1 Putnam

1. Divide and conquer to understand better, i.e. to check if something is divisible, break it up, $\frac{10+5}{2} = \frac{10}{2}\frac{5}{2}$, now check, if one is fraction and other is not, then sum is integer + fraction.

2 Predicates

1. A predicate can either be quantified to always be true or sometimes true, the former is known as universally quantified, while the latter as existentially quantified.

2. NOT ($\exists\, x, P(x)$) IFF $\forall\, x$. NOT ($P(x)$).

3 Patterns Of Proofs

3.1 Proof by Contradiction

1. A proof by contracdiction is essentially proving the contrapositive of T $\implies$ P, which is, $\neg P \implies F$, this means if we can prove that $\neg P \implies F$, then P must be true.

2. We have to assume the initial statement is false, and take the negation to be true.

3. If a sequance of deduction contradicts the hypothesis then we have an inderect proof.

4. If it contradicts a fact to be known true we have reductio ad absurdum.

3.2 Proofs about Sets

1. Informally, a set is just a collection of objects, which are called elements.

2. A set can contain a set.

3. $\{x, x\} = \{x\}$.

Symbol	Set	Elements
$\varnothing$	empty set	
$\mathbb{N}$	non-negative integers	{0, 1, 2, ...}
$\mathbb{Z}$	integers	{..., -1, 0, 1, ...}
$\mathbb{Q}$	rational numbers	0.5, -9, 33.33, ect
$\mathbb{R}$	real numbers	$\pi, \sqrt{2}, 9.9$, ect.
$\mathbb{C}$	complex numbers	i, 34, ect.

1. $\mathbb{R}^+$ is only positive real numbers.