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Abstract

Requirements for different Definitions of the gamma function The (complete) gamma function $\Gamma(n)$ is defined to be an extension of the factorial to complex and real number arguments. It is related to the factorial by $\Gamma(n) = (n-1)!$.

1 $\Gamma(z)$

$$\Gamma(z) = ((-1)^k)/(k!).$$

1.1 $\Gamma(5)$ Requirements:

It is analytic everywhere except at $z = 0, -1, -2, \dots$, and the residue at $z = -k$, There are no points z at which $\Gamma(z)=0$.

2 Definite integral

$$\Gamma(z) = \int_0^\infty (z-1)e^{-t} dt$$

2.1 $\Gamma(5)$ Requirements:

everywhere except at $Z \leq 0$

3 Multiplication Formula

There are three formulas,

$$\Gamma(x) = \sqrt{2\pi} x^{x-1/2} e^{-x+\mu(x)} \text{ where}$$

$$\mu(x) = \sum_{n=0}^{\infty} (x+n+\frac{1}{2}) \log(1+\frac{1}{x+n}) - 1 = \frac{\theta}{12x} \quad n! = \sqrt{2\pi} x^{x-1/2} e^{-n+\frac{\theta}{12n}}$$

3.1 $\Gamma(5)$ Requirements:

θ everywhere except $\theta < 0$, $\theta > 1$

3.2 $\Gamma(5)$ Requirements:

$$\pi = 3.141592653589793$$

3.3 $\Gamma(5)$ Requirements:

in $\log(x)$, except $x < 0$

3.4 $\Gamma(5)$ Requirements:

e^x everywhere except $x < 0$

References

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- [4] Young, Robert M. Excursions in Calculus: An Interplay of the Continuous and Discrete. United States of America: The Mathematical Association of America, 1992.