State Estimation - Assignment 5

Haniyeh Altafi
haniyeh.altafi@torontomu.ca

***Abstract*—This summary provides an overview of different estimation methods and algorithms, namely the Kalman Filter, Extended Kalman Filter (EKF), Particle Filter, and RANSAC. Each algorithm has unique characteristics and applications in various fields. Below is a concise description of their key differences, advantages, and disadvantages.**

I. Kalman Filter

The Kalman Filter is a mathematical tool used to estimate the state of a dynamic system based on measurements. It incorporates both prediction and measurement update steps iteratively to refine the state estimate. The advantages of the Kalman Filter include its efficiency in tracking linear systems and its ability to handle Gaussian noise. However, it assumes linearity and Gaussian distributions, limiting its effectiveness in nonlinear or non-Gaussian scenarios. The Kalman Filter algorithm consists of several steps:

Initialization: The initial state of the system and its uncertainty are defined, including relevant variables and a covariance matrix representing uncertainty.

Prediction: The Kalman Filter predicts the next state based on the previous state and system dynamics, incorporating information about how the system is expected to evolve over time. Uncertainty in the prediction is updated using the covariance matrix.

Measurement Update: When a measurement is obtained, the Kalman Filter combines the predicted state with the measurement to refine the state estimate. The Kalman Gain determines the weight given to the predicted state and measurement, resulting in an updated state and reduced uncertainty.

Repeat: Steps 2 and 3 are iteratively performed for each new measurement. The Kalman Filter continuously refines its estimate by incorporating measurements and updating the state and uncertainty.

II. Extended Kalman Filter

The Extended Kalman Filter (EKF) is an extension of the Kalman Filter that addresses nonlinear systems by linearizing them using the Jacobian matrix. The EKF algorithm follows these steps:

1. Initialization: Similar to the Kalman Filter, the initial state estimate, error covariance matrix, process noise covariance, and measurement noise covariance are initialized.

2. Prediction Step: The state estimate is predicted using the nonlinear system dynamics function and linearized using the Jacobian matrix. The error covariance is updated based on the linearized dynamics and process noise.

3. Update Step: A measurement is obtained, and the measurement residual is computed. The measurement model is linearized using the Jacobian matrix. The Kalman gain is computed, and the state estimate and error covariance are updated.

4. Repeat: Steps 2 and 3 are iterated for each subsequent time step.

However, linearization introduces approximation errors, and the EKF may struggle with highly nonlinear systems or large uncertainties. The EKF is capable of estimating the state of nonlinear systems, making it more versatile than the Kalman Filter.

III. Particle Filter

The Particle Filter, also known as Sequential Monte Carlo or the bootstrap filter, is a recursive filtering technique for estimating the state of sequential or time-varying systems. It represents the posterior distribution using a set of random samples called particles. The Particle Filter algorithm can be described as follows:

Initialization: An initial set of particles is generated, representing possible states, with equal weights assigned.

Prediction: Each particle is advanced in time using a stochastic equation that considers system dynamics and external factors.

Update (Correction): Measurements are used to evaluate the likelihood of particles based on their predicted and actual values. Particle weights are adjusted accordingly.

Resampling: Particles with higher weights are selected and duplicated, while those with lower weights may be eliminated, improving the accuracy of the posterior distribution representation.

Repeat: Steps 2 to 4 are iterated for subsequent time steps, adapting the particle set to track the system's changing state.

By iteratively predicting, updating, and resampling, the Particle Filter algorithm provides an approximation of the posterior distribution of the system's state. The final set of particles represents the estimated state of the system at each time step, allowing for tracking and inference in dynamic environments where the underlying system evolves over time. The Particle Filter excels in nonlinear and non-Gaussian scenarios where other methods may fail. However, its main disadvantage is the computational complexity, which increases with the number of particles used.

IV. RANSAC

The RANSAC (Random Sample Consensus) algorithm is a robust method for estimating the parameters of a mathematical model from a set of observed data points that may contain outliers. It is commonly used in image processing tasks, such as estimating geometric transformations like the homography matrix.

The RANSAC algorithm can be summarized as follows:

Random Sample Selection: A subset of data points is randomly chosen, typically consisting of corresponding points from the source and destination images.

Model Estimation: The selected points are used to compute the model parameters, involving solving linear equations, such as estimating a homography matrix.

Model Validation: The computed model is applied to transform the remaining data points from the source image to the destination image. The error between the transformed points and the actual points in the destination image is measured, often using the Euclidean distance.

Inlier Determination: Points with errors below a predefined threshold are considered inliers, indicating potential matches that fit the estimated transformation.

Iteration and Consensus: Steps 1 to 4 are repeated a sufficient number of times to identify the set of inliers that best fit the model. The set with the highest number of inliers is considered the consensus set.

After the iterations, the RANSAC algorithm returns the set of inliers that produced the highest number of inliers. The RANSAC algorithm robustly estimates the parameters of a mathematical model by repeatedly sampling and fitting the model, while mitigating the impact of outliers. However, it may struggle with datasets containing a high percentage of outliers or when multiple models need to be estimated simultaneously.

V. Conclusion

In conclusion, the choice of estimation method or algorithm depends on the characteristics of the data. The Kalman Filter and EKF are suitable for linear and moderately nonlinear systems, while the Particle Filter is well-suited for highly nonlinear and non-Gaussian systems. RANSAC is a robust algorithm for parameter estimation in the presence of outliers. Understanding the strengths and limitations of each algorithm is crucial for selecting the appropriate method for a given task.