State Estimation - Assignment 1.1

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1 Question 2

e) Simulate d) using the Monte Carlo simulation, i.e. assume

$$\mathbf{x} = \begin{bmatrix} 1 \text{ m} \\ 0.5^\circ \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.005 \end{bmatrix}.$$

Sample 1000 points from this distribution and plot the transformed results on $\mathrm{x} - \mathrm{y}$ coordinates. Plot the uncertainty ellipse, calculated from part **d**). Overlay the ellipse on the point samples.

Answer: First, I derived the covariance in the x-y coordinates analytically. I used it in a Python program to plot 1000 points from this distribution and plot the uncertainty ellipse on the point samples. I found information about the uncertainty ellipse from the following website:

https://www.visiondummy.com/2014/04/draw-error-ellipse-representing-covariance-matrix/

$$\begin{bmatrix} \cos 0.5 & -\sin 0.5 \\ \sin 0.5 & \cos 0.5 \end{bmatrix} \begin{bmatrix} 0.01 & 0 \\ 0 & 0.005 \end{bmatrix} \begin{bmatrix} \cos 0.5 & \sin 0.5 \\ -\sin 0.5 & \cos 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.01\cos 0.5 & -0.005 & \sin 0.5 \\ 0.01\sin 0.5 & 0.005 & \cos 0.5 \end{bmatrix} \begin{bmatrix} \cos 0.5 & \sin 0.5 \\ -\sin 0.5 & \cos 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.01\cos^2 0.5 + 0.005\sin^2 0.5 & 0.005\sin 0.5\cos 0.5 \\ 0.005\sin 0.5\cos 0.5 & 0.01\sin^2 0.5 + 0.005\cos^2 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 9.9996 \times 10^{-3} & 4.3631 \times 10^{-3} \\ 4.3631 \times 10^{-3} & 5.0004 \times 10^{-3} \end{bmatrix}$$

f) Repeat part e), for the following values:

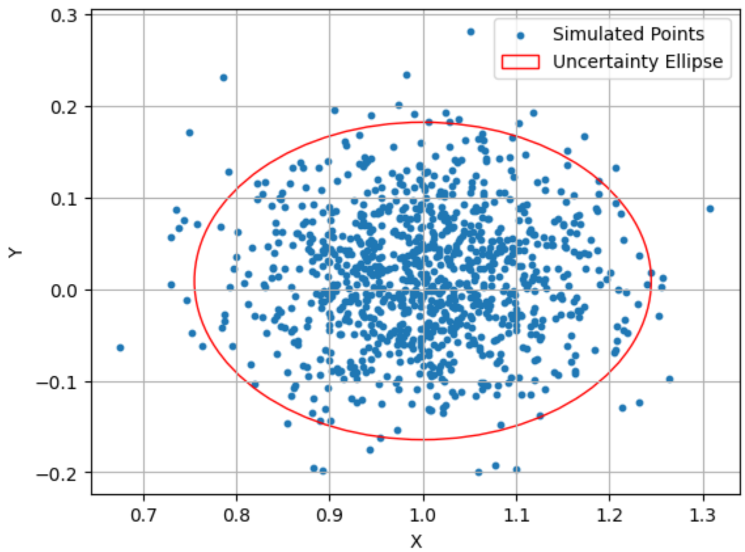

Figure 1: Uncertainty ellipse and the sample points

$$\mathbf{x} = \begin{bmatrix} 1 \text{ m} \\ 0.5^\circ \end{bmatrix}, \quad \mathbf{\Sigma} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.005 \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} 1 \text{ m} \\ 0.5^\circ \end{bmatrix}, \quad \mathbf{\Sigma} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.1 \end{bmatrix},$$

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$$\mathbf{x} = \begin{bmatrix} 1 \text{ m} \\ 0.5^\circ \end{bmatrix}, \quad \mathbf{\Sigma} = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix}.$$

Answer: The covariance matrix cov x contains elements that represent the relationship between the r and theta coordinates of the original samples. Larger values indicate greater

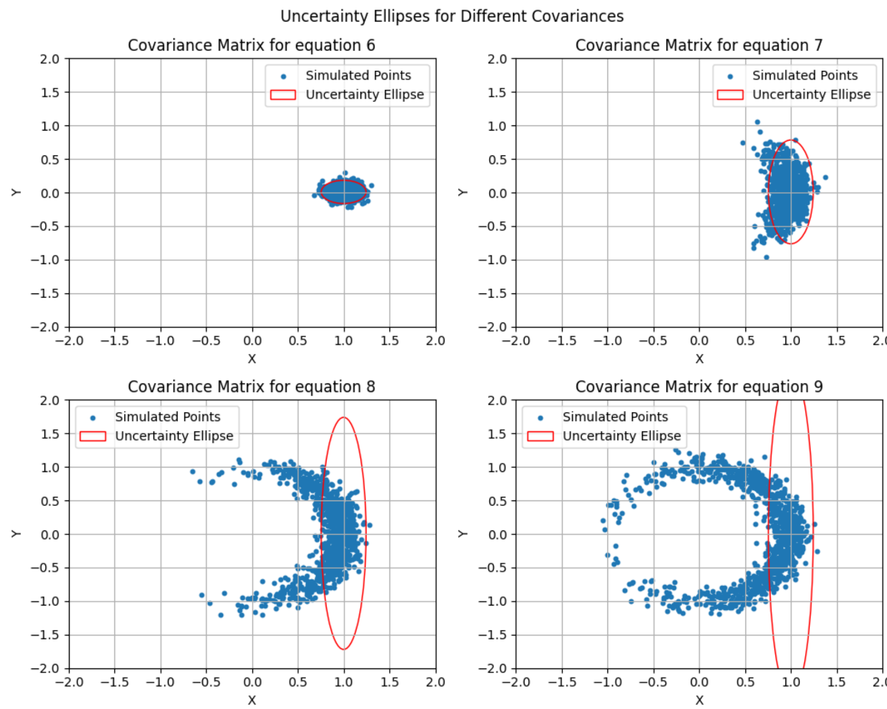

Figure 2: Different values of covariance x

variance or stronger covariance, while smaller values indicate lower variance or weaker covariance. When cov x has larger values, the transformed distribution will exhibit a broader dispersion along the r and theta axes.

The ellipses depicted in each plot convey the shape and orientation of the transformed distribution. The width and height of the ellipse are determined by the eigenvalues of the covariance matrix cov y. Larger eigenvalues indicate greater variability along the corresponding eigenvectors, resulting in a wider ellipse. The angle of the ellipse indicates the orientation of the transformed distribution.