Assignment_3

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1. Formulate and solve this transportation problem using R

Converting all details into table format

```
tab <- matrix(c(22,14,30,600,100,
                16,20,24,625,120,
                80,60,70,"-","-") , ncol=5 , byrow=TRUE)

colnames(tab) <- c("Warehouse1","Warehouse2","Warehouse3","Prod cost","Prod Capacity")
row.names(tab) <- c("Plant A","Plant B","Demand")
tab <- as.table(tab)
tab
```

```
##         Warehouse1 Warehouse2 Warehouse3 Prod cost Prod Capacity
## Plant A 22         14         30         600       100
## Plant B 16         20         24         625       120
## Demand  80         60         70         -         -
```

$$\text{Min } TC = 622X_{11} + 614X_{12} + 630X_{13} + 641X_{21} + 645X_{22} + 649X_{23}$$

/text{subject to}

#Production Capacity consntraints Production plant A :

$$X_{11} + X_{12} + X_{13} + \leq 100$$

Production Plant B :

$$X_{21} + X_{22} + X_{23} + \leq 120$$

#Demand Consntraints

Demand Warehouse 1 :

$$X_{11} + X_{21} \geq 80$$

Demand Warehouse 2 :

$$X_{12} + X_{22} \geq 60$$

Demand Warehouse 3 :

$$X_{13} + X_{23} \geq 70$$

Non-negativity of the variables

$$X_{ij} \geq 0$$

Where

$$i = 1, 2, 3$$

And

$$j = 1, 2, 3$$

It is unbalanced since Demand not equal to supply so we have created the dummy row as warehouse_4

```
library(lpSolveAPI)
```

```
## Warning: package 'lpSolveAPI' was built under R version 4.1.3
```

```
library(lpSolve)
```

```
## Warning: package 'lpSolve' was built under R version 4.1.3
```

```
# Set up cost matrix
Trans_cost <- matrix(c(622,614,630,0,
                641,645,649,0) , ncol=4 , byrow=TRUE)
#defyning rows and coloumns
colnames(Trans_cost) <- c("Warehouse_1","Warehouse_2","Warehouse_3","Dummy")
rownames(Trans_cost) <- c("Plant_A", "Plant_B")
Trans_cost
```

```
##         Warehouse_1 Warehouse_2 Warehouse_3 Dummy
## Plant_A         622         614         630     0
## Plant_B         641         645         649     0
```

```
#setting up constraint signs and right-hand sides(Production side)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

#Demand side constraints#
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)

#solve the model
lptrans <- lp.transport(Trans_cost,"min",row.signs,row.rhs,col.signs,col.rhs)

lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

When solved the transportation problem, I got the values of the variables as

$$x_{12} = 60$$

$$x_{13} = 40$$

$$x_{21} = 80$$

$$x_{23} = 30$$

$$x_{24} = 10$$

```
lptrans$objval
```

```
## [1] 132790
```

2)Formulate the dual of the transportation problem

As we know the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA).

```
cost_2 <- matrix(c(622,614,630,100,"u1",
641,645,649,120,"u2",
80,60,70,220,"-",
"v1","v2","v3","-","-"),ncol = 5,nrow = 4,byrow = TRUE)
colnames(cost_2) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","Production Capacity","Supply(Dual)")
rownames(cost_2) <- c("Plant_A","Plant_B","Demand","Demand(Dual)")
```

p and q will be the variables for the dual.

$$\text{Max } \ Z = 100p_1 + 120p_2 + 80q_1 + 60q_2 + 70q_3$$

Subject to the following constraints

$$p_1 + q_1 \leq 622$$

$$p_1 + q_2 \leq 614$$

$$p_1 + q_3 \leq 630$$

$$p_2 + q_1 \leq 641$$

$$p_2 + q_2 \leq 645$$

$$p_2 + q_3 \leq 649$$

Where q1 = Warehouse_1

q2 = Warehouse_2

q3 = Warehouse_3

p1 = Plant_1

p2 = Plant_2

```r
#Objective function

f.obj <- c(100,120,80,60,70)

#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                  1,0,0,1,0,
                  1,0,0,0,1,
                  0,1,1,0,0,
                  0,1,0,1,0,
                  0,1,0,0,1), nrow = 6, byrow = TRUE)

f.dir <- c("<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=")

f.rhs <- c(622,614,630,641,645,649)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 139120
```

```r
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

So Z=139,120 dollars and variables are:

$$p_1 = 614$$

which represents Plant A

$$p_2 = 633$$

which represents Plant B

$$q_1 = 8$$

which represents Warehouse 1

$$q_3 = 16$$

which represents Warehouse 3

3) Economic Interpretation of the dual

Observations:

The maximum combined costs of shipping and production will be 139,120 dollars.

There is a minimal Z=132790 (Primal) and a maximum Z=139120 (Dual).This problem is trying to find a maximum and a minimum.As a result, we discovered that we should not be shipping from Plant(A/B) to all three warehouses at the same time. We should be shipping from:

$$60x_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40x_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80x_{21}$$

which is 80 Units from Plant B to Warehouse 1.

$$30x_{23}$$

which is 30 Units from Plant B to Warehouse 3. We will Max the profit from each distribution to the respective capacity.

We have the following:

$$p_1^0 - q_1^0 \leq 622$$

then we subtract

$$q_1^0$$

to the other side to get

$$p_1^0 \leq 622 - q_1^0$$

To compute it would be $614 <= (-8+622) which is correct. we would continue to evaluate these equations:

$$p_1 \leq 622 - q_1 => 614 \leq 622 - 8 = 614 => correct$$

$$p_1 \leq 614 - q_2 => 614 \leq 614 - 0 = 614 => correct$$

$$p_1 \leq 630 - q_3 => 614 \leq 630 - 16 = 614 => correct$$

$$p_2 \leq 641 - q_1 => 633 \leq 614 - 8 = 633 => correct$$

$$p_2 \leq 645 - q_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

$$p_2 \leq 649 - q_3 => 633 \leq 649 - 16 = 633 => correct$$

From the Duality-and-Sensitivity, By updating each of the columns we can test for the shadow price. In our LP Transportation problem , we change 100 to 101 and 120 to 121.Here we can see it R.

```
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=",2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=",4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=",2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=",4)

lp.transport(Trans_cost,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```
lp.transport(Trans_cost,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```r
lp.transport(Trans_cost,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

By taking the minimum of this specific function, seeing the number go down by 19 means the shadow price is 19, which was calculated by adding 1 to every plant.The Plant B does not have a shadow price. We also

From the dual variable

$$q_2$$

where Marginal Revenue <= Marginal Cost. The equation was

$$p_2 \leq 645 - q_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

and this was found by using

$$p_1^0 - q_1^0 \leq 622$$

then we subtract

$$q_1^0$$

to the other side to get

$$p_1^0 \leq 622 - q_1^0$$

```r
lp("max", f.obj,f.con, f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

$$q_2 = 0$$

.

The interpretation from above: from the primal:

$$60x_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40x_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80x_{21}$$

which is 80 Units from Plant B to Warehouse 1.

$$30x_{23}$$

which is 60 Units from Plant B to Warehouse 3.

From the dual

The agenda is to get MR=MC.In five of the six cases, MR<=MC.Only Plant B to Warehouse 2 fails to satisfy this requirement.From the primal, we can see that no AED devices will be shipped there.