Intro to Combinatorial Problems

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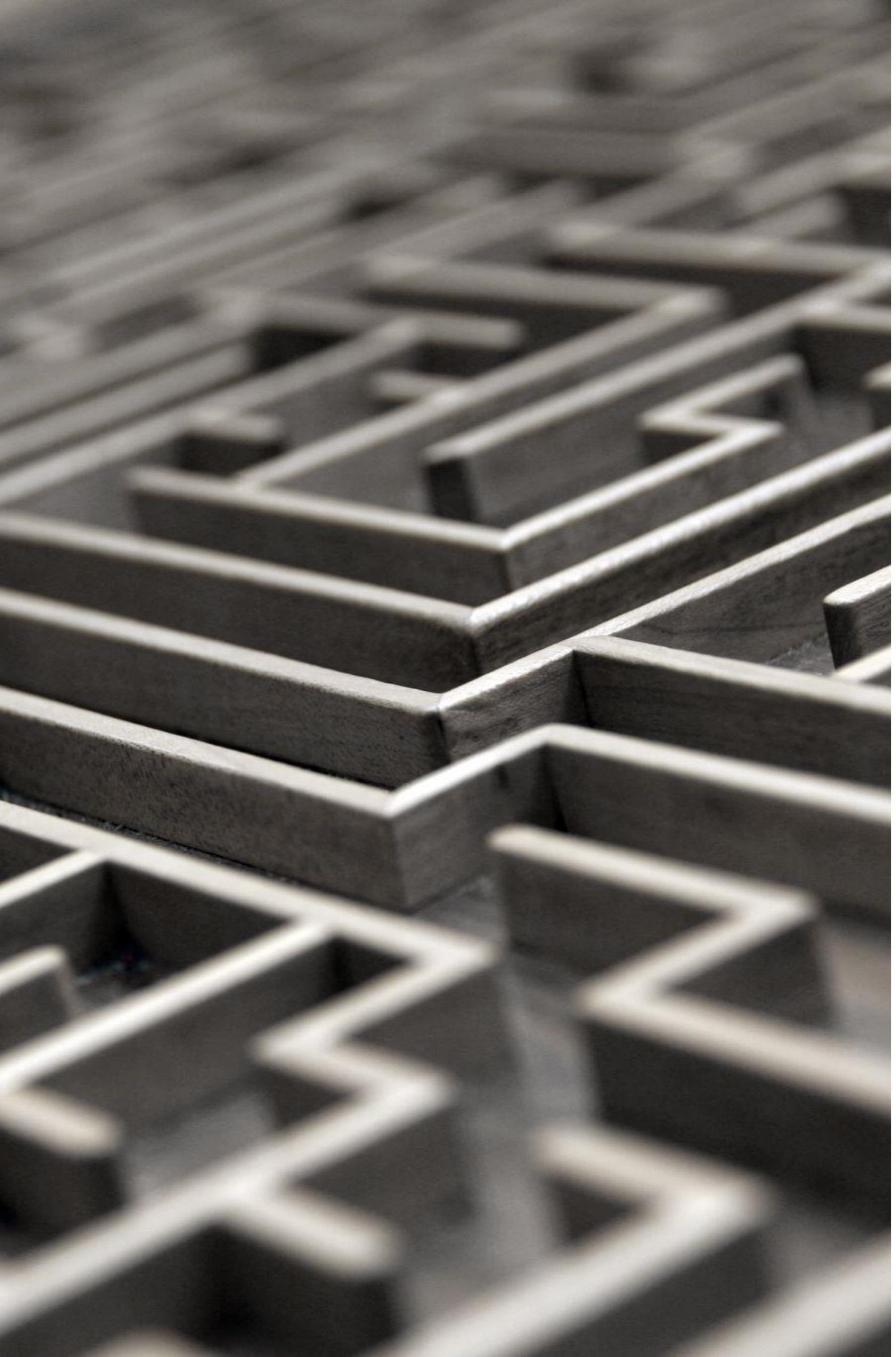
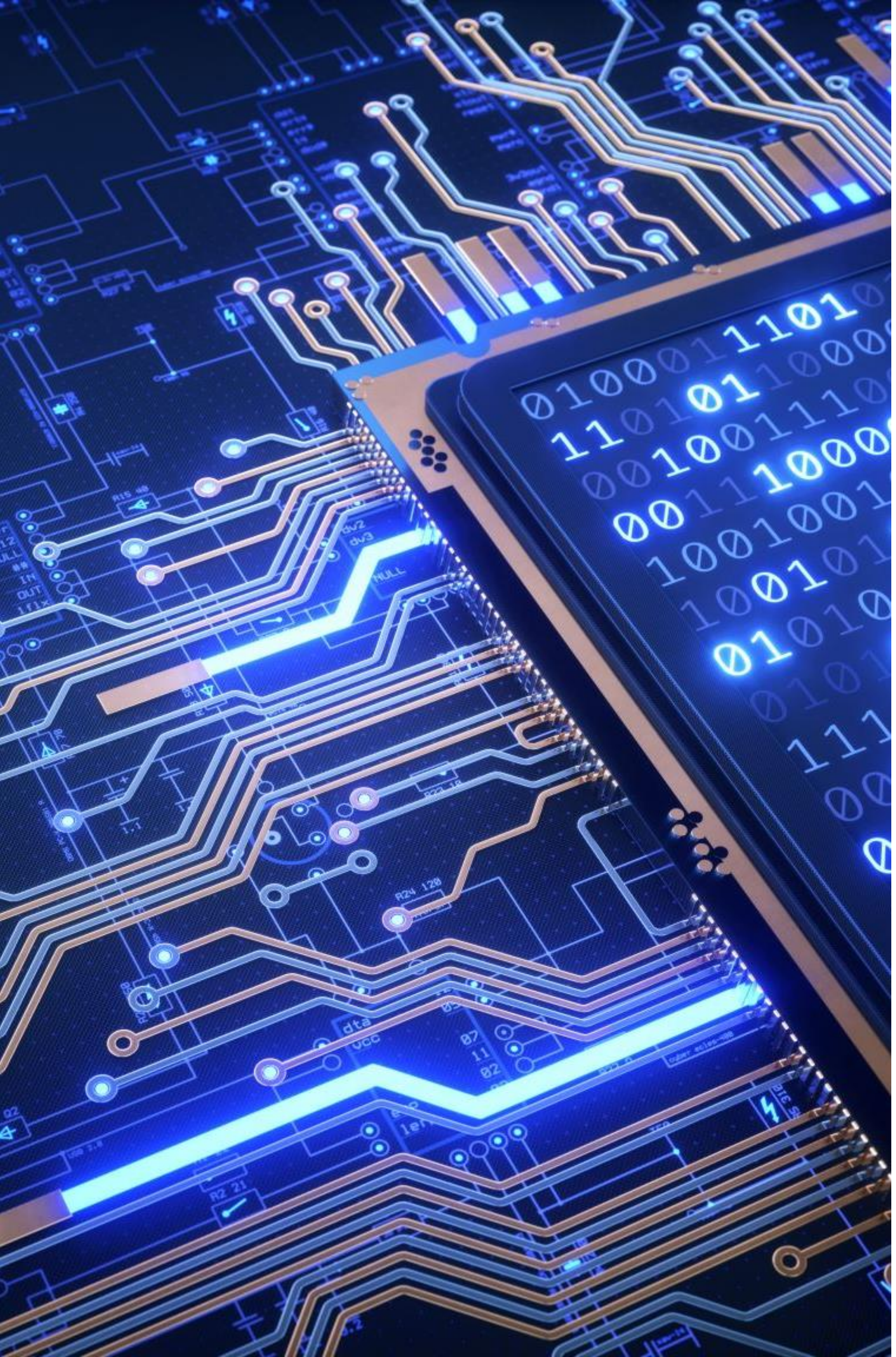


Combinatorial Problems

- **Combinatorial problems** involve finding a grouping, ordering, or assignment of a **discrete**, **finite** set of objects that **satisfies** given **conditions**.
- **Solutions** are candidate solutions that satisfy all **given conditions.**
- **Conditions:** can be rules or constraints.



Example

- **Data center resource managements:**
  - The company receives some requested cpus and memories for some computations.
  - There are bunch of machines with different cpus, memory resource.
  - We want to assign jobs or services or requests to these machines in a way that optimizes some objectives
  - Objectives:
    - smallest number of machines
  - Constraint:
    - each service on one machine only
    - don't exceed memory capacity
    - don't exceed cpu capacity !

Combinatorial Algorithms

- Combinatorial algorithms are ones which deal with a system of **discrete objects**.
- These objects need to be **arranged or selected** to minimize or maximizing a **cost function** depending on the problem.
- For example: Cost function can be performance of algorithms in terms of runtime.
- Mostly, computation behind these algorithms
  - too difficult
  - too expensive to compute
  - faster solutions
  - number of alternatives to be explored grows very fast with the increase in problem size

Sudoku

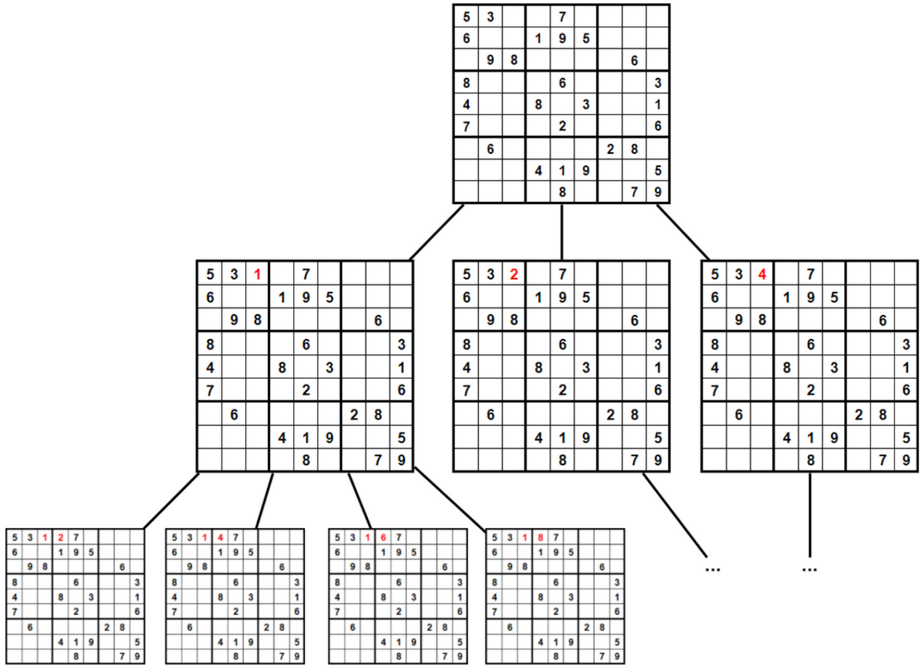
- Constraints:
  - numbers 1-9 exactly once in each row, column, and square.
- Constraints: make the search space smaller and make the problem trackable.
- Less constraints: we have bigger search spaces for each cell so it will be harder problem to solver the sudoku.
- Number of solutions in a sudoku depend on the number of empty cells or the size of the search space; if we have more empty cells or more search spaces, we will have more solutions for a sudoku. There are different ways and algorithms that solve sudokus and based on a cost function we can do optimization!

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	4,7
			4	1	9		3	5
				8			7	9

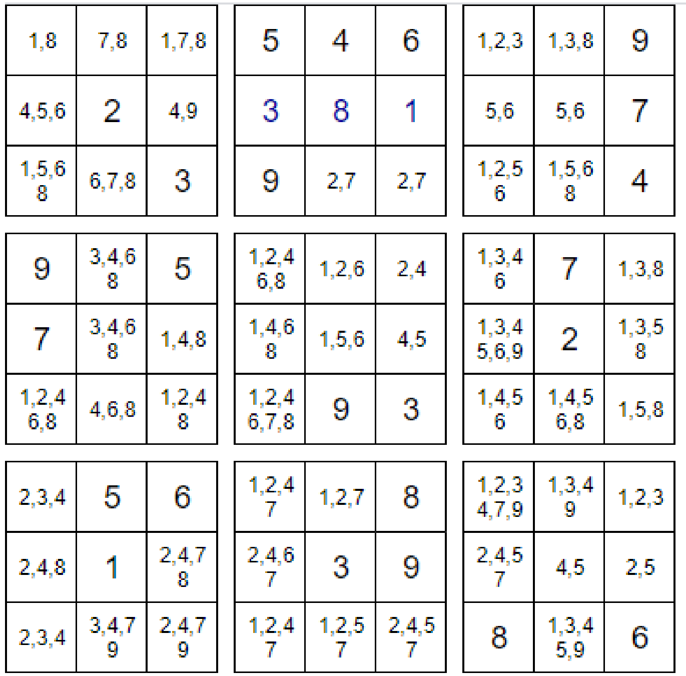
5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Strategies

Solving Sudoku with Backtracking



Solving Sudoku with Constraint Propagation



1,8	7,8	1,7,8	5	4	6	1,2,3	1,3,8	9
4,5,6	2	4,9	3	8	1	5,6	5,6	7
1,5,6 8	6,7,8	3	9	2,7	2,7	1,2,5 6	1,5,6 8	4
9	3,4,6 8	5	1,2,4 6,8	1,2,6	2,4	1,3,4 6	7	1,3,8
7	3,4,6 8	1,4,8	1,4,6 8	1,5,6	4,5	1,3,4 5,6,9	2	1,3,5 8
1,2,4 6,8	4,6,8	1,2,4 8	1,2,4 6,7,8	9	3	1,4,5 6	1,4,5 6,8	1,5,8
2,3,4	5	6	1,2,4 7	1,2,7	8	1,2,3 4,7,9	1,3,4 9	1,2,3
2,4,8	1	2,4,7 8	2,4,6 7	3	9	2,4,5 7	4,5	2,5
2,3,4	3,4,7 9	2,4,7 9	1,2,4 7	1,2,5 7	2,4,5 7	8	1,3,4 5,9	6

Combinatorial Optimization

Subset of mathematical optimization that is related to operations research, algorithm theory and computational complexity.

Find the optimal object from finite set of objects in terms of the desired cost function.

Which solution is better using cost function optimization

Solving Combinatorial Problems with ML

Not inventing new algorithms

✓ Using ML techniques

Using the existing algorithms intelligently and automatically.

This can be selection between these algorithms, automatically configurating the algorithms, or ordering the algorithms in a desired order!

Other Popular Combinatorial Problems

- Traveling Salesman Problem (TSP)
- Boolean Satisfiability (SAT)
- Planning
- Time-tabling

- **Some Application Examples:**
  - Optimal way to deliver packages
  - Scheduling rescue units in response to disasters like earthquakes
  - Internet data packet routing

References and Additional Resources

- http://www.cs.ubc.ca/labs/beta/Courses/CPSC532D-05/Slides/ch1-slides.pdf
- https://www.sciencedirect.com/science/article/pii/B9781558608726500184
- https://www.youtube.com/watch?v=3z7TIxRA3RE
- https://ojs.aaai.org/index.php/aimagazine/article/view/2460
- https://www.youtube.com/watch?v=PkLjIvv-H-Q&t=554s
- https://www.youtube.com/watch?v=XVLd7hf6y6M&t=0s