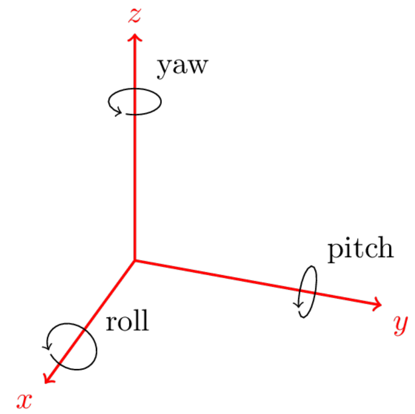
Rotation Matrices for Principal Axes

1. Coordinate Systems



2. Attitude Representation

2.1 The Three Angles

- **Roll (ϕ)**: Rotation around X-axis (banking left/right)
- **Pitch (θ)**: Rotation around Y-axis (nose up/down)
- **Yaw (ψ)**: Rotation around Z-axis (heading/turning)

2.2 ZYX Convention (Aerospace Standard)

Applied in sequence:

1. Yaw rotation (around Z)
2. Pitch rotation (around new Y)
3. Roll rotation (around newest X)

3. Rotation Matrices: Connecting the Frames

3.1 Basic Concept

Rotation about X-axis by angle $\emptyset$ (Roll, $\emptyset$)**:** plane of effect YZ

$$R(x,\emptyset) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\emptyset & -\sin\emptyset \\ 0 & \sin\emptyset & \cos\emptyset \end{bmatrix} \tag{1}$$

Let say a point $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is rotated by $\emptyset$ about X-axis

$$\vec{v}' = R(x,\emptyset) \cdot \vec{v}$$

$$\vec{v}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\emptyset & -\sin\emptyset \\ 0 & \sin\emptyset & \cos\emptyset \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y\cos\emptyset - z\sin\emptyset \\ y\sin\emptyset + z\cos\emptyset \end{bmatrix} \tag{2}$$

Think of a wheel spinning around the X-axis:

- Any point on the wheel has changing Y and Z values.
- Its X-position stays fixed.
- The angle θ determines how far the point has rotated.

Rotation about Y-axis by angle θ (Pitch, θ)**:** plane of effect XZ

$$R(y,\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{3}$$

Let say a point $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is rotated by θ about Y-axis

$$\vec{v}' = R(y,\theta) \cdot \vec{v}$$

$$\vec{v}' = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ z\cos\theta - x\sin\theta \end{bmatrix} \tag{4}$$

Rotation about Z-axis by angle ψ (Yaw, ψ)**:** plane of effect XY

$$R(z,\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Let say a point $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is rotated by ψ about Z-axis

$$\vec{v}' = R(z,\psi) \cdot \vec{v}$$

$$\vec{v}' = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos\psi - y\sin\psi \\ x\sin\psi + y\cos\psi \\ z \end{bmatrix} \tag{4}$$

3.2 Key Applications

In control systems and robotic kinematics:

- This is used to describe joint rotations or orientation adjustments.
- Example: If a robot arm rotates its shoulder about the X-axis, the elbow's position is affected in the YZ-plane.

In aerospace:

- A pitch maneuver is a rotation about the X-axis (if X is forward).