1 Putnam

1. Divide and conquer to understand better, i.e. to check if something is divisible, break it up, $\frac{10+5}{2} = \frac{10}{2}\frac{5}{2}$, now check, if one is fraction and other is not, then sum is integer + fraction.

2. A rational + an irrational is irrational.

3. A rational $\times$ irrational is irrational.

4. Sometimes breaking up something is not the way to go, instead write it in simplete form, example, $\sqrt{2} + \sqrt{3} + \sqrt{5} = r$.

5. Sum of two rationals is always rational.

6. Apparently either or in math has a beyond terrible definition.

7. Try to reduce the complexity of the problem by starting from the middle, instead of n, n + 1 , n + 2, try, n - 1, n, n + 1.

8. The distance from the origin is a very important function.

2 Predicates

1. A predicate can either be quantified to always be true or sometimes true, the former is known as universally quantified, while the latter as existentially quantified.

2. NOT $(\exists\, x, P(x))$ IFF $\forall\, x$. NOT $(P(x))$.

3 Patterns Of Proofs

3.1 Proof by Contradiction

1. A proof by contracdiction is essentially proving the contrapositive of T $\implies$ P, which is, $\neg P \implies F$, this means if we can prove that $\neg P \implies F$, then P must be true.

2. We have to assume the initial statement is false, and take the negation to be true.

3. If a sequance of deduction contradicts the hypothesis then we have an inderect proof.

4. If it contradicts a fact to be known true we have reductio ad absurdum.

3.2 Proofs about Sets

1. $\in$ means is an element of.
2. Order does not matter in sets, nor number of times an element appears.
3. Informally, a set is just a collection of objects, which are called elements.
4. A set can contain a set.
5. $\{x, x\} = \{x\}$.

Symbol	Set	Elements
$\varnothing$	empty set	
$\mathbb{N}$	non-negative integers	{0, 1, 2, ...}
$\mathbb{Z}$	integers	{..., -1, 0, 1, ...}
$\mathbb{Q}$	rational numbers	0.5, -9, 33.33, ect
$\mathbb{R}$	real numbers	$\pi, \sqrt{2}, 9.9$, ect.
$\mathbb{C}$	complex numbers	i, 34, ect.

1. $\mathbb{R}^+$ is only positive real numbers.