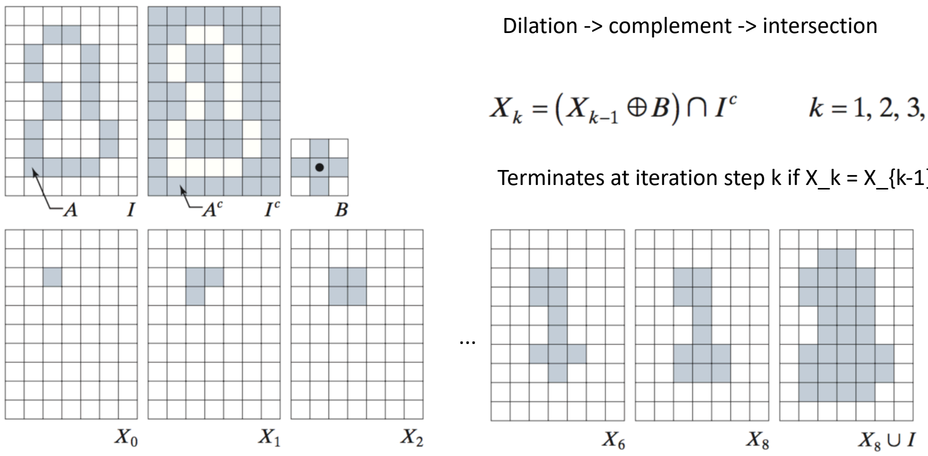
Hole filling



Dilation -> complement -> intersection

$$X_k = (X_{k-1} \oplus B) \cap I^c \qquad k = 1, 2, 3, \ldots$$

Terminates at iteration step k if X_k = X_{k-1}

Ref: Gonzalez et.al., Fig. 9.17

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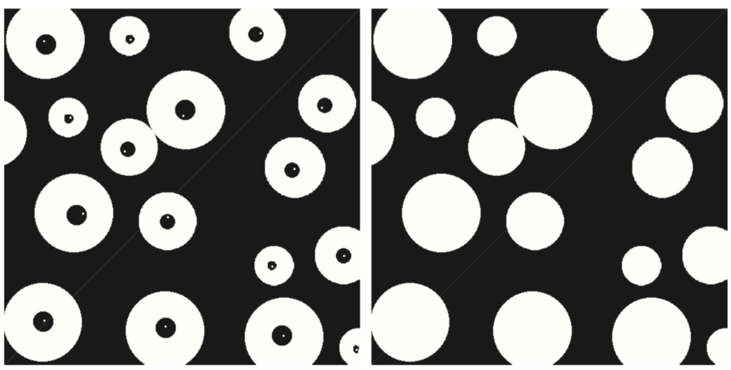
Hole filling example

a b

FIGURE 9.18 (a) Binary image. The white dots inside the regions (shown enlarged for clarity) are the starting points for the hole-filling algorithm. (b) Result of filling all holes.



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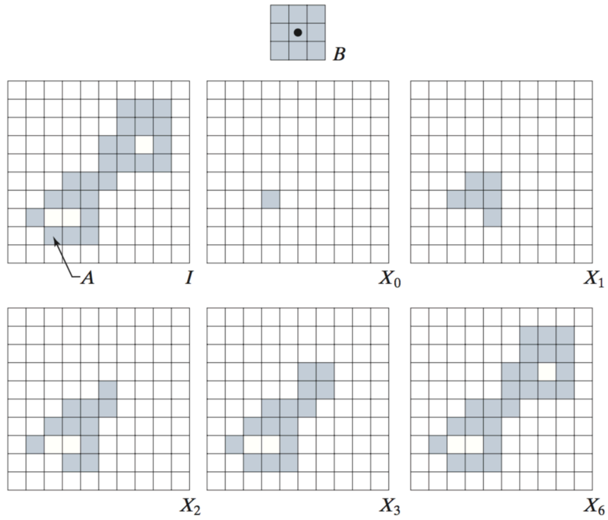
Connected components



Dilation -> intersection

$$X_k = \left(X_{k-1} \oplus B\right) \cap I$$

Ref: Gonzalez et.al., Fig. 9.19

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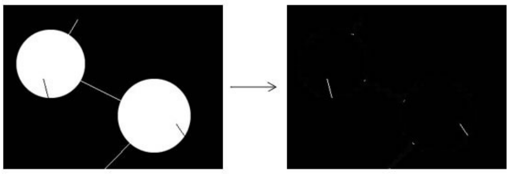
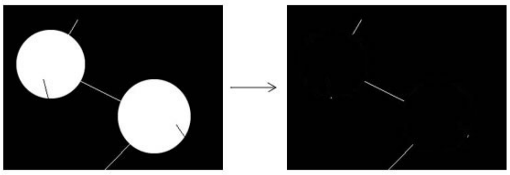
Other algorithms

- Convex hull
- Top hat
- Black hat
- Morphological reconstruction
  - Geodesic dilation
  - Geodesic erosion
- Morphology for grayscale images



Morphological Black hat



Morphological Top hat

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Applications of morphological operations

- Surface scanning
- Form approximation
- Identifying contact points
- Counting
- ...

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License plate detection using morphology

(1) Threshold to make binary image

(2) Opening to remove small regions

(3) Dilation to connect close regions

(4) Connected components to detect highest region

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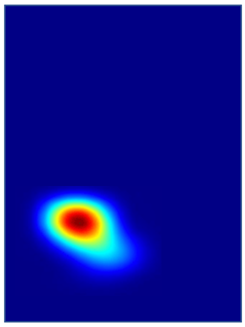
Homework : Detect QR code

- Description: Detect QR code region in an image and build a heatmap for detected region

Input:

Detected QR code

QR heatmap



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