Assignment 03

Formal Methods

Anum Sajid
i21-1233
Se-P

Q1 #

1. Bubble Sort

arr[4] = arr 0, arr 1, arr 2, arr 3

n = 4

$i_1 = 0$

$j_1 = 0$

$\phi_1 = i < n-1$

$\phi_2 = j_1 < n - i_1 - 1$

$\phi_3 = arr_0 > arr_1$

Swap:

temp 1 = arr_0

$arr_4 = arr_1$

$arr_5 = temp\ 1$

$j_2 = j_1 + 1$

$arr_6 = \phi_3 ?\ arr_4 : arr_0$

$arr_7 = \phi_3 ?\ arr_5 : arr_1$

$arr_8 = \phi_2 ?\ arr_6 : arr_0$

$arr_9 = \phi_2 ?\ arr_7 : arr_1$

$j_3 = \phi_2 ?\ j_2 : j_1$

$\phi_4 = j_3 < n - i_1 - 1$

$\phi_5 = arr_9 > arr_2$

Prince

CamScanner

swap:-

$temp_2 = arr_9$

$arr_{10} = arr_2$

$arr_{11} = temp_2$

$j_4 = j_3 + 1$

$arr_{12} = \phi_5 \; ? \; arr_{10} : arr_9$

$arr_{13} = \phi_5 \; ? \; arr_{11} : arr_2$

$arr_{14} = \phi_4 \; ? \; arr_{12} : arr_9$

$arr_{15} = \phi_4 \; ? \; arr_{13} : arr_2$

$j_5 = \phi_4 \; ? \; j_4 : j_3$

$\phi_6 = j_5 < n - i_1 - 1$

$\phi_7 = arr_{15} > arr_3$

swap:

$temp_3 = arr_{15}$

$arr_{16} = arr_3$

$arr_{17} = temp_3$

$j_6 = j_5 + 1$

$arr_{18} = \phi_7 \; ? \; arr_{16} : arr_{15}$

$arr_{19} = \phi_7 \; ? \; arr_{17} : arr_3$

$arr_{20} = \phi_6 \; ? \; arr_{16} : arr_{15}$

$arr_{21} = \phi_6 \; ? \; arr_{19} : arr_3$

$j_7 = \phi_6 \; ? \; j_6 : j_5$

$i_2 = i_1 + 1$

$i_3 = \phi_1 \; ? \; i_2 : i_1$

$arr_{22} = \phi_1 \; ? \; arr_8 : arr_0$

$arr_{23} = \phi_1 \; ? \; arr_{14} : arr_1$

$arr_{24} = \phi_1 \; ? \; arr_{20} : arr_2$

Print

$\phi_8 = i_3 < n-1$

$j_8 = 0$

$\phi_9 = j_8 < n - i_3 - 1$

$\phi_{10} = arr_{22} > arr_{23}$

swap:-

$temp_4 = arr_{22}$

$arr_{22} = arr_{23}$

$arr_{27} = temp_4$

$j_9 = j_8 + 1$

$arr_{28} = \phi_{10} ? arr_{26} : arr_{22}$

$arr_{29} = \phi_{10} ? arr_{26} : arr_{23}$

$arr_{30} = \phi_9 ? arr_{28} : arr_{22}$

$arr_{31} = \phi_9 ? arr_{29} : arr_{23}$

$j_{10} = \phi_9 ? j_9 : j_8$

$\phi_{11} = j_{10} < n - i_3 - 1$, $\phi_{12} = arr_{31} > arr_{24}$

swap:-

$temp_5 = arr_{31}$

$arr_{32} = arr_{24}$

$arr_{33} = temp_5$

$j_{11} = j_{10} + 1$

$arr_{34} = \phi_{12} ? arr_{32} : arr_{31}$

$arr_{35} = \phi_{12} ? arr_{33} : arr_{24}$

$arr_{36} = \phi_{11} ? arr_{34} : arr_{31}$

$arr_{37} = \phi_{11} ? arr_{35} : arr_{24}$

$j_{12} = \phi_{11} ? j_{11} : j_{10}$

Prince

CamScanner

$i_4 = i_2 + 1$

$i_5 = \phi_8 \; ? \; i_4 : i_3$

$arr_{36} = \phi_8 \; ? \; arr_{30} : arr_{22}$

$arr_{39} = \phi_8 \; ? \; arr_{36} : arr_{25}$

$arr_{40} = \phi_8 \; ? \; arr_{37} : arr_{24}$

$j_{13} = 0$

$\phi_{13} = i_5 < n-1$

$\phi_{14} = j_{13} < n - i_5 - 1$

$\phi_{15} = arr_{38} > arr_{39}$

Swap:

$temp_6 = arr_{38}$

$arr_{41} = arr_{39}$

$arr_{42} = temp_6$

$j_{14} = j_{13} + 1$

$arr_{43} = \phi_{15} \; ? \; arr_{41} : arr_{38}$

$arr_{44} = \phi_{15} \; ? \; arr_{42} : arr_{39}$

$arr_{45} = \phi_{14} \; ? \; arr_{43} : arr_{38}$

$arr_{46} = \phi_{14} \; ? \; arr_{44} : arr_{39}$

$j_{15} = \phi_{14} \; ? \; j_{14} : j_{13}$

$i_6 = i_5 + 1$

$i_7 = \phi_{13} \; ? \; i_6 : i_5$

$arr_{47} = \phi_{13} \; ? \; arr_{45} : arr_{38}$

$arr_{48} = \phi_{13} \; ? \; arr_{46} : arr_{39}$

CamScanner

DATE

2. Insertion Sort

$x_1 = arr_0.length$
$i_1 = 1$
$\phi_1 = i_1 < x_1$

$key_1 = a_1$
$j_1 = i_1 - 1$
$\phi_{11} = j_1 >= 0$ and $a0_0 > key_1$

$al_1 = a0_0$
$j_3 = j_1 - 1$
$al_2 = \phi_{11} ? al_1 : al_0$

$j_3 = \phi_{11} ? j_3 : j_1$

$a0_1 = \phi_{11} ? key_1 : a0_0$
$al_3 = \phi_{11} ? al_2 : key_1$
$i_2 = i_1 + 1$
$\phi_2 = i_2 < x_1$
$key_2 = a2_0$
$j_4 = i_2 - 1$
$\phi_{21} = j_4 >= 0$ and $al_3 > key_2$

$a2_1 = al_3$
$j_5 = j_4 - 1$

$\phi_{22} = j_5 >= 0$ and $a0_1 > key_2$
$al_4 = a0_1$

Prince

CamScanner

$j_6 = j_5 - 1$

$al_5 = \phi_{22} \; ? \; al_4 : al_3$

$j_7 = \phi_{22} \; ? \; j_6 : j_5$

$a_{22} = \phi_{21} \; ? \; a2_1 : a2_0$

$j_8 = \phi_{21} \; ? \; j_7 : j_4$

$aO_2 = \phi_{22} \; ? \; key_2 : aO_1$

$al_6 = (!\phi_{22} \; \&\& \; !\phi_{20}) \; ? \; key_2 : al_5$

$a2_3 = \phi_{24} \; ? \; a2_2 : key_2$

$i_3 = i_2 + 1$

$\phi_3 = i_3 < x_1$

$key_3 = a3_0$

$j_9 = i_3 - 1$

$\phi_{31} = j_9 >= 0$ and $a2_3 > key_3$

$a3_1 = a2_3$

$j_{10} = j_9 - 1$

$\phi_{32} = j_{10} >= 0$ and $al_6 > key_3$

$a2_4 = al_6$

$j_{11} = j_{10} - 1$

$\phi_{33} = j_{11} >= 0$ and $aO_2 > key_3$

Prince

CamScanner

$al_7 = aO_2$

$j_{17} = \phi_{33}\ ?\ j_{11} : j_{10}$

$al_8 = \phi_{33}\ ?\ al_7 : al_6$

$j_{13} = \phi_{33}\ ?\ j_{12} : j_{11}$

$a2_5 = \phi_{32}\ ?\ a2_4 : a2_3$

$j_{14} = \phi_{32}\ ?\ j_{13} : j_{10}$

$a3_2 = \phi_{31}\ ?\ a3_1 : a3_0$

$j_{15} = \phi_{31}\ ?\ j_{14} : j_9$

$aO_3 = \phi_{33}\ ?\ key_3 : aO_2$

$al_9 = (!\phi_{33}\ \&\&\ \phi_{32}\ \&\&\ \phi_{31})\ ?\ key_3 = al_8$

$a2_6 = (!\phi_{32}\ \&\&\ \phi_{31})\ ?\ key_3 : a2_5$

$a3_3 = \phi_{31}\ ?\ a3_2 : key_3$

$i_4 = i_3 + 1$

- $aO_4 = \phi_3\ ?\ aO_3 : aO_2$
  $al_{10} = \phi_3\ ?\ al_9 : al_6$
  $a2_7 = \phi_3\ ?\ a2_6 : a2_3$
  $a3_4 = \phi_3\ ?\ a3_3 : a3_0$
  $i_5 = \phi_3\ ?\ i_4 : i_3$

$aO_5 = \phi_2\ ?\ aO_4 : aO_1$

$al_{11} = \phi_2\ ?\ al_{10} : al_3$

$a2_8 = \phi_2\ ?\ a2_7 : a2_0$

$i_6 = \phi_2\ ?\ i_5 : i_2$

$aO_0 = \phi_1\ ?\ aO_5 : aO_0$

$al_{12} = \phi_1\ ?\ al_{11} : al_0$

$i_7 = \phi_1\ ?\ i_6 : i_1$

Prince

CamScanner

DATE

3. Selection Sort

$x_1 = arr_0.length$

$i_1 = 0$

$\phi_1 = i_1 < x_1 - 1$

$min\_idx_1 = i_1$

$a\_min_1 = a0_0$

$j_1 = i_1 + 1$

$\phi_{11} = j_1 < x_1$

$\phi_{111} = a_1 < a\_min_1$

$min\_idx_2 = j_1$

$min\_idx_3 = \phi_{111} \; ? \; min\_idx_2 : min\_idx_1$

$a\_min_2 = \phi_{111} \; ? \; a1_0 : a\_min_1$

$j_2 = j_1 + 1$

$\phi_{12} = j_2 < x_1$

$\phi_{121} = a_2 < a\_min_2$

$min\_idx_4 = j_2$

$min\_idx_5 = \phi_{121} \; ? \; min\_idx_4 : min\_idx_3$

$a\_min_3 = \phi_{121} \; ? \; a2_0 : a\_min_2$

$j_3 = j_2 + 1$

$\phi_{13} = j_3 < x_1$

$\phi_{131} = a3 < a\_min_3$

$min\_idx_6 = j_3$

$min\_idx_7 = \phi_{131} \; ? \; min\_idx_6 : min\_idx_5$

Prince

CamScanner

$a\_min_4 = \phi_{131} ? a3_0 : a\_min_3$

$j_4 = j_3 + 1$

$min\_idx_8 = \phi_{13} ? min\_idx_7 : min\_idx_5$

$a\_min_5 = \phi_{13} ? a\_min_4 : a\_min_3$

$j_5 = \phi_{13} ? j_4 : j_3$

$min\_idx_9 = \phi_{12} ? min\_idx_8 : min\_idx_3$

$a\_min_6 = \phi_{12} ? a\_min_5 : a\_min_2$

$j_6 = \phi_{12} ? j_5 : j_2$

$min\_idx_{10} = \phi_{11} ? min\_idx_9 : min\_idx_1$

$a\_min_7 = \phi_{11} ? a\_min_6 : a\_min$

$j_7 = \phi_{11} ? j_6 : j_1$

$Temp_1 = a0_0$

$a0_1 = a\_min_7$

$a\_min_8 = temp_{81}$

$a1_1 = (\phi_{111} \&\& !\phi_{121} \&\& !\phi_{131}) ? a\_min_8 : a1$

$a2_1 = (\phi_{121} \&\& !\phi_{131}) ? a\_min_8 : a2_0$

$a3_1 = \phi_{131} ? a\_min_8 : a3_0$

$i_2 = i_1 + 1$

$\phi_2 = i_2 < x_1 - 1$

$min\_idx_{11} = i_2$

$a\_min_9 = a1_1$

$j_8 = i_2 + 1$

$\phi_{21} = j_8 < x_1$

$\phi_{211} = a2_1 < a\_min_9$

$min\_idx_{12} = j8$

$min\_idx_{13} = \phi_{211}\ ?\ min\_idx_{12} : min\_idx_{11}$

$a\_min_{10} = \phi_{211}\ ?\ a_{21} : a\_min_9$

$j9 = j8 + 1$

$\phi_{12} = j_9 < n_1$

$\phi_{221} = a3_1 < a\_min_{10}$

$min\_idx_{14} = j9$

$min\_idx_{15} = \phi_{221}\ ?\ min\_idx_{14} : min\_idx_{13}$

$a\_min_{11} = \phi_{221}\ ?\ a_{31} : a\_min_{10}$

$j_{10} = j_9 + 1$

$min\_idx_{16} = \phi_{22}\ ?\ min\_idx_{15} : min\_idx_{13}$

$a\_min_{12} = \phi_{22}\ ?\ a\_min_{11} : a\_min_{10}$

$j_{11} = \phi_{22}\ ?\ j_{10} : j_9$

$min\_idx_{17} = \phi_{21}\ ?\ minidx_{16} : min\_idx_{11}$

$a\_min_{13} = \phi_{21}\ ?\ a\_min_{12} : a\_min_9$

$j_{12} = \phi_{21}\ ?\ j_{11} : j_8$

$temp_2 = a1_1$

$a1_2 = a\_min_{13}$

$a\_min_{14} = temp_2$

$a2_2 = (\phi_{211} \;\&\; !\phi_{221})\ ?\ a\_min_{14} : a2_1$

$a3_2 = \phi_{22}\ ?\ a\_min_{14} : c3_1$

$i_3 = i_2 + 1$

$\phi_3 = i_3 < n_1$

$min\_idx_{18} = i_3$

$a\_min_{15} = a2_2$

$j_{13} = i_3 + 1$

$\phi_{31} = j_{13} < n_1$

$\phi_{311} = a3_2 < a\_min_{15}$

$min\_idx_{19} = j_{13}$

$min\_idx_{20} = \phi_{311}\ ?\ min\_idx_{19} : min\_idx_{18}$

$a\_min_{16} = \phi_{311}\ ?\ a3_2 : a\_min_{15}$

$j_{15} = \phi_{31}\ ?\ j_4 : j_{13}$

$temp_3 = a2_2$

$a2_3 = a\_min_{17}$

$a\_min_{16} = temp_3$

$a3_3 = \phi_{311}\ ?\ a\_min_{18} : a3_2$

$i_4 = i_3 + 1$

$a2_4 = \phi_3\ ?\ a2_3 : a2_2$

$a3_4 = \phi_3\ ?\ a3_3 : a3_2$

$i_5 = \phi_3\ ?\ i_4 : i_3$

CamScanner

$al_3 = \phi_2 \; ? \; al_2 : al_1$

$a2_5 = \phi_2 \; ? \; a2_4 : a2_1$

$a3_5 = \phi_2 \; ? \; c3_4 : a3_1$

$i_6 = \phi_2 \; ? \; i_5 : i_2$

$aO_2 = \phi_2 \; ? \; aO_1 : aO_0$

$al_4 = \phi_1 \; ? \; al_3 : al_0$

$a2_6 = \phi_1 \; ? \; a2_5 : a2_0$

$a3_6 = \phi_1 \; ? \; a3_5 : a3_0$

$i_7 = \phi_1 \; ? \; i_6 : i_1$

CamScanner