

Example 8.4 Design a TM to accept the language $L = \{\text{set of all palindromes over } a, b\}$. Show the IDs for the null string, 'a', 'aba', and 'baab'.

i) Odd palindromes, where the number of characters is odd

A string which starts with 'a' or 'b' must end with 'a' or 'b', respectively, if the string is a palindrome. If a string starts with 'a', that 'a' is replaced by a blank symbol 'B' upon traversal with a state change from q1 to q2 and the right shift of the read–write head. The transitional function is

Then, the machine needs to search for the end of the string. The string must end with 'a', and that 'a' exists before a blank symbol B at the right hand side. Before getting that blank symbol, the machine needs to traverse all the remaining 'a' and 'b' of the string. The transitional functions are

$$\delta(q_3, a) \rightarrow (q_4, B, L)$$

$$\delta(q_4, B) \rightarrow (q_1, B, R)$$
$$\delta(q_1, b) \rightarrow (q_5, B, R)$$

$$\delta(q_5, a) \rightarrow (q_5, a, R)$$

$$\delta(q_5, b) \rightarrow (q_5, b, R)$$

$$\delta(q_5, B) \rightarrow (q_6, B, L)$$

$$\delta(q_6, b) \rightarrow (q_4, B, L)$$

When all 'a' and 'b' are traversed and replaced by B, the states may be one of q3 or q6, if the last symbol traversed is 'a' or 'b', respectively. Transitional functions for acceptance are

$$\delta(q_3, B) \rightarrow (q_7, B, H)$$

$$\delta(q_6, B) \rightarrow (q_7, B, H)$$

A null string is also a palindrome. On the tape, a null symbol means blank B. In state q1, the machine gets the symbol. The transitional function is

$$\delta(q_1, B) \rightarrow (q_7, B, H)$$

The transitional functions in tabular form are represented as follows.

State	A	b	B
q ₁	(q ₂ , B, R)	(q ₅ , B, R)	(q ₇ , B, H)
q ₂	(q ₂ , a, R)	(q ₂ , b, R)	(q ₃ , B, L)
q ₃	(q ₄ , B, L)	–	(q ₇ , B, H)
q ₄	(q ₄ , a, L)	(q ₄ , b, L)	(q ₁ , B, R)
q ₅	(q ₅ , a, R)	(q ₅ , b, R)	(q ₆ , B, L)
q ₆	–	(q ₄ , B, L)	(q ₇ , B, H)

ID for Null String

$$(q_1, B) \rightarrow (q_7, B) \text{ [Halt]}$$

ID for the String 'a'

$$(q_1, aB) \rightarrow (q_2, BB) \rightarrow (q_3, BB) \rightarrow (q_7, BB) \text{ [Halt]}$$

ID for the String 'aba'

$$(q_1, abaB) \rightarrow (q_2, BbaB) \rightarrow (q_2, BbaB) \rightarrow (q_2, BbaB) \rightarrow (q_3, BbaB) \rightarrow (q_4, BbBB) \rightarrow (q_4, BbBB) \rightarrow (q_1, BbBB) \rightarrow (q_1, BBBB) \rightarrow (q_7, BBBB) \text{ [Halt]}$$

ID for the String 'baab'

$$(q_1, baabB) \rightarrow (q_5, BaabB) \rightarrow (q_5, BaabB) \rightarrow (q_5, BaabB) \rightarrow (q_5, BaabB) \rightarrow (q_6, BaabB) \rightarrow (q_4, BaaBB) \rightarrow (q_4, BaaBB) \rightarrow (q_4, BaaBB) \rightarrow (q_1, BaaBB) \rightarrow (q_2, BBaBB) \rightarrow (q_2, BBaBB) \rightarrow (q_3, BBaBB) \rightarrow (q_3, BBBBB) \rightarrow (q_7, BBBBB) \text{ [Halt]}.$$

Example 8.5 Design a TM to accept the language $L = anbncn$, where $n \geq 1$.

Solution: This is a context-sensitive language. The language consists of a, b, and c. Here, the number of 'a' is equal to the number of 'b' which is equal to the number of 'c'. n number 'c' is followed by n number of 'b' which are followed by n number of 'a'. The TM is designed as follows. For each 'a', search for 'b' and 'c' and again traverse back to the left to search for the leftmost 'a'. 'a' is replaced by 'X', 'b' is replaced by 'Y', and 'c' is replaced by 'Z'. When all the 'a' are traversed and replaced by 'X', the machine traverses the right side to find if any untraversed 'b' or 'c' is left or not. If not, the machine gets Y and Z and at last a blank. Upon getting the blank symbol, the machine halts. The transitional functions are

$\delta(q_1, a) \rightarrow (q_2, X, R)$ // 'a' is traversed

$\delta(q_2, a) \rightarrow (q_2, a, R)$ // the remaining 'a's are traversed

$\delta(q_2, b) \rightarrow (q_3, Y, R)$ // 'b' is traversed

$\delta(q_3, b) \rightarrow (q_3, b, R)$ // the remaining 'b's are traversed

$\delta(q_3, c) \rightarrow (q_4, Z, L)$ // 'c' is traversed

$\delta(q_4, b) \rightarrow (q_4, b, L)$ // the remaining 'b's are traversed from right to left

$\delta(q_4, Y) \rightarrow (q_4, Y, L)$ // Y, replacement for 'b' is traversed

$\delta(q_4, a) \rightarrow (q_4, a, L)$ // the remaining 'a's are traversed from right to left

$\delta(q_4, X) \rightarrow (q_1, X, R)$ // rightmost X, replacement of 'a' is traversed

$\delta(q_2, Y) \rightarrow (q_2, Y, R)$ // this is used from the second time onwards

$\delta(q_3, Z) \rightarrow (q_3, Z, R)$ // this is used from the second time onwards

$\delta(q_4, Z) \rightarrow (q_4, Z, L)$ // this is used from the second time onwards

$\delta(q_1, Y) \rightarrow (q_5, Y, R)$ // this is used when all the 'a' are traversed

$\delta(q_5, Y) \rightarrow (q_5, Y, R)$ // this is used when all the 'a' are traversed and the machine is traversing right

$\delta(q_5, Z) \rightarrow (q_6, Z, R)$ // this is used when all the 'a' and 'b' are traversed.

$\delta(q_6, Z) \rightarrow (q_6, Z, R)$ // this is used when all the 'a' and 'b' are traversed and the machine is traversing right to search for the remaining 'c'

$\delta(q_6, B) \rightarrow (q_f, B, H)$ // this is used when all the 'a', 'b', and 'c' are traversed

Upon executing the last transitional function, the machine halts.

The transitional functions in tabular form are represented as follows.

State	a	b	c	B	X	Y	Z
q ₁	(q ₂ , X, R)	–	–	–	–	(q ₅ , Y, R)	–
q ₂	(q ₂ , a, R)	(q ₃ , Y, R)	–	–	–	(q ₂ , Y, R)	–
q ₃	–	(q ₃ , b, R)	(q ₄ , Z, L)	–	–	–	(q ₃ , Z, R)
q ₄	(q ₄ , a, L)	(q ₄ , b, L)	–	–	(q ₁ , X, R)	(q ₄ , Y, L)	(q ₄ , Z, L)
q ₅	–	–	–	–	–	(q ₅ , Y, R)	(q ₆ , Z, R)
q ₆	–	–	–	(q _f , B, H)	–	–	(q ₆ , Z, R)

Example 8.6 Design a TM to perform the concatenation operation on string of '1'. Show an ID for w₁ = 111 and w₂ = 1111.

Solution: The TM does the concatenation operation on two strings w₁ and w₂. These two strings are placed on the tape of the TM separated by the blank symbol B. After traversal, the blank symbol is removed and the two strings are concatenated.

The transitional functions are

$\delta(q_1, 1) \rightarrow (q_1, 1, R)$ // traversing '1' of the first string

$\delta(q_1, B) \rightarrow (q_2, 1, R)$ // traversing the separating blank symbol and converting it to '1'

$\delta(q_2, 1) \rightarrow (q_2, 1, R)$ // traversing '1' of the second string

$\delta(q_2, B) \rightarrow (q_3, B, L)$ // blank symbol traversal means the end of the second string

$\delta(q_3, 1) \rightarrow (q_4, B, L)$ // the last '1' of the second string is converted to 'B' to keep the number of '1' the same

$\delta(q_4, 1) \rightarrow (q_4, 1, L)$ // traversing the remaining '1' at the left side

$\delta(q_4, B) \rightarrow (q_5, B, H)$ // halts

[This machine is also applicable for performing $Z = X + Y$, where $X = |w_1|$ and $Y = |w_2|$.]

ID for w₁ = 111 and w₂ = 1111 and the result is 1111111

(q₁, 111B1111B) → (q₁, 111B1111B) → (q₁, 111B1111) → (q₁, 111B1111B) → (q₂, 1111111B) →

(q₂, 11111111B) → (q₂, 11111111B) → (q₂, 11111111B) → (q₂, 11111111B) → (q₃, 11111111B) →

(q₄, 11111111BB) → (q₄, 11111111BB) → (q₄, 11111111BB) → (q₄, 11111111BB) → (q₄, B11111111BB) →

(q₄, B11111111BB) → (q₄, B11111111BB) → (q₄, B11111111BB) → (q₅, B11111111BB) [Halt]

Example 8.7 Design a TM to perform the following operation

$f(x, y) = x - y$, where $x > y$

Show an ID for $4 - 2 = 2$.

Solution: This performs the subtraction operation. Here, x and y both are integer numbers. Let $x = |W1|$ and $y = |W2|$, where $W1$ and $W2$ are the strings of '1'. The two strings are placed on the input tape separated by B .

Starting traversal from the left hand side '1', the machine moves towards the right. It traverses B with a state changed from q_0 to q_1 . In state q_1 , it again traverses right to find B , that is, the end of the second string. It changes its state and traverses left and changes the rightmost '1' by B (with state change and traversing left). It traverses the separating ' B ' symbol and all '1's of the first string and finds the beginning of the first string, which starts after B at the left hand side. It changes the first '1' by B and again traverses right. This process continues till all the '1's of the second string are replaced by ' B '. The transitional functions are

$\delta(q_0, 1) \rightarrow (q_0, 1, R)$ // all the '1's of the first string are traversed

$\delta(q_0, B) \rightarrow (q_1, B, R)$ // the separating B is traversed

$\delta(q_1, 1) \rightarrow (q_1, 1, R)$ // all the '1's of the second string are traversed

$\delta(q_1, B) \rightarrow (q_2, B, L)$ // to signify the end of the second string