# Entrepreneurial Heterogeneity and Aggregate Dynamics in Emerging Markets

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May 30, 2024

**Abstract.** This paper seeks to understand how the heterogeneity in entrepreneurs affects aggregate dynamics in emerging markets. Using Argentinian microdata, I document the need to distinguish between self-employed and employer entrepreneurs and highlight two facts. First, the composition of entrepreneurs varies across the income distribution, as the majority of poorer entrepreneurs are self-employed, while the majority of employer entrepreneurs exist at the upper tail of the income distribution. Second, the population share of entrepreneurs is countercyclical with respect to GDP, driven by the large population share of low-income, self-employed entrepreneurs. Motivated by these findings, I develop a small open-economy, heterogeneous-agent model with aggregate shocks in which households endogenously choose to become workers, self-employed entrepreneurs, or employer entrepreneurs. The model is calibrated to match income inequalities within and across occupations in the stationary equilibrium and to replicate business cycle dynamics. Model simulations suggest that loosening financial frictions amplifies business cycles, while self-employment helps stabilize business cycles by providing households with a means to mitigate income declines during recessions.

*Keywords:* Small open economy; Entrepreneurs; Emerging markets business cycles; Heterogeneous agents

*JEL Classification:* E32, F41

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# 1 Introduction

Entrepreneurship plays a vital role in economic development while also significantly accounting for the growing income and wealth inequalities. The entrepreneur landscape in emerging markets differs from the richer countries' because in emerging economies, where formal employment opportunities may be limited, entrepreneurship often acts as a crucial avenue for individuals to generate any economic activity. These entrepreneurial activities are far from homogeneous, with a diverse range of entrepreneurs operating at different scales and facing unique challenges. Understanding the heterogeneity among entrepreneurs and its implications for aggregate dynamics is essential for broadening our understanding of emerging market fluctuations and designing effective policies that foster inclusive growth.

This paper sheds light on the importance of entrepreneurial heterogeneity in shaping the business cycles of emerging economies, focusing on the case of Argentina. I begin by documenting two key empirical facts using Argentinian microdata: first, the share of entrepreneurs does not monotonically increase with income, and second, the population share of entrepreneurs are countercyclical with respect to GDP. These findings highlight the need to distinguish between different types of entrepreneurs, particularly self-employed individuals and those who employ others (employer entrepreneurs). Self-employed entrepreneurs operate small establishments and are mostly found at the bottom end of the income distribution. They also make up the majority of the entrepreneurial group. The rest of the group is made up of employer-entrepreneurs who are mostly found at the top end of the income distribution, operate large establishments, and have a high income relative to workers and self-employed entrepreneurs.

To capture these empirical patterns and investigate their implications, I develop a dynamic general equilibrium model of occupational choice in a small open economy setting with aggregate shocks. The baseline model incorporates key features such as the differentiation between self-employed and employer entrepreneurs, uninsurable idiosyncratic risks, and aggregate shocks to productivity and the fixed costs of production for employers. This paper distinguishes between the two types of entrepreneurs by modeling the choice of labor inputs in the production function as the share of time spent on managing versus working.[2] Additionally, I provide an analytical framework for examining the transmission channels through which aggregate shocks and changes in the entrepreneurial composition propagate through the economy. By decomposing the effects into intensive and extensive margins, I shed light on how individual-level decisions and aggregate dynamics interact with each other.

The model is calibrated to match salient features of the Argentinian economy, including income inequalities within and across occupations and business cycle dynamics. Using the quantitative framework, I vary two key parameters that changes the population distribution of workers, self-employed entrepreneurs, and employer entrepreneurs to study how it affects business cycles. First parameter is the fixed cost, which mostly shifts the composition of entrepreneurs while relatively keeping the overall population of entrepreneurs stable. Specifically, increasing the fixed cost discourages employer entrepreneurship relative to self-employed entrepreneurship. I find that this shift in composition reduces aggregate volatilities. These results indicate that self-employment serves as a stabilizing force during economic downturns, providing a buffer for households to mitigate income losses. This finding underscores the importance of supporting self-employment as a coping mechanism during recessions. Second parameter that I experiment with is the parameter that impacts the degree of financial frictions faced by entrepreneurs. I find that loosening financial frictions encourages all forms of entrepreneurship and raise the level of aggregate output in the stationary equilibrium. However, such policies also amplify business cycle fluctuations in emerging economies. This suggests that policies aimed to increase business dynamism could potentially exacerbate economic volatility. In the simulation results, there is a distinct trade-off in the economy between the long-run economy and business cycles. Higher levels of output, consumption, and capital in the stationary equilibrium are almost always accompanied by larger fluctua-

[2]Given the paper's focus on entrepreneurs, the unemployment state is outside the scope of this study. However, this can have important implications for my model. As self-employment is the only absorbing state for individuals, the contribution of self-employed entrepreneurs to aggregate dynamics is likely overstated in my model. Furthermore, I emphasize production heterogeneity through labor inputs as a way to differentiate self-employed and employer entrepreneurs, which means I abstract from fiscal considerations (i.e., informality and tax evasion).

tions of business cycle.

This paper builds upon and contributes to several strands of literature in entrepreneurship, financial frictions, and business cycles. First, the paper contributes to the literature on financial frictions and entrepreneurship. Evans and Jovanovic (1989) and Quadrini (2000) demonstrate that financial constraints can significantly impact entrepreneurial decisions and outcomes. More recently, Buera et al. (2011) and Midrigan and Xu (2014) have explored the macroeconomic implications of financial frictions on entrepreneurship and economic development. Lopez-Martin (2018) studies entrepreneurs with informality. Other papers, such as Allub and Erosa (2019), argue that self-employment significantly distort the role of financial frictions in emerging markets. The baseline model of this paper focuses on the production heterogeneity of entrepreneurs, which builds upon the work of Allub and Erosa (2019).

Next, the paper draws insights from the literature on occupational choice and income inequality. Lucas (1978) develop models of occupational choice between wage work and entrepreneurship based on individual characteristics and market conditions. Cagetti and De Nardi (2006) investigates the role of entrepreneurship in shaping wealth and income inequality. Recent literature, such as Kwark and Ma (2020), have found that entrepreneurs help explain important income inequality dynamics in the United States. This study extends these frameworks by incorporating aggregate shocks in a small open economy setting and exploring the implications of occupational choices for business cycle dynamics.

My paper also contributes to a growing literature on exploring various aspects of heterogeneity in emerging markets. There has been a number of papers on studying distributional impact during sudden stops using heterogeneous-agent models. For example, de Ferra et al. (2019) examines how wealth heterogeneity can have differential impact to a sudden stop in foreign capital flow. Villalvazo (2020) explores a classical Fisherian deflation channel can have significant impact on inequality. Cugat (2019) develops a two-sector Two Agent New Keynesian (TANK) model to explore the distributional impact through sectoral heterogeneity. Liu et al. (2020) study how the capital account policy can affect the income distribution between entrepreneurs and workers.

Finally, my work relates to the literature on business cycles in emerging economies.

Mendoza (1994) and Aguiar and Gopinath (2007) document key features of business cycles in emerging markets, such as the higher volatility of consumption relative to output. Self-employment is often used as one of the proxy to measure the size of the informal sector. There has been a number of papers that study the relationship between informal employment and business cycle dynamics, such as Restrepo-Echavarria (2014), Horvath (2018), and Fernández and Meza (2015). McKiernan (2021) quantitatively study how informal sector affects welfare from pension reforms. Recently, papers such as Hong (2020) have worked to incorporate heterogeneous-agent framework into emerging market business cycle literature. His paper develops a heterogeneous-agent model with several idiosyncratic and aggregate shocks, but with the focus on explaining the consumption volatility puzzle in emerging markets. My paper provides a complementary perspective by studying how occupational heterogeneity can affect aggregate dynamics.

The outline of the paper is as follows: Section 2 presents the empirical evidence on entrepreneur heterogeneity. Section 3 presents a baseline model of occupational choice with heterogeneous agents in a small open-economy setting. Section 4 presents the calibration. Section 5 discusses the model's ability to match the population distribution in the stationary equilibrium and aggregate dynamics. Section 6 concludes.

## 2 Empirical Evidence on Entrepreneur Heterogeneity

In this section, I document the heterogeneity in the entrepreneurial group by highlighting two findings. First, the population shares of entrepreneurs do not uniformly increase with income. Second, the population shares of entrepreneurs are countercyclical. Both of these findings are driven by the fact that self-employed entrepreneurs make up the majority of entrepreneurs and are relatively poorer than the rest of the population. Because of this, I emphasize the need to distinguish households based on income sources to capture distributional and aggregate dynamics in emerging markets.

**Data Description and Summary Statistics**

The microdata utilized in this empirical analysis is derived from the Encuesta Permanente de Hogares (Permanent Household Survey, or EPH), which is conducted by the National

Institute of Statistics and Censuses (INDEC) of Argentina. The reason for using this data is two-fold. First, Argentina has a high share of self-employed entrepreneurs, which is one of the main focus of this paper. Thus, focusing on Argentina gives us an insight into how the differences in the population distribution affect business cycles. Second, the Permanent Household Survey has been available quarterly since 2004 with detailed data on individuals, occupations, and income.[3] Thus, the availability of the data is appealing for studying business cycle dynamics at a relatively higher frequency than yearly, which is the case for many of the other individual-level surveys.

The microdata utilized in this empirical analysis is derived from the Encuesta Permanente de Hogares (Permanent Household Survey, or EPH), which is conducted by the National Institute of Statistics and Censuses (INDEC) of Argentina. The selection of this particular dataset was done for two reasons. First, Argentina is characterized by a significant prevalence of self-employed entrepreneurs, a demographic that constitutes the focal point of this study. Analyzing this dataset provides a nuanced understanding of the influence of population distribution of occupations on business cycle dynamics. Second, the EPH, offering comprehensive data on individual characteristics, occupations, and incomes, has been available on a quarterly basis since 2004, providing a rich, relatively higher frequency dataset that is preferable for analyzing business cycle fluctuations. This level of detail and frequency presents a stark contrast to the annual data compilation common in many individual-level surveys.[4]

This study focuses on the period spanning from the first quarter of 2004 to the second quarter of 2015 to account for the methodological and questionnaire revisions introduced in the EPH post-2015. It is pertinent to note that the EPH data for the third quarter of 2007 is absent due to an EPH staff strike; in instances where this gap impacts the analysis, I linearly interpolate data. The survey reports individual characteristics related to employment, sectors, housing, occupation, and income. In particular, I focus on variables

[3] The survey is also available twice a year from 1998 to 2003. However, due to the differences in survey frequency, I chose not to include the pre-2004 data. However, as a robustness check, the empirical analysis was also done with the pre-2004 sample, and the results were very similar.

[4] While the EPH data extends back to 1998, available biannually till 2003, this study excludes the pre-2004 data due to inconsistencies in survey frequency. However, to ensure robustness, analyses incorporating the pre-2004 data were conducted, yielding similar results.

called *P47T* and *CAT_OCUP*. *P47T* reports an individual's total income as a sum of earned and unearned incomes. *CAT_OCUP* lists an individual's primary occupation as either a self-employed entrepreneur, employer entrepreneur, or an employee. As the model does not feature unemployment and is outside the scope of this paper, I only kept individuals whose total income was positive and who reported themselves as either workers or entrepreneurs.

Finally, the data on aggregate variables such as GDP and consumption come from the International Financial Statistics (IFS) database of the International Monetary Fund (IMF). All of the series are logged and linearly filtered. All results in this paper are robust to using alternative filtering methods.

Table 1: **Summary Statistics**

| | Workers (1) | Entrepreneurs (2)=(3)+(4) | Self-Employed (3) | Employer (4) |
|---|---|---|---|---|
| Income Share | 78 | 22 | 15 | 7 |
| Population Share | 77 | 23 | 19 | 4 |
| Income Share/Population Share | 1.01 | 0.96 | 0.78 | 1.75 |
| Mean | 2465 | 2290 | 1906 | 4109 |
| Median | 1980 | 1584 | 1384 | 3096 |
| P20 | 969 | 686 | 614 | 1544 |
| P80 | 3499 | 3299 | 2749 | 5767 |
| Gini | 0.56 | 0.54 | 0.53 | 0.55 |

**Note**: These are time-averaged statistics from 2004Q1 to 2015Q2. The first two rows are in percentages, P20 and P80 imply 20th and 80th percentiles. Mean, median, P20, and P80 are real monthly income deflated by the GDP deflator, which are in units of Argentine peso.

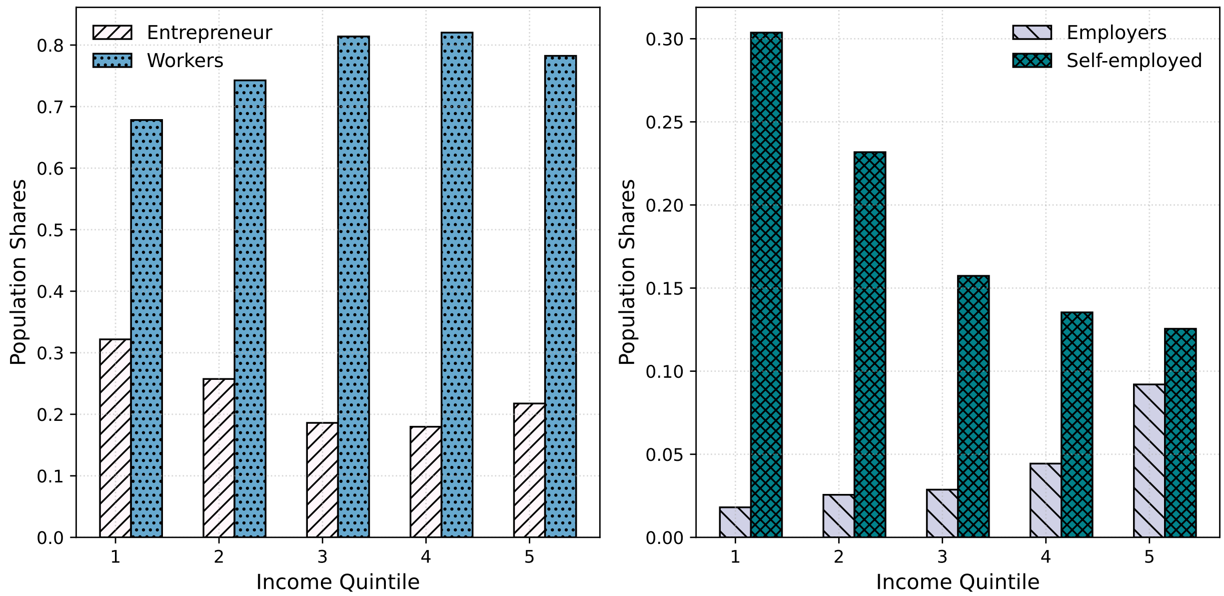
Table 1 presents summary statistics on the income distribution across occupations. The first three rows of the table highlights income and population share disparities of occupations. First, workers constitute a majority of both income (78%) and population share (77%). Entrepreneurs, which include both self-employed and employer entrepreneurs, have a smaller share of the total income (22%) compared to their population share (23%). This might suggest that, on average, entrepreneurs are earning slightly less relative to their proportion in the population. The income share-to-population share measures the level of concentration in each group. This ratio is close to 1 for workers, but slightly less than 1 for entrepreneur. When the entrepreneurial group is further decomposed into self-employed and employers, I find that the lower income share relative to the population share is driven by the self-employed entrepreneurs. Self-employed individuals are the majority of entrepreneurs (19% out of 23% of all entrepreneurs), but only makes up 15% of the total income. On the other hand, employers make up only 4% of the population, but command a significantly higher income share (7%).

The last 4 rows of the table show how the income is distributed within groups. The mean and median incomes show that employers earn substantially more (mean of 4109 pesos and a median of 3096 pesos) than the other groups. The standard entrepreneurship literature focuses on advanced economies and emphasizes the role of rich entrepreneurs in explaining income and wealth inequality. However, in emerging economies such as Argentina, self-employment is prevalent. The self-employed group shows the lowest earnings among the occupations, with a mean income of 1906 pesos and a median income of 1384 pesos. This particular group of entrepreneurs tend to become entrepreneurs not out of economic opportunity, but out of necessity.

The Gini coefficients, a measure of income inequality, are relatively similar across all groups, hovering around 0.53 to 0.56. This suggests that income distribution within each occupational group is somewhat unequal, but the degree of inequality is consistent across different types of employment. Finally, the 20th and 80th percentiles (P20 and P80) provide insights into the lower and upper ends of the income distribution. For instance, the P20 income for employers (1544 pesos) is higher than the median income of self-employed individuals (1384 pesos), highlighting the substantial income disparities within these occupational categories. Next, I move onto highlighting two main empirical facts that motivate the baseline model.

**Fact 1: Population shares of entrepreneurs initially fall as income rises.** Previous literature finds that the share of entrepreneurs increases as the income rises. For example, in Cagetti and De Nardi (2006), they find that entrepreneurs (self-employed and employers) make up 39% of the population in the top 20% of wealth distribution, but in the top 5%, the population share rises to 68% in the US. This directly contrasts with the Argentinian data. The left panel of Figure 1 plots the population distribution of workers and entrepreneurs across each income quintile. The top and the bottom of the income distribution are characterized by relatively higher population shares of entrepreneurs within its income groups. In particular, 30 percent of individuals in the bottom income group are entrepreneurs, which is higher than the share of population in the top income quintile. Workers, on the other hand, show a much more uniform distribution.

Figure 1: **Population Distribution Across Income Quintiles**




**Note**: The left panel depicts the shares of workers and entrepreneurs within each income quintile, while the right panel illustrates the entrepreneurial shares of employers and self-employed individuals within each income quintile. These statistics are time-averaged from 2004Q1 to 2015Q2.

The right panel of Figure 1 distinctly separates entrepreneurs into two categories: employers and self-employed. The proportion of self-employed individuals shows a decreasing trend as income increases. This suggests that self-employment is more prevalent among lower-income groups, potentially indicating necessity-driven entrepreneurship in these segments. In contrast, the share of employers within the population appears to increase with higher income, reflecting that employer entrepreneurship is more common among higher-income groups, possibly indicative of opportunity-driven entrepreneurship.

**Fact 2: Population shares of entrepreneurs are countercyclical in Argentina contrary to the United States.**

While the previous figure focuses on the time-averaged behavior of population distribution across the income distribution, Table 2 presents how the population shares of occupations behave with respect to GDP across time. This presents an intriguing insight into the relationship between GDP and the population shares of entrepreneurs and workers in both the United States and Argentina. In both economies, the population share of workers are highly procyclical with respect to GDP. In the US, the correlation is 0.77 and in Argentina, the correlation is 0.88. The population share of entrepreneurs is slightly procyclical in the US (0.76), but highly countercyclical in Argentina (-0.88).

Table 2: **Population Share and GDP**

| | $\rho(Y, W)$ | $\rho(Y, E)$ | $\rho(Y, Se)$ | $\rho(Y, Em)$ |
|---|---|---|---|---|
| United States | 0.76 | 0.24 | - | - |
| Argentina | 0.88 | -0.88 | -0.93 | 0.66 |

**Note**: $\rho(Y, X)$ denotes correlation of variable X with respect to $Y$, where $Y$ is GDP, $E$ is the population share of entrepreneurs (self-employed and employers combined), $Se$ is the population share of self-employed, $Em$ is the population share of employers. Moments using the U.S. data is taken from Kwark and Ma (2020).

When the entrepreneurial group is decomposed into self-employed and employers, it can be seen that the main driver of negative correlation in Argentina is driven by self-employed entrepreneurs. This indicates that, during economic downturns, individuals turn to self-employed entrepreneurship out of necessity. However, the population share of employer entrepreneurs is highly procyclical. During economic booms, individuals may find it more optimal or less difficult to start their own businesses. The empirical findings highlight the importance of modeling occupational choice with entrepreneur heterogeneity, aggregate dynamics, and its role in shaping business cycles.

# 3 Baseline Model

In this section, I develop a dynamic general equilibrium model of occupational choice in a small open economy. Motivated by the empirical results, the baseline model incorporates two crucial features. First, the model differentiates self-employed and employer entrepreneurs. Second, the model introduces aggregate shocks that drive the dynamics away from its steady state.

Time is discrete, and the horizon is infinite. There is a continuum of infinitely-lived households normalized to a unit measure. At the beginning of each period, households with an asset level ($a$) observe their skills as a worker ($z_w$) and as a manager ($z_m$). Each household is subject to uninsurable idiosyncratic shocks to their skills. In each period, after households observe their individual states, they decide whether to be a worker, self-employed entrepreneur, or an employer entrepreneur. The biggest difference between the two types of entrepreneurship is that while employer entrepreneur hires external labor and self-employed entrepreneurs, labor inputs must come from their own; employers pay a fixed cost denoted by $\kappa$ that are time-varying. Then, households choose their amount of final goods consumption ($c$) and decide how much to invest their assets in a domestic mutual fund with a risk-free return denoted by $r_t^a$. The rest of the model follows a standard small open economy setup. There is a corporate sector that behaves in a perfectly competitive manner and a domestic mutual fund that collects households' savings and invests them in foreign bonds and capital.

## 3.1 Preferences

The household problem extends Lucas (1978) to include self-employed entrepreneurs. Following the formulation in Allub and Erosa (2019), this setting allows the heterogeneity of entrepreneurs in the household problem to be cast as a time allocation problem. It also replicates key income inequality statistics within and across occupations in the stationary equilibrium. This makes their model a great starting point to study population dynamics away from the steady state. Households derive utility from the consumption of a final good, denoted by $c_t$. Individuals maximize the expected utility of the form

$$U(c_t) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma} - 1}{1-\gamma}\right], \tag{1}$$

where $\beta$ is the discount factor between 0 and 1, and $\gamma$ denotes the parameter that relates to the household's risk aversion.

## 3.2 Evolution of Idiosyncratic Skills

At time $t$, a household's skills as a worker and an entrepreneur are determined by their labor productivity $z_{wt}$ and managerial/entrepreneurial[5] abilities $z_{mt}$. Their skills are a sum of persistent and permanent components in logs. Specifically,

$$\log z_{wt} = \alpha_{wt} + \theta_{wt}, \tag{2}$$

$$\log z_{mt} = \alpha_{mt} + \theta_{mt} + \mu_m, \tag{3}$$

where $\alpha_{wt}$ and $\alpha_{mt}$ denote fixed effects of working and entrepreneurial abilities, $\theta_{wt}$ and $\theta_{mt}$ denote persistent components. $\mu_m$ is a parameter that shifts the mean of managerial abilities that is not time-varying. Fixed effects are denoted with a $t$ subscript because they are re-drawn from a fixed bivariate distribution $F(\alpha_w, \alpha_m)$ with a small probability $p_\alpha$. Fixed effect of managerial abilities $\alpha_{mt}$ are assumed to be distributed Pareto while the fixed effect of working is assumed to be distributed normal conditional on the values of $\alpha_{mt}$. The persistent components of idiosyncratic skills are assumed to follow AR(1) process in logs.

$$\theta_{wt+1} = \rho_w \theta_{wt} + \epsilon_{wt+1}, \tag{4}$$

$$\theta_{mt+1} = \rho_m \theta_{mt} + \epsilon_{mt+1}, \tag{5}$$

where $\rho_w$ and $\rho_m$ are persistence parameters. The vector of innovation terms, denoted by $\epsilon_t = (\epsilon_{wt}, \epsilon_{mt})$, are assumed to be drawn from a joint normal distribution with standard deviations represented by $\sigma_w$, $\sigma_m$, and the correlation coefficient $\rho_{\epsilon_{w,m}}$.

## 3.3 Technology

Production takes place in two different types of sectors: corporate and entrepreneurial. The corporate sector behaves in a perfectly competitive manner. Given the market wage and rental rate, the corporate firm maximizes profit by choosing how much labor to hire and capital to rent. The entrepreneurial sector also takes market wages and rental rates as given. However, departing from the previous literature, there are two types of

[5] I use managerial skills and entrepreneurial abilities interchangeably throughout the rest of the text.

entrepreneurs. Entrepreneurs split their time between working and managing. If entrepreneurs' labor inputs do not hire additional labor outside their own, they are considered self-employed entrepreneurs. On the other hand, if entrepreneurs hire external labor to operate their business, they are employers.

**Corporate Production** The corporate sector produces final goods that can either be used for consumption or investment using a constant-returns-to-scale Cobb-Douglas production technology:

$$Y_t^c(K_t^c, L_t^c, Z_t) = Z_t K_t^{c,\alpha} L_t^{c,1-\alpha}, \tag{6}$$

where $K_t^c$ and $L_t^c$ denote capital and labor used in the corporate production. $\alpha$ is the share of capital used and is bounded between 0 and 1. $Z_t$ is the aggregate productivity process that follows AR(1) in logs:

$$\log Z_{t+1} = (1-\rho_z)\log \bar{Z} + \rho_z \log Z_t + \varepsilon_{z,t+1}. \tag{7}$$

Here, $\bar{Z}$ is the level of aggregate productivity in stationary equilibrium, $\rho_z$ is the persistence, and $\varepsilon_{zt}$ is the innovation term that follows a normal distribution with a standard deviation $\sigma_z$.

**Entrepreneurial Production** Following Lucas (1978) and Allub and Erosa (2019), entrepreneurs' production uses three inputs: managerial ability, labor, and capital. There are no markets for managerial ability, so entrepreneurs must only use their own manager skills as input. The total effective managerial input is determined by the level of managerial skill $z_m$ and time devoted to managing their businesses $t_m$. The production function of an entrepreneur can be written as follows:

$$Y(m_t, k_t, n_t; Z_t) = Z_t m_t^{\nu} k_t^{\xi} n_t^{\eta}, \tag{8}$$

where $\nu + \xi + \eta = 1$ and $m_t = t_{mt} z_{mt}$ represents effective managerial supply. Likewise, $k_t$ represents capital inputs for an entrepreneur, and $n_t$ denotes effective units of labor input. The time allocation decision is used to differentiate two types of entrepreneurship. $t_m$ is the share of time an entrepreneur spends on managerial activities, which is assumed to be bounded between 0 and 1. When $t_m$ is equal to 0, then the household uses all their time to work. When $t_m$ is equal to 1, then the household fully devotes its time to managing. If $t_m$ lies between 0 and 1, then entrepreneurs supply managerial and labor inputs to their businesses. An entrepreneur's own labor inputs are written as $(1 - t_m)z_w$. The total effective labor inputs $n_t$ can be written as $n_t = (1 - t_{mt})z_{wt} + n_{dt}$, which is the sum of labor supplied by entrepreneurs and external labor hired. Finally, both types of entrepreneurs face a collateral constraint of the following form.

$$k_t \leq \phi_a a_t, \tag{9}$$

where $\phi_a \geq 1$. This functional form follows the previous literature, such as Buera et al. (2015), and the parameter $\phi_a$ captures the degree of financial friction in the economy. This parameter is assumed to be constant across time and occupations, meaning that self-employed and employer entrepreneurs face the same degree of financial friction. Later, I drop this assumption and study how this parameter affects aggregate and population dynamics. When the capital market is perfect, $\phi_a = \infty$ and when an entrepreneur has no access to credit, $\phi_a = 1$. Given the market wage and rental rates $(W_t, U_t)$, the profit maximization problem for a self-employed entrepreneur with idiosyncratic state variables $(a_t, z_{wt}, z_{mt})$ can be written as

$$\pi^{se}(a_t, z_{wt}, z_{mt}; Z_t) = \max_{t_{mt}, k_t} Z_t(z_{mt}t_{mt})^{\nu} k_t^{\xi}((1 - t_{mt})z_{wt})^{\eta} - U_t k_t, \tag{10}$$

subject to (9). For a self-employed entrepreneur, their only choice is how much time they spend on managerial activities and the amount of capital input. For an employer entrepreneur, their profit maximization problem can be written as

$$\pi^{em}(a_t, z_{wt}, z_{mt}; Z_t, \kappa_t) = \max_{t_{mt}, k_t, n_{dt}} Z_t(z_{mt}t_{mt})^{\nu} k_t^{\xi}((1 - t_{mt})z_{wt} + n_{dt})^{\eta} - U_t k_t - W_t n_{dt} - \kappa_t, \tag{11}$$

subject to (9). Employer entrepreneurs must not only choose capital and time spent on managing, but they also decide how many workers to hire. Finally, $\kappa_t$ denotes the fixed cost of production for an employer entrepreneur that varies over time. Like the aggregate productivity shock, the fixed cost shock follows AR(1) in logs.

$$\log \kappa_{t+1} = (1-\rho_\kappa)\log \bar{\kappa} + \rho_\kappa \log \kappa_t + \varepsilon_{\kappa,t+1}. \tag{12}$$

$\bar{\kappa}$ represents the barrier to entry for employers in stationary equilibrium, and $\rho_\kappa$ is the persistence of the second aggregate shock. Finally, $\varepsilon_{\kappa,t}$ is the innovation term that follows a normal distribution with a standard deviation $\sigma_\kappa$. I also assume that the innovation terms of the aggregate productivity and the fixed cost are correlated, denoted by the correlation parameter $\rho_{Z,\kappa}$.

## 3.4 Occupational Choice

Household's budget constraint can be written as:

$$c_t + a_{t+1} = \max_{o_t}\{W_t z_{wt}, \pi_t^{se}, \pi_t^{em}\} + (1+r_t^a)a_t, \quad a_{t+1} \geq 0, \tag{13}$$

where $r_t^a$ is the risk-free rate of return of liquid assets. The max operator in the budget constraint denotes the choice of occupation of households, $o_t$. The constraint $a_{t+1} \geq 0$ implies that households are not able to borrow in this economy. Let $s_t = (a_t, z_{wt}, z_{mt})$ be the tuple of idiosyncratic variables and $S_t = (\Lambda_t, Z_t, \kappa_t)$ be the tuple of economy-wide or aggregate state variables where $\Lambda_t$ denotes the joint distribution of assets and idiosyncratic skills at time $t$. The value function for the household problem is

$$\begin{aligned} v(s_t, S_t) &= \max_{a_{t+1}, c_t} \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \beta \mathbb{E}[v(s_{t+1}, S_{t+1}) | s_t, S_t] \right\}, \\ \text{s.t. } c_t &= \max_{o_t}\{W_t z_{wt}, \pi_t^{se}, \pi_t^{em}\} + (1+r_t^a)a_t - a_{t+1}, \\ a_{t+1} &\geq 0, \\ \Lambda_{t+1} &= \Gamma(\Lambda_t), \end{aligned} \tag{14}$$

where $\Gamma(.)$ is an equilibrium object that specifies the evolution of the distribution $\Lambda_t$.

## 3.5 Domestic Mutual Fund

The structure of the domestic mutual fund follows existing heterogeneous-agent literature such as Hagedorn et al. (2020) and Auclert et al. (2021). The mutual fund collects households savings $A_{t+1} = \int a_{it} d\Lambda_{t+1}$, pays a real return $\tilde{r}^a_t$. They invest savings into capital $K_{t+1}$ and additionally borrow (or save) into foreign bonds $B^*_{t+1}$. If $B^*_{t+1}$ is negative (positive), then the mutual fund borrows (saves). Their optimization problem can be written as

$$V^{MF}(K_t) = \max_{K_{t+1}, B^*_{t+1}} (1 + U_{t+1} - \delta)K_{t+1} + (1 + r^*_t)B^*_{t+1} - (1 + \tilde{r}^a_t)A_{t+1} + \mathbb{E}_t \left[ \frac{V^{MF}(K_{t+1})}{1 + \tilde{r}^a_{t+1}} \right], \tag{15}$$

$$\text{s.t.} \quad A_{t+1} = K_{t+1} + B^*_{t+1} + \frac{\phi_k}{2}(\frac{K_{t+1}}{K_t} - 1)^2 K_t, \tag{16}$$

where they take prices and capital stock $K_t$ as given. $r^*_t$ denotes the interest rate on foreign bonds, $\delta$ is the depreciation rate of capital, and $\phi_k$ is the adjustment cost of accumulating capital. The equilibrium first-order conditions are

$$r^*_t = \tilde{r}^a_t, \tag{17}$$

$$(1 + \tilde{r}^a_t)(1 + \phi_k(\frac{K_{t+1}}{K_t} - 1)) = \mathbb{E}_t \left[ 1 + U_{t+1} - \delta + \phi_k \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} - \frac{\phi_k}{2} \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 \right], \tag{18}$$

$$A_{t+1} = K_{t+1} + B^*_{t+1} + \frac{\phi_k}{2}(\frac{K_{t+1}}{K_t} - 1)^2 K_t. \tag{19}$$

The total dividend of the mutual fund can be expressed as

$$D^{MF}_{t+1} = (1 + U_{t+1} - \delta)K_{t+1} + (1 + r^*_t)B_{t+1} - (1 + \tilde{r}^a_t)A_{t+1}, \tag{20}$$

and the dividend per unit of investment can be written as $d^{MF}_{t+1} = D^{MF}_{t+1}/A_{t+1}$. Households receive the total dividends of investment such that the households' real return equals

$$1 + r^a_t = 1 + \tilde{r}^a_t + d^{MF}_{t+1}. \tag{21}$$

Finally, the foreign interest rate is assumed to be debt-elastic following Schmitt-Grohé and Uribe (2003).

$$r_t^* = r_{ss}^* + \omega_r(\exp^{-B_{t+1}^*+\bar{B}^*} - 1), \tag{22}$$

where $\omega_r$ is the parameter that determines the elasticity of debt to interest rate. This value is set close to 0 to have a minimal role in equilibrium dynamics.

### 3.6 Market Clearing and Recursive Competitive Equilibrium

In equilibrium, the labor demanded by the corporate sector and employer entrepreneurs must equal the labor supplied by workers. Similarly, the amount of capital demanded by the corporate sector and entrepreneurs must equal the capital supplied by the mutual fund. The total amount of assets households provide must equal the demand for them by the mutual fund.

$$\int z_{wt} d\Lambda_{w,t} = L_t^c + \int n_{dt} d\Lambda_{em,t}, \tag{23}$$

$$K_t = \int k_t d\Lambda_{se,em,t} + K_t^c, \tag{24}$$

$$A_{t+1} - K_{t+1} = B_{t+1}^* + \frac{\phi_k}{2}(\frac{K_{t+1}}{K_t} - 1)^2 K_t), \tag{25}$$

where $d\Lambda_{j,t}$ is the distribution over households of occupation $j$, where $j \in \{w, se, em\}$. The above market clearing conditions lead to the final goods market clearing condition, which can be stated as follows:

$$C_t + I_t + B_{t+1}^* = (1 + r_t^*)B_t^* + Y_t + \frac{\phi_k}{2}(\frac{K_{t+1}}{K_t} - 1)^2 K_t + \kappa_t \int d\Lambda_{em,t}, \tag{26}$$

where the aggregate investment $I_t$ is defined as $K_{t+1} - (1 - \delta)K_t$. The last term is the deadweight loss that stems from the fixed cost of production for households that choose to be employer entrepreneurs. From the above condition, one can verify that the current account identity holds.

$$TB_t = Y_t - C_t - I_t + \frac{\phi_k}{2}(\frac{K_{t+1}}{K_t} - 1)^2 K_t + \kappa_t \int d\Lambda_{em,t}, \tag{27}$$

$$CA_t = TB_t - r_t^* B_t^*. \tag{28}$$

A *recursive competitive equilibrium* for this economy is given by a set of pricing functions $\{W(S), U(S), r^*(S), r^a(S), \tilde{r}^a(S)\}$, corporate capital and labor decisions $\{K^c(S), L^c(S)\}$, mutual fund decisions $\{K(S), B^*(S)\}$, households' policy functions $\{c(s,S), a'(s,S), o(s,S), t_m(s,S), n_d(s,S), k(s,S)\}$, value functions $v(s,S)$, and law of motion for the distribution $\Gamma(.)$ such that the following conditions hold:

1. Households maximize their value functions by choosing policy functions and their choices of occupations given prices;

2. Corporate firms maximize profit by choosing capital and labor given prices;

3. Mutual fund maximizes profits taking prices as given;

4. Labor market, capital market, and goods market clear;

5. Aggregate law of motion is generated by savings decisions by households.

The baseline model consists of heterogeneity at the individual level and two aggregate shocks at the macro level. A standard computational method to solve these types of models is Krusell and Smith (1998), which approximates the aggregate law of motion by simulations. However, the simulation-based method takes a long time and can quickly become infeasible once multiple shocks are introduced. Therefore, Reiter (2009) recommends solving heterogeneous-agent models with aggregate shocks by solving for the stationary equilibrium using nonlinear methods and only linearizing the equilibrium dynamics with respect to aggregate shocks. Papers such as Bayer and Luetticke (2018) and Terry (2017) find that this method provides the best balance in terms of accuracy and speed. Thus, I solve the household problem by iterating on the value function and linearizing the equilibrium dynamics only with respect to aggregate shocks given the paper's focus on business cycle properties.

# 4 Calibration

The goal of the calibration is to target key occupation-level statistics as well as aggregate business cycle moments. First, I externally calibrate a subset of parameters that is consistent with the literature. Second, I internally calibrate the rest of the parameters to match several key moments in the data. All moments in the stationary equilibrium are time-averaged from 2004Q1 to 2015Q4 unless noted otherwise. All business cycle moments are calculated using quarterly data. As noted above, the EPH in Argentina is used to calibrate micro moments in the baseline model while IFS is used to calibrate business cycle moments.

**External Calibration** Table 3 presents values that are externally calibrated. First, the risk aversion parameter in the utility function is set to a value of 2, which is a standard value used in the literature. Next, I turn to calibration of idiosyncratic shocks, which resembles Allub and Erosa (2019). There are 5 parameters that relate to the fixed effects of shocks. They are the tail parameter of managerial skills ($\eta_m$), the variance of fixed effects of workers ($\sigma_{\alpha_w}$), the correlation between fixed effects of working and managerial skills ($\rho_{\alpha_{w,m}}$), the probability of drawing new fixed effects ($p_\alpha$), and the mean shift parameter of managerial skills ($\mu_m$). The variance of fixed effects of workers is externally calibrated, which is set to 0.55, while the rest are internally calibrated. Five parameters characterize the persistent component of idiosyncratic shocks. They are persistence of skills ($\rho_w, \rho_m$), standard deviations ($\sigma_w, \sigma_m$), and the correlation between working and managing ($\rho_{\epsilon_w,m}$). The variance of log wages grows approximately linearly with age, implying a high autocorrelation of wage shocks. Thus, the persistence of worker's idiosyncratic is set to 0.96.

Turning to the production, I set the capital share of the corporate production, $\alpha$, to 0.33, following Schmitt-Grohé and Uribe (2003). The manager, capital, and labor shares in the entrepreneur production is set to 0.2, 0.33, and 0.47, respectively, based on the parameters outlined by Guner et al. (2008). The elasticity of interest rate to debt is set to 0.0001 so that it plays a minimal role in equilibrium dynamics. The quarterly capital depreciation rate is set to 0.025, which is a standard value in the literature. Finally, the correlation between fixed cost and productivity shocks are assumed to be negatively correlated and set to a

Table 3: **Externally Calibrated Parameters**

| Parameter | Description | Value |
|---|---|---|
| $\gamma$ | Risk aversion | 2 |
| $\rho_w$ | Persistence of worker productivity | 0.96 |
| $\sigma_{\alpha_w}$ | Standard deviation of worker productivity | 0.55 |
| $\alpha$ | Capital share in corporate production | 0.33 |
| $\nu$ | Managerial share in entrepreneur production | 0.20 |
| $\xi$ | Capital share in entrepreneur production | 0.33 |
| $\eta$ | Labor share in entrepreneur production | 0.47 |
| $\omega^r$ | Elasticity of interest rate to debt | 0.0001 |
| $\delta$ | Capital depreciation rate | 0.025 |
| $\rho_{Z,\kappa}$ | Correlation between productivity and fixed cost shock | -0.99 |

baseline value of -0.99. Later, I study the aggregate implications of this value. I find that a high negative correlation is necessary to get the procyclicality of the population share of employers with respect to GDP.

**Internal Calibration** I choose 11 parameters in Table 4 to match 15 empirical moments in the data. It is beneficial to identify which parameters have the greatest impact on specific empirical moments. The discount factor, $\beta$, is set to match the time-averaged capital-to-output ratio. The collateral constraint, $\phi_a$, is set to match the external debt-to-GDP ratio in the steady state. The standard deviation of the innovation term of persistent working skills, $\sigma_w$, and the tail parameter of entrepreneurial fixed effects, $\eta_m$, are used to match the variance of log income of workers and entrepreneurs. The shift parameter of entrepreneurial fixed effects, $\mu_m$, and the fixed cost of production for employers in the steady state, $\bar{\kappa}$, are set to match the population shares of self-employed and employer entrepreneurs. The standard deviation of the innovation term and the serial correlation of the persistent component of managerial skills, $\sigma_m$ and $\rho_m$, the correlation to the innovation, $\rho_{\epsilon_{m,w}}$, and the probability of drawing a new fixed effects, $p_\alpha$, are set to match the quarterly transition probabilities of workers, self-employed entrepreneurs, and employers. The correlation of fixed effects, $\rho_{\alpha_{m,w}}$, is set to match the average income of workers over the average income of employers, the average income of self-employed over the average income of workers, and the average income of self-employed over the average income of employers. Next, there are 3 parameters chosen to match 3 moments from aggregate dynamics. The adjustment cost of capital, $\phi_k$, is chosen to match the relative

Table 4: **Internally Calibrated Parameters**

| Parameter | Description | Value |
|---|---|---|
| **Stationary Equilibrium** | | |
| $\beta$ | Discount factor | 0.85 |
| $\phi_a$ | Collateral constraint | 1.25 |
| $\sigma_w$ | Standard deviation of working abilities | 0.07 |
| $\eta_m$ | Tail parameter of managerial fixed effects | 6.49 |
| $\mu_m$ | Shift parameter of managerial fixed effects | -1.85 |
| $\bar{\kappa}$ | Steady state fixed cost for employers | 0.91 |
| $\sigma_m$ | Standard deviation of managerial abilities | 0.79 |
| $\rho_m$ | Serial correlation of managerial abilities | 0.90 |
| $\rho_{\epsilon_{w,m}}$ | Correlation between working and managing | -0.96 |
| $\rho_{\alpha_{w,m}}$ | Correlation of fixed effects between working and managing | 0.11 |
| $p_\alpha$ | Probability of a new fixed effects | .03 |
| | | |
| **Aggregate Parameters** | | |
| $\phi_k$ | Adjustment cost of capital | 3.75 |
| $\rho_Z$ | Persistence of TFP shock | 0.77 |
| $\sigma_Z$ | Standard deviation of TFP shock | 0.03 |
| $\rho_\kappa$ | Persistence of fixed cost shock | 0.64 |
| $\sigma_\kappa$ | Standard deviation of fixed cost shock | 0.02 |

**Note**: This table lists parameter values that are chosen to match the moments in Table 5.

volatility of investment to output. The persistence and standard deviation of TFP shock, $\rho_z$ and $\sigma_z$, are chosen to match the serial correlation and standard deviation of aggregate output.

Table 5 shows the results of the internally calibrated parameters. The upper panel compares moments in the data and the model in the stationary equilibrium, whereas the latter part of the table compares business cycle moments in the data and the model. Overall, the model does well in capturing various aggregate statistics. For example, the model is able to capture the capital-to-output ratio (2.54 in the data, 2.56 in the model) and the external debt-to-GDP ratio (19 percent in the data and the model).

The model also does fairly well in capturing inequalities within and between occupations. Looking within the occupations, the model captures the variance of log income of workers and entrepreneurs (0.72 and 0.99 in the data, 0.79 and 0.96 in the model). This is consistent with the empirical finding that the income distribution of entrepreneurs tend to be more unequal compared to the income distribution of workers. Looking across occupations, the model matches the data both in terms of the population shares and the average income differences between occupations. In the data, the majority of the population are workers (76.50%), and the majority of entrepreneurs are self-employed (19.40%). This is also the case in the model, where 77.40% of the population are workers, followed by 19.70% of self-employed entrepreneurs. Looking at relative income differences across occupations, The model captures the average income of workers to the average income of self-employed very well (0.80 in the data, 0.83 in the model). Finally, the model does a good job of matching the average income rankings (employers $>$ workers $>$ self-employed) although the model seems to overstate the average income of employers relative to the other groups. The model also does well in capturing the transitions between occupations in the steady state, such as high persistence of being a worker, lower persistence of being an entrepreneur, and the exit rates from both forms of entrepreneurships. Lastly, there are five parameters that do not influence the stationary equilibrium and are used to only match five aggregate business cycle moments. They are relative volatility of investment to output, persistence of output, volatility of output, persistence of population share of employers, and volatility of employer shares, which are matched exactly.

To externally validate the model, I compare several key moments in the model with the data. Table 6 presents the results. In the stationary equilibrium, I compare the distribution of self-employed to employers at the top and bottom income quintiles. The model does particularly well capturing the fact that a significant part of the employer entrepreneurs reside in the top of the income distribution. Particularly, in the top income quintile, the share of employer entrepreneurs is 1.84% in the data, and 2.51% in the model. The share of self-employed entrepreneurs is 2.51% in the data, and 2.88% in the model. In contrast, according to the data, 6.07% of the population are self-employed in the bottom income quintile, whereas it is 4.58% in the model. For employers, it's 0.36% in the data and 0.20% in the model. The model also does well in capturing the overall income inequality, as measured by the Gini coefficient (0.56 in the data vs. 0.52 in the model). Outside the stationary equilibrium, I focus on the cyclicality of occupations with respect to output. Across all occupations, the model does well in capturing these moments. In particular,

Table 5: **Calibration Results**

| Moment | Data | Model |
|---|---|---|
| **Stationary Equilibrium** | | |
| Capital-output ratio | 2.54 | 2.56 |
| External debt-output ratio | -0.19 | -0.19 |
| Population share of workers | 76.50 | 77.33 |
| Population share of self-employed | 19.40 | 19.53 |
| Population share of employers | 4.10 | 3.14 |
| Average income of X to the average income of Y | | |
| Workers/employers | 0.60 | 0.36 |
| Self-employed/workers | 0.80 | 0.83 |
| Self-employed/employers | 0.46 | 0.28 |
| Variance of log income of workers | 0.72 | 0.79 |
| Variance of log income of entrepreneurs | 0.99 | 0.96 |
| Occupation transition | | |
| Worker to worker | 0.93 | 0.93 |
| Worker to self-employed | 0.06 | 0.07 |
| Self-employed to self-employed | 0.70 | 0.73 |
| Self-employed to workers | 0.20 | 0.22 |
| Employers to employers | 0.62 | 0.75 |
| Employers to workers | 0.10 | 0.10 |
| **Aggregate Dynamics** | | |
| Volatility of investment to output | 2.40 | 2.40 |
| Serial correlation of output | 0.83 | 0.83 |
| Standard deviation of output | 5.10 | 5.10 |
| Serial correlation of employer shares | 0.71 | 0.71 |
| Standard deviation of employer shares | 0.04 | 0.04 |

the model captures the procyclicality of workers (0.86 in the model vs. 0.74 in the data) and employers (0.67 in the model vs. 0.69 in the data) as well as the countercyclicality of self-employed entrepreneurs (-0.88 in the model vs. -0.79 in the data).

# 5 Aggregate Dynamics of Occupational Heterogeneity

Now that the model is calibrated to match the occupational structure and business cycle dynamics in Argentina, this section performs several simulations to study its properties. First, I study the occupational choice of households in stationary equilibrium. Next, I analytically study how aggregate shocks or changes to parameters can influence aggregate

Table 6: **Untargeted Moments**

| Moment | Data | Model |
|---|---|---|
| **Stationary Equilibrium** | | |
| First income quintile | | |
| Share of self-employed (% of total) | 6.07 | 4.58 |
| Share of employers (% of total) | 0.36 | 0.20 |
| Fifth income quintile | | |
| Share of self-employed (% of total) | 2.51 | 2.88 |
| Share of employers (% of total) | 1.84 | 2.51 |
| Income Gini | 0.56 | 0.52 |
| | | |
| **Aggregate Dynamics** | | |
| Correlation between X and aggregate output | | |
| Share of workers | 0.74 | 0.86 |
| Share of self-employed | -0.79 | -0.88 |
| Share of employers | 0.69 | 0.66 |

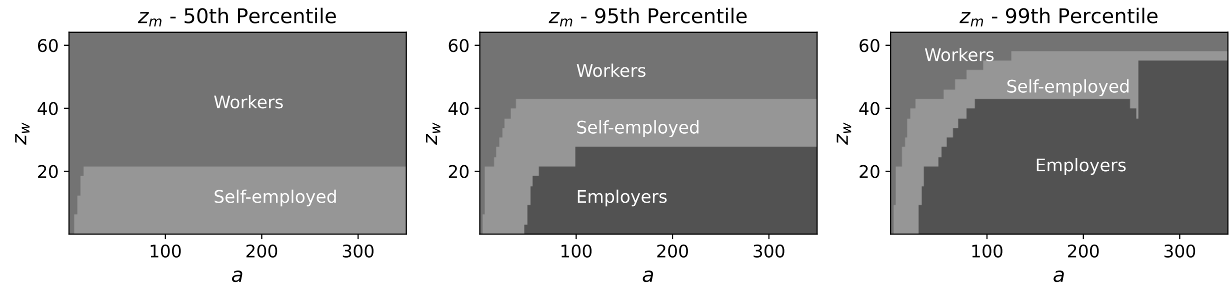
dynamics. Afterwards, I conduct an impulse response analysis to study how aggregate shocks can be transmitted through the economy and affect the aggregate dynamics as well as population shares across occupations of households. Next, I study the business cycle properties implied by the model and find that the baseline model does well in capturing the cyclicality of entrepreneurs with respect to output. Finally, I vary several key parameters that influence the occupational structure in the steady state and ask how these affect business cycles as a counterfactual exercise.

## 5.1 Occupational Choice in Stationary Equilibrium

This section delves into how the wealth and productivity levels of households influence their occupational decisions, as depicted in Figure 2. Each plot within the figure is aligned with distinct levels of managerial productivity, mapping household wealth on the x-axis against labor skills on the y-axis.

We find that the choice set to become employers increases with wealth and managerial abilities. The distribution of individuals choosing to become employers exhibits a pronounced rightward skew, characterized by an absence of the choice to be an employer at the 50th percentile and a substantial increase in the choice at both the 95th and 99th

Figure 2: Occupational Choice




percentiles. Due to the sizable fixed cost of operating as an employer entrepreneur, only the most productive and wealthy entrepreneurs choose to be employers. Also, the proportion of self-employed within entrepreneurs declines as managerial productivity rises, and entrepreneurs with higher working ability are more likely to become self-employed rather than employers. For self-employed entrepreneurs, the only source of labor input is their own labor supply. Thus, the region of the decision to be self-employed lies above the region of choosing to be an employer. Finally, as is common in settings with imperfect capital markets, the choice of occupation depends not only on relative skills but also on the amount of wealth that the individual has accumulated.

## 5.2 From (to) Micro to (from) Macro

It is informative to see how the model's macro and micro variables interact with one another before presenting the quantitative results. To do this, I ask two questions. First, how do aggregate shocks affect macroeconomic variables? Second, how do changes to the population distribution at the steady state affect business cycle dynamics?

To answer these questions, I first examine the output response of entrepreneurs to a sudden change in the macroeconomy (for example, a sudden rise in aggregate productivity, $Z_t$). Throughout this section, all variables are denoted with time subscripts to represent that variables are changing in response to aggregate shocks over time. Furthermore, individual-level variables are written as a function of the idiosyncratic states, denoted by $s$. Thus, the level of output of a particular entrepreneur with an idiosyncratic variable $s$ at time $t$ is denoted as $y_t(s)$, the log deviation from its steady state value is denoted by $\hat{y}(s)_t$, and $\bar{y}(s)$ represents the value in the stationary equilibrium.

The log-linearized output of a particular entrepreneur, whose idiosyncratic state is $s$, can be written as

$$\hat{y}_t(s) = \hat{Z}_t + \nu \hat{t}_{mt}(s) + \xi \hat{k}_t(s) + \nu \hat{n}_t(s), \tag{29}$$

$$\hat{n}_t(s) = \begin{cases} \hat{t}_{mt}(s), & \text{if } o = se, \\ -\theta_{t_m(s)} \hat{t}_{mt}(s) + \theta_{n_d} \hat{n}_{dt}(s), & \text{if } o = em, \end{cases} \tag{30}$$

where $\theta t_m(s)$ is the share of time spent on managing over total labor inputs, $\frac{t_m(s)}{n(s)}$, and $\theta_{n_d(s)}$ is the share of external labor over total labor inputs, $\frac{n_d(s)}{n(s)}$. Idiosyncratic shocks, $z_w$ or $z_m$, do not appear in the previous equations because the model is linearized only with respect to aggregate shocks, which are uncorrelated with the idiosyncratic productivity process. The largest difference in output responses between the two types of entrepreneurship will depend on how labor (both the share of time spent on managing and the external labor demand for employers) adjusts in response to aggregate shocks.

One can show that the time spent on managing for a self-employed entrepreneur is constant in stationary equilibrium, which implies that $\hat{t}_{mt}(s) = 0$.[6] In the aggregate, when prices are changing, a large part of the relative output change between self-employed and employer entrepreneurs will depend on whether the labor inputs of employers amplify or dampen output from the general equilibrium feedback.

Additionally, the total output produced by each type of entrepreneur in the economy can be expressed as

$$Y_t^j = \int y_t^j(s) d\Lambda_{j,t}(s), \qquad j \in \{se, em\}. \tag{31}$$

The above equation, after log-linearization at each $s$, can be decomposed into the follow-

[6]This can be derived by taking the first-order conditions of a self-employed entrepreneur's profit function with respect to $t_m$. This is done in the appendix.

ing.

$$\hat{Y}_t^j = \frac{\int \hat{y}_t^j(s)\bar{y}^j(s)d\bar{\Lambda}_j(s)}{\bar{Y}^j} + \frac{\int \bar{y}^j(s)d\bar{\Lambda}_j(s)d\hat{\Lambda}_{j,t}(s)}{\bar{Y}^j}, \tag{32}$$

where $\bar{Y}^j = \int \bar{y}_t^j d\bar{\Lambda} j, t$ is the total value of output by an entrepreneur of type $j$ in the stationary equilibrium. Equation (32) states that the percent change in the total output of a self-employed or employer entrepreneur is the sum of the percent change in the individual-level output (intensive margin) and the percent change in the wealth distribution (extensive margin).

Finally, the aggregate output of the economy can be written as follows:

$$Y_t^{agg} = Y_t^c + Y_t^{se} + Y_t^{em}, \tag{33}$$

where $Y_t^c$ is the total output of the corporate sector, $Y_t^{se}$ is the total output produced by self-employed entrepreneurs, and $Y_t^{em}$ is the total output produced by employer entrepreneurs. After log-linearization, eq. (33) can be written as a function of each term's steady state shares multiplied by the percentage deviations:

$$\hat{Y}_t^{agg} = \theta_c \hat{Y}_t^c + \theta_{se}\hat{Y}_t^{se} + \theta_{em}\hat{Y}_t^{em}, \tag{34}$$

where $\theta_j = \frac{Y_t^j}{Y_t^{agg}}$ is the steady state share of total output of component $i$, and $\hat{y}_t$ represents the log deviation of $y$ from its steady state value.

Equations (29), (32), and (34) provide separate pieces to answer the questions posed earlier in this section. Suppose one wants to examine the impact of a productivity shock on aggregate output. First, according to equation (29), there will be a microeconomic effect on entrepreneurs at each point of the state space, which will differ depending on whether an entrepreneur is self-employed or an employer. Second, the total change in group $j$'s total income depends on the individual's output changes as well as the endogenous changes in the wealth distribution. Lastly, the aggregate output will depend on the share-weighted sum of the percentage changes across different components of output.

Suppose there is a decline in the fixed cost of production, $\kappa$, due to institutional changes. Changes to the parameter values affect the population distribution even in the stationary equilibrium, which impacts steady state shares, $\theta_j$, in equation (34). The decline in the fixed cost will increase (decrease) the income share of employer (self-employed) entrepreneurs, and employer (self-employed) entrepreneurs will have a higher (lower) contribution to the change in aggregate output.

In summary, this section provides an analytical framework for understanding how aggregate shocks and changes in the stationary equilibrium propagate through the economy due to heterogeneous responses of entrepreneurs.

### 5.3 Transmission Mechanisms of Aggregate Shocks

To examine the transmission mechanisms of the baseline model, I study how aggregate variables and population dynamics behave in response to a one percent rise in aggregate productivity and the fixed cost of an employer entrepreneur. Figure 3 and Figure 4 plot the results of the simulation. Prices and other main aggregate variables of interest are shown in panel (a), and population dynamics are shown in panel (b). Output, consumption, investment, wage, and rental rate are expressed as percentage deviations from their steady state values, while trade balance-to-GDP and population shares across occupations are expressed as percentage point deviations from their steady state values.

Turning first to Figure 3, responses of main aggregates are mostly consistent with the representative agent model. Quantitatively, a one percent rise in aggregate productivity leads to a 0.9 percent rise in aggregate output on impact, while consumption and investment rise by 0.4 percent and 2.5 percent, respectively. The trade balance-to-GDP experiences a moderate rise on impact by 0.4 percent but falls below 0 after 4 quarters following the shock. As the trade balance and the current account are closely linked by equation 28, one way to interpret the response is through the equilibrium conditions of the capital market. The amount of foreign borrowing, denoted by $B_t^*$, is determined by the difference between domestic savings, $A_t$, and capital, $K_t$. A productivity boom induces the corporate firm and entrepreneurs to accumulate more capital. Not only that, but it also leads to a rise in prices, both interest rates and wages, which also lead to more savings by households. However, the rise in household savings is larger than the capital market, which leads to a current account surplus and the improvement in the trade balance.

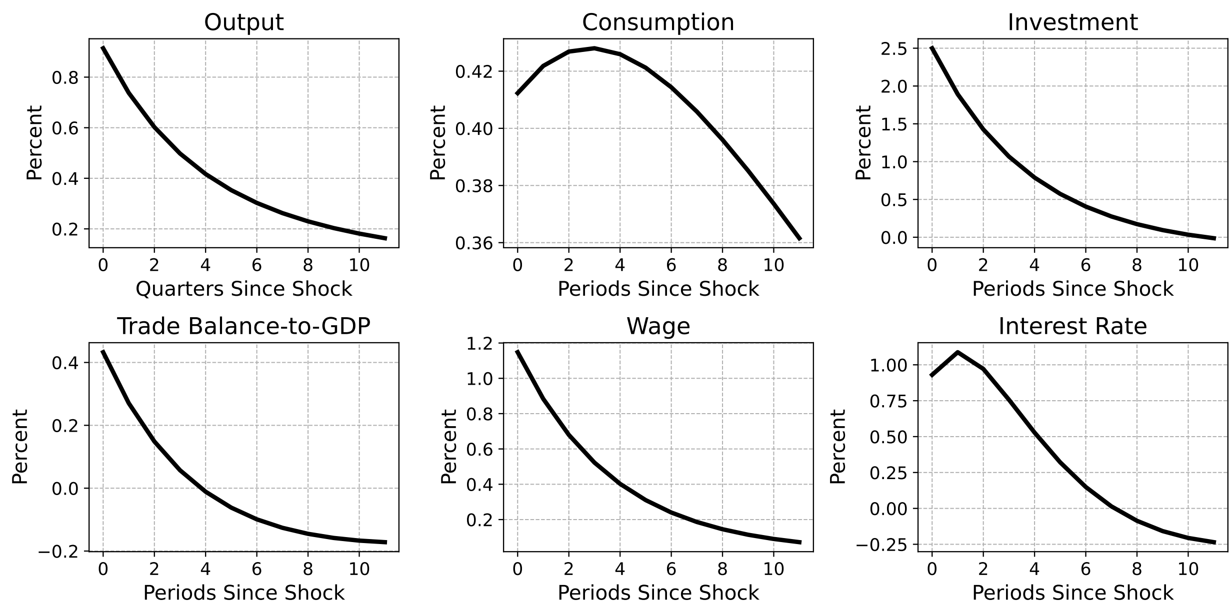
Panel (b) of Figure 3 plots the population dynamics following an aggregate productivity increase. While incomes for all occupations rise, the income of workers rises more than both types of entrepreneurs, relatively speaking. The decreasing returns to scale in the entrepreneurial production function and the factor adjustments from rising prices dampen the direct impact of the productivity rise. Meanwhile, workers' labor income is directly impacted by changes to the wage rates, which rise by 1.2 percent. Thus, it is more beneficial for individuals to exit entrepreneurship and become workers, leading to an overall decline in the population shares of entrepreneurs. By 10 quarters after the shock, population shares settle back into the steady state as aggregate productivity also goes back to its long-run levels.

Figure 4 plots the baseline economy's response to the rise in the fixed cost for employers. I find that a one percent rise in the fixed cost of operations of employer entrepreneurs leads to a moderate decline in aggregate output (-0.2%) and a substantial decline in investment (-2.0%). The investment decline is driven by the declining share of employers, as they hire more labor and demand more capital relative to self-employed entrepreneurs. Thus, as entrepreneurs exit employer entrepreneurship and enter self-employment, aggregate capital demand and aggregate output decline, leading to recessions. Interest rates, due to the decline in demand, also fall. However, this induces households to save less and consume more, which leads to a slight rise in consumption on impact (0.06%), but quickly falls below 0 as the decline in investment and output persist even after 10 quarters.

I now turn to the discussion of the labor market response. When the fixed cost of being an employer rises, its direct impact initially leads to a lower share of employers, which initially shifts the labor demand curve inwards, causing wages to decline briefly. However, two additional factors indirectly impact the labor market through general equilibrium. First, the corporate sector dampens the inward shift of the labor demand curve, as it is cheaper to hire labor. Second, due to the decline in interest rates, capital is cheaper. Therefore, workers who are at the margin of being self-employed pursue entrepreneurship, which causes the labor supply to decline. The end result is that, on impact, worker

Figure 3: **Impulse Response to a Positive Productivity Shock**

(a) **Aggregates and Prices**




(b) **Population Shares**

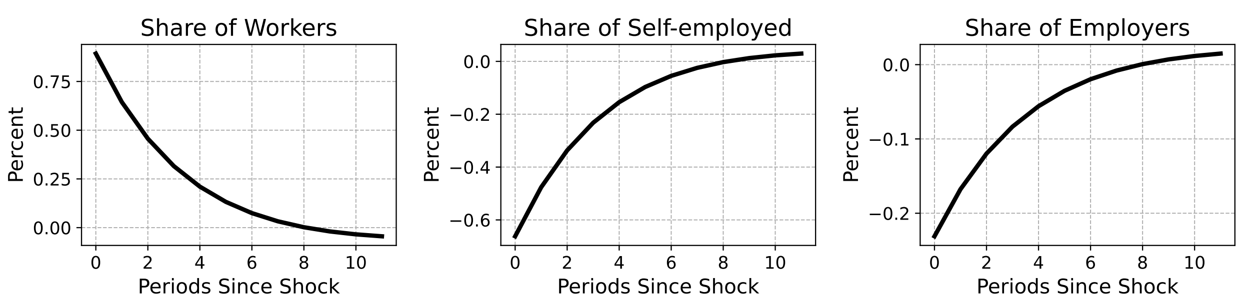
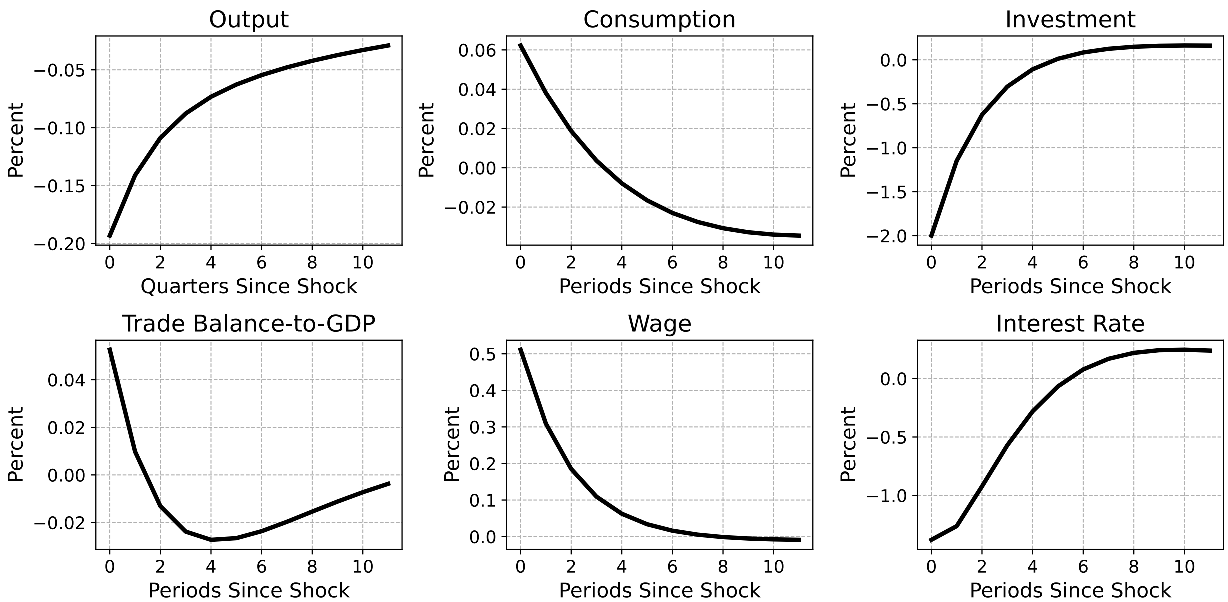
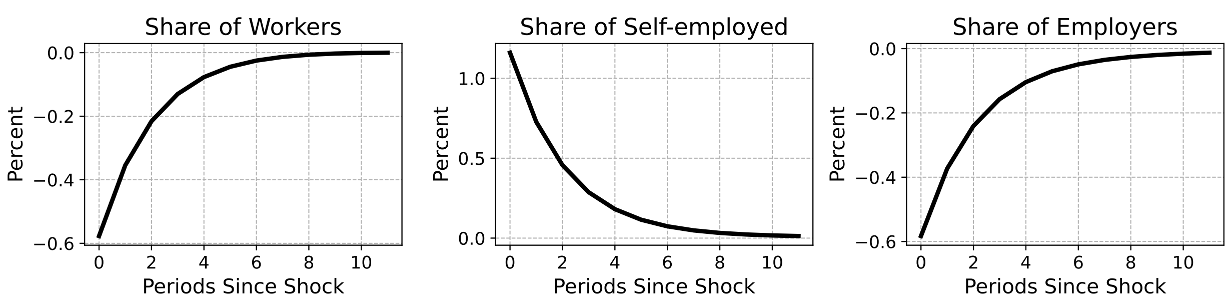


Figure 4: **Impulse Response to a Positive Fixed Cost Shock**

(a) **Aggregates and Prices**




(b) **Population Shares**




and employer entrepreneurs exit to become self-employed, and wage rates increase moderately.

## 5.4 Business Cycles in Presence of Entrepreneurial Heterogeneity

This section explores how business cycles in emerging markets are impacted when individuals are making occupational choices at the microeconomic level. First, we study how aggregate shocks change the business cycles and occupation dynamics. Second, we study how changes to the occupational structure in the stationary equilibrium change the business cycle dynamics.

Table 7 presents long-run business cycle moments implied by the model. The first and second columns present various second moments implied by the data and the model under the baseline calibration. The model captures perfectly the volatility of output of 5.12 percent and investment of 12.27 percent as intended from the calibration. Furthermore, one prominent feature of emerging market business cycles is the fact that consumption is more volatile than output. The model captures this feature very well, as the volatility of consumption under the baseline calibration is 5.45 percent. Lastly, as was also highlighted in the untargeted dimension of the calibration, the model also does well in capturing the population dynamics of workers, self-employed, and employer entrepreneurs.

Starting from the third column, I examine how the covariance structures of aggregate shocks impact business cycles and population dynamics. The first two scenarios ask the relative importance of the shocks in explaining aggregate fluctuations. First, the majority of variations in output, consumption, and investment come from the aggregate TFP shock rather than the fixed cost shock. This is intuitive as the fluctuations in the TFP impact all occupations directly whereas the latter shock only impacts a small subset of the population. In terms of the population dynamics, the productivity shock is unable to solely explain the procyclicality of employers. This implies that, at the very least, an employer-specific shock in the form of fixed cost helps capture shifts in the occupational structure during booms and busts. The last two columns study the quantitative properties of the assumption that the two shocks are strongly and negatively correlated. Reducing the negative correlation from -0.99 to -0.5 slightly reduces the volatility of aggregate variables and makes the population share of employers slightly countercyclical. When the correlation between the aggregate TFP and the fixed cost innovation is positive, the volatility of investment declines by half and the share of entrepreneurs continues to be at odds with the data. These simulations highlight the importance of including the fixed cost shock and the negative correlations of the shocks to both capture business cycle and population dynamics.

Table 8 provides a comparative analysis of how varying key parameters affects the population shares and business cycle moments in the model. The first three rows list the population share of workers, self-employed entrepreneurs, and employer entrepreneurs,

Table 7: **Second Moment Table I - Varying Covariance Structure**

| | (1) Data | (2) Baseline | (3) $\sigma_\kappa = 0$ | (4) $\sigma_z = 0$ | (5) $\rho_{Z,\kappa} = -0.5$ | (6) $\rho_{z\kappa} = 0.99$ |
|---|---|---|---|---|---|---|
| $\sigma(Y)$ | 5.12 | 5.12 | 4.65 | 0.48 | 4.91 | 4.18 |
| $\sigma(C)$ | 5.58 | 5.45 | 5.29 | 0.26 | 5.38 | 5.15 |
| $\sigma(I)$ | 12.27 | 12.27 | 9.23 | 3.29 | 11.12 | 6.44 |
| | | | | | | |
| $\rho(W,Y)$ | 0.74 | 0.87 | 0.85 | 0.91 | 0.85 | 0.82 |
| $\rho(Se,Y)$ | -0.79 | -0.88 | -0.86 | -0.93 | -0.81 | -0.35 |
| $\rho(Em,Y)$ | 0.69 | 0.66 | -0.83 | 0.94 | -0.17 | -0.87 |

**Note**: The table above presents how the second moments change in response to various shock structures of the baseline economy. The rows denote the variance of aggregate output $Y$ (percent), consumption $C$ (percent), investment $I$ (percent), and the correlation of output to the population share of workers $W$, self-employed $Se$, and employers $Em$, respectively.

respectively. The last three rows show the standard deviations of aggregate output, consumption, and investment. Specifically, I focus on the fixed cost parameter ($\kappa$) in the second and third columns and the collateral constraint parameter ($\phi_a$) in the fourth, fifth, and sixth columns. Lower (Higher) $\kappa$ implies a value of 0.46 (1.82), which is half of (3 times) the value in the baseline calibration. Lower (Higher) $\phi_a$ implies a value of 1.1 (1.5) whereas it was 1.25 in the baseline calibration.[7]

I first focus on varying the fixed cost parameter, $\kappa$. When $\kappa$ is lower (column 2), the share of employer entrepreneurs increases from 3.14% to 5.97%, while the share of self-employed decreases. Conversely, when $\kappa$ is higher (column 3), the share of employer entrepreneurs falls to 0.70%, and the share of self-employed rises to 21.42%. This makes sense as the fixed cost of being an employer rises (falls), individuals at the margin find it more optimal to exit (enter) employer entrepreneurship and turn either towards working or becoming self-employed. The change in the population distribution affects steady state income shares across groups and the aggregate output in the steady state. Higher share of employer entrepreneurs lead to a 3% increase in the output while the lower share of employer entrepreneurs lead to a 4% increase in output. The changes to the population distribution has a moderate impact on business cycle dynamics. In the economy with a larger share of employer entrepreneurs, the volatility of investment rises from 12.27% to

[7] Results in Table 8 are consistent for various values that I consider.

13.56% and the volatility of output increases to 5.18%. In the economy with a lower share of employer entrepreneurs, the volatility of output declines below 5% to 4.91% while the volatility of consumption falls to 5.18%. The volatility of investment also falls to 11.62%. This is from the fact that the contribution of employer entrepreneurs to the aggregate volatility falls from smaller population shares.

The table also showcases the impact of financial frictions, captured by the collateral constraint parameter ($\phi_a$), on business cycle moments. When $\phi_a$ is lower (column 4), implying tighter capital constraints, the population shares of both types of entrepreneurs decline. The share of employer entrepreneurs declines from 3.14% to 2.40%, while the share of self-employed entrepreneurs declines from 19.53% to 18.31%. This leads to lower aggregate output in the stationary equilibrium, but also leads to a lower volatility of output, consumption, and investment. On the other hand, when $\phi_a$ is higher (column 6), indicating looser capital constraints, the volatility of output, consumption, and investment rises substantially as there are more entrepreneurs. The level of output, consumption, and investment are also moderately higher in the stationary equilibrium. One implication that arises from this exercise is that in the model with aggregate shocks and occupational choice, financial frictions may inhibit economic growth, but also serve as a stabilizing mechanism in the business cycle sense.

Finally, the last column relaxes the assumption that both types of entrepreneurs face an equivalent degree of financial friction. This is not a very realistic assumption as self-employed entrepreneurs typically find it harder to raise capital, especially in emerging markets. When $\phi_{a,se} = 1$, it implies that self-employed entrepreneurs do not have access to external finances. In this case, the economy with such a setting, I find that the population share of self-employed entrepreneurs declines moderately from 19.53% to 16.90%, and the population share of employer entrepreneurs declines slightly from 3.14% to 3.08%. The second moments are also impacted due to the changes in the population distribution as the volatilities of output, consumption, and investment decline compared to the baseline calibration.

Overall, this section finds that the higher aggregate output in the long-run, defined by the stationary equilibrium, lead to more volatile business cycles. This has important policy implications because when certain policies are implemented to promote economic growth in the long-run, it may also lead to higher volatility in the short-run.

Table 8: **Second Moment Table II - Varying Parameters**

| | (1) Baseline | (2) Lower $\kappa$ | (3) Higher $\kappa$ | (4) Lower $\phi_a$ | (5) Higher $\phi_a$ | (6) $\phi_{a,se} = 1$ |
|---|---|---|---|---|---|---|
| Stationary Equilibrium | | | | | | |
| $W$ | 77.33 | 76.86 | 77.86 | 79.31 | 76.01 | 80.0 |
| $Se$ | 19.53 | 17.17 | 21.42 | 18.31 | 20.3 | 16.90 |
| $Em$ | 3.14 | 5.97 | 0.70 | 2.40 | 3.61 | 3.08 |
| $Y/Y^{baseline}$ | 1.00 | 1.03 | 0.96 | 0.97 | 1.02 | 0.98 |
| Business Cycle Moments | | | | | | |
| $\sigma(Y)$ | 5.12 | 5.18 | 4.91 | 5.06 | 5.29 | 5.08 |
| $\sigma(C)$ | 5.45 | 5.58 | 5.18 | 5.30 | 5.62 | 5.40 |
| $\sigma(I)$ | 12.27 | 13.56 | 11.62 | 10.53 | 12.25 | 11.70 |

**Note**: The table above presents how varying parameters affect the population shares and the second moments of the economy. The first three rows represent the population shares of occupations, the fourth row lists the ratio of aggregate output in the steady state, and the next 3 rows represent the variance of aggregate output $Y$ (percent), consumption $C$ (percent), and investment $I$ (percent).

# 6 Conclusion

This paper provides valuable insights into the role of entrepreneurial heterogeneity in shaping business cycles of emerging economies, focusing on the case of Argentina. I document two key empirical facts - the non-monotonic relationship between entrepreneurial shares and income, and the countercyclicality of entrepreneurial shares with respect to GDP. These empirical findings emphasized the need to distinguish between self-employed and employer entrepreneurs.

To capture these empirical findings, the paper develops a dynamic general equilibrium model of occupational choice in a small open economy setting. The model incorporates crucial features such as the differentiation between self-employed and employer entrepreneurs, and the introduction of aggregate shocks to productivity and fixed costs of being an employer. By calibrating the model to match income inequalities within and across occupations in the stationary equilibrium, as well as business cycle moments, the study provides a quantitative framework for analyzing the interactions between microeconomic occupational choices and macroeconomic dynamics.

The paper provides an analytical framework for understanding how aggregate shocks and changes in the stationary equilibrium propagate through the economy via the heterogeneous responses of entrepreneurs. By decomposing the effects into intensive and extensive margins, the study sheds light on the channels through which individual-level decisions and aggregate dynamics interact.

Finally, the model simulations yield several important implications. First, the paper finds that loosening financial frictions, represented by the collateral constraint parameter, amplifies business cycles. This suggests that policies aimed at improving access to credit for entrepreneurs could potentially exacerbate economic fluctuations in emerging markets. Second, the study reveals that self-employment acts as a stabilizing force during recessions, as it provides an alternative avenue for households to mitigate income declines.

Policymakers should consider the heterogeneity within the entrepreneurial sector when designing policies to promote entrepreneurship and foster economic growth. Future research could explore additional dimensions of heterogeneity, such as sectoral or other occupations. Furthermore, the current setting could be extended to study fiscal policy implications of self-employment.

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# A Entrepreneurs Production Decisions

First, I re-write the profit maximization problem of the self-employed entrepreneurs.

$$\pi^{se}(a_t, z_{wt}, z_{mt}; Z_t) = \max_{t_{mt}, k_t} Z_t (z_{mt} t_{mt})^{\nu} k_t^{\xi} ((1 - t_{mt}) z_{wt})^{\eta} - U_t k_t, \tag{A1}$$

$$\text{subject to } k_t \leq \phi_a a_t. \tag{A2}$$

Taking the first order conditions with respect to $t_{mt}$ and $k_t$ and letting $\lambda_t$ be the multiplier on the collateral constraint,

$$\text{FOC}[t_{mt}] : \nu Z_t (z_{mt} t_{mt})^{(\nu-1)} k_t^{\xi} ((1 - t_{mt}) z_{wt})^{\eta} z_{mt} = \eta Z_t (z_{mt} t_{mt})^{\nu} k_t^{\xi} ((1 - t_{mt}) z_{wt})^{\eta - 1} z_{wt} \tag{A3}$$

$$\implies t_{mt} = \frac{\nu}{\nu + \eta}, \tag{A4}$$

$$\text{FOC}[k_t] : \xi Z_t (z_{mt} t_{mt})^{\nu} k_t^{(\xi - 1)} ((1 - t_{mt}) z_{wt})^{\eta} = U_t + \lambda_t, \tag{A5}$$

$$\implies k_t = \left(\frac{Z_t (z_{mt} t_{mt})^{\nu} \xi (z_{wt} (1 - t_{mt}))^{\eta}}{U_t + \lambda_t}\right)^{\frac{1}{1-\xi}}. \tag{A6}$$

For employer entrepreneurs, their profit maximization problem can be written as

$$\pi^{em}(a_t, z_{wt}, z_{mt}; Z_t, \kappa_t) = \max_{t_{mt}, k_t, n_{dt}} Z_t (z_{mt} t_{mt})^{\nu} k_t^{\xi} (t_{wt} z_{wt} + n_{dt})^{\eta} - U_t k_t - W_t n_{dt} - \kappa_t, \tag{A7}$$

$$\text{subject to} \quad k_t \leq \phi_a a_t, \quad t_{mt} = 1 - t_{wt}, \quad t_{wt} \geq 0. \tag{A8}$$

Taking the first order conditions with respect to $t_{mt}$, $k_t$, $n_{dt}$ and letting $\lambda_t$, $\lambda_{t_m,t}$, and $\lambda_{t_w,t}$

be multipliers on the collateral, time, and non-negativity constraints,

$$\text{FOC}[t_{mt}] : Z_t z_{mt} \nu (z_{mt} t_{mt})^{(\nu-1)} k_t^{\eta} (n_{dt} + z_{wt} t_{wt})^{\eta} = \lambda_{t_m,t}, \tag{A9}$$

$$\text{FOC}[t_{wt}] : Z_t (z_{mt} t_{mt})^{\nu} k_t^{\xi} \eta (n_{dt} + z_{wt} t_{wt})^{(\eta-1)} z_{wt} = \lambda_{t_m,t} - \lambda_{t_w,t}, \tag{A10}$$

$$\text{FOC}[n_{dt}] : Z_t (z_{mt} t_{mt})^{\nu} k_t^{\xi} \eta (n_{dt} + t_{wt} z_{wt})^{(\eta-1)} = W_t, \tag{A11}$$

$$\text{FOC}[k_t] : Z_t (z_{mt} t_{mt})^{\nu} \xi k_t^{(\xi-1)} (n_{dt} + t_{wt} z_{wt})^{\eta} = U_t + \lambda_t, \tag{A12}$$

Following the analysis from Allub and Erosa (2019) (AE), given prices $W_t, U_t$ and shocks $Z_t, \kappa_t$, one can show that i.) if the borrowing constraint does not bind ($\lambda_t = 0$), then the entrepreneur devotes all his time to managing, (i.e. $t_{wt} = 0, t_{mt} = 1$), ii.) if the borrowing constraint does bind ($\lambda_t > 0$), then there exists a threshold level of assets, $a^*$, where $t_{wt} > 0$ if $a < a^*$ and $t_{wt} = 0$ if $a > a^*$.[8] The main difference from the AE economy is that prices are changing due to aggregate shocks.

As a final note, the total income is defined to be the sum of labor (business) income, $W_t z_{wt}$ ($\pi$), for workers (entrepreneurs) plus the interest income, $r_t^a a_t$.

# B Computational Appendix

## B.1 Details on Managing and Working Fixed Effects

I discuss the joint distribution of fixed effects, $(\alpha_w, \alpha_m)$ in this section, which follows Allub and Erosa (2019). Managing fixed effect, $\alpha_m$, is distributed Pareto, and the fixed effects on working ability are assumed to be normally distributed conditional on each realization of the managerial fixed effects abilities. In particular, the fixed effects on working ability can be written as

$$\alpha_w = \varepsilon_{wf} + \rho_{\alpha_{w,m}} \frac{(\alpha_m - E(\alpha_m))}{std(\alpha_m)} \sigma_{\alpha_w}, \quad \text{where } \varepsilon_{wf} \sim N(0, \sigma_{wf}^2), \tag{A13}$$

where $\rho_{\alpha_{w,m}}$ represents the correlation of fixed effects, $\sigma_{wf}^{w}$ is the variance of fixed effects on working ability, and $std(\alpha_m), E(\alpha_m)$ are the standard deviation and mean of managerial

[8] In particular, one can find $a^*$ by combining the first-order conditions related to labor demand and time allocations to find that $a^* = \frac{1}{\phi_a}[\frac{W_t}{Z_t \xi z_{mt}^{\nu}}(\frac{z_{wt}\eta}{\nu})^{(1-\eta)}]^{\frac{1}{\xi}}$. For $a < a^*$, $t_{mt} = [\frac{\xi Z_t z_{mt}^{\nu}(\phi_a a)^{\xi}}{W_t}(\frac{\nu}{z_{wt}\eta})^{1-\eta}]^{\frac{1}{1-\eta-\nu}}$.

fixed effects. Given $\sigma_{\alpha_w}$ (standard deviation of the working ability shock) and $\rho_{\alpha_{m,w}}$ from Table 3 and Table 4, one can find $\sigma^2_{wf}$ from the following formula.

$$\sigma^2_{wf} = (1 - \rho^2_{\alpha_{mw}}) \times \sigma^2_{\alpha_w}. \tag{A14}$$

There are 7 grid points each for the persistent components of working and managing skills, 5 grid points for the fixed effects of managing, and 3 grid points for the fixed effects of working. In total, there are 735 possible states for $(z_w, z_m)$.

## B.2 General Steps

The baseline model contains multiple idiosyncratic productivity shocks, multiple aggregate shocks, and discrete choice at the household level. The general steps to solve the model is as follows:

1. Solve for the stationary equilibrium given a fixed supply of foreign bonds, $\bar{B}^*$.
   (a) Guess the level of interest rate, $r^*_{ss}$.
   (b) Find factor prices $(U, W)$. Furthermore, $r^a = \tilde{r}^a = r^*_{ss}$.
   (c) Given prices, solve the household problem, which is solved using a value function iteration with a linear spline to evaluate the value function off the grid points. The asset grid is set from 0.001 to 350. The number of asset grid points is 120. Increasing the number of grid points did not substantially change the results. In order to capture the high degree of nonlinearity at the lower end of the grid, I include more grid points towards the bottom of grid. There are 35 grid points in the managerial productivity shock, 21 grid points in the working productivity shock, and 120 grid points in the asset state space. In total, there are 88,200 individual state space.
   (d) Given the solution to the household problem, simulate the distribution as in Young (2010).
   (e) Check that equation (19) holds.
2. Linearize the model with only respect to aggregate shocks

(a) Jacobian is calculated by taking numerical differentiations

(b) Use the dimension reduction techniques from Bayer and Luetticke (2018) to make the model computationally feasible. Using this technique reduces the number of state variables from 88,204 (88,200 individual states plus two aggregate shocks, and two aggregate state variables) to 163 (130 asset grids plus 7 managerial productivity (persistent component) plus 7 working productivity (persistent component) plus 15 fixed effects components plus 2 aggregate state variables plus 2 aggregate shocks). The number of control variables decline from 88,211 (88,200 individual states plus 12 aggregates) to 2001.

3. Solve the linearized model using Klein's method.

### B.2.1 Details on Solving the Model

There are two infinite dimensional objects in the model that need to be approximated, which are the value functions and the distribution of households over the idiosyncratic states. The value functions with both idiosyncratic and aggregate shocks can be written as follows.

$$v(s_t, S_t) = \max_{a_{t+1}, c_t} \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \beta \mathbb{E}[v(s_{t+1}, S_{t+1}) | s_t, S_t] \right\} \tag{A15}$$

$$\text{s.t. } c_t = \max_{o_t} \{W_t z_{wt}, \pi_t^{se}, \pi_t^{em}\} + (1 + r_t^a) a_t - a_{t+1}$$

$$a_{t+1} \geq 0, \quad k_t \geq \phi_a a_{t+1},$$

$$\pi^{se}(a_t, z_{wt}, z_{mt}; Z_t) = \max_{t_{mt}, k_t} Z_t (z_{mt} t_{mt})^\nu k_t^\xi ((1 - t_{mt}) z_{wt})^\eta - U_t k_t, \tag{A16}$$

$$\pi^{em}(a_t, z_{wt}, z_{mt}; Z_t, \kappa_t) = \max_{t_{mt}, k_t, n_{dt}} Z_t (z_{mt} t_{mt})^\nu k_t^\xi ((1 - t_{mt}) z_{wt} + n_{dt})^\eta - U_t k_t - W_t n_{dt} - \kappa_t. \tag{A17}$$

Value functions are approximated by a linear spline. I first calculate the prices in the steady state and turn off all aggregate shocks. In this step, the conditional expectations for the value functions are only taken with respect to idiosyncratic shocks. Given that the value function can be approximated with a linear spline, one can maximize each value functions on the RHS using Brent's method. And then the household's equilibrium dynamics with aggregate shocks can be characterized by the above set of equations, where $s_t = (a_t, z_{wt}, z_{mt})$ are idiosyncratic state variables at time $t$ while $S_t = (Z_t, \kappa_t, B_t, K_t, \Lambda_t)$ will be aggregate state variables at time $t$.

The high dimensionality of this system makes the computation nearly infeasible. Therefore, I pursue a dimension reduction technique as Bayer and Luetticke (2018) for the distribution of households. To reduce the dimension of the value function, compression algorithm is used. To be more specific, I write the value function as some form of sparse polynomial expansions around its stationary equilibrium values.

$$v_t(s) = \bar{v}(s) + g_v(s; \theta_v^s), \tag{A18}$$

where $g_v$ is the discrete cosine transformation of the stationary equilibrium of the value function. I shrink all but the largest elements without losing too much information. That is, I only keep the nodes of the value function where it is most informative in response to aggregate shocks. For more technical details, I refer the readers to Bayer and Luetticke (2018).

The second inifinite dimensional object in the model to be approximated is the distribution of the idiosyncratic state. This is done with a histogram method as in Young (2010). Let $a'(s, S)$ be the savings function for the household's that maximizes their value functions. Then the distribution over households can be summarized by a transition matrix $Q$, where each element $Q_{i,i'}$ is the probability that a type $i$ will be type $i'$. This can be obtained by

$$Q_{i,i'} = \mathbb{P}[(a^{i'} = a_j, \epsilon^{i'} = \epsilon_s, \theta^{i'} = \theta_k)|(a^i, \epsilon^i, \theta^i)] \tag{A19}$$

$$= w_{ij}\mathbb{P}(\epsilon_s, \theta_k|\epsilon^i\theta^i) \tag{A20}$$

In the case of the stationary equilibrium, the steady state distribution over households is a histogram $\Lambda(s)$ that satisfies the following condition:

$$\Lambda = Q\Lambda \tag{A21}$$

With aggregate shocks, the equilibrium dynamic must satisfy the following:

$$\Lambda_{t+1} = Q_t \Lambda_t \tag{A22}$$

where $Q_t$ is generated by the savings function $a'(s_t, S_t)$.

Furthermore, we need equations that describe the aggregate capital stock, bonds, two aggregate shocks, and the interest rate in order to make the model stationary. They are

$$(1+\tilde{r}_t^a)(1+\phi_k(\frac{K_{t+1}}{K_t}-1)) = \mathbb{E}_t\left[1+U_{t+1}-\delta+\phi_k\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\frac{K_{t+2}}{K_{t+1}}-\frac{\phi_k}{2}\left(\frac{K_{t+2}}{K_{t+1}}-1\right)^2\right], \tag{A23}$$

$$A_{t+1} = K_{t+1} + B^*_{t+1} + \frac{\phi_k}{2}(\frac{K_{t+1}}{K_t}-1)^2 K_t, \tag{A24}$$

$$\log Z_{t+1} = (1-\rho_z)\log \bar{Z} + \rho_z \log Z_t + \varepsilon_{z,t+1}, \tag{A25}$$

$$\log \kappa_{t+1} = (1-\rho_\kappa)\log \bar{\kappa} + \rho_\kappa \log \kappa_t + \varepsilon_{\kappa,t+1}, \tag{A26}$$

$$Y_t^c = Z_t (K^c)_t^\alpha (L_t^c)^\alpha, \tag{A27}$$

$$Y_t = Y_t^c + \int y_t(s) d\Lambda_{em,se,t}(s), \tag{A28}$$

$$W_t = Z_t(1-\alpha)(K_t^c/L_t^c)^\alpha, \tag{A29}$$

$$U_t = Z_t \alpha (K_t^c/L_t^c)^{(1-\alpha)}, \tag{A30}$$

$$I_t = K_{t+1} - (1-\delta)K_t, \tag{A31}$$

$$C_t = \int c_t(s) d\Lambda_t, \tag{A32}$$

$$K_t = K_t^c + \int k_t(s) d\Lambda_{se,em,t}, \tag{A33}$$

$$\int z_{wt}(s) d\Lambda_{w,t} = L_t^c + \int n_{dt}(s) d\Lambda_{em,t}, \tag{A34}$$

$$r_t^* = r_{ss}^* + \omega_r(\exp^{-B^*_{t+1}-\bar{B}^*} - 1), \tag{A35}$$

$$D_t = (1+U_t-\delta)K_t + (1+r_t^*)B_t - (1+\tilde{r}_t^a)A_{t+1}, \tag{A36}$$

$$r_t^a = \tilde{r}_t^a + D_{t+1}/A_{t+1}, \tag{A37}$$

$$\tilde{r}_t^a = r_a^*. \tag{A38}$$

This completes the minimum number of equations in order to fully characterize the equilibrium dynamics in my model.

Given the distribution and the value functions, all other auxiliary aggregate variables such as output, consumption, and investment can be calculated from the value functions (and its resulting savings/consumption functions) and the resulting distribution.

The equilibrium dynamic can be represented by a set of nonlinear equations (shown above) which then can be written as:

$$\mathbb{E}_t F(X_t, X_{t+1}, Y_t, Y_{t+1}) = 0, \tag{A39}$$

where $Y_t$ is the set of control variables (such as value functions or aggregate output), and $X_t$ is the set of state variables (such as the distribution $\Lambda_t$ or aggregate capital $K_t$). Thus, linearizing the model with respect to aggregate shocks gives the following linear dynamic system.

$$X_{t+1} = H_x X_t + \eta \epsilon_{t+1}, \tag{A40}$$

$$Y_t = G_x X_t, \tag{A41}$$

which then can simulate the model, calculate second moments, and perform impulse response analysis.