

Homework 5

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Due on: Wednesday Dec 5th (40 points)

Name: _____ PID: _____

Instructions

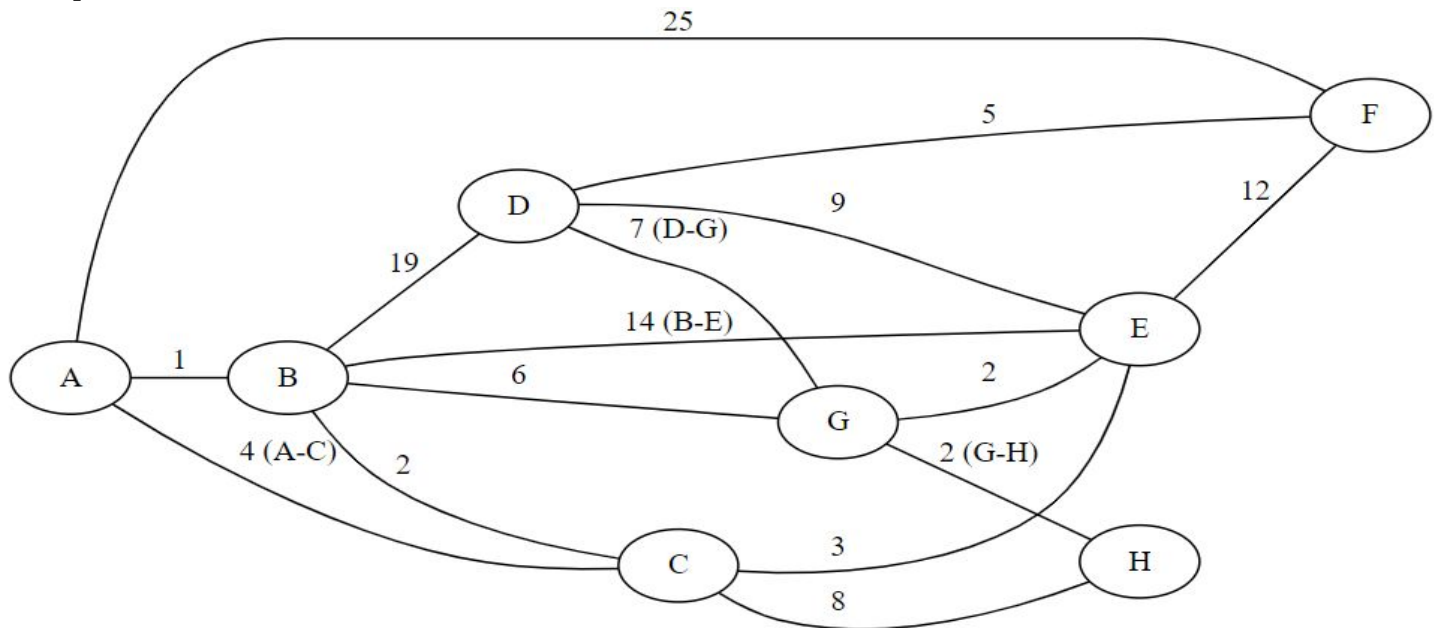
1. Answer each problem in the boxes provided. Any writing outside of the boxes *will NOT be graded*. Do not turn in responses recorded on separate sheets.
2. The size of boxes does *not* necessarily correspond to the amount of space needed to solve the problem.
3. Handwritten or typed responses are accepted. In either case, make sure all answers are in the appropriate boxes.
4. All responses *must* be neat and legible. Illegible answers will result in zero points.
5. Make sure to *scan in portrait mode* and to *select the corresponding pages* on Gradescope for each question.
6. You may use code from any of the class resources, including Stepik. You may not use code from other sources.

1. (5 points - **Correctness**) Shortest Paths

Let A be the shortest path from some vertex s to some other vertex t in a directed graph with nonnegatively weighted edges. If the weight of each edge in the graph is increased by one, will A still be the shortest path from s to t ? If true, prove it (remember, the proof should be comprehensive - i.e it should exhaust all scenarios). If false, provide a counterexample.

2. (2 points - **Completeness**) **Dijkstra's**

Walk-through Dijkstra's algorithm to compute the shortest paths from A to every other node in the given graph. Show your steps in the table below. Do this by crossing out old values and writing in new ones as the algorithm proceeds.



Vertex	Visited	Cost	Previous
A			
B			
C			
D			
E			
F			
G			
H			

(4 points - **Correctness**) All Vertices, In Order Visited: (Visited == Found the Shortest Path to)

(4 points - **Correctness**) Shortest Path from A to F:

3. Disjoint Sets // Union-Find

$\text{Union}(x,y) :=$ Make the sentinel of x point to the sentinel of y . Does not perform the weighting optimization.

You are given a set of unconnected nodes 0 through 7. You now perform the following operations in the given order on the nodes.

Union (0,1), Union (2,3), Union (0,2), Union (4,0), Union(5,6), Union(7,5), Union (5,4), Find (5)

Give the array representation of this disjoint set data structure if the above operations use ...

- (a) (5 points - **Correctness**) ... no path compressions

- (b) (5 points - **Correctness**) ... path compressions

4. P vs NP

- (a) (5 points - **Correctness**) Suppose you have a problem that you believe is in NP-Complete. What is the most common way to go about showing that your intuition is correct? Explain what steps you would take, and why those steps show that the problem is in NP-Complete.

- (b) (5 points - **Correctness**) Suppose you have a problem that you believe is in P. What is the most common way to go about showing that your intuition is correct? Explain what steps you would take, and why those steps show that the problem is in P.

5. (5 points - **Completeness**) **Islands**

You are given a 2D array consisting of '1's (land) and '0's (water). Write pseudocode to count the number of islands in the 2D array. An island is surrounded by water and is formed by lands adjacent to each other either horizontally or vertically, not diagonally. You may assume all four edges of the grid are all surrounded by water.

