Byzantine Agreement with Incomplete Views

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December 2, 2019

The Byzantine Agreement (BA) Problem

Definition (Binary Byzantine Agreement)

Suppose there are N parties

- $P_1, \ldots, P_N$. Each P_i holds an initial input $v_i \in \{0,1\}$,
- t of the parties are corrupted by an adversary.

A protocol achieves Byzantine Agreement if the following conditions hold after termination:

- (Validity) If all honest parties begin with the same input v, they also output v.
- (Agreement) All honest parties output the same value

Standard Model

Communication

- All parties communicate in rounds over point-to-point channels
- The communication network is *complete*

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- The communication network is *complete*

Adversary

- Adaptively corrupt honest parties at the start of any round
- Corrupted parties deviates from the protocol arbirarily
- Receives honest messages before sending corrupted messages

Our Model

Communication

- All parties communicate in rounds over point-to-point channels
- The communication network is *sparse*

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Parametrization

(The inclusive neighborhood Γ_i of P_i is called its *view*)

- α, the maximum fraction of corruption in an honest view
- δ, the minimum fraction of overlapping between any pair of honest views

BA in the plain model

Corruption

- BA is possible if and only if less than $1/3$ parties are corrupted. [PSL80], [LSP].

Round Complexity

- There exists a BA protocol of $t+1$ rounds [GM98].
- No deterministic protocol of less than $t+1$ rounds exists [FL82].

BA with PKI and randomization

w/ PKI

- tolerates 1/2 corruption
- requires $t + 1$ rounds [DS83]

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- runs in expected *constant* round [FM97]

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w/ PKI and Randomization

- tolerates $1/2$ corruption
- runs in expected *constant* round [FG03], [KK06], [MV17]

Our Result

- assumes a PKI setup
- uses randomization
- recall δ = overlapping, α = corruption

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Theorem (Positive)

If $\delta > 2\alpha$, there exists an expected O(n) round Byzantine Agreement protocol. Further if $\alpha = 1/2 - \epsilon$ for any constant ϵ, there exists an expected constant round Byzantine Agreement protocol in our model.

Our Result

- assumes a PKI setup
- uses randomization
- recall δ = overlapping, α = corruption

Theorem (Positive)

If $\delta > 2\alpha$, there exists an expected O(n) round Byzantine Agreement protocol. Further if $\alpha = 1/2 - \epsilon$ for any constant ϵ, there exists an expected constant round Byzantine Agreement protocol in our model.

Theorem (Negative)

If $\alpha \geq 1/2$ or $\delta \leq 2\alpha$, there does not exist a Byzantine Agreement protocol in our model

Unique Digital Signature: Definition

Definition

A Digital Signature scheme is a triple of polynomial time computable algorithms $(\texttt{Gen}, \texttt{Sign}, \texttt{Verify})$.

- $(S_k, P_k) \xleftarrow{R} \texttt{Gen}(1^k)$.
- $\sigma = \texttt{Sign}(S_k, x)$.
- $b \xleftarrow{R} \texttt{Verify}(P_k, x, \sigma)$.

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Correctness

With overwhelming probability over $(S_k, P_k) \overset{R}{\longleftarrow} \texttt{Gen}(1^k)$:
$1 \overset{R}{\longleftarrow} \texttt{Verify}(P_k, x, \texttt{Sign}(S_k, x))$.

Unique Digital Signature: Uniqueness

Uniqueness

- There do not exist values $(P_k, x, \sigma_1, \sigma_2)$ such that $\sigma_1 \neq \sigma_2$, and $\texttt{Verify}(P_k, x, \sigma_1) = \texttt{Verify}(P_k, x, \sigma_2) = 1$.
- Construction given in [MRVil] under the RSA assumption.

Unique Digital Signature: Security

Security (existentially unforgeable)

Let T be any polynomial time algorithm. The probability that T wins the following game must be negligible:

- Run $(P_k, S_k) \overset{R}{\leftarrow} \texttt{Gen}(1^k)$:
- Run $(x, \sigma) \overset{R}{\leftarrow} T^{\texttt{Sign}(S_k, \cdot)}(1^k, P_k)$
- T wins if $1 \overset{R}{\leftarrow} \texttt{Verify}(P_k, x, \sigma)$

PKI for our model

Public-key Infrastructure

- P_i is assigned Pk_i, Sk_i
- P_i is assigned Pk_j for all P_j in its view
- The adversary knows Pk_i for all P_i

Random Oracle

- We assume a public random oracle function $H(\cdot)$ for simplicity
- In our protocol, $H(\cdot)$ can replaced by a Verifiable Random Function [MRVil]

Definition (Random Oracle)

A random oracle H is a map from $\{0,1\}^*$ to $\{0,1\}^k$ chosen by selecting every bit of $H(x)$ uniformly and independenty, for every x.

Road Map

- Graded Broadcast: Prevents a party from sending different values in the same round.
- Leader Selection: Guarantees honest parties to agree on an honest leader with some chance.
- Main protocol: Guarantees honest parties to either agree on the same value or follow its leader.

The size of honest views

For simplicity, we assume that all honest views have the same size n. However, all our results hold without this assumption.

Graded Broadcast: Definition

Definition (Graded Broadcast)

For N parties $P_1, ..., P_N$, one of which is the dealer P_d holding a message m, the following must hold after the protocol:

- Each honest $P_i \in \Gamma_d$ outputs (m_i, g_i), where $g_i \in \{0, 1\}$.
- (Validity) If P_d is honest, then $m_i = m$, and $g_i = 1$ for all honest $P_i \in \Gamma_d$.
- (Agreement) If two honest parties $P_i, P_j \in \Gamma_d$ outputs $(m_i, 1)$, and $(m_j, 1)$, then $m_i = m_j$.

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Graded Broadcast: Protocol

Protocol: Graded Broadcast w/ dealer P_d

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1: $P_d$ sends $(m, \sigma_d(m))$ to all $P_i \in \Gamma_d$
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Graded Broadcast: Protocol

Protocol: Graded Broadcast w/ dealer P_d

1: P_d sends $(m, \sigma_d(m))$ to all $P_i \in \Gamma_d$

2: Every honest $P_i \in \Gamma_d$ receives $(m, \sigma_d(m))$

- if $\sigma_d(m)$ is valid, then P_i forwards it to all $P_j \in \Gamma_i$
- else P_i does nothing.

Graded Broadcast: Protocol

Protocol: Graded Broadcast w/ dealer P_d

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2: Every honest $P_i \in \Gamma_d$ receives $(m, \sigma_d(m))$

- if $\sigma_d(m)$ is valid, then P_i forwards it to all $P_j \in \Gamma_i$
- else P_i does nothing.

3: Every honest $P_i \in \Gamma_d$ receives $(m_j, \sigma_d(m_j))$ from some $P_j \in \Gamma_i$

- if there are contradicting valid $\sigma_d(m), \sigma_d(m')$ and $m \neq m'$, then P_i outputs $(\phi, 0)$.
- if there are at least $(\delta - \alpha)n$ valid signatures on some m, then P_i outputs $(m, 1)$
- else, P_i outputs $(\phi, 0)$.

Graded Broadcast: Proof

Claim (Validity)

If the dealer P_d is honest with message m, then all honest $P_i \in \Gamma_d$ output $(m, 1)$.

Graded Broadcast: Proof

Claim (Validity)

If the dealer P_d is honest with message m, then all honest $P_i \in \Gamma_d$ output $(m, 1)$.

- no contradicting siganature can be forged
- enough honest parties in $\Gamma_d \cap \Gamma_i$ for all honest $P_i \in \Gamma_d$

Graded Broadcast: Proof

Claim (Agreement)

If two honest $P_i, P_j \in \Gamma_d$ *outputs* $(m_i, 1)$ *and* $(m_j, 1)$, *and if* $\delta > 2\alpha$, *then* $m_i = m_j$.

Graded Broadcast: Proof

Claim (Agreement)

If two honest $P_i, P_j \in \Gamma_d$ outputs $(m_i, 1)$ and $(m_j, 1)$, and if $\delta > 2\alpha$, then $m_i = m_j$.

- P_i receives forwarding $(m_i, \sigma_d(m_i))$ from at leat one honest party $P_k \in \Gamma_d$
- P_k must have forwarded $(m_i, \sigma_d(m_i))$ to P_j. Contradiction.

Graded Broadcast: Summary

Lemma (Graded Broadcast)

If $\delta > 2\alpha$, there exists a three round Graded Broadcast protocol.

Leader Selection: Definition

Definition (Leader Selection)

For a set of parties $P_1, \ldots, P_N$ and fairness γ, when the protocol terminates, the following conditions must hold with probability at least γ:

- Every honest P_i outputs P_l and P_l is honest by the end of the protocol.

When such an event happens, we say that an honest leader P_l is elected.

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Leader Selection: Protocol

Protocol: Leader Selection w/ input r

1: Every honest P_i sends $m_i = (i, \sigma_i(r))$ to all $P_j \in \Gamma_i$

2: Every honest P_i forwards valid messages to all $P_j \in \Gamma_j$

3: Every honest P_i accepts n forwardings from each $P_j \in \Gamma_i$ and ignores the rest. P_i computes a set S_i of messages forwarded by at least $(\delta - \alpha)n$ parties, and send S_i to all $P_j \in \Gamma_i$

4: Every honest P_i receives a set S_j from every $P_j \in \Gamma_i$

- P_i computes a set S_i^* of messages appear in at least $(1 - \alpha)n$ received sets.
- P_i computes $H(m_k)$ for every $m_k \in S_i^*$, and outputs P_l where l is the smallest id such that $\forall m_k \in S_i^*, H(m_l) \leq H(m_k)$.

Leader Selection: Proof

Claim (All honest parties are accepted)

In any iteration r, if P_i, P_j are any two honest participants, then $m_j = (j, Sig_j(r)) \in S_i^*$ in Step 4.

- For all honest P_k: enough honest parties in $\Gamma_k \cap \Gamma_j$ hence $P_j \in S_k$ in Step 3.
- P_j is in at least $(1 - \alpha)n$ sets sent to P_i, hence $P_j \in S_i^*$.

Leader Selection: Proof

Claim (Bound corrupt messages in S_i)

In any iteration r, if P_i is honest, and if $\delta > 2\alpha$, then S_i contains at most $2n$ messages from corrupted participants.

- # corrupted message forwared to an honest P_i in Step 3: (*blue* from corruped; *red* from honest)

$$\alpha n^2 + (1 - \alpha)\alpha n^2 = \alpha(2 - \alpha)n^2$$

Leader Selection: Proof

Claim (Bound corrupt messages in S_i)

In any iteration r, if P_i is honest, and if $\delta > 2\alpha$, then S_i contains at most $2n$ messages from corrupted participants.

- # corrupted message forwarded to an honest P_i in Step 3: (*blue* from corruped; *red* from honest)

$$\alpha n^2 + (1-\alpha)\alpha n^2 = \alpha(2-\alpha)n^2$$

- Since a corrupted message is only accepted if it is forwarded $(\delta - \alpha)n$ times, the total # corrupted messages accepted by P_i (into S_i) is at most:

$$\frac{\alpha(2-\alpha)n^2}{(\delta-\alpha)n} \leq \frac{\alpha}{\delta-\alpha}2n < 2n$$

Leader Selection: Summary

Lemma (Fairness of the protocol)

If $\delta > 2\alpha$, there exists a three round Oblivious Leader Selection protocol with fairness $1/(5n)$. Further if $\alpha < 1/2 - \epsilon$ for some positive constant ϵ, then there exists a three round Oblivious Leader Selection protocol with constant fairness.

(Let C be the set of honest parties. $S^* = \cup_{P_i \in C} S_i^*$.)

- A corrupted message accepted into S^* in Step 4 has to appear in at least $(1 - 2\alpha)n$ honest sets.
  We bound # of corrupted messages in S^*:

$$\frac{2n|C|}{(1-2\alpha)n} = \frac{2}{1-2\alpha}|C|$$

Leader Selection: Summary

Lemma (Fairness of the protocol)

If $\delta > 2\alpha$, there exists a three round Oblivious Leader Selection protocol with fairness $1/(5n)$. Further if $\alpha < 1/2 - \epsilon$ for some positive constant ϵ, then there exists a three round Oblivious Leader Selection protocol with constant fairness.

(Let C be the set of honest parties. $S^* = \cup_{P_i \in C} S_i^*$.)

- Since $1 - 2\alpha > 1/(2n)$, the probability that an honest leader is elected is at least:

$$\frac{|C|}{|S^*|} \geq \frac{|C|}{|C| + \frac{2}{1/(2n)}|C|} = \frac{1}{1+4n} \geq \frac{1}{5n}$$

Leader Selection: Summary

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(Let C be the set of honest parties. $S^* = \cup_{P_i \in C} S_i^*$.)

- If $\alpha < 1/2 - \epsilon$, then $1 - 2\alpha > 2\epsilon$ and the probability that an honest leader is elected is at least:

$$\frac{|C|}{|S^*|} \geq \frac{|C|}{|C| + \frac{2}{2\epsilon}|C|} = \frac{1}{1 + 1/\epsilon}$$

Simplified Main Protocol

- We present only the simplified version
- refer to the thesis for the full version.

Full Version:

- Honest parties terminates after reaching agreement
- Run until agreement is reached
- Agreement guaranteed after termination

Simplified Version:

- Honest parties keeps the agreement after reaching one
- Run predetermined rounds and stop
- May fail with negligeable chance

Main Protocol

Protocol: Simplified Main w/ input $r \leftarrow 0$ and v_i

```
1: Every honest $P_i$ sends a random bit $b \overset{R}{\leftarrow} \{0,1\}$ to all $P_j \in \Gamma_i$,
   and then forwards received bits with ids to all $P_j \in \Gamma_i$.
2: Every honest $P_i$ counts the forwarded bits: $\forall j$, if exists a
   unique value $b_j^*$ with at least $(\delta - \alpha)n$ votes, then set $b_{ij} \leftarrow b_j^*$.
   Otherwise, set $b_{ij} \leftarrow 0$
3: Every honest $P_i$ runs Leader Selection with $r$, and outputs $P_{l_i}$.
4: Every honest $P_i$ runs Graded Broadcast as the dealer with
   message $v_i$. $P_i$ outputs $(v_j, g_j)$ for every $P_j \in \Gamma_i$.
     • if at least $(1 - \alpha)n$ 1s have grades 1, then set $v_i \leftarrow 1$.
     • if at least $(1 - \alpha)n$ 0s have grades 1, then set $v_i \leftarrow 0$.
     • else set $v_i \leftarrow b_{il_i}$.
5: Increament $r$ and *repeat*
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Main Protocol: Proof

Claim (Honest random bit reaches all)

If P_i is honest and samples $b_i \overset{R}{\leftarrow} \{0,1\}$ in Step 1, then every honest P_j sets $b_{ji} \leftarrow b_i$ in Step 2.

- enough honest parties in $\Gamma_i \cap \Gamma_j$ for all honest P_j, hence b_i has enough votes
- at most $\alpha n < (\delta - \alpha)n$ votes forwarded by corrupted parties, hence b_i is unique.

Main Protocol: Proof

Claim (Agreement remains agreement)

If all honest P_i start with $v_i = v$ at Step 1, then they still have $v_i = v$ at Step 5.

Claim (Honest leader leads to agreement)

If all honest P_i outputs the same P_l in Step 3 (Leader Selection), and P_l is honest, then with probability $1/2$ they reach agreement.

- Note if an honest P_i sets $v_i \leftarrow 1$ in Step 4, then every other honest P_j either also sets $v_j \leftarrow 1$ or sets $v_j \leftarrow b_{jl_j}$

Road Map

- Byzantine Agreement implies Broadcast
- Broadcast is impossible for $\alpha \geq 1/2$
- Broadcast is impossible for $\delta \leq 2\alpha$

Broadcast: Definition

Definition (Broadcast)

For N parties $P_1, \ldots, P_N$, one of which is a distinguished dealer $P_d \in S$ holding a message m, the following conditions must hold after the protocol:

- (Agreement) Every honest participant P_i outputs the same m^*, for some m^*.
- (Validity) If the P_d is honest, then $m^* = m$.

Note: if $P_i \notin \Gamma_d$, P_i enters the protocol with the player id P_d, but not its public key Pk_d.

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Broadcast: Protocol

Protocol: Broadcast

1: The dealer P_d runs a Graded Broadcast protocol with message $m \in \{0, 1\}$. Every honest $P_i \in \Gamma_d$ outputs (m_i, g_i).

2: For every honest $P_i \in \Gamma_d$:

- if $g_i = 1$, then send m_i to all $P_j \in \Gamma_i$ and set $v_i \leftarrow m_i$.
- else, set $v_i \leftarrow 0$

3: For every honest $P_i \notin \Gamma_d$:

- if P_i receives at least $(\delta - \alpha)n$ messages of a unique message m, then set $v_i \leftarrow m_i$
- Otherwise, set $v_i \leftarrow 0$.

4: Every honest P_i runs Byzantine Agreement with input v_i, and use its output v^* as the broadcast output.

$\alpha \geq 1/2$

Example ($\mathcal{C}_1$)

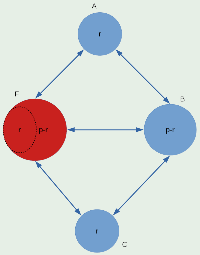


Figure: A configuration $\mathcal{C}_1$ with $\alpha = 1/2$, and $\delta = (2p - r)/2p$.

Claim ($\mathcal{C}_1$ is valid)

$\mathcal{C}_1$ represents a valid configuration with $\alpha = 1/2$ and any $\delta = (2p - r)/(2p)$.

- $\Gamma_A = A \cup B \cup F$
- $\Gamma_B = A \cup B \cup C$
- $\Gamma_C = B \cup C \cup F$

$\alpha \geq 1/2$

Example ($\mathcal{C}_1$)

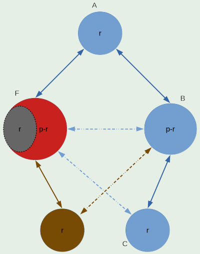


Figure: An adversarial strategy for configuration $\mathcal{C}_1$.

Claim ($\mathcal{C}_1$ is impossible)

Let $P_d \in C$ be some honest participant in group C with message $m \in \{0, 1\}$. We now claim that broadcast is impossible for P_d in $\mathcal{C}_1$.

(Adversary strategy:)

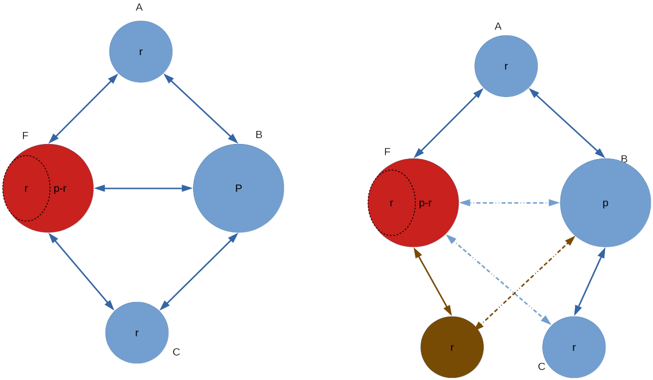
- simulates C' locally
- disables r corrupted ones in F
- F, C' ignores B, C

$\alpha \geq 1/2$: Summary

Lemma ($\alpha \geq 1/2$)

Assuming $\alpha \geq 1/2$, for any $0 < \delta < 1$, there does not exist a Broadcast protocol in our model.

$\delta \leq 2\alpha$



(a) A configuration $\mathcal{C}_2$ with $\alpha = p/(2p + r) < 1/2$, and $\delta = 2\alpha$.

(b) An adversarial strategy for configuration $\mathcal{C}_2$.

Figure: A counter example for the case of $\alpha < 1/2, \delta \leq 2\alpha$.

$\delta \leq 2\alpha$: Proof

Claim ($\mathcal{C}_2$ is valid)

$\mathcal{C}_2$ represents a valid configuration with $\alpha = p/(2p + r) < 1/2$ and $\delta = 2\alpha$.

Claim ($\mathcal{C}_2$ is impossible)

Broadcast is impossible for P_d in configuration $\mathcal{C}_2$.

Lemma ($\delta \leq 2\alpha$)

Assuming $\alpha < 1/2$ and $\delta \leq 2\alpha$, there does not exist a Broadcast protocol in our model

Recap

Theorem (Positive)

If $\delta > 2\alpha$, there exists an expected O(n) round Byzantine Agreement protocol. Further if $\alpha = 1/2 - \epsilon$ for any constant ϵ, there exists an expected constant round Byzantine Agreement protocol in our model.

Recap

Theorem (Positive)

If $\delta > 2\alpha$, there exists an expected O(n) round Byzantine Agreement protocol. Further if $\alpha = 1/2 - \epsilon$ for any constant ϵ, there exists an expected constant round Byzantine Agreement protocol in our model.

Theorem (Negative)

If $\alpha \geq 1/2$ or $\delta \leq 2\alpha$, there does not exist a Byzantine Agreement protocol in our model

Future Work

Question 1:

Note that our protocol runs in expected *constant* round if $\alpha < 1/2 - \epsilon$ for any positive constant ϵ. Does there exist an expected *constant* round protocol for $\alpha < 1/2$?

Question 2:

Note that our protocol somewhat defends against sybil attack, in that only corrupted parties connected to an honest party have effect. Can we make it fully resistant to sybil attack (possibly using *proof of work*)?

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