

## 1 Putnam

1. Divide and conquer to understand better, i.e. to check if something is divisible, break it up,  $\frac{10+5}{2} = \frac{10}{2} \frac{5}{2}$ , now check, if one is fraction and other is not, then sum is integer + fraction.

## 2 Patterns Of Proofs

### 2.1 Proof by Contradiction

1. A proof by contradiction is essentially proving the contrapositive of  $T \implies P$ , which is,  $\neg P \implies F$ , this means if we can prove that  $\neg P \implies F$ , then  $P$  must be true.

### 2.2 Proofs about Sets

1. Informally, a set is just a collection of objects, which are called elements.
2. A set can contain a set.
3.  $\{x, x\} = \{x\}$ .

| Symbol       | Set                   | Elements                     |
|--------------|-----------------------|------------------------------|
| $\emptyset$  | empty set             |                              |
| $\mathbb{N}$ | non-negative integers | $\{0, 1, 2, \dots\}$         |
| $\mathbb{Z}$ | integers              | $\{\dots, -1, 0, 1, \dots\}$ |
| $\mathbb{Q}$ | rational numbers      | 0.5, -9, 33.33, ect          |
| $\mathbb{R}$ | real numbers          | $\pi, \sqrt{2}, 9.9$ , ect.  |
| $\mathbb{C}$ | complex numbers       | i, 34, ect.                  |

1.  $\mathbb{R}^+$  is only positive real numbers.