MEC781 – ADVANCED ROBOTICS

# Robotic System Design
## Mobile Robot Kinematic


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# Robot Locomotion

- **Purpose**: A mobile robot needs locomotion mechanisms to enable it to move through its environment.
- **Mechanism to accomplish** legged/wheeled/blade
- Influenced by the type of environment

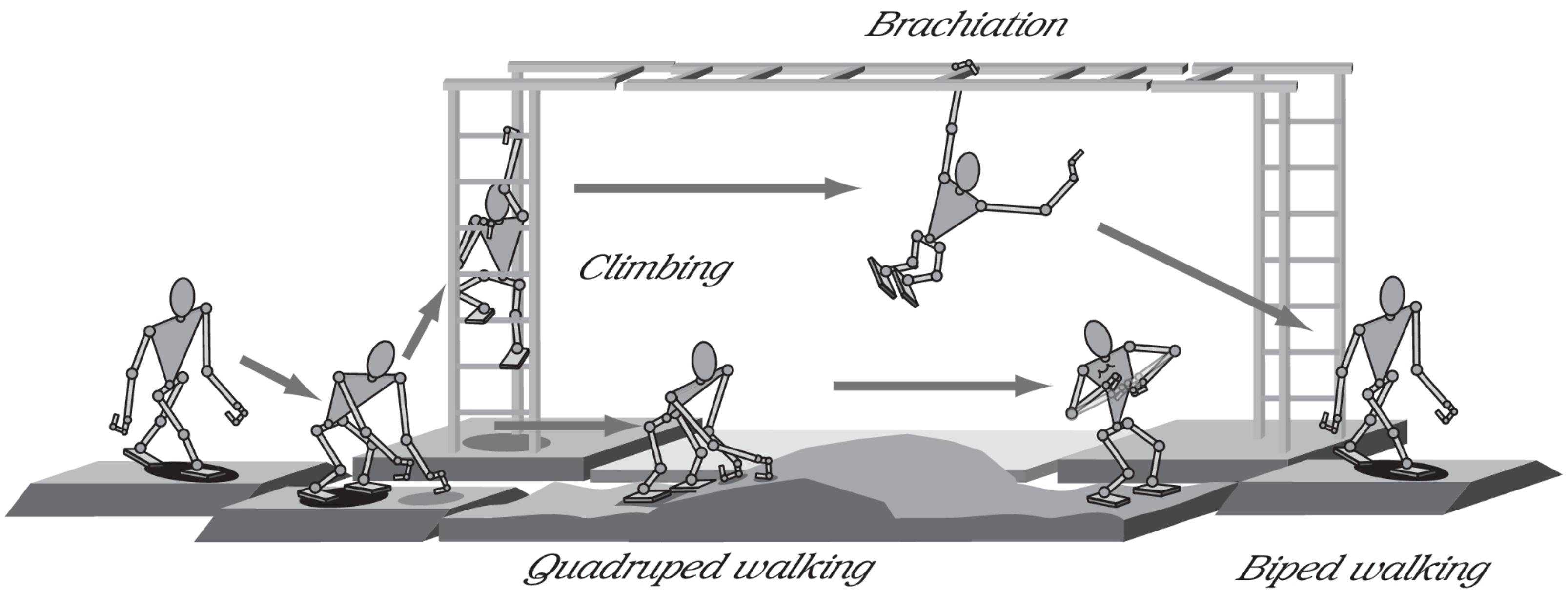
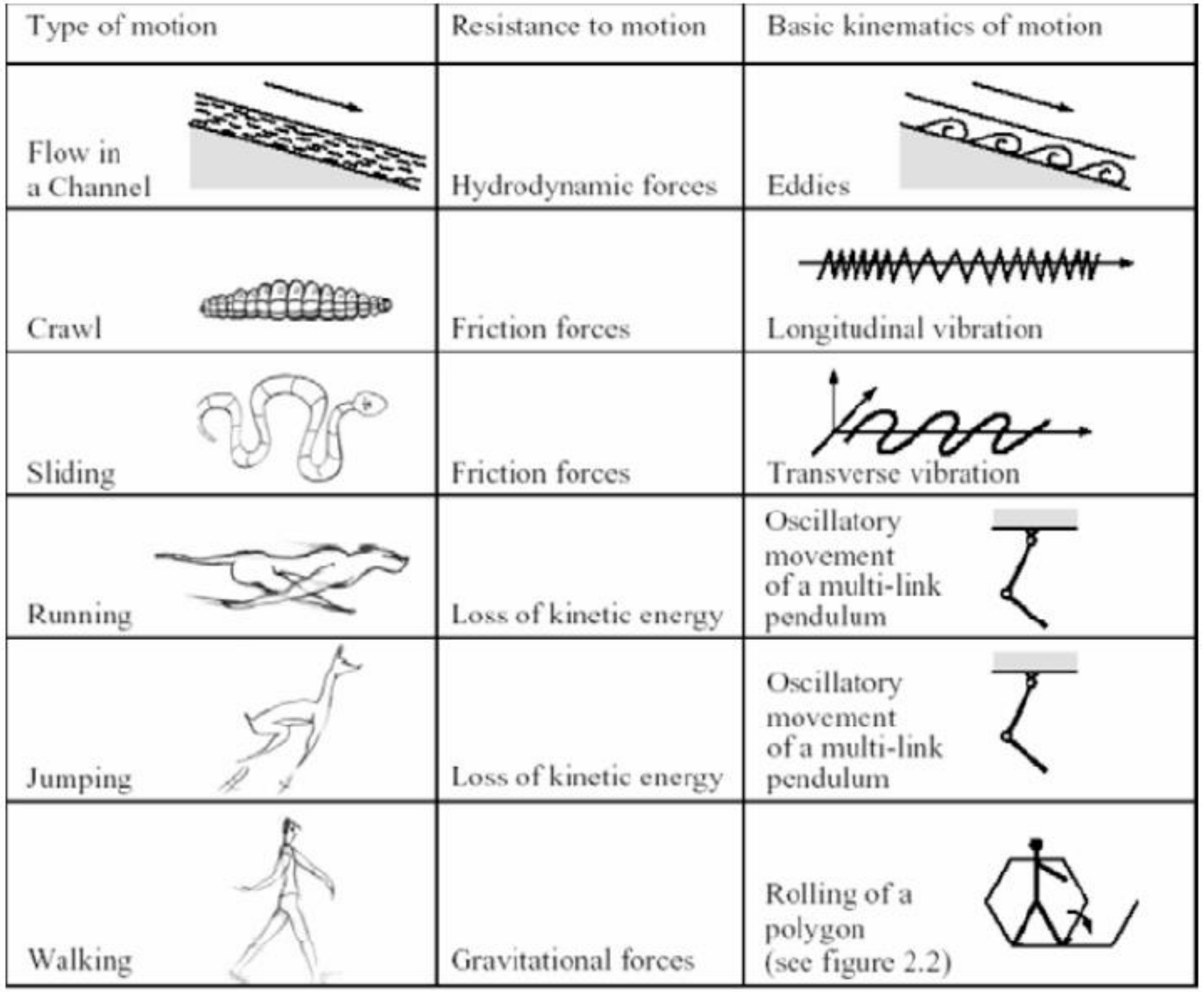



Fig. 1
Example of Locomotion

- **Bio-inspired locomotion** is a common approach, drawing from human and animal movement.
- **Direct replication is challenging** due to several key factors:
  - **Mechanical complexity** of articulated leg structures.
  - **Stability issues** during dynamic motion and terrain adaptation.
  - **High power consumption** required for actuation and control.



| Type of motion | Resistance to motion | Basic kinematics of motion |
|---|---|---|
| Flow in a Channel | Hydrodynamic forces | Eddies |
| Crawl | Friction forces | Longitudinal vibration |
| Sliding | Friction forces | Transverse vibration |
| Running | Loss of kinetic energy | Oscillatory movement of a multi-link pendulum |
| Jumping | Loss of kinetic energy | Oscillatory movement of a multi-link pendulum |
| Walking | Gravitational forces | Rolling of a polygon (see figure 2.2) |

Fig. 2
The type of Locomotion found in nature.

- **Human bipedal walking** can be approximated using simplified mechanical models.
- One such model is a **rolling polygon**,
  - where each side represents a leg segment during stance phase.
  - the **length of each side (d)** corresponds to the **step span**.
  - This abstraction aids in analyzing walking dynamics and stability.

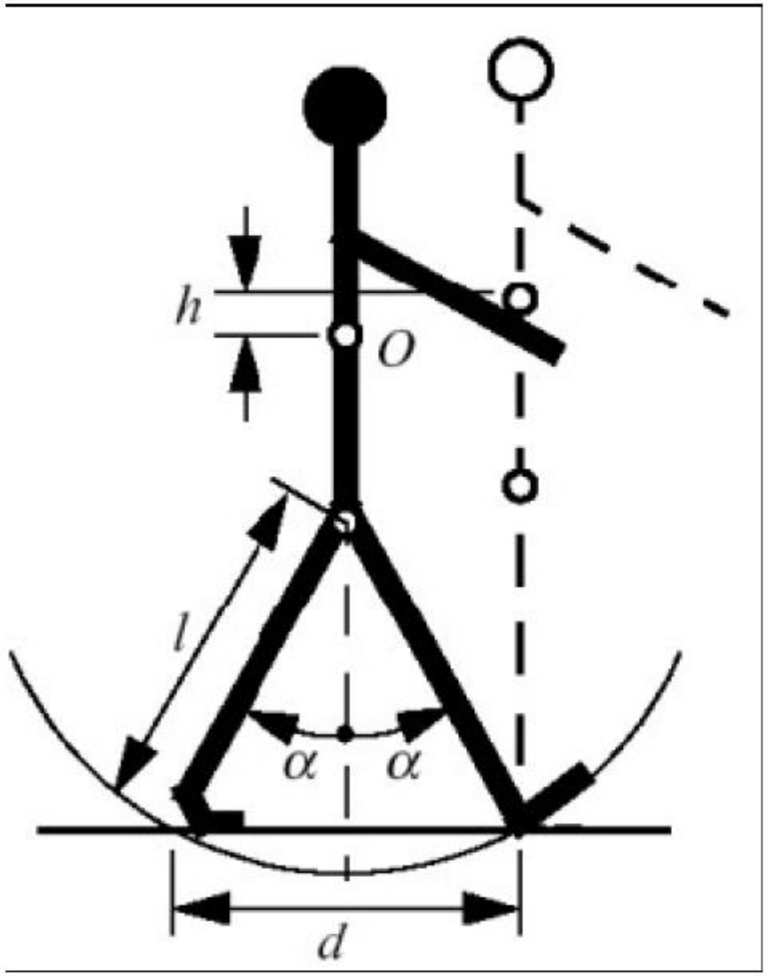



Fig. 3
Approximation of the Human walking by rolling polygon.

# Factors that determine an effective locomotion

1. **Stability**
   - the number and geometry of contact points
   - the robot's center of gravity
   - if the robot is static or dynamic stable and the terrain inclination.
2. **Characteristics of the ground**
   - the type of the contact point
   - the angle of contact to the ground
   - the friction between the robot and the surface.
3. **The type of environment**
   - the structure of the medium (flat/rough etc)
   - the medium itself (water/air/ground)

# Legged Locomotion

- Well-suited for rough terrain; it can climb steps, cross gaps that are as large as its stride, and walk on extremely rough terrain which, due to ground irregularities, the use of wheels would not be feasible.
- To make a legged robot mobile each leg must have at least two degrees of freedom (DOF) or two joints. For each DOF one joint is needed, which is usually powered by one servo motor.

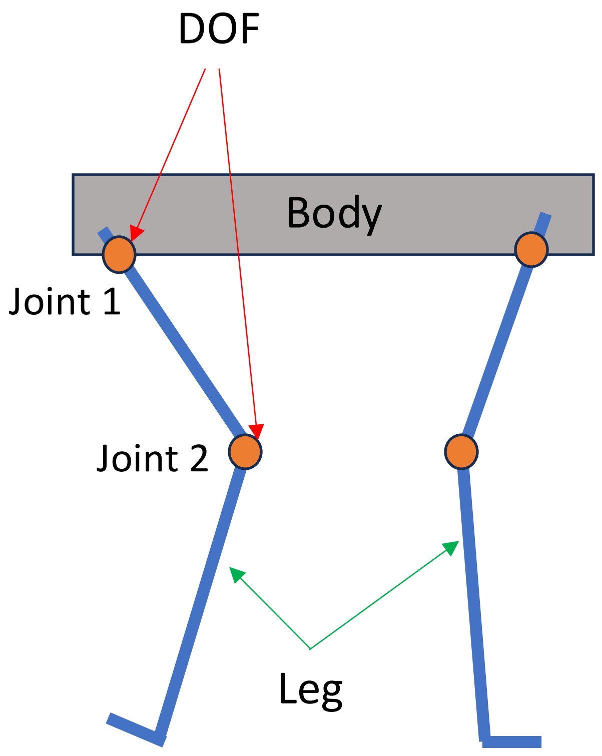



Fig. 4
One leg must have at least two DOF/joints.

## Question

How many servo motors are required to actuate a quadruped (four-legged) robot?

Each leg typically requires **multiple degrees of freedom (DoF)**, leading to a higher motor count.

- As a result, **walking robots consume significantly more energy** than wheeled robots.
- This increased energy demand is due to:
  - Multiple actuators per leg,
  - Complex coordination, and
  - Frequent dynamic adjustments (see Fig. 5).

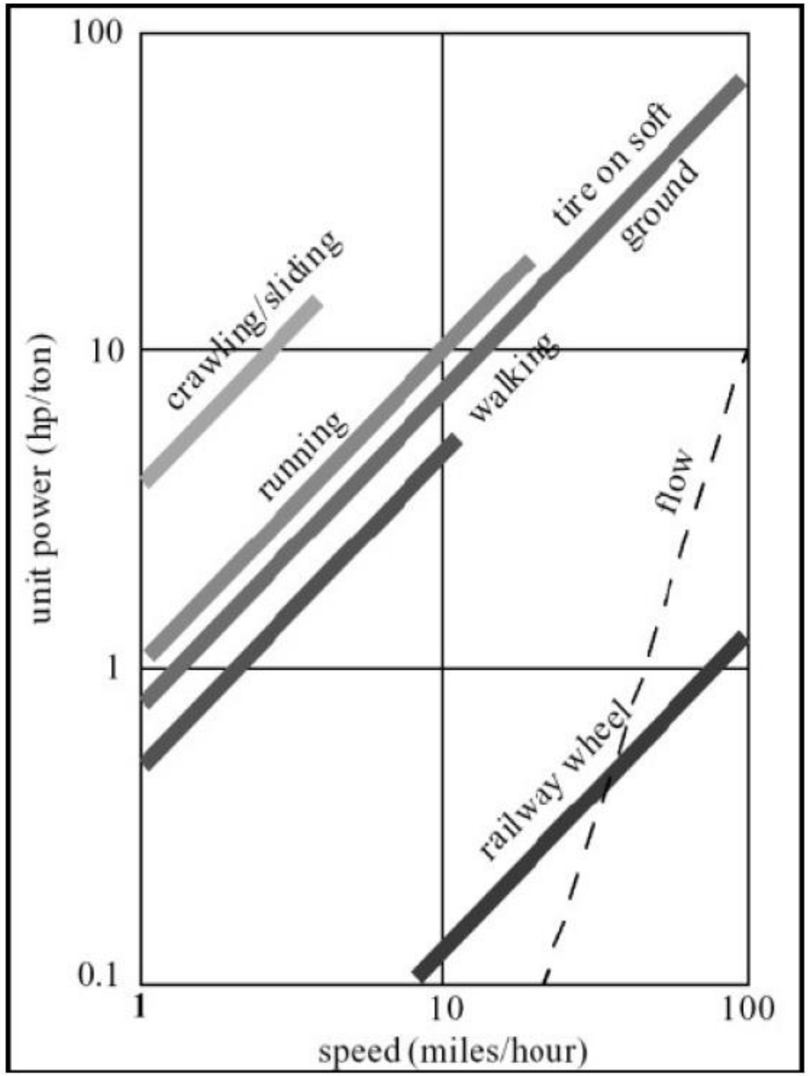



Fig. 5
Power consumption of several locomotion mechanisms

# Wheeled Locomotion

- **Most widely used locomotion mechanism** in mobile robots.
- Why is it preferred**?**
  - **Simplified mechanical design** — fewer moving parts and easy to implement.
  - **No balance control required** — stable with more than two wheels.
  - **High power efficiency** — effective at maintaining high linear and angular velocities.
  - **Improved stability on uneven terrain** — higher **contact fraction** ensures better traction.

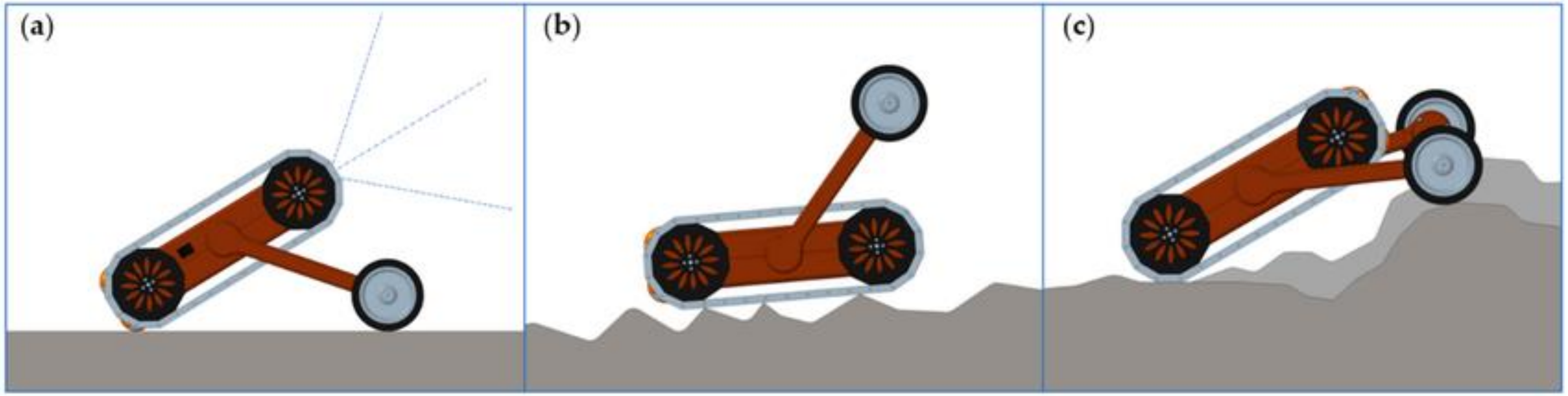



Fig. 6 Example of Wheeled locomotion on rough or uneven surface

- In general there are four major classes of wheels.

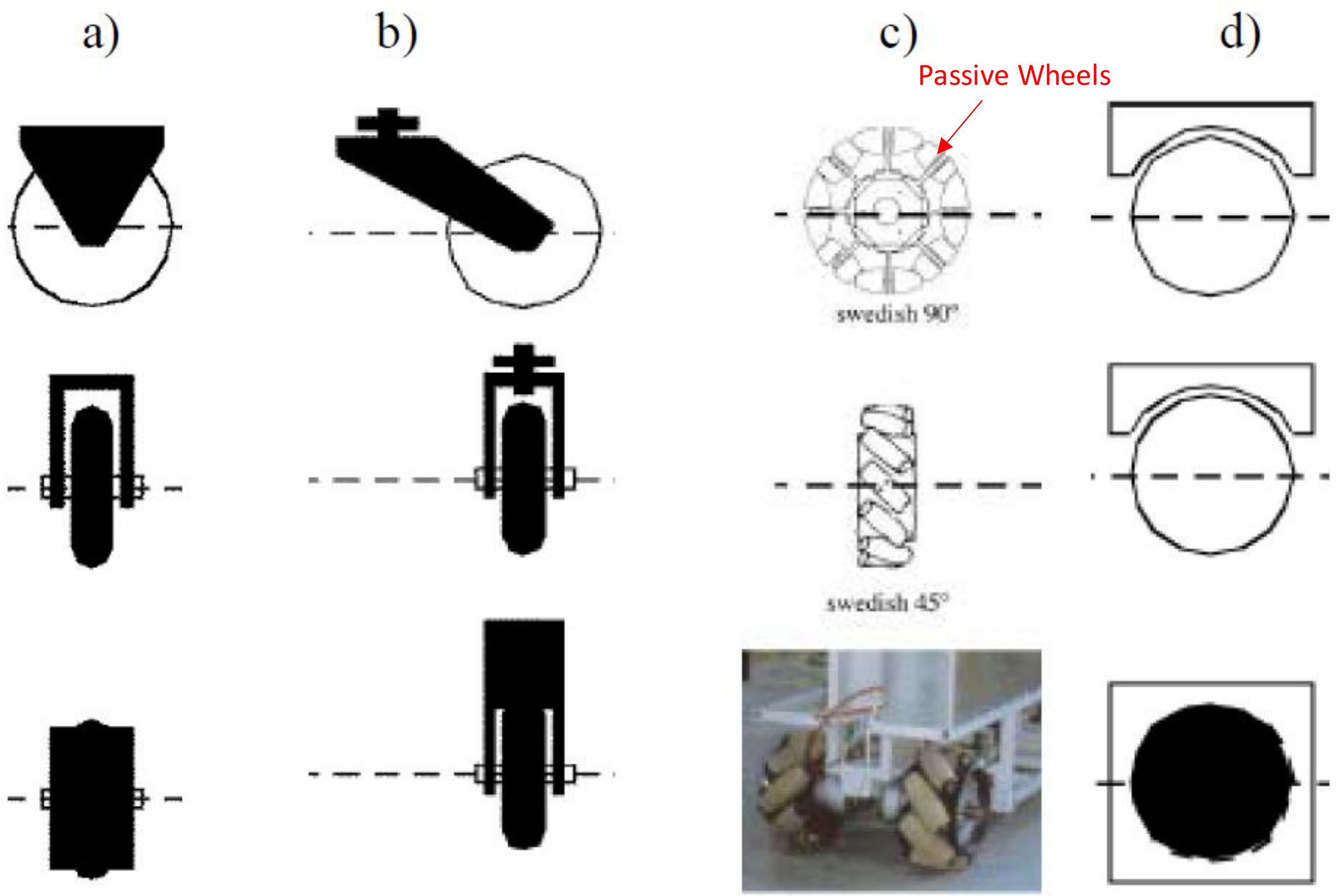



Fig. 7
Four basic wheel types

Figure 18 a) shows the standard wheel with two degrees of freedom, these are rotation around the wheel axle and around the contact;

Figure 18 b) shows the castor wheel with two degrees of freedom, rotation around the wheel axle, and the offset steering joint;

Figure 18 c) shows the Swedish 45° and Swedish 90° or omni wheel, which has three degrees of freedom: rotation around the contact point, around the wheel axle and around the rollers;

Figure 18 d) shows the Ball or spherical wheel, this wheel is omnidirectional, but it is technically difficult to implement.

# Swedish Wheel (Mecanum/Omni Wheel)

## The movement

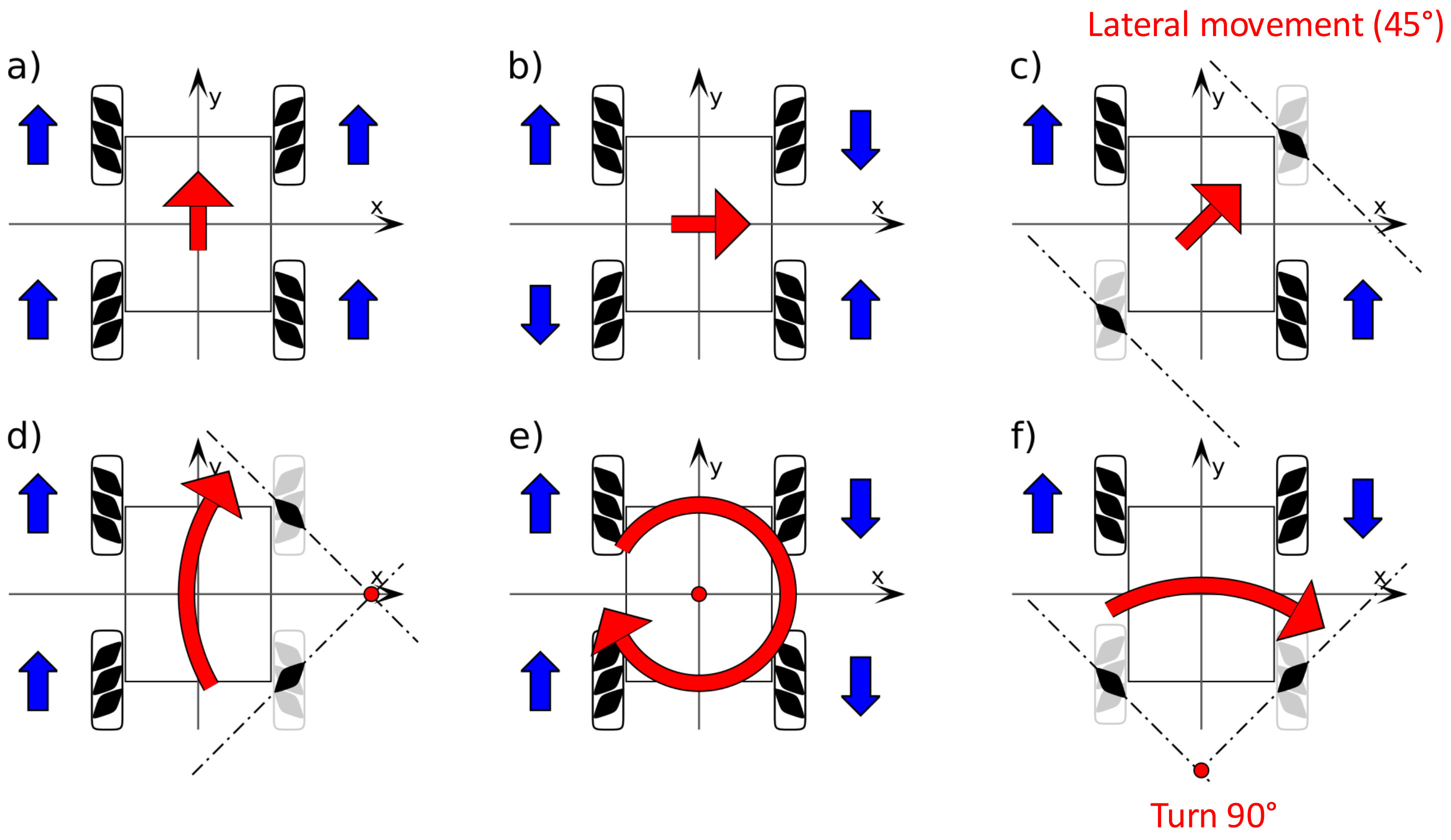



Fig. 8
Example of Swedish wheel

Fig. 9
Movements to any directions: blue: wheel drive direction; red: vehicle moving direction. a) Moving straight ahead, b) Moving sideways, c) Moving diagonally, d) Moving around a bend, e) Rotation, f) Rotation around the central point of one axle

# Issues with Wheeled Locomotion

1. Stability
   - minimum number of wheels required for static stability is two.
   - More stable – if more than two wheels
   - DOF – 3 (two translational, one orientation at plane)
2. Manoeuvrability
   - When a robot can move in any direction of the ground plane (x,y) it is omnidirectional.
   - Ackermann steering configuration, which is used by cars, is not omnidirectional.
   - What are the advantages of Ackermann compared to the Omnidirectional robot?
3. Controllability
   - omnidirectional designs is the high maneuverability of the robot, but this advantage makes it more difficult to control the robot.
   - Why controlling the Ackermann robot is easier than the omnidirectional robot?

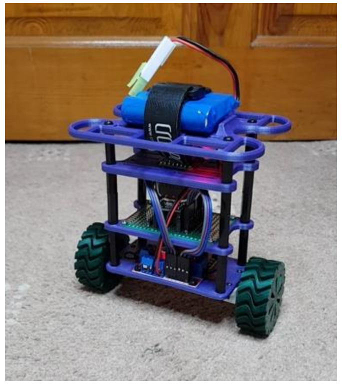


Fig 10
The center of the weight is below the axle of the wheel.

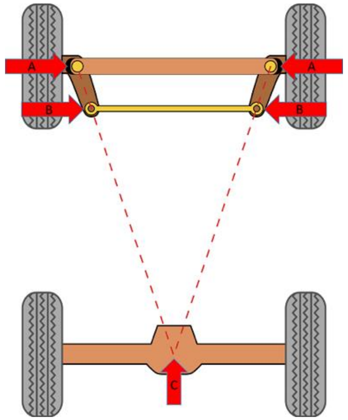


Fig 11
Ackermann driving/steering.

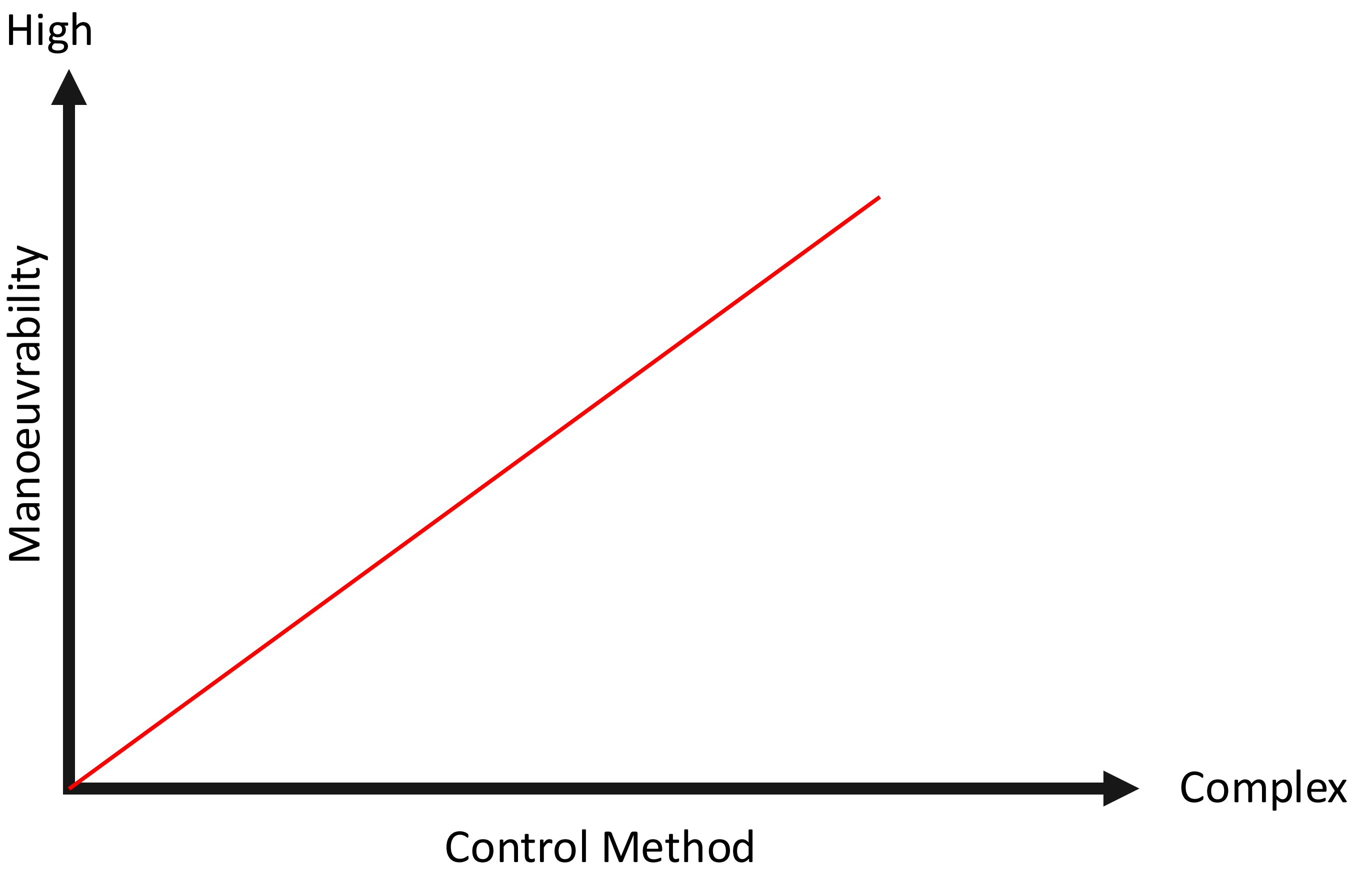
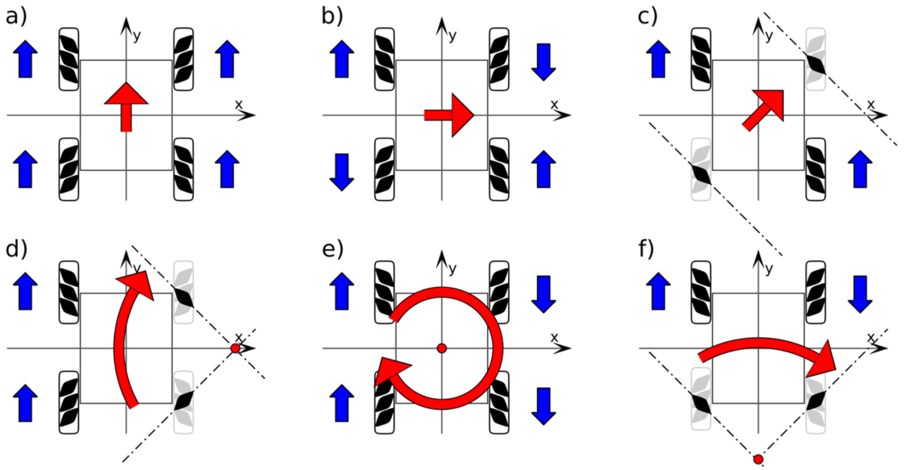
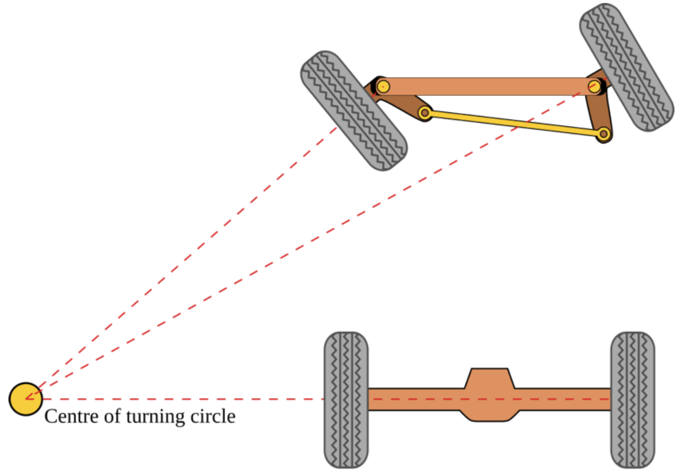



Fig 12
Relationship between manoeuvrability
and Motion control method for mobile robot



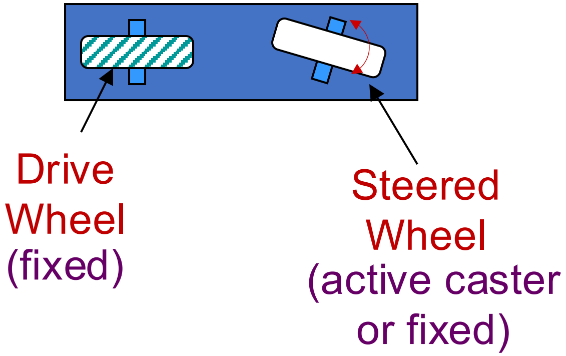

High Manoevrability

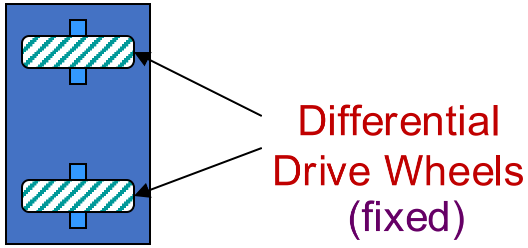



Low Manoevrability

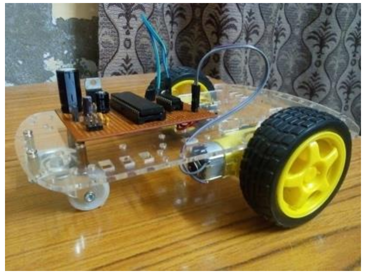
# Example of Wheel Configurations (Recall from previous topic)

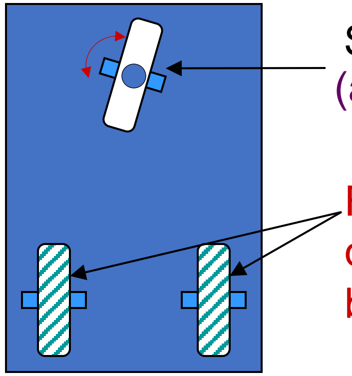
1. Two wheels configuration



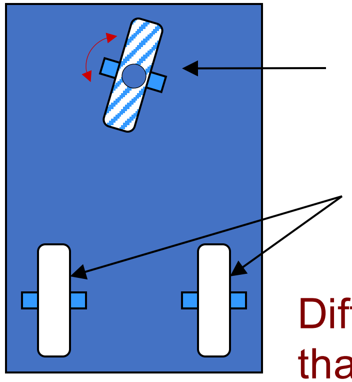
## 2. Three wheels configuration





Steer Wheel (active caster or fixed)

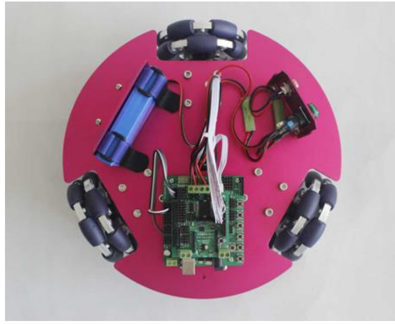
Fixed wheels as drive Wheels. Driven by common actuator.

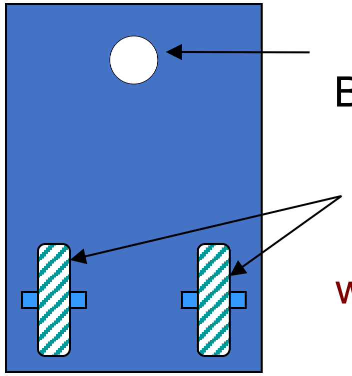


Steer & Drive Wheel (active caster or fixed)

Balance wheels

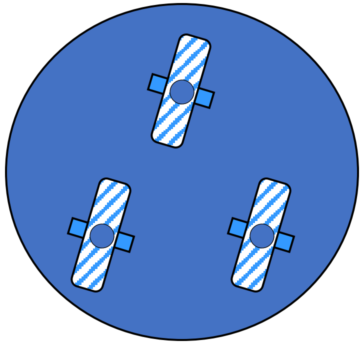
Difficult to build a mechanism that steers and drives at the same time





Caster as Balance Wheel

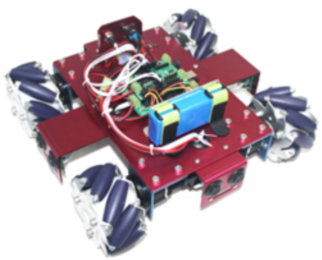
Differential Drive Wheels (Fixed wheels driven by two actuators)

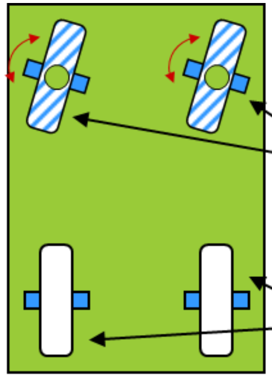


<u>Synchro drive system</u>

Two actuators (motors) are required, one motor rotates all wheels together to produce motion and the other motor turns all wheels to change direction.

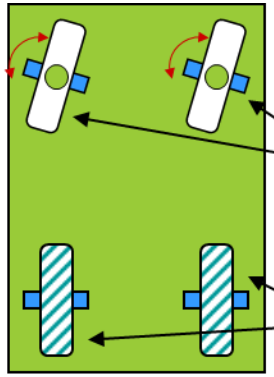
# 3. Four wheels configuration





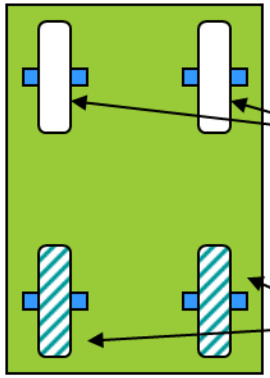
Front Driven & Steered Wheels (Ackerman Steering)

Balancing Rear Wheels (fixed passive)



Front Steered Wheels (Ackerman Steering)

Rear Driven Wheels (fixed active)



Balancing Front Wheels (Free running) (fixed passive)

Differential Drive Rear Wheels (fixed active)

# Other Wheels Configuration

## 1. Tracked slip/skid locomotion

Fig. 13
Nanokhod, developed by Hoerner and Sulger GMBH and the Max Planck Institute

- In tracked slip/skid locomotion vehicles using tracks like a tank, A tracked vehicle is steered by moving the tracks with different speed in the same direction or in opposite direction.
- much larger area of ground contact (loose ground)
- change their direction by skidding
- vehicle needs a lot of space to change the orientation of the chassis

What is the surface is hard? Is skidding efficient for hard surface?

## 2. Walking wheels

- Legged robots can climb stairs and travel through rough terrain, but they offer some inefficiencies on flat surface and controlling the robots is difficult.
- Wheeled robots are very energy efficient on hard surface, even at high speed, but most of them are surely not able to climb stairs.
- Hybrid solution which combines the advantages of legged and wheeled locomotion.

Fig. 14
Example of walking wheels robot

# Mobile Robot Kinematic

- Focuses on the **motion behaviour of mechanical systems** without considering forces.
- In mobile robotics, kinematic analysis is essential to:
  - **Design effective mobile platforms** for specific tasks.
  - **Develop control algorithms** tailored to hardware constraints.
- **Mobile robot kinematics** are generally simpler than those of manipulators,
- since manipulators involve **multiple joints and higher degrees of freedom.**
- Kinematic modeling allows us to:
  - **Define feasible paths and trajectories** within the robot's workspace.

# Mobile Robot Motion and Position Estimation

- A **mobile robot** is a **self-contained system** capable of moving relative to its environment.
- **Direct position measurement is not feasible** in real-time.
  - Instead, position must be **estimated by integrating motion over time**.
- **Motion estimation is error-prone** due to factors such as:
  - **Wheel slippage**,
  - **Surface irregularities**, and
  - **Sensor inaccuracies**.
- Understanding robot motion starts with analyzing **individual wheel contributions**.
  - Each wheel plays a **specific role** in producing translational and rotational movement.

# Kinematic Modeling and Reference Frames

- Introduce a consistent **notation system** for expressing robot motion in:
  - A **global (world) reference frame**, and
  - The **robot's local reference frame**.
- This notation enables the development of **forward kinematic models**, which:
  - Describe how the **robot's overall motion** emerges from
    - its **geometric configuration**, and
    - the **behavior of individual wheels**.
- These models form the foundation for motion planning and control in mobile robotics.

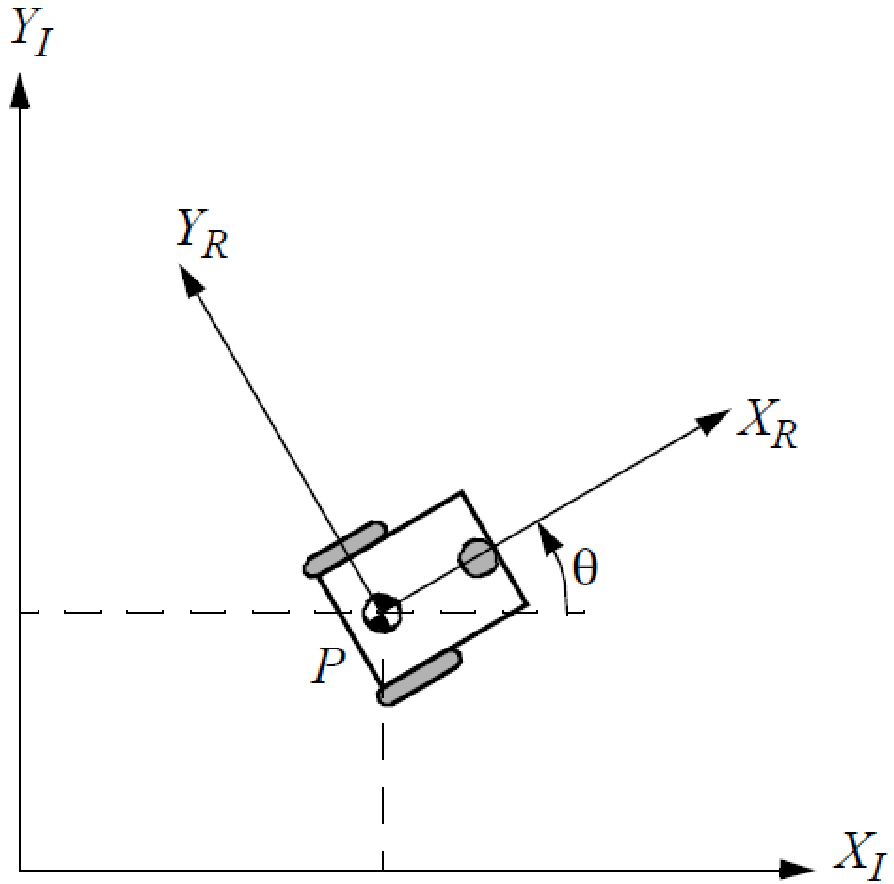



Fig. 15
The global reference frame ($X_I$,$Y_I$) and the robot local reference frame ($X_R$, $Y_R$).

# Kinematic Models and Constraints

## 1. Representing robot position

- The robot is modeled as a **rigid body on wheels**, constrained to a **horizontal plane**.
- The **robot chassis** refers solely to the rigid body,
  - excluding internal joints and degrees of freedom of wheels or actuators.
- The **global (inertial) reference frame** is defined by axes **{$X_I$, $Y_I$}**, originating from a fixed point, **O**.
- A point **P** on the chassis is selected as the **position reference point** of the robot.
- The **local reference frame** is defined by axes **{$X_R$, $Y_R$}**, fixed to the robot at point **P**.
- The robot's pose is described by:
  - **(X, Y)** — coordinates of point **P** in the global frame.
  - **θ (theta)** — the orientation angle between the global and local frames.

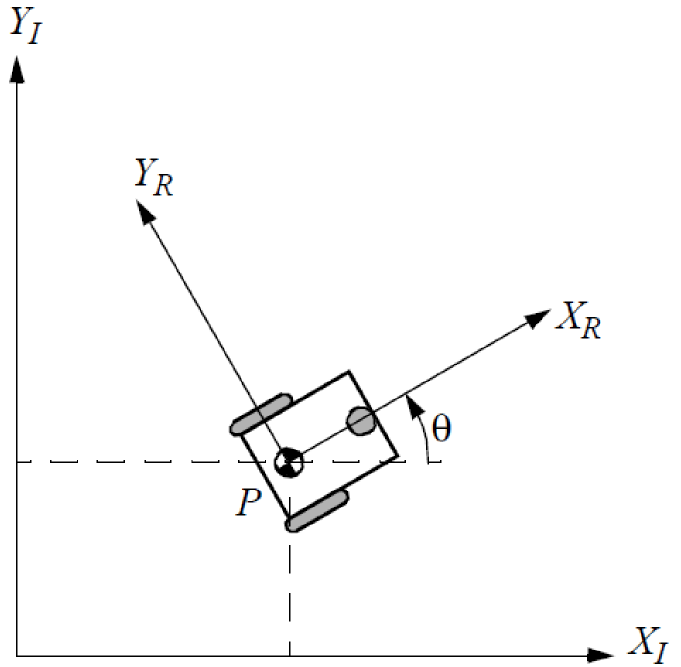
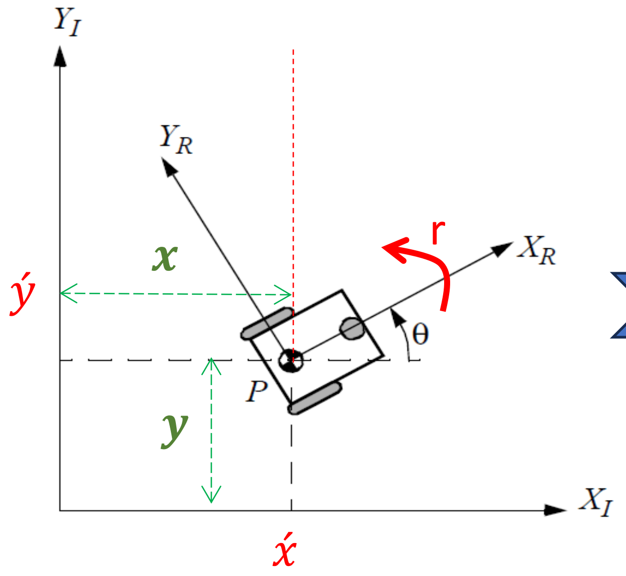



Fig. 15
The global reference frame ($X_I$,$Y_I$) and the robot local reference frame ($X_R$, $Y_R$).

# Basic Mobile Robot Kinematic Transformation




x: Forward displacement of the mobile robot w.r.t. *I*

*y:* Lateral displacement of the mobile robot w.r.t. *I*

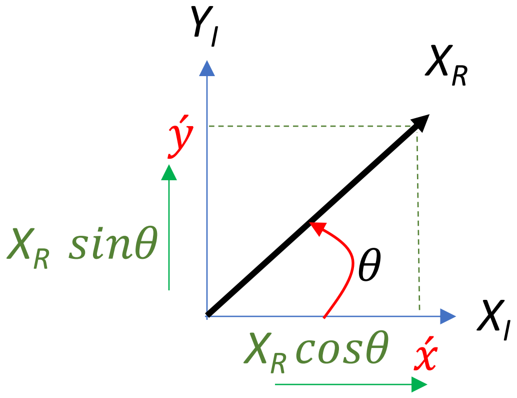
$\theta$*:* Angular displacement of the mobile robot w.r.t *I*

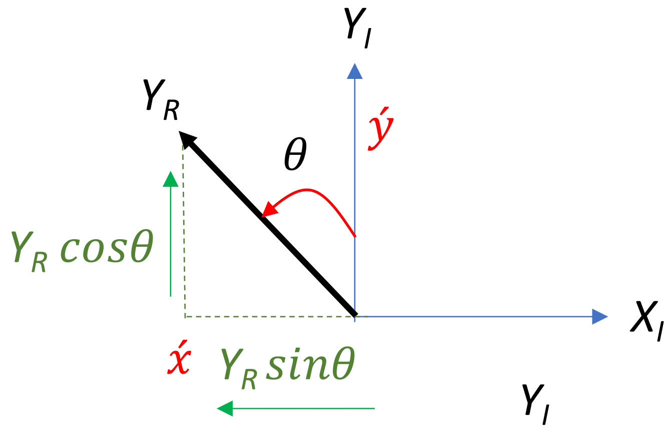
$X_R$*:* Forward velocity of the mobile robot w.r.t. *R*

$Y_R$*:* Lateral velocity of the mobile robot w.r.t. *R*

*r:* Angular velocity of the mobile robot w.r.t. *R*

Rotation matrix of global frame (*I*) to robot frame (*R*)
(Rotation about z-axis)

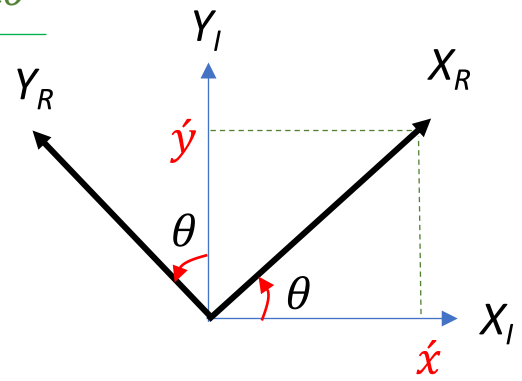



We will get,

$$\acute{x} = X_R \cos\theta - Y_R \sin\theta$$

$$\acute{y} = X_R \sin\theta + Y_R \cos\theta$$




Then,

$$\begin{bmatrix} \acute{x} \\ \acute{y} \\ \acute{\theta} \end{bmatrix} = \begin{bmatrix} X_R \cos\theta - Y_R \sin\theta \\ X_R \sin\theta + Y_R \cos\theta \\ r \end{bmatrix}$$

Then, we can transform the equation into 3x3 matrix transformation

$$\begin{bmatrix} \acute{x} \\ \acute{y} \\ \acute{\theta} \end{bmatrix} = \begin{bmatrix} X_R \cos\theta - Y_R \sin\theta \\ X_R \sin\theta + Y_R \cos\theta \\ r \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_R \\ Y_R \\ r \end{bmatrix}$$

$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\theta})\boldsymbol{\zeta}$ → It describes the relation between the velocity input command ($\boldsymbol{\zeta}$) and the derivatives of generalized coordinates ($\dot{\boldsymbol{\eta}}$)

# Kinematic Models and Constraints
## 2. Forward Differential Kinematics

For given velocity input commands, finding the derivatives generalized coordinates (finding the system's motion)

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\theta})\boldsymbol{\zeta}$$

*Simulating and analyzing the system in velocity level

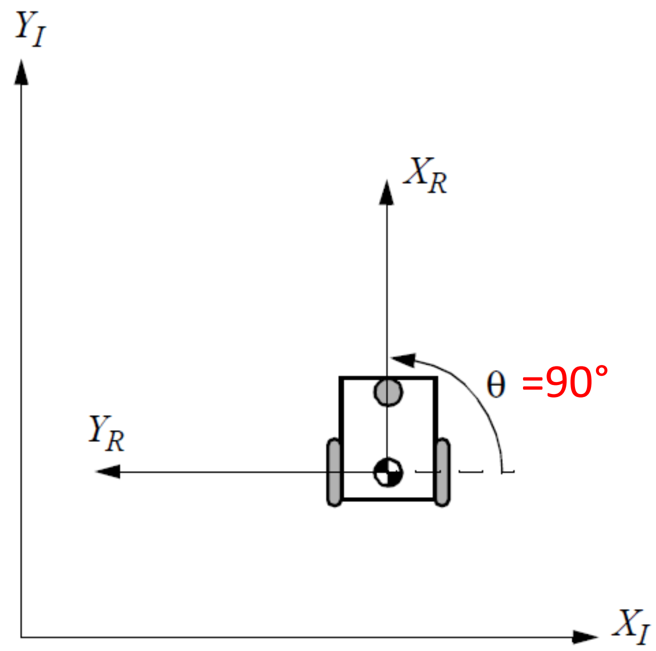
# Kinematic Models and Constraints
## 3. Inverse Differential Kinematics

For the desired (given) derivatives of generalized coordinates (or given position trajectory), finding the corresponding input velocity command.

$$\boldsymbol{\zeta} = \boldsymbol{J}^{-1}(\boldsymbol{\theta})\dot{\boldsymbol{\eta}}$$

*Controlling the system in velocity level. You wanted the mobile robot to move in a certain manner to arrive at the desired derivatives of generalized coordinates.

- For example, let say the robot is rotated at 90°, therefore the final position of the mobile robot is,




$$\dot{\boldsymbol{\eta}}(\frac{\pi}{2}) = \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_R \\ Y_R \\ r \end{bmatrix}$$

$$\dot{\boldsymbol{\eta}}(\frac{\pi}{2}) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_R \\ Y_R \\ r \end{bmatrix}$$

In this case, we can say that motion along $X_R$ *is equal to* $-\acute{y}$ and Motion along $Y_R$ is equal to $\acute{x}$

# Kinematic Models and Constraints

# 4. Modelling a Differential Drive Mobile Robot

This topic will only focus on a differential drive robot with the third wheel is the castor wheel.

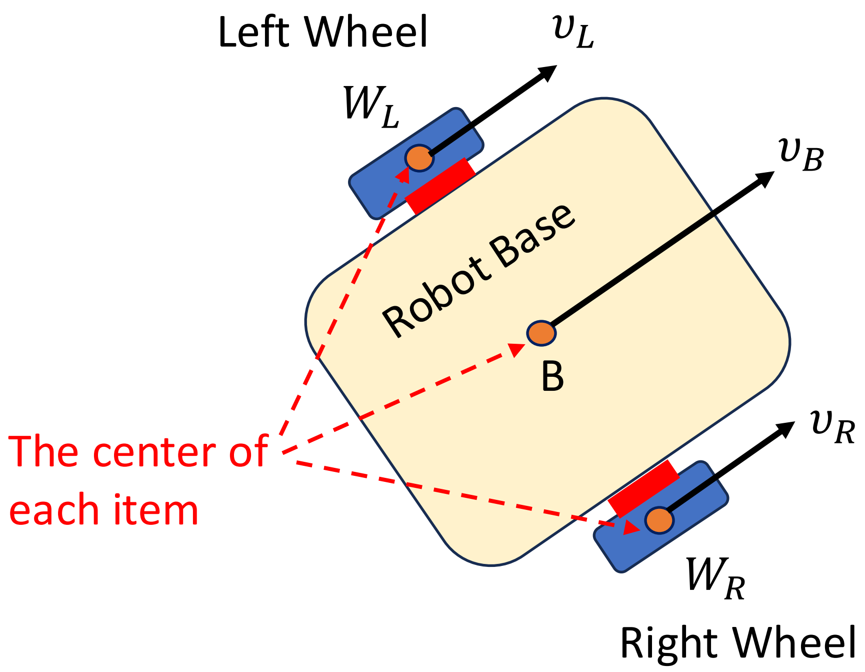
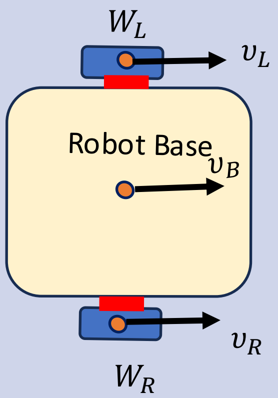
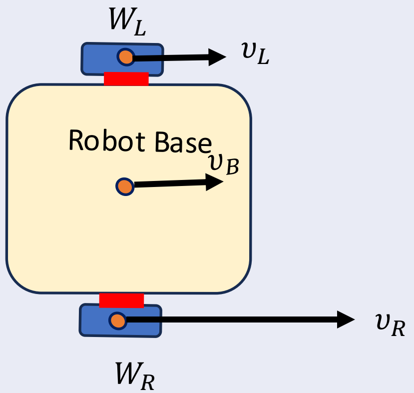
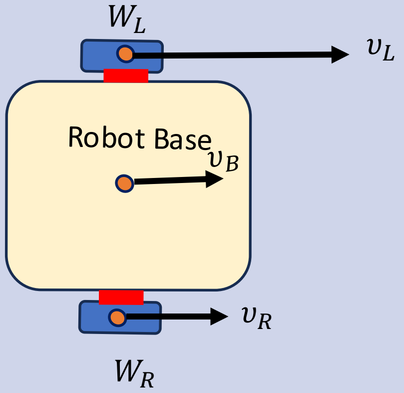



Fig. 16
A differential drive robot in its global reference frame

- The points $W_L$ and $W_R$ are located at the corresponding centers of the two wheels.
- Point B is the point in the middle of the line connecting the points L and R.
- The velocity $v_L$ is the velocity of the center point of the left wheel. Similarly, the velocity $v_R$ is the velocity of the center point of the right wheel. These velocities are a direct consequence of the wheels spinning due to the torques exerted by the motors.
- We can completely control the robot's motion by controlling the right and left wheel angular velocities or the right and left wheel rotational angles. The velocities $v_L$ and $v_R$ are linearly proportional to the angular velocities of the right and left wheels.

| Robot Motion | Illustration | Detail Description |
|---|---|---|
| Moving straight |   | $v_L = v_R$ |
| Turn Left |   | $v_L < v_R$ |
| Turn Right |   | $v_L > v_R$ |

**Question**:
What are the outputs of $v_L$ and $v_R$ if the mobile robot is made to spin?

Table 1
Motion control of the differential drive mobile robot.

# Kinematic Equation for Differential Drive Mobile Robot

- In this topic, we must establish the kinematic equation for differential drive mobile robots.
- **Objective**: To establish the equations that will relate the angular velocities of two wheels with the velocity of the center of the robot, and the angular velocity of the robot rotation.

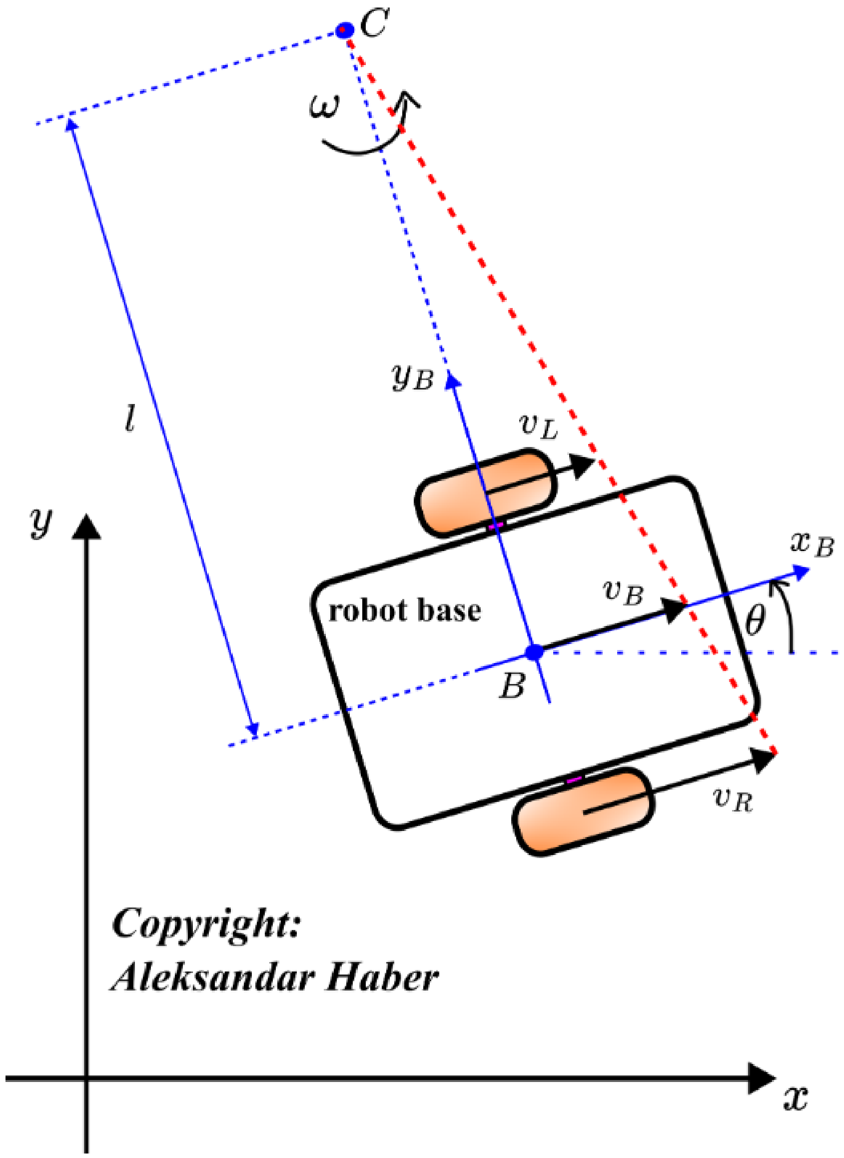



Fig. 17
Kinematic diagram of a differential drive robot.

- In this case,
  - $x - y$ is the global coordinate system
  - $x_B$ - $y_B$ is the robot coordinate system at point B is the coordinate system rigidly attached to the robot body. It translates and rotates together with the robot. It is also called the robot coordinate system (frame).
  - Point C is the robot's instantaneous center of rotation. From the velocity analysis perspective, during a short time interval, the robot seems to rotate around the instantaneous center of rotation. This point is constructed by finding an intersection of the line connecting the top of the velocity arrows with the line passing through the centers of the wheels. The symbol $\omega$ denotes the instantaneous angular velocity.
  - The angle $\theta$ is the rotation angle of the robot body. This angle is at the same time the rotation of the body frame with respect to the inertial frame $x - y$.

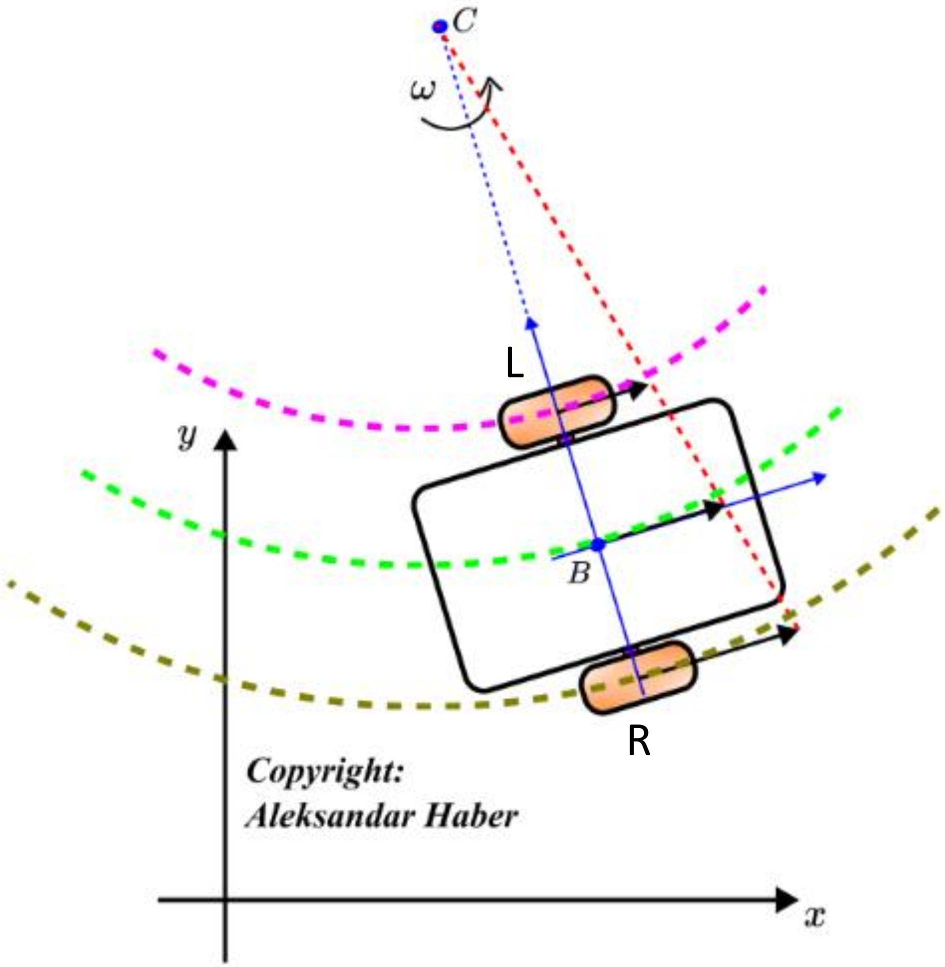



Fig. 18
Trajectories are described by the points L, B, and R during a time interval.

- Under the assumption that the intensities of the velocities are not changing during a time interval, the points L, B, and R describe concentric circles centered at point C during the considered time interval. This is shown in Fig. 18.

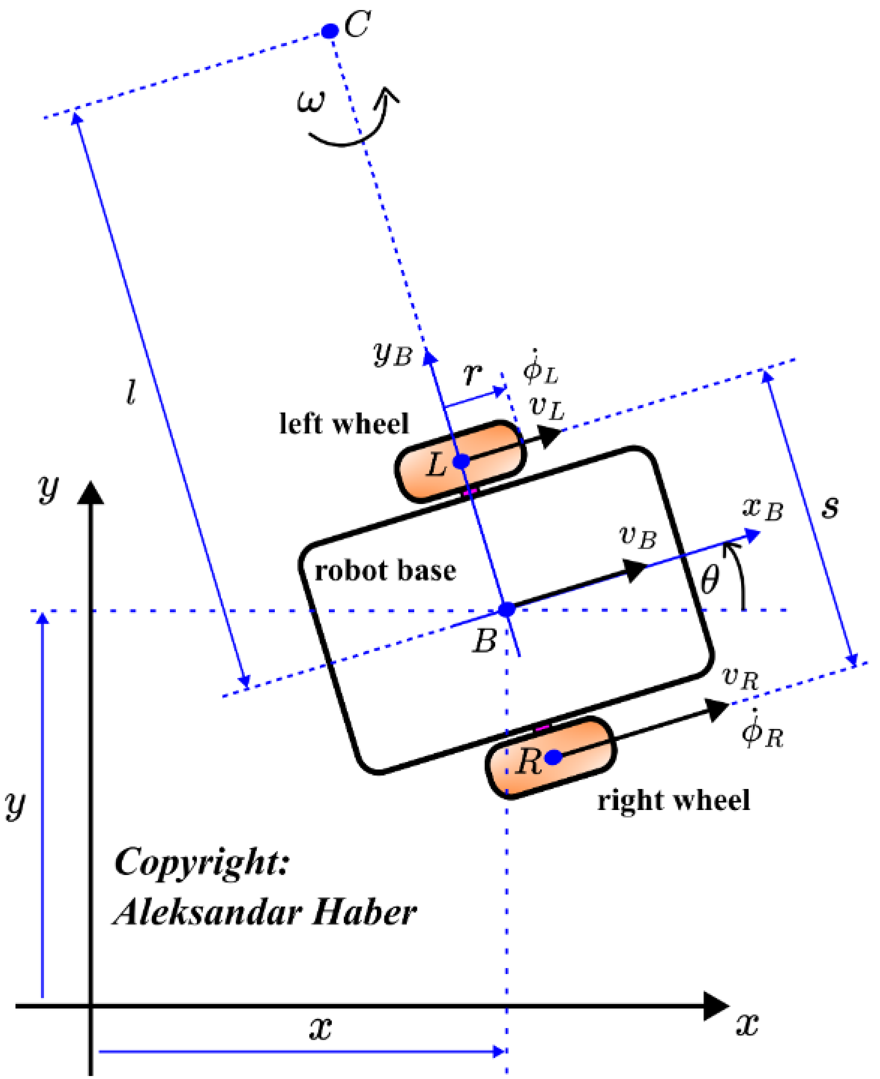



Fig. 19
Detailed kinematic diagram with all the parameters.

To derive the kinematic equation, we need to consider the diagram in Fig. 19.

- $x$ and $y$ are the translation coordinates of the body frame attached to the point B with respect to frame $x - y$
- $\theta$ is the angle rotation of the robot which is at the same time the angle between the robot body frame and the inertial (global) frame.
- $C$ is the instantaneous center of rotation
- $x_B$ and $y_B$are the coordinates of the robot body frame and at the same time denote the axes of the body frame.
- $\omega$ is the instantaneous angular velocity of the robot body.
- $L$ and $R$ are the center point of the left and right wheel.
- $B$ is the middle point between the points $L$ and $R$.
- $v_L$ and $v_R$is the velocity of the center of the left and right wheels.
- $v_B$ is the velocity of the point $B$.
- $\dot{\phi}_L$ and $\dot{\phi}_R$ is the angular velocity of the left and right wheels.
- $l$ is the distance between the point $B$ and the point $C$.

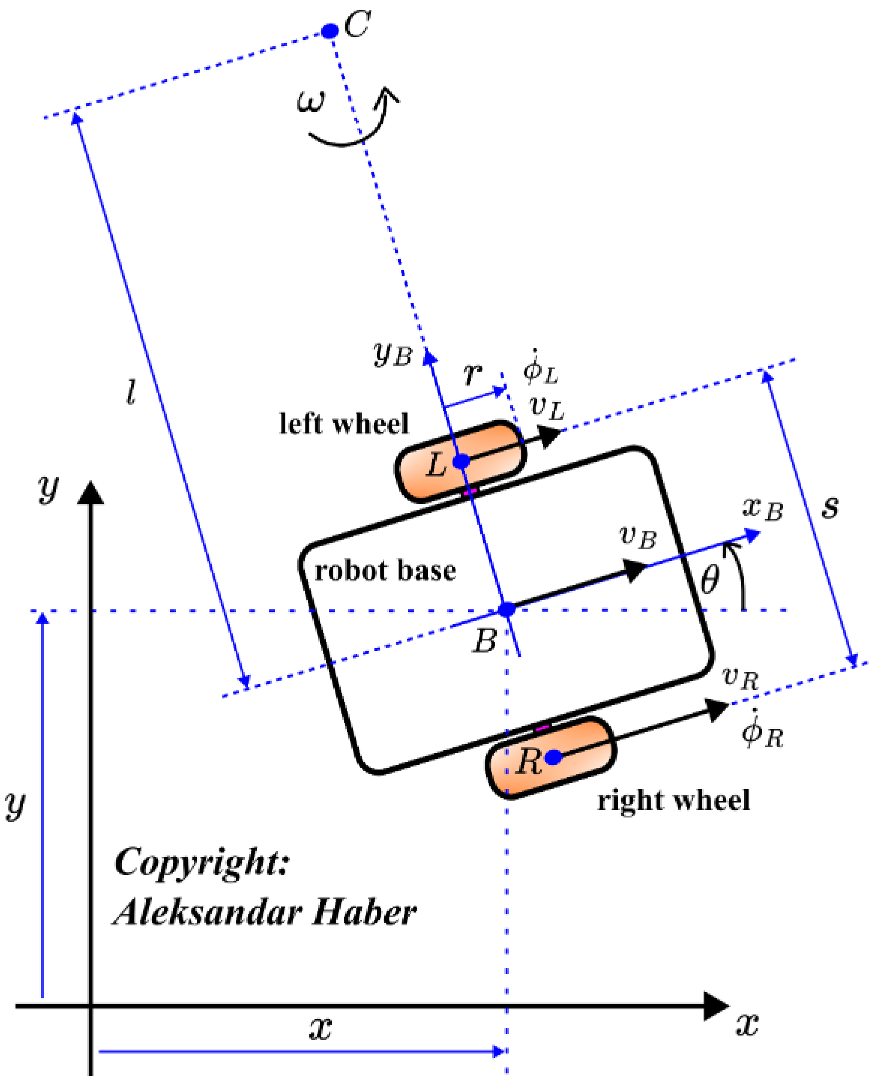



Fig. 19
Detailed kinematic diagram with all the parameters.

## Continue..

- $r$ is the radius of the wheels.
- $s$ is the distance between the points $L$ and $R$.
- $\dot{x}$ and $\dot{y}$ are the projection of the $v_B$ on the $x$-$y$ axes.

In the sequel, we will derive the equations that will relate $\dot{\phi}_L$ and $\dot{\phi}_R$ with $\dot{x}$, $\dot{y}$ , and $\dot{\theta}$. These equations will enable us to predict the robot center point velocity and angular velocity as the function of control variables $\dot{\phi}_L$ and $\dot{\phi}_R$.

## In another words,

We start from the assumption that the following quantities and parameters are known $\dot{\phi}_L$, $\dot{\phi}_R$, $s$, and $r$, and we want to determine $\dot{x}$, $\dot{y}$ , and $\dot{\theta}$.

## Let's start the derivation now

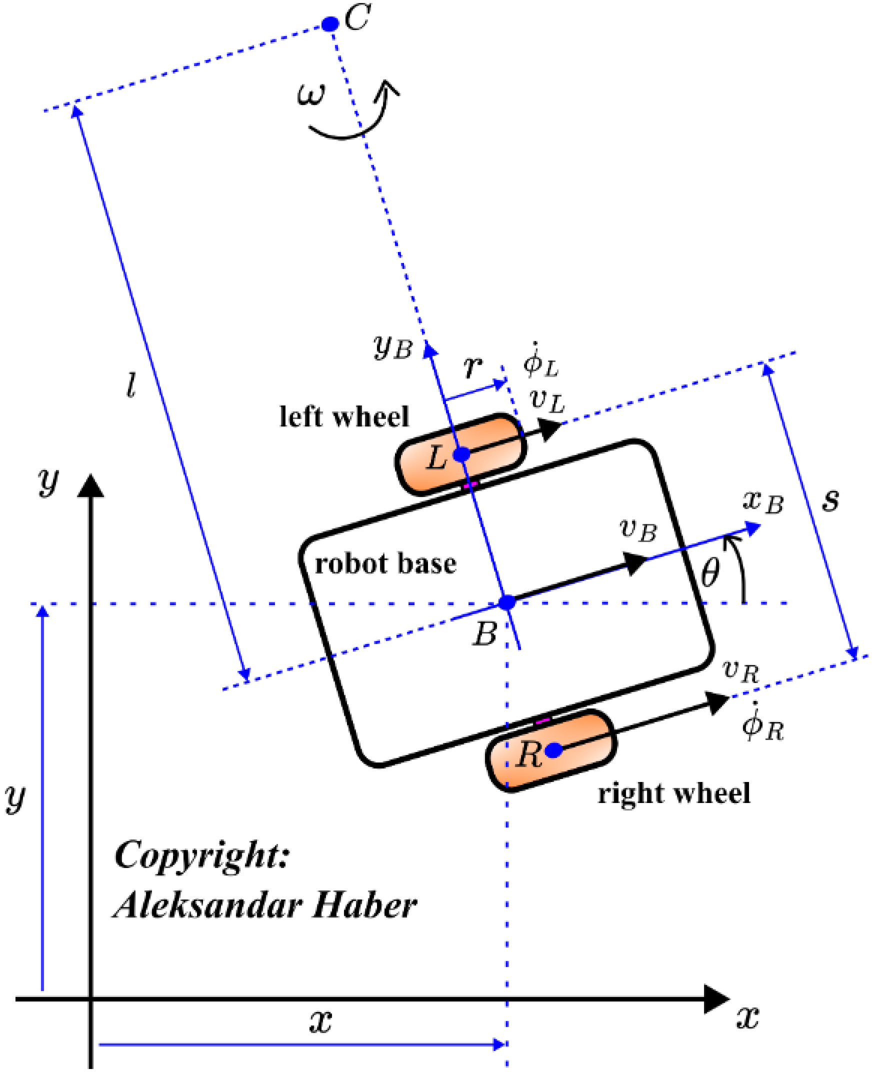



Fig. 19
Detailed kinematic diagram with all the parameters.

Based on Fig. 19,

First, we obtain function for $v_L$ and $v_R$ from the function of $l$ at point $B$ and $\omega$ at point $C$. Point B is equivalent to $l - \frac{s}{2}$ as $B$ is located at the center of the robot body. Therefore, the equations of $v_L$ and $v_R$

$$v_L = \omega\left(l - \frac{s}{2}\right) \qquad (1)$$

$$v_R = \omega\left(l + \frac{s}{2}\right)$$

However, the issue now is that the values of $\omega$ and $l$ are unknown. We need to solve these two equations for $\omega$ and $l$. From the first equation in (1), we can express the variable $\omega$ as follows,

$$\omega = \frac{v_L}{l - \frac{s}{2}} \qquad (2)$$

By substituting equation (2) into (1), we will get,

$$v_R = \frac{v_L}{l - \frac{s}{2}}\left(l + \frac{s}{2}\right)$$

$$v_R\left(l - \frac{s}{2}\right) = v_L(l + \frac{s}{2}) \qquad (3)$$

$$v_R l \; - \; v_R\frac{s}{2} = v_L(l + \frac{s}{2})$$

$$l(v_R - v_L) = v_R\frac{s}{2} + \; v_L\frac{s}{2}$$

From Equation (3), we can obtain the equation for $l$

$$l = \frac{s(v_R + v_L)}{2(v_R - v_L)} \qquad (4)$$

Now, substituting equation (4) to (1) for $v_L$, you will get,

$$
\begin{aligned}
v_L &= \omega\left(l - \frac{s}{2}\right) \\
v_L &= \omega\left(\frac{s(v_R + v_L)}{2(v_R - v_L)} - \frac{s}{2}\right) \\
v_L &= \omega\frac{s}{2}\left(\frac{(v_R + v_L)}{(v_R - v_L)} - 1\right) \\
v_L &= \frac{\omega s}{2}\left(\frac{2v_L}{(v_R - v_L)}\right)
\end{aligned}
\tag{5}
$$

Simplify equation (5) to get $\omega$,

$$\omega = \frac{(v_R - v_L)}{s} \tag{6}$$

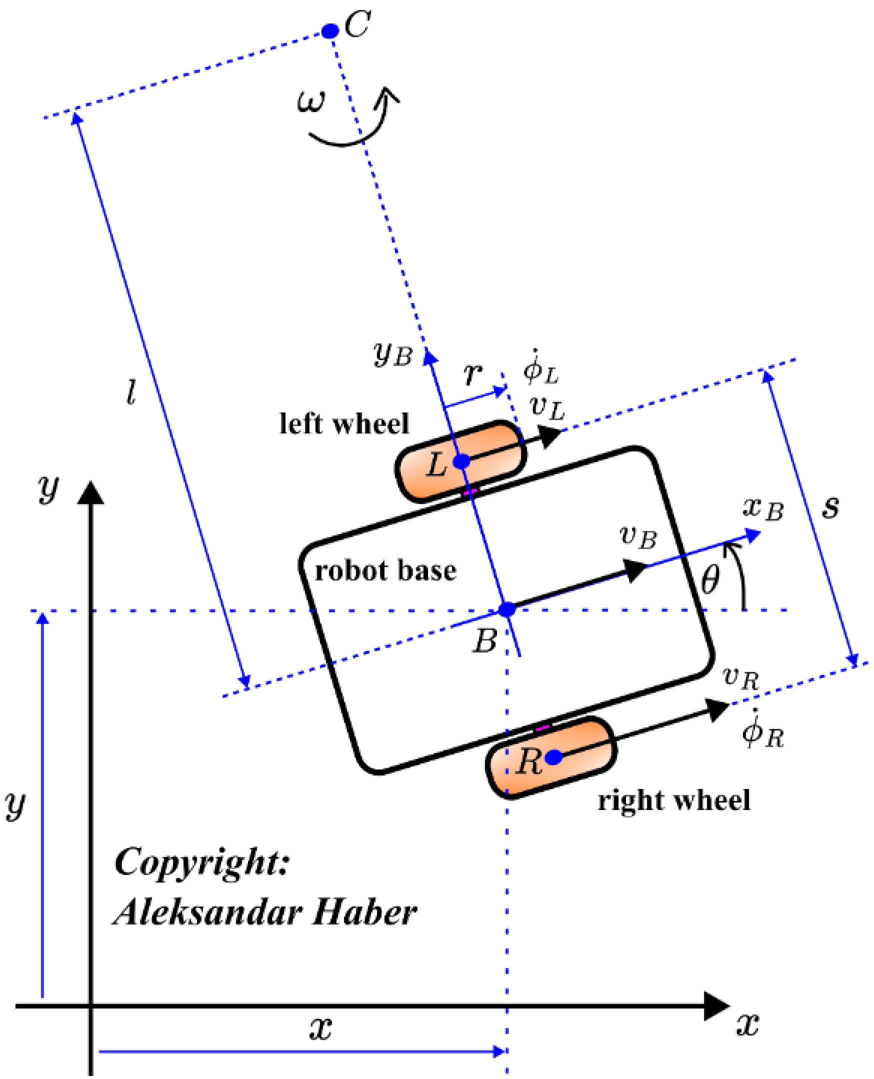



Fig. 19
Detailed kinematic diagram with all the parameters.

To clarify, we have obtained equation for $\omega$ and $l$

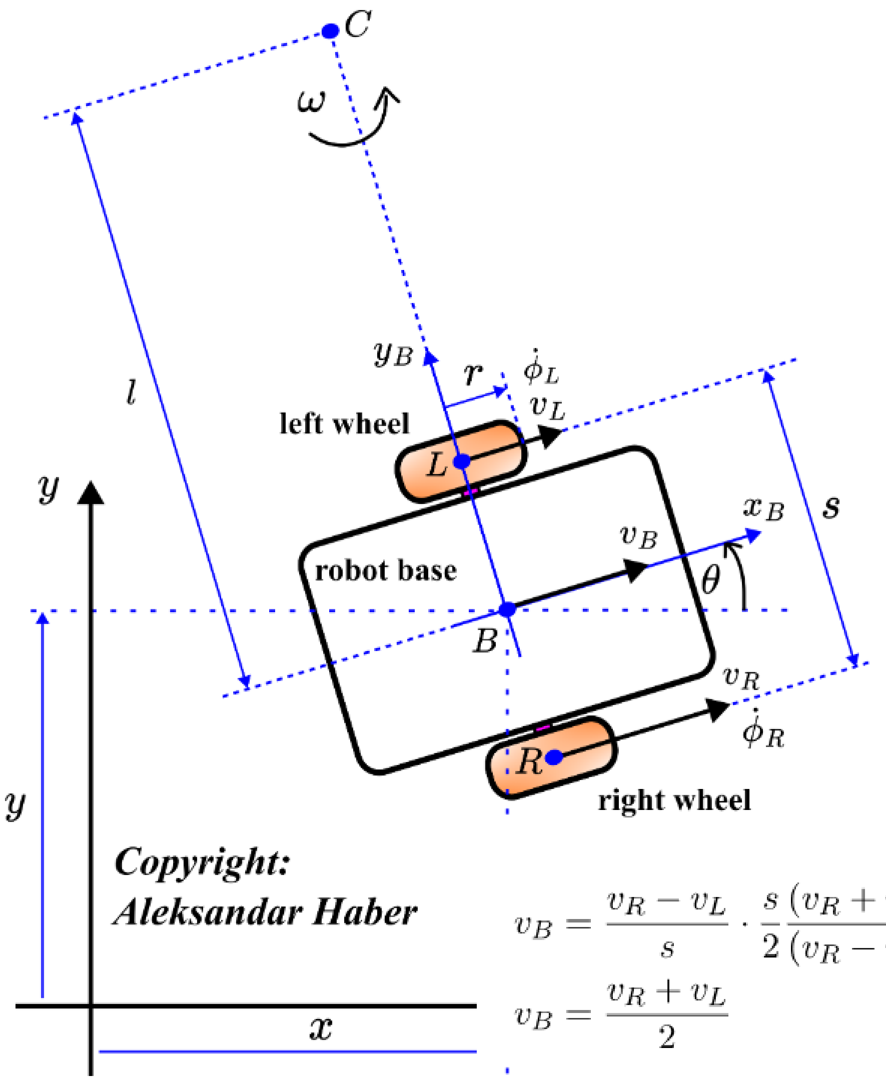
$$l = \frac{s(v_R+v_L)}{2(v_R-v_L)} \quad \text{and} \quad \omega = \frac{(v_R-v_L)}{s}$$

Now, we need to solve for $\dot{x}$, $\dot{y}$ , and $\dot{\theta}$. From Fig. 19, we have

$$\begin{aligned} \dot{x} &= v_B \cos\theta \\ \dot{y} &= v_B \sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{7}$$

Equation (7) can be written in vector-matrix form,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_B \\ \omega \end{bmatrix} \tag{8}$$




$$v_B = \frac{v_R - v_L}{s} \cdot \frac{s}{2}\frac{(v_R + v_L)}{(v_R - v_L)}$$

$$v_B = \frac{v_R + v_L}{2}$$

Fig. 19
Detailed kinematic diagram with all the parameters.

On the other hand, we have the intensity of the velocity $v_B$ is given by,

$$v_B = \omega \cdot l \tag{9}$$

By substituting $\omega$ and $l$ into equation (9), we obtain,

$$v_B = \frac{v_R + v_L}{2} \tag{10}$$

By combining this equation, with the equation for $\omega$ from (6), we obtain the following equations,

$$v_B = \frac{v_R + v_L}{2} = \frac{v_R}{2} + \frac{v_L}{2}$$

$$\omega = \frac{(v_R - v_L)}{s} = \frac{v_R}{s} - \frac{v_L}{s}$$

We can obtain equation for $\begin{bmatrix} v_B \\ \omega \end{bmatrix}$

The equation for $\begin{bmatrix} v_B \\ \omega \end{bmatrix}$ is,

$$\begin{bmatrix} v_B \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{s} & \frac{1}{s} \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix} \tag{11}$$

By substituting equation (11) into (8),

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{s} & \frac{1}{s} \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{\cos\theta}{2} & \frac{\cos\theta}{2} \\ \frac{\sin\theta}{2} & \frac{\sin\theta}{2} \\ \frac{-1}{s} & \frac{1}{s} \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix} \tag{13}$$

The system can be expanded into equations as follows,

$$\dot{x} = \frac{v_L}{2}\cos\theta + \frac{v_R}{2}\cos\theta$$
$$\dot{y} = \frac{v_L}{2}\sin\theta + \frac{v_R}{2}\sin\theta \qquad (14)$$
$$\dot{\theta} = -\frac{1}{s}v_L + \frac{1}{s}v_R$$

The system of equations (14) relates the controlled wheel $v_R$ and $v_L$ with the velocity projections of the center $B$ of the robot and the angular velocity of the robot.

However, we know that the wheel velocities are functions of the wheel angular velocities $\dot{\emptyset}_L$ and $\dot{\emptyset}_R$ :

$$v_L = r\dot{\emptyset}_L \qquad (15)$$
$$v_R = r\dot{\emptyset}_R$$

We can write equation (15) in vector-matrix form,

$$\begin{bmatrix} v_L \\ v_R \end{bmatrix} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}\begin{bmatrix} \dot{\emptyset}_L \\ \dot{\emptyset}_R \end{bmatrix} \qquad (16)$$

By substituting equation (16) into (13),

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{\cos\theta}{2} & \frac{\cos\theta}{2}\\ \frac{\sin\theta}{2} & \frac{\sin\theta}{2}\\ \frac{-1}{s} & \frac{1}{s}\end{bmatrix}\begin{bmatrix}r & 0\\ 0 & r\end{bmatrix}\begin{bmatrix}\dot{\emptyset}_L\\ \dot{\emptyset}_R\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{\mathrm{r}\cos\theta}{2} & \frac{\mathrm{r}\cos\theta}{2}\\ \frac{\mathrm{r}\sin\theta}{2} & \frac{\mathrm{r}\sin\theta}{2}\\ -\frac{r}{s} & \frac{r}{s}\end{bmatrix}\begin{bmatrix}\dot{\emptyset}_L\\ \dot{\emptyset}_R\end{bmatrix} \tag{17}$$

In the form of equations,

$$\dot{x} = \frac{r\dot{\emptyset}_L}{2}\cos\theta + \frac{r\dot{\emptyset}_R}{2}\cos\theta$$
$$\dot{y} = \frac{r\dot{\emptyset}_L}{2}\sin\theta + \frac{r\dot{\emptyset}_R}{2}\sin\theta \tag{18}$$
$$\dot{\theta} = -\frac{r}{s}\dot{\emptyset}_L + \frac{r}{s}\dot{\emptyset}_R$$

The equation (18) is the final equation derived in this example. It relates the angular velocities of the wheels with the velocity projections of the center of the robot and the robot's angular velocity. This equation enables us to predict the robot's motion.

# The End