# LinearRegression

## Tran Thi Hanh

### October 2022

**Bài 1.** Biến đổi lại Linear Regression

$$t = y(x, w) + \epsilon \rightarrow w = (X^T X)^{-1} X^T t$$

We have:

$$L = \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2, \hat{y}_i = w_0 + w_1 x_i$$

$$w = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \ x_1 \\ 1 \ x_2 \\ ... \\ 1 \ x_n \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ ... \\ y_n \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ ... \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} w_0 + w_1 x_1 \\ w_0 + w_1 x_2 \\ ... \\ w_0 + w_1 x_n \end{bmatrix} = x_w$$

$$L = \parallel \hat{y} - y \parallel_2^2 = \parallel X_w - y \parallel_2^2 = (X_w - y)^T (X_w - y)$$

$$\rightarrow \frac{L'}{w'} = 2X^T(X_w - y) = 0$$

$$\Leftrightarrow X^T X_w = X^T y$$

$$\Leftrightarrow w = (X^T X)^{-1} X^T y$$

**Bài 4** Prove that $X^t X$ is invertable when X of full rank
**Solution**
If X is full rank, X is linear independent.

$$\Rightarrow \vec{v}^T X^T X \vec{v} = \vec{v}^T \vec{0} = 0$$
$$\Rightarrow (X\vec{v})^T X\vec{v} = 0$$
$$\Rightarrow (X\vec{v}) \cdot (X\vec{v}) = 0$$
$$\Rightarrow X\vec{v} = \vec{0}$$

So we have: if $\vec{v} \in N\left(X^T X\right) \Rightarrow \vec{v} \in N(X)$

$$\Rightarrow \vec{v} \text{ can only be } \vec{0}$$
$$\Rightarrow N\left(X^T X\right) = N(X) = \{\vec{0}\}$$

$\Rightarrow X^T X$ is linearly independent; and $X^T X$ is a square matrix $\Rightarrow X^T X$ is invertible