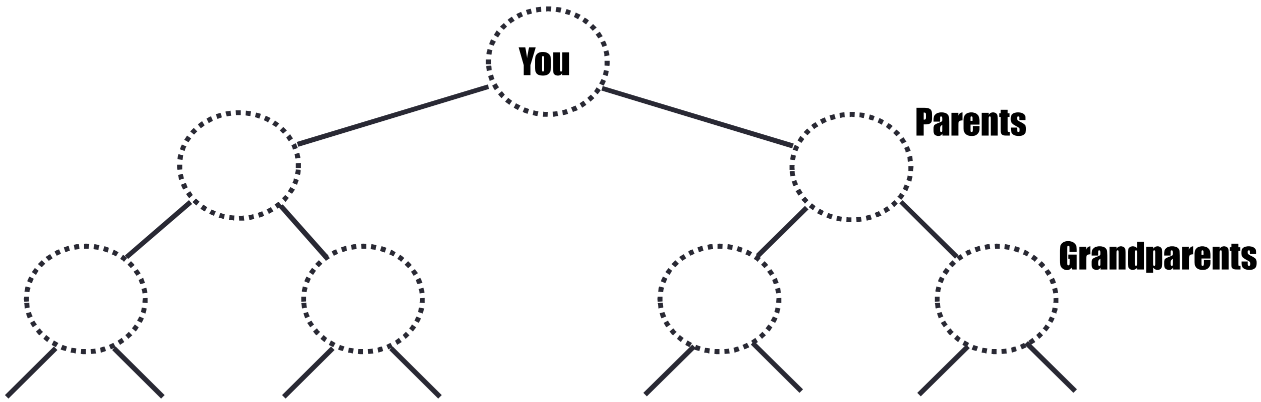
CSE 100: GRAPH

Announcements

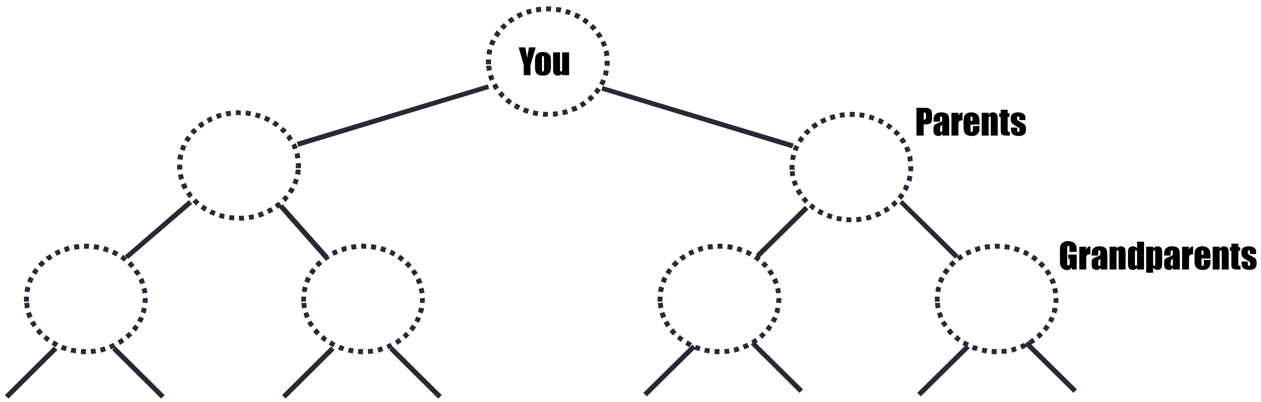
- PA3
  - Checkpoint deadline 11:59pm on Thursday, November 29 (No slip days)
  - Final submission deadline 11:59pm on Thursday, December 6 (slip days allowed)
- No class on Friday
  - Happy Thanksgiving!!

From Trees to Graphs



Is this a tree, or…?

From Trees to Graphs

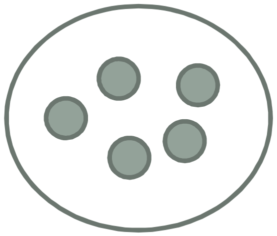


Is this a tree, or…?

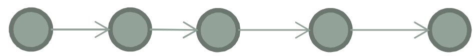
1 generation = 30 years ➔ 100 generations over the last 3000 years

2^100 = 1.267x10^30 (How many people are on earth?)

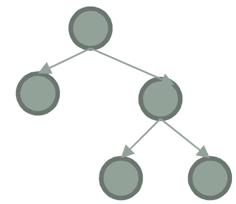
Kinds of Data Structures



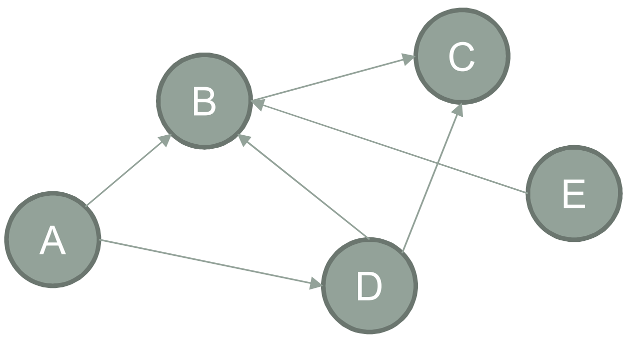
Unstructured structures
(sets)



Sequential, linear structures
(arrays, linked lists)



Hierarchical structures
(trees)

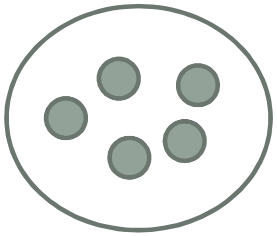


Graphs

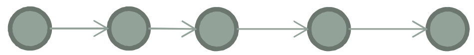
Which of the following is NOT true about graphs?

A. They consist of both vertices and edges
B. They have an inherent order
C. Edges may be weighed or unweighted
D. Edges may be directed or undirected
E. They may contain cycles

Kinds of Data Structures



Unstructured structures
(sets)



Sequential, linear structures
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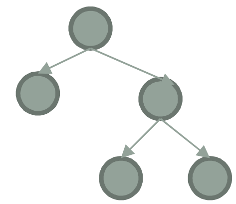
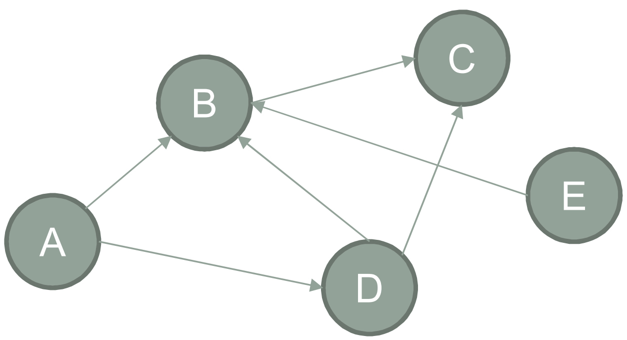


Hierarchical structures
(trees)

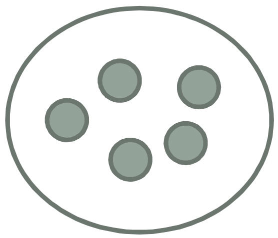


<u>Graphs</u>

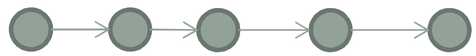
Which of the following is ALWAYS a graph:

A. A list
B. A tree
C. Both
D. Neither

Kinds of Data Structures



Unstructured structures
(sets)



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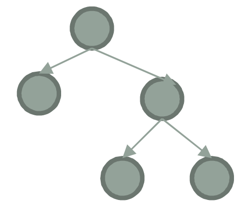
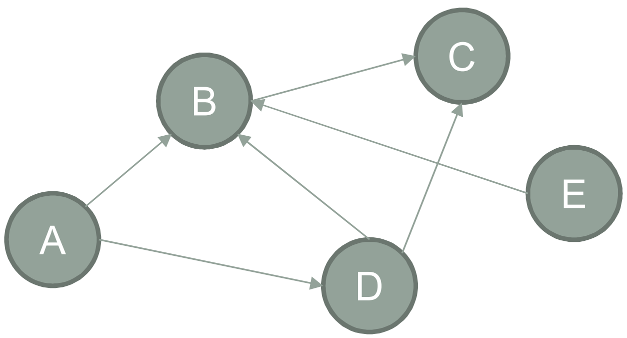


Hierarchical structures
(trees)



<u>Graphs</u>
Consist of:

- A collection of elements (“nodes” or “vertices”)
- A set of connections (“edges” or “links” or “arcs”) between pairs of nodes.
  - Edges may be directed or undirected
  - Edges may have weight associated with them

Graphs are not hierarchical or sequential,
no requirements for a “root” or “parent/child”
relationships between nodes

Note that trees are special cases of graphs; lists are special cases of trees.

Graphs

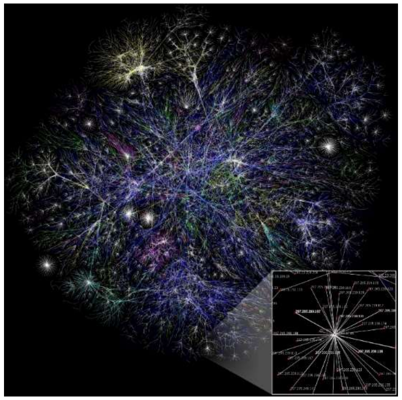
Basic objects : **vertices, nodes**

Relationships between them : **edges, arcs, links**

Graphs

Basic objects : websites

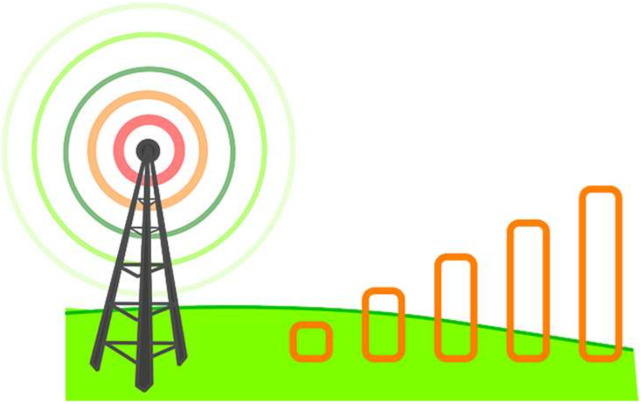
Relationships between them : hyperlinks



Graphs

Basic objects : cell phone towers

Relationships between them : coverage area overlaps



Graphs

Basic objects : **game units**

Relationships between them : **paths on map**

Graphs

Basic objects : people

Relationships between them : friends

Graphs

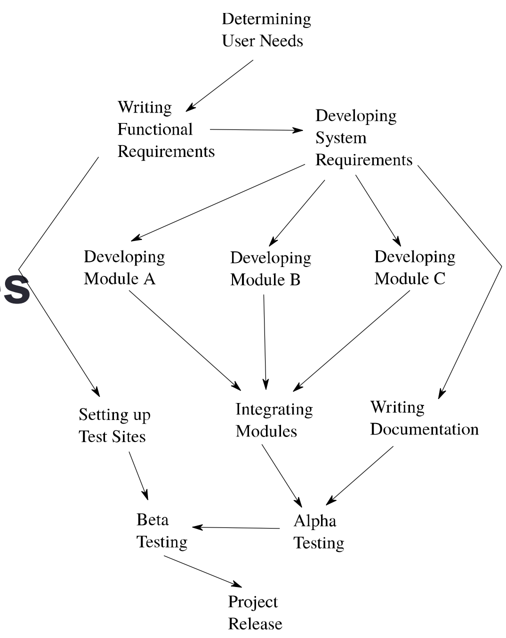
Basic objects : **cities**

Relationships between them : **nonstop flights OR roads**

Graphs

Basic objects : tasks

Relationships between them : dependencies



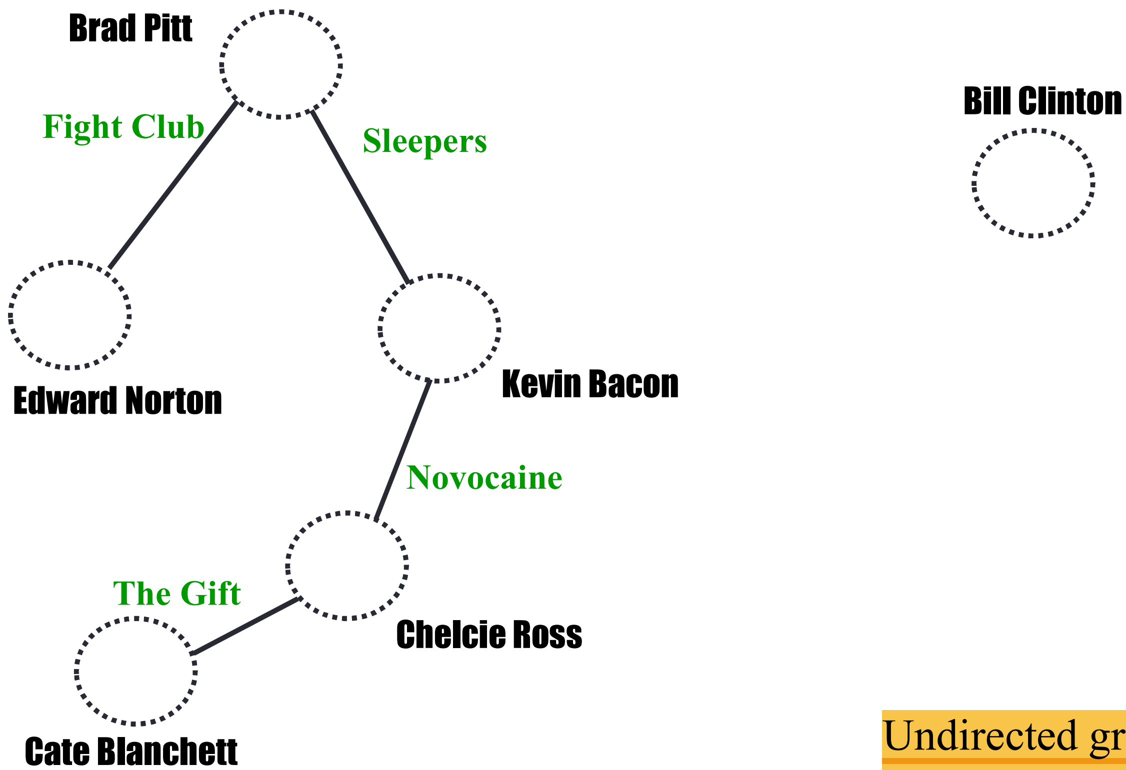
Why Graphs?

But don't just take my word for it…

https://www.coursera.org/learn/advanced-data-structures/lecture/3ovpb/in-the-real-world-graphs-at-google

https://www.coursera.org/learn/advanced-data-structures/lecture/ACQAt/in-the-real-world-more-graphs-at-google

Another (Important?) Application of Graphs

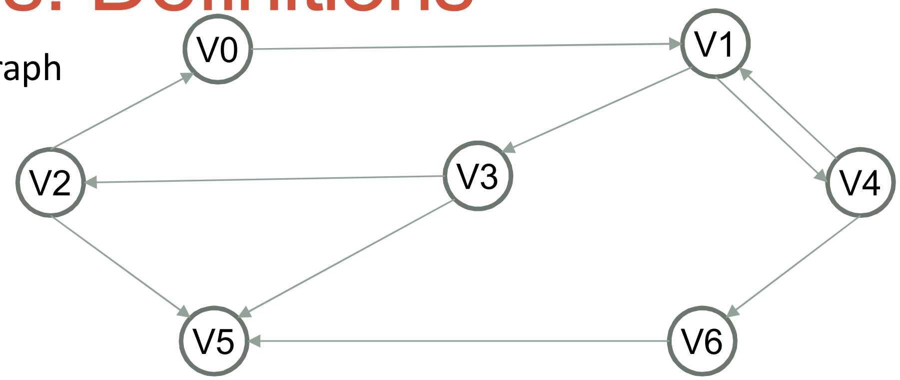


The "Oracle of Bacon" at **oracleofbacon.org/**

Undirected graphs model relationships in which all connections are two-way.

Graphs: Definitions

A directed graph

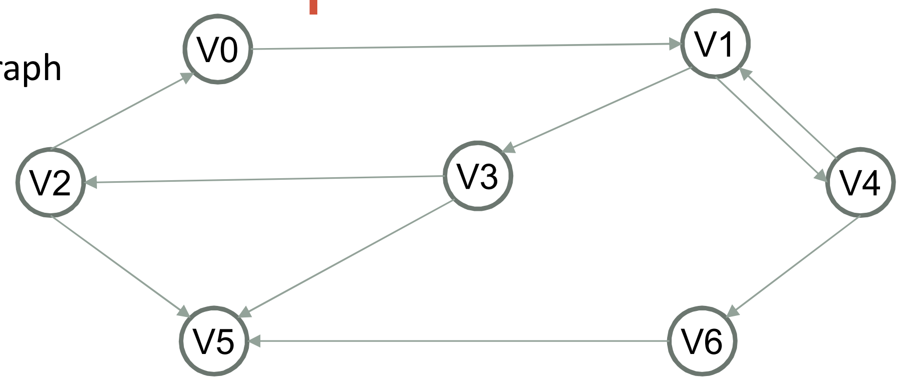


A graph G = (V,E) consists of a set of vertices V and a set of edges E

- Each edge in E is a pair (v,w) such that v and w are in V.
- If G is an *undirected* graph, (v,w) in E means vertices v and w are connected by an edge in G. This (v,w) is an unordered pair
- If G is a *directed* graph, (v,w) in E means there is an edge going from vertex v to vertex w in G. This (v,w) is an ordered pair; there may or may not also be an edge (w,v) in E
- In a *weighted* graph, each edge also has a “weight” or “cost” c, and an edge in E is a triple (v,w,c)
- When talking about the size of a problem involving a graph, the number of vertices |V| and the number of edges |E| will be relevant

Graphs: Example

A directed graph



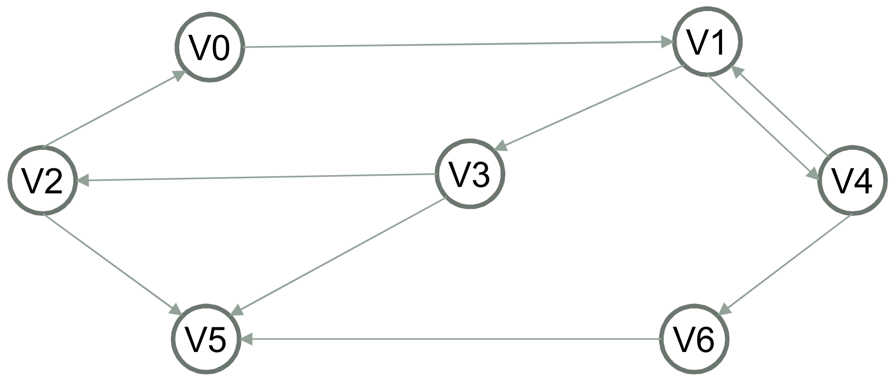
V = {

|V| =

E = {

|E|

Representing Graphs: Adjacency Matrix

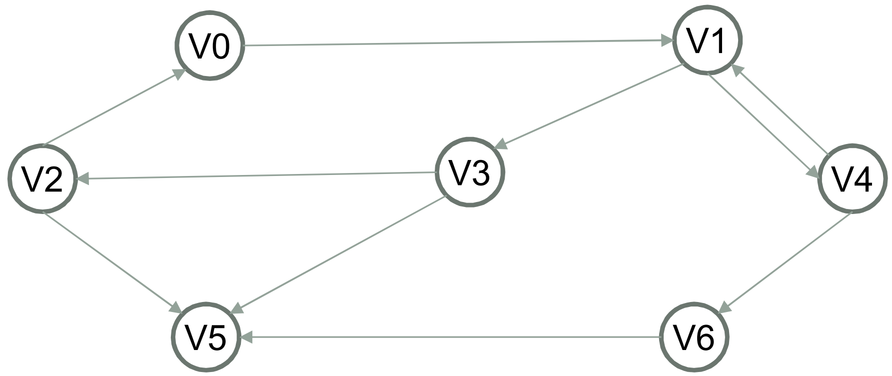


	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

A 2D array where each entry [i][j] encodes connectivity information between i and j

- For an unweighted graph, the entry is 1 if there is an edge from i to j, 0 otherwise
- For a weighted graph, the entry is the weight of the edge from i to j, or “infinity” if there is no edge
- Note an undirected graph’s adjacency matrix will be symmetrical

Representing Graphs: Adjacency Matrix



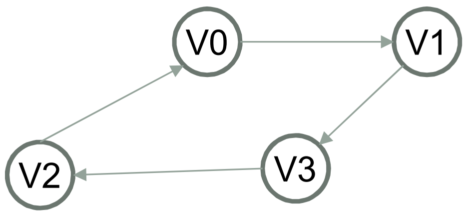
	0	1	2	3	4	5	6
0		1					
1				1	1		
2	1					1	
3			1			1	
4		1					1
5							
6						1	

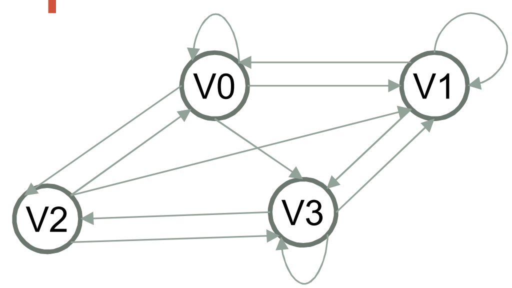
How big is an adjacency matrix in terms of the number of nodes and edges (BigO, tightest bound)?

A. |V|
B. |V|+|E|
C. |V|^2
D. |E|^2
E. Other

When is that OK? When is it a problem?

Sparse vs. Dense Graphs





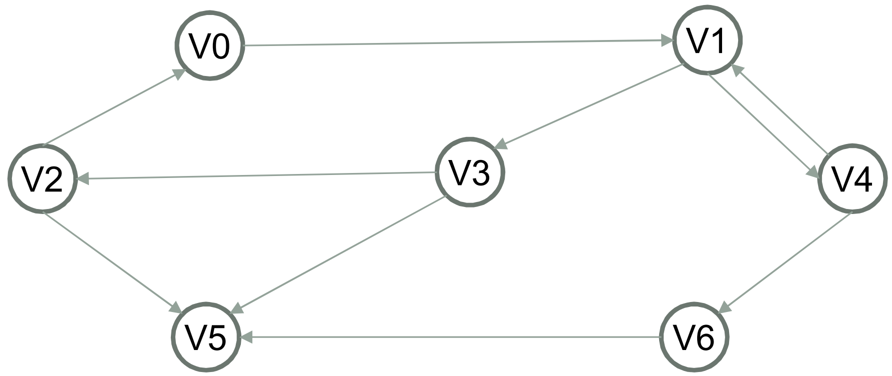
	0	1	2	3
0	0	1	0	0
1	0	0	0	1
2	1	0	0	0
3	0	0	1	0

	0	1	2	3
0	1	1	1	1
1	1	1	0	1
2	1	1	0	1
3	0	1	1	1

A dense graph is one where $|E|$ is "close to" $|V|^2$.
A sparse graph is one where $|E|$ is "closer to" $|V|$.

Adjacency matrices are space inefficient for sparse graphs

Representing Graphs: Adjacency Lists



V0:

V1:

V2:

V3:

V4:

V5:

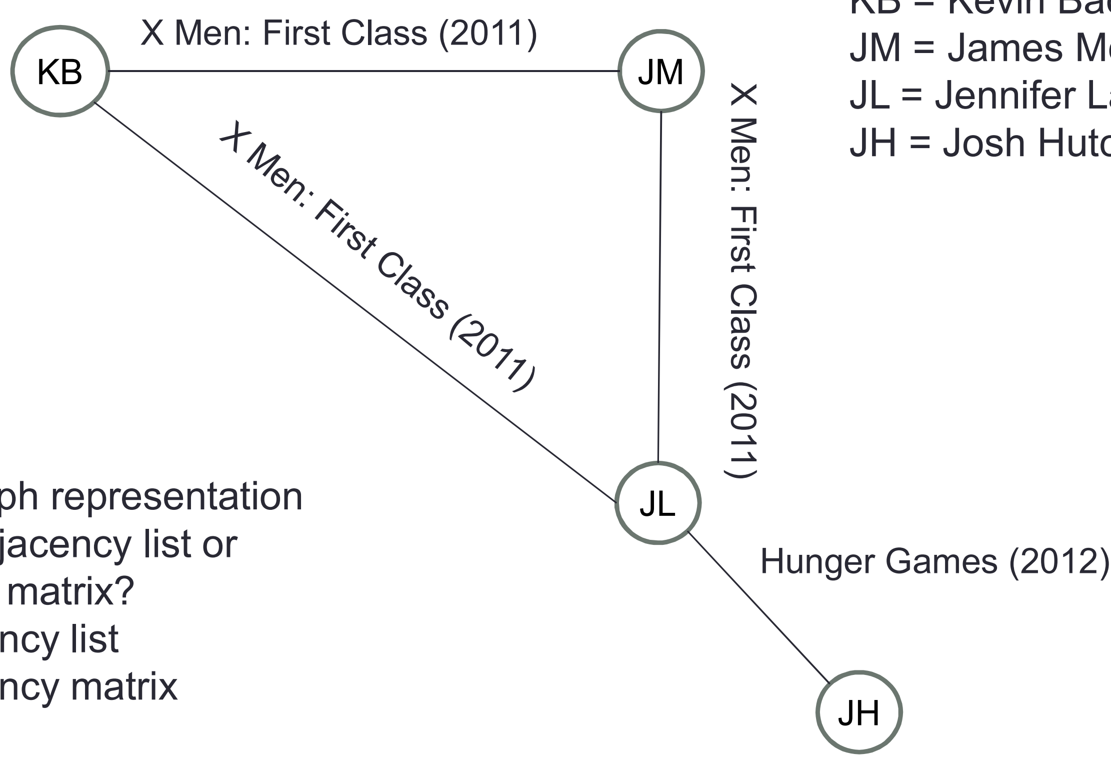
V6:

Each vertex has a list with the vertices adjacent to it.
In a weighted graph this list will include weights.

How much storage does this representation need?
(BigO, tightest bound)

A. |V|
B. |E|
C. |V|+|E|
D. |V|^2
E. |E|^2

Movie graphs: Matrix vs Lists

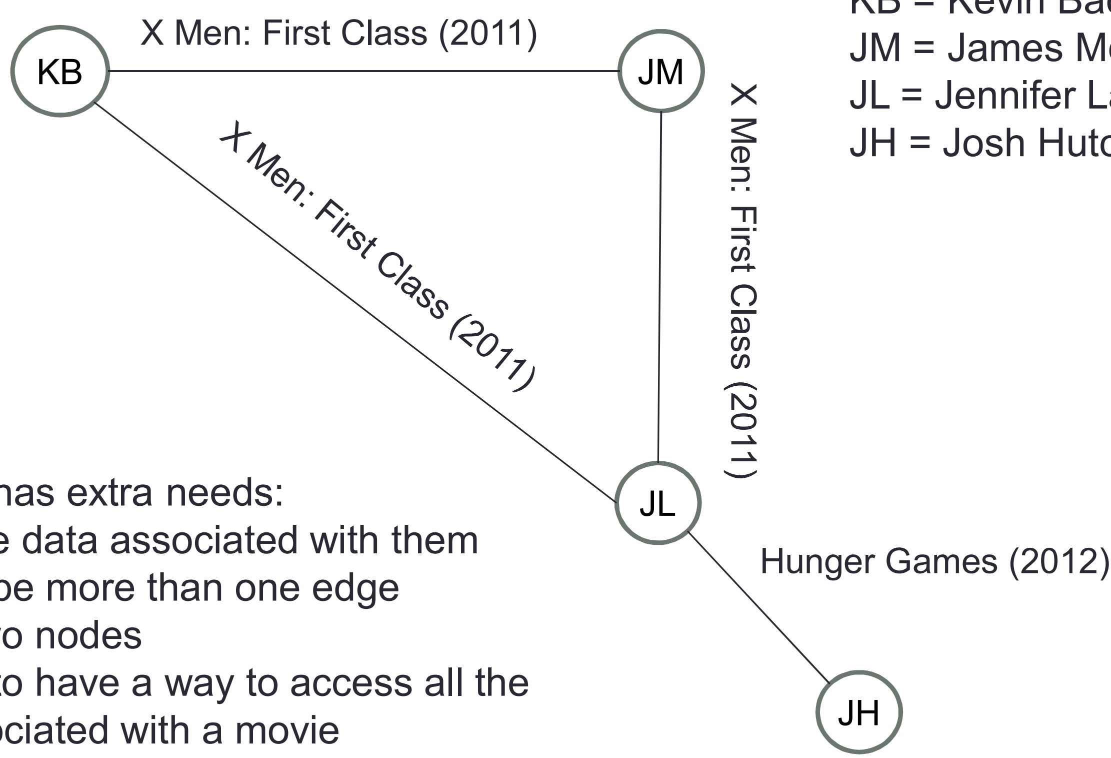


KB = Kevin Bacon
JM = James McAvoy
JL = Jennifer Lawrence
JH = Josh Hutcherson

Which graph representation is best: adjacency list or adjacency matrix?

A. Adjacency list
B. Adjacency matrix

Movie graphs: Representation hints

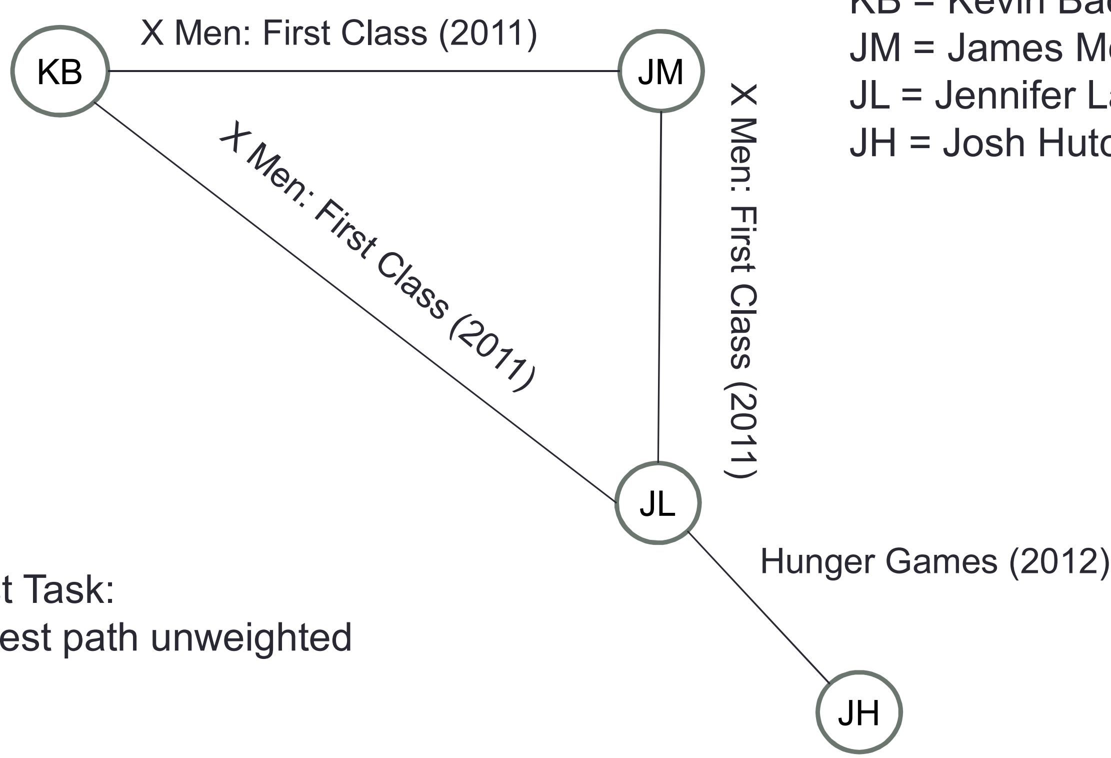


KB = Kevin Bacon
JM = James McAvoy
JL = Jennifer Lawrence
JH = Josh Hutcherson

This problem has extra needs:

- Edges have data associated with them
- There can be more than one edge between two nodes
- It is useful to have a way to access all the actors associated with a movie
- You will need to adapt the data structures we look at in class to meet your needs for PA3

Movie graphs: Representation hints



KB = Kevin Bacon
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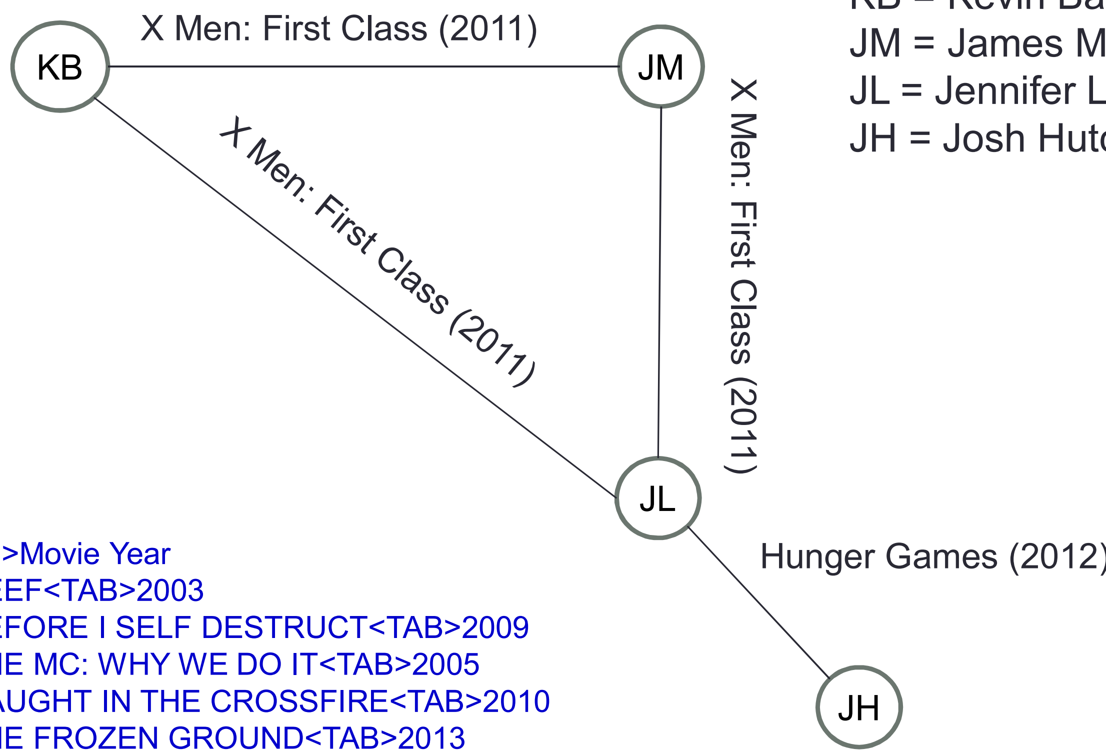
- PA3's First Task:
  - Shortest path unweighted

Movie graphs: Representation hints



KB = Kevin Bacon
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Actor/Actress<TAB>Movie Year
50 CENT<TAB>BEEF<TAB>2003
50 CENT<TAB>BEFORE I SELF DESTRUCT<TAB>2009
50 CENT<TAB>THE MC: WHY WE DO IT<TAB>2005
50 CENT<TAB>CAUGHT IN THE CROSSFIRE<TAB>2010
50 CENT<TAB>THE FROZEN GROUND<TAB>2013
50 CENT<TAB>BEEF III<TAB>2005
50 CENT<TAB>LAST VEGAS<TAB>2013
50 CENT<TAB>GUN<TAB>2010

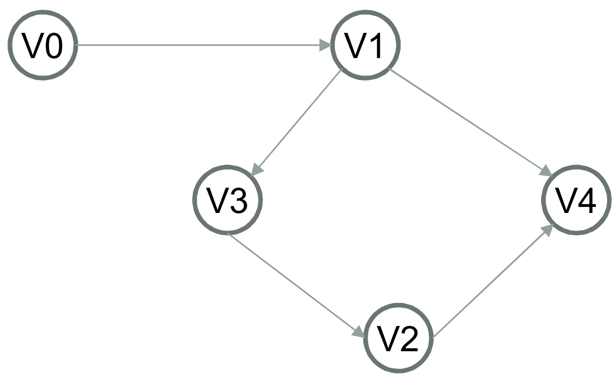
- Take time to come up with at least two ways to store this structure.

Movie graphs: Representation

Actor/Actress<TAB>Movie Year
50 CENT<TAB>BEEF<TAB>2003
50 CENT<TAB>BEFORE I SELF DESTRUCT<TAB>2009
50 CENT<TAB>THE MC: WHY WE DO IT<TAB>2005
50 CENT<TAB>CAUGHT IN THE CROSSFIRE<TAB>2010
50 CENT<TAB>THE FROZEN GROUND<TAB>2013
50 CENT<TAB>BEEF III<TAB>2005
50 CENT<TAB>LAST VEGAS<TAB>2013
50 CENT<TAB>GUN<TAB>2010

Depth First Search for Graph Traversal

- Search as far down a single path as possible before backtracking

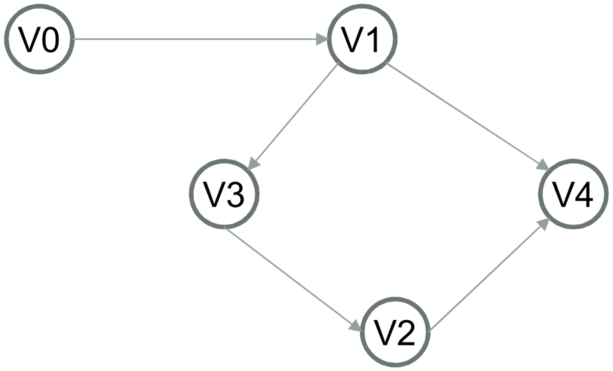


Assuming DFS chooses the lower number node to explore first,
in what order does DFS visit the nodes in this graph (start at V0)?

A. V0, V1, V2, V3, V4
B. V0, V1, V3, V4, V2
C. V0, V1, V3, V2, V4
D. Other

Depth First Search for Graph Traversal

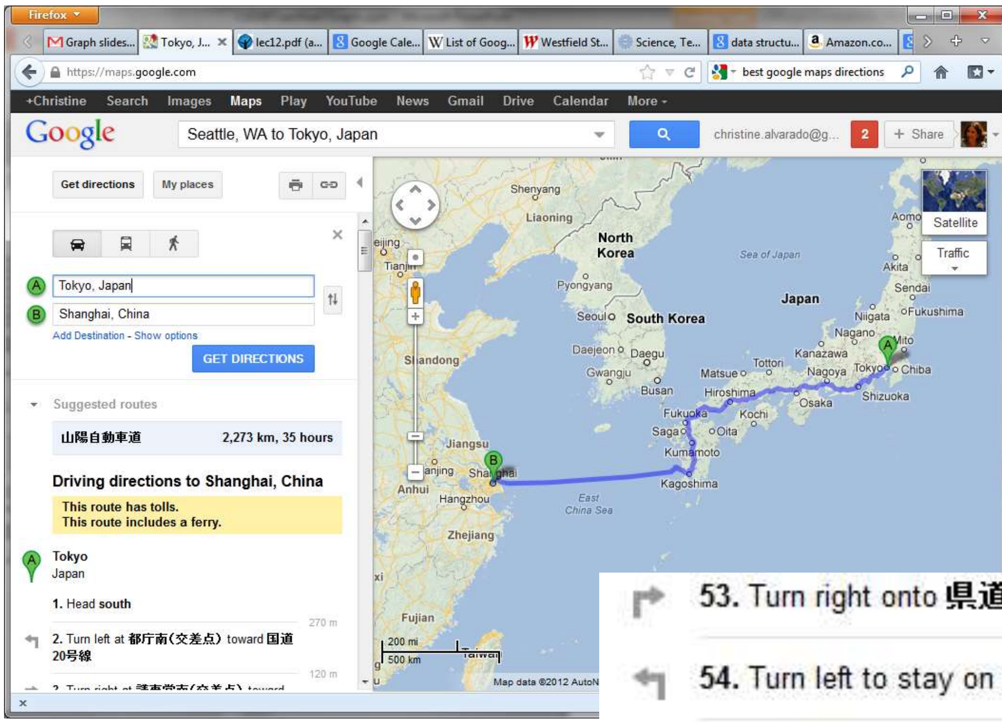
- Search as far down a single path as possible before backtracking



Does DFS always find the shortest path between nodes the first time it encounters a node in its search?

A. Yes
B. No

Shortest Path Algorithms



Finding the shortest route from one city to another is a natural application of graph algorithms!

(Of course there are many other examples)

Shortest Path Algorithms

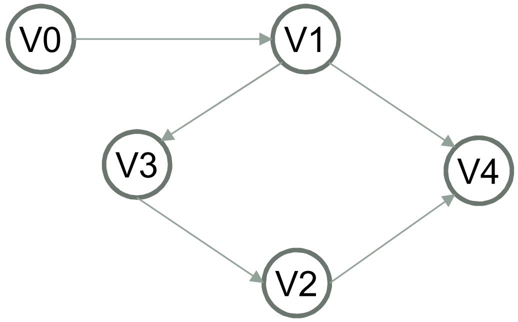
- We'll look at shortest path algorithms in unweighted and weighted graphs
- These algorithms find the shortest path from a "source" (or start) vertex to every other vertex in the graph (it's no slower than finding a path to just one destination)
- You will implement some of these algorithms in your PA3

Unweighted Shortest Path

- Input: an unweighted directed graph G = (V, E) and a source vertex *s* in V
- Output: for each vertex *v* in V, a representation of the shortest path in G that starts in *s* and ends at *v*
- This is really just a search problem. We'll look at three algorithms:
  - Depth First Search – inefficient to produce the shortest path
  - Breadth First Search
  - Best-First Search (for weighted graphs)

Breadth First Search

- Explore all the nodes reachable from a given node before moving on to the next node to explore

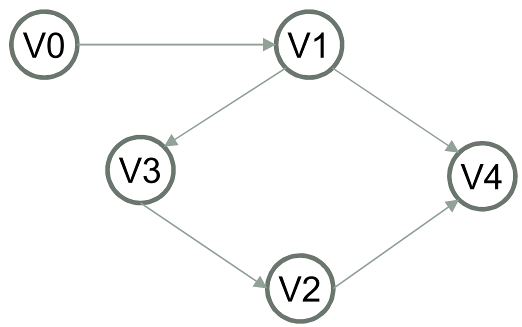


Assuming BFS chooses the lower number node to explore first, in what order does BFS visit the nodes in this graph?

A. V0, V1, V2, V3, V4
B. V0, V1, V3, V4, V2
C. V0, V1, V3, V2, V4
D. Other

BFS Traverse: Idea

- Input: an unweighted directed graph G = (V, E) and a source vertex *s* in V
- Output: for each vertex *v* in V, a representation of the shortest path in G that starts in *s* and ends at *v*

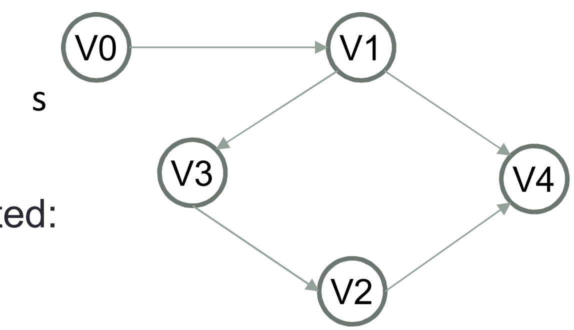


Start at *s*. It has distance 0 from itself.
Consider nodes adjacent to s. They have distance 1. Mark them as visited.
Then consider nodes that have not yet been visited
adjacent to those at distance 1. They have distance 2. Mark them as visited.
Etc. etc. until all nodes are visited.

BFS Traverse: Sketch of Algorithm

The basic idea is a breadth-first search of the graph, starting at source vertex *s*

- Initially, give all vertices in the graph a distance of INFINITY
- Start at *s*; give *s* distance = 0
- Enqueue *s* into a queue
- While the queue is not empty:
  - Dequeue the vertex *v* from the head of the queue
  - For each of *v*'s adjacent nodes that has not yet been visited:
    - Mark its distance as 1 + the distance to *v*
    - Enqueue it in the queue



Queue:

BFS Traverse: Sketch of Algorithm

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- Start at *s*; give *s* distance = 0
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  - For each of *v*'s adjacent nodes that has not yet been visited:
    - Mark its distance as 1 + the distance to *v*
    - Enqueue it in the queue

Questions:

- What data do you need to keep track of for each node?
- How can you tell if a vertex has been visited yet?
- This algorithm finds the length of the shortest path from s to all nodes. How can you also find the path itself?

BFS Traverse: Details

source

```
V0: dist=          prev=       adj: V1

V1: dist=          prev=       adj: V3, V4

V2: dist=          prev=       adj: V0, V5

V3: dist=          prev=       adj: V2, V5, V6

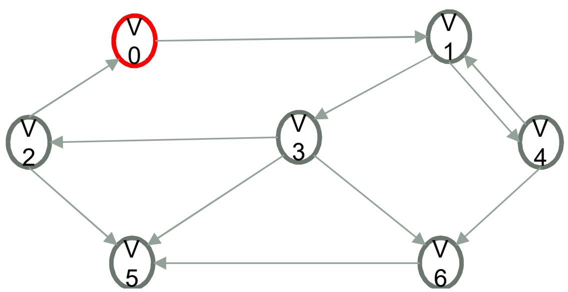
V4: dist=          prev=       adj: V1, V6

V5: dist=          prev=       adj:

V6: dist=          prev=       adj: V5
```

The queue (give source vertex dist=0 and prev=-1 and enqueue to start):

HEAD **TAIL**



Representing the graph with `structs`

```
#include <iostream>
#include <limits>
#include <vector>
#include <queue>

using namespace std;

struct Vertex {
  vector<int> adj;  // The adjacency list
  int dist;         // The distance from the source
  int index;  // The index of this vertex
  int prev;   // The index of the vertex previous in the path
};

vector<Vertex*> createGraph() {
…
}
```

Unweighted Shortest Path: C++ code

```
/** Traverse the graph using BFS */
void BFSTraverse( vector<Vertex*> theGraph, int from )
{
  // assume code to initialize each Vertex's dist to INFINITY
  queue<Vertex*> toExplore;
  Vertex* start = theGraph[from];
  // finish the code…

}
```

```
struct Vertex
{
  vector<int> adj;
  int dist;
  int index;
  int prev;
};
```

Unweighted Shortest Path: C++ code

```
/** Traverse the graph using BFS */
void BFSTraverse( vector<Vertex*> theGraph, int from )
{
  // assume code to initialize each Vertex's dist to INFINITY
  queue<Vertex*> toExplore;
  Vertex* start = theGraph[from];
  start->dist = 0;
  toExplore.push(start);
  while ( !toExplore.empty() ) {

    Vertex* next = toExplore.front();
    toExplore.pop();
    vector<int>::iterator it = next->adj.begin();
    for ( ; it != next->adj.end(); ++it ) {
      Vertex* neighbor = theGraph[*it];
      if (next->dist+1 < neighbor->dist) {
        neighbor->dist = next->dist + 1;
        neighbor->prev = next->index;
        toExplore.push(neighbor);
      }
    }
  }
}
```

Unweighted Shortest Path: Running Time

The basic idea is a breadth-first search of the graph, starting at source vertex *s*

- Initially, give all vertices in the graph a distance of INFINITY
- Start at *s*; give *s* distance = 0
- Enqueue *s* into a queue
- While the queue is not empty:
  - Dequeue the vertex *v* from the head of the queue
  - For each of *v*'s adjacent nodes that has not yet been visited:
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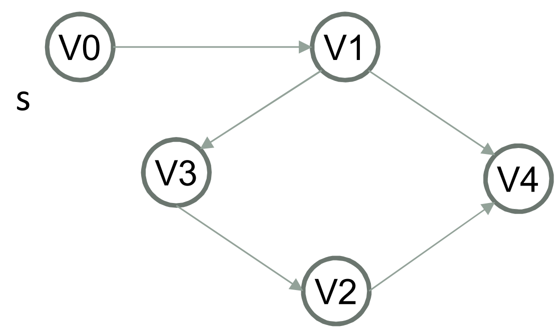

What is the tightest worst-case time complexity (in terms of |V| and |E|) of this algorithm?

A. O(|V|) B. O(|E|) C. O(|V|+|E|)

D. O(|V|^2) E. Other

Representing the graph with `structs`

```
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#include <limits>
#include <vector>
#include <queue>

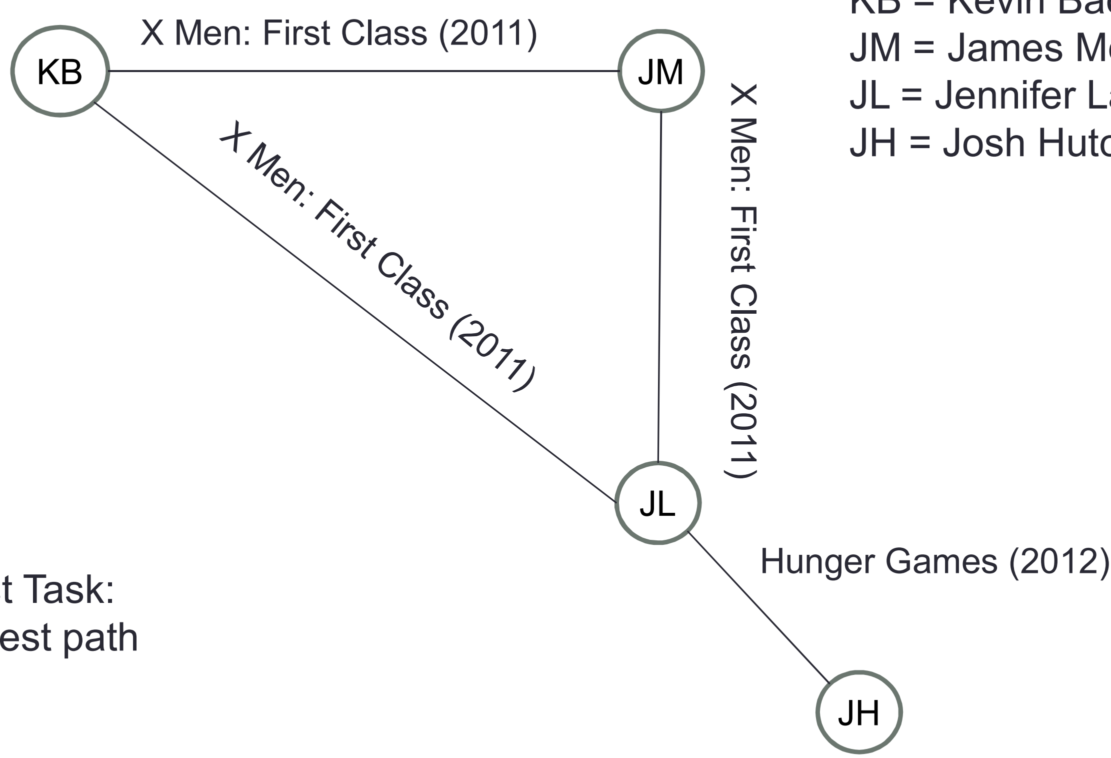
using namespace std;

struct Vertex
{
  vector<int> adj;  // The adjacency list
  int dist;         // The distance from the source
  int index;  // The index of this vertex
  int prev;   // The index of the vertex previous in the path
};

vector<Vertex*> createGraph()
{ … }
```

Your representation for PA3 will have some similarities and probably some differences.

Movie graphs: Representation hints



KB = Kevin Bacon
JM = James McAvoy
JL = Jennifer Lawrence
JH = Josh Hutcherson

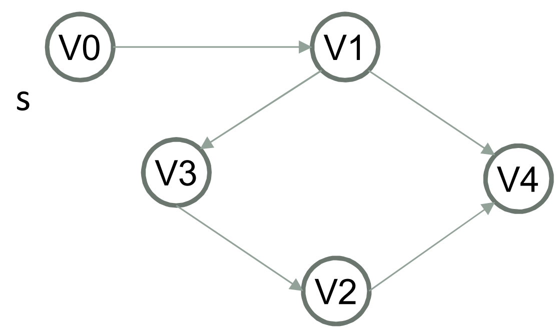
- PA3's First Task:
  - Shortest path

What is this algorithm??

The basic idea is a breadth-first search of the graph, starting at source vertex *s*

- Initially, give all vertices in the graph a distance of INFINITY
- Start at *s*; give *s* distance = 0
- ~~Enqueue~~ Push *s* into a ~~queue~~ stack
- While the ~~queue~~ stack is not empty:
  - ~~Dequeue~~ pop the vertex *v* from the ~~head of the queue~~ top of the stack
  - For each of *v*'s adjacent nodes that has not yet been visited:
    - Mark its distance as 1 + the distance to *v*
    - ~~Enqueue~~ Push it on the ~~queue~~ stack

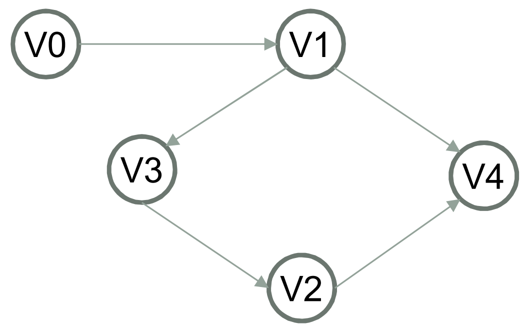
Stack:



A. BFS B. DFS shortest path C. DFS not shortest path D. Dijkstra's algorithm

Breadth First Search

- Explore all the nodes reachable from a given node before moving on to the next node to explore

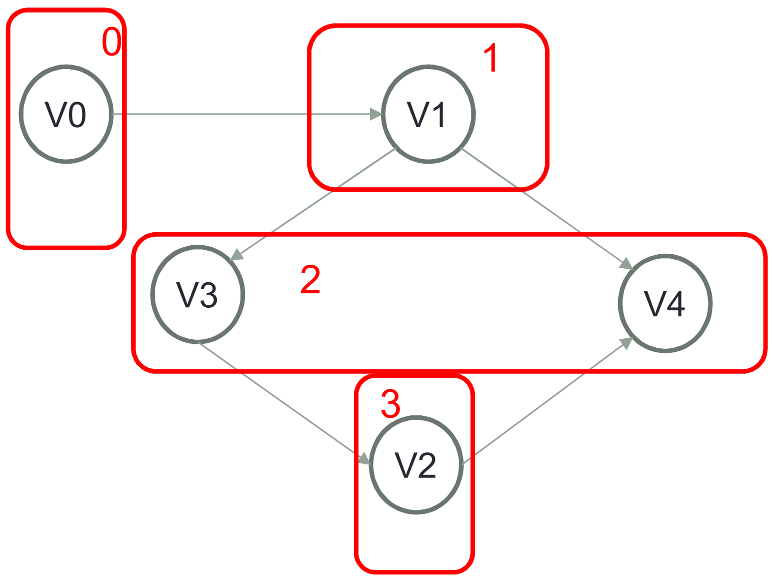


Does BFS always find the shortest path from the source to any node?

A. Yes for unweighted graphs
B. Yes for all graphs
C. No

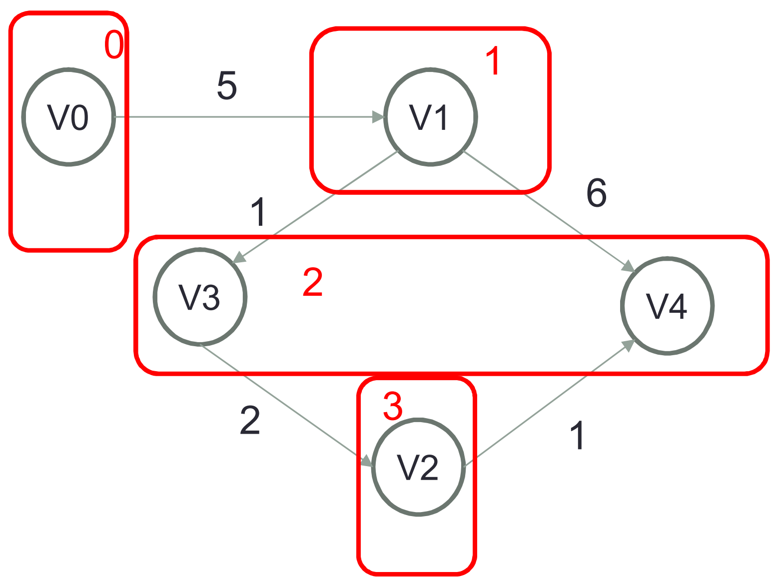
Why Did BFS Work for Shortest Path?

- Vertices are explored in order of their distance away from the source. So we are guaranteed that the first time we see a vertex we have found the shortest path to it.
- The queue (FIFO) assures that we will explore from all nodes in one level before moving on to the next.



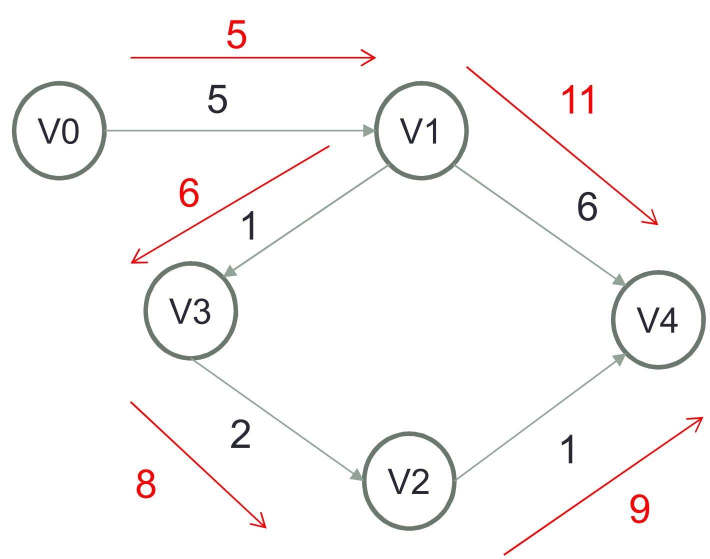
BFS on weighted graphs?

- Run BFS on this weighted graph. What are the weights of the paths that you find to each vertex from v0? Are these the shortest paths?



BFS on weighted graphs?

- In a weighted graph, the number of edges no longer corresponds to the length of the path. We need to decouple path length from edges, and explore paths in increasing *path length* (rather than increasing number of edges).
- In addition, the first time we encounter a vertex may, we may not have found the shortest path to it, so we need to delay committing to that path.



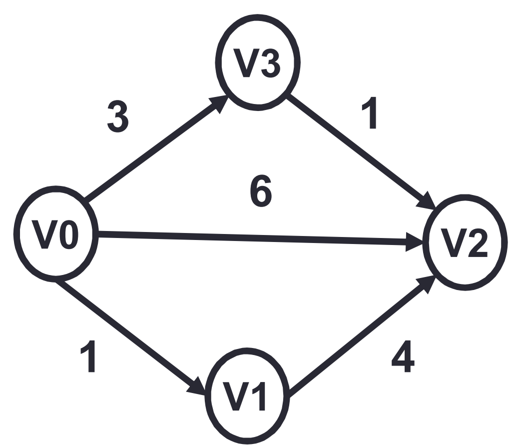
Dijkstra’s Algorithm: Data Structures

- Maintain a sequence (e.g. an array) of vertex objects, indexed by vertex number
  - Vertex objects contain these 3 fields (and others):
    - “dist”: the cost of the best (least-cost) path discovered so far from the start vertex to this vertex
    - “prev”: the vertex number (index) of the previous node on that best path
    - “done”: a boolean indicating whether the “dist” and “prev” fields contain the final best values for this vertex, or not
- Maintain a priority queue
  - The priority queue will contain (*pointer-to-vertex*, *path cost*) pairs
  - *Path cost* is priority, in the sense that low cost means high priority
  - Note: multiple pairs with the same “*pointer-to-vertex*” part can exist in the priority queue at the same time. These will usually differ in the “*path cost*” part

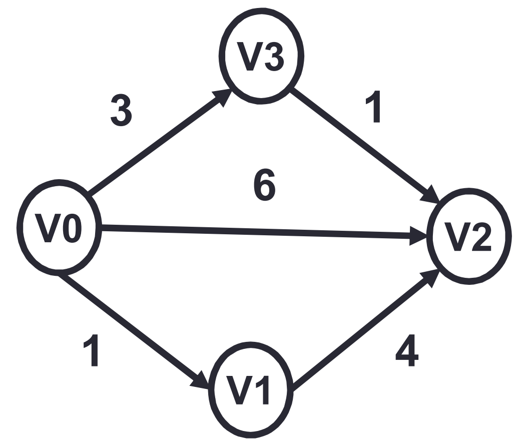
Dijkstra's Algorithm

Nodes have:
prev
dist
done

```
Dijkstra(S):
  Initialize: Priority queue (PQ), dist fields to infinity,
              prev fields to -1, done fields to false
  Set S's dist to 0
  Enqueue {S, 0} onto the PQ
  while PQ is not empty:
    dequeue node v from front of queue
    if (v is not done)
      set v.done to true
      for each of v's neighbors, w:
        //distance to w through v is:
        c = v.dist + edgeWeight(v, w)
        if c is less than w.dist:
          set w.prev = v and w.dist = c
          enqueue {w, c} into the PQ
```



The array of vertices, which include dist, prev, and done fields (initialize dist to ‘INF’ and done to ‘f’):



V0: dist= **prev=** **done=**

V1: dist= **prev=** **done=**

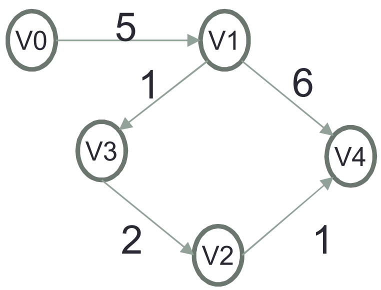
V2: dist= **prev=** **done=**

V3: dist= **prev=** **done=**

Initial PQ:	(0, V0)					
PQ after expanding V0:						

Dijkstra’s Algorithm: Questions

```
Dijkstra(S):
   Initialize: Priority queue (PQ), dist fields to infinity, prev fields to -1, done fields to false
   Enqueue {S, 0} onto the PQ
   while PQ is not empty:
      dequeue node v from front of queue
      if (v is not done)
         set v.done to true
         for each of v's neighbors, w:
             distance to w through v, c = v.dist + edgeWeight(v, w)
                if c is less than w.dist:
                set w.prev = v and w.dist = c
                enqueue {w, c} into the PQ
```



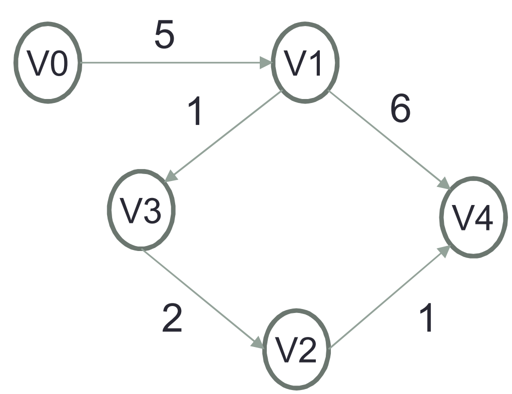
When a node comes out of the priority queue, how do you know you’ve found the shortest path to the node?

Dijkstra’s Algorithm: Running time

```
Dijkstra(S):
    Initialize: Priority queue (PQ), dist fields to infinity, prev fields to -1, done fields to false
    Enqueue {S, 0} onto the PQ
    while PQ is not empty:
        dequeue node v from front of queue
        if (v is not done)
            set v.done to true
            for each of v's neighbors, w:
                distance to w through v, c = v.dist + edgeWeight(v, w)
                if c is less than w.dist:
                    set w.prev = v and w.dist = c
                    enqueue {w, c} into the PQ
```

How long does the step in red take?

A. O(1)
B. O(|V|)
C. O(|E|)
D. O(|V|+|E|)
E. Other



Dijkstra's Algorithm: Running time

```
Dijkstra(S):
  Initialize: Priority queue (PQ), dist fields to infinity, prev fields to -1, done fields to false
  Enqueue {S, 0} onto the PQ
  while PQ is not empty:
    dequeue node v from front of queue
    if (v is not done)
      set v.done to true
      for each of v's neighbors, w:
        distance to w through v, c = v.dist + edgeWeight(v, w)
        if c is less than w.dist:
          set w.prev = v and w.dist = c
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```

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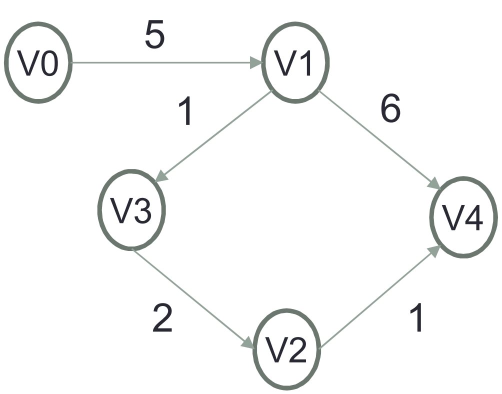

Walkthrough

```
Dijkstra(S, G):
    Initialize: Priority queue (PQ), dist fields to infinity,
                prev fields to -1, done fields to false
    Enqueue {S, 0} onto the PQ
    while PQ is not empty:
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                distance to w through v, c = v.dist + edgeWeight(v, w)
                if c is less than w.dist:
                    set w.prev = v and w.dist = c
                    enqueue {w, c} into the PQ
```

O(|V|)

O(|1|)

Walkthrough

Dijkstra(S, G):

```
Dijkstra(S, G):
    Initialize: Priority queue (PQ), dist fields to infinity,
                prev fields to -1, done fields to false
    Enqueue {S, 0} onto the PQ
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                distance to w through v, c = v.dist + edgeWeight(v, w)
                if c is less than w.dist:
                    set w.prev = v and w.dist = c
                    enqueue {w, c} into the PQ
```

$O(|V|)$

$\cancel{O(|1|)}$

$O(|V|+\cancel{1})$

Walkthrough

Dijkstra(S, G):

Initialize: Priority queue (PQ), dist fields to infinity,
prev fields to -1, done fields to false

Enqueue {S, 0} onto the PQ

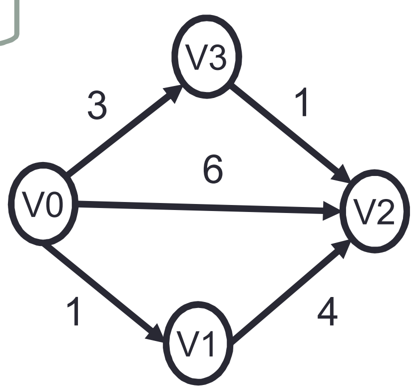
while PQ is not empty:
dequeue node v from front of queue
if (v is not done)
set v.done to true
for each of v's neighbors, w:
distance to w through v, c = v.dist + edgeWeight(v, w)
if c is less than w.dist:
set w.prev = v and w.dist = c
enqueue {w, c} into the PQ

O(|V|)

O(?)

Pairs of (node, cost) go into the priority queue. Can a node go into the priority queue more than once?

A. Yes
B. No



Walkthrough

Dijkstra(S, G):

Initialize: Priority queue (PQ), dist fields to infinity, prev fields to -1, done fields to false

Enqueue {S, 0} onto the PQ

```
while PQ is not empty:
    dequeue node v from front of queue
    if (v is not done)
        set v.done to true
        for each of v's neighbors, w:
            distance to w through v, c = v.dist + edgeWeight(v, w)
            if c is less than w.dist:
                set w.prev = v and w.dist = c
                enqueue {w, c} into the PQ
```

O(|V|)

O(?)

The total number of pairs that go into the priority queue is approximately which of the following (in the worst case):

A. |V| (the number of nodes in the graph)
B. |E| (the number of edges in the graph)
C. |V| + |E|
D. |V| * |E|

Walkthrough

Dijkstra(S, G):

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            prev fields to -1, done fields to false
Enqueue {S, 0} onto the PQ
while PQ is not empty:
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    if (v is not done)
        set v.done to true
        for each of v's neighbors, w:
            distance to w through v, c = v.dist + edgeWeight(v, w)
            if c is less than w.dist:
                set w.prev = v and w.dist = c
                enqueue {w, c} into the PQ
```

O(|V|)

O(?)

So the while loop is making O(|E|) insertions into a priority queue with size at most O(|E|).
What is the total running time for the while loop?

A. O(|E|)
B. O(|E| log |E|)
C. O(|E| * |E|)
D. Other

Walkthrough

Dijkstra(S, G):

```
Dijkstra(S, G):
    Initialize: Priority queue (PQ), dist fields to infinity,
                prev fields to -1, done fields to false
    Enqueue {S, 0} onto the PQ
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            set v.done to true
            for each of v's neighbors, w:
                distance to w through v, c = v.dist + edgeWeight(v, w)
                if c is less than w.dist:
                    set w.prev = v and w.dist = c
                    enqueue {w, c} into the PQ
```

`O(|V|)`

whole loop `O(|E| log |E|)`

Walkthrough

Dijkstra(S, G):

```
Initialize: Priority queue (PQ), dist fields to infinity,
            prev fields to -1, done fields to false
Enqueue {S, 0} onto the PQ
while PQ is not empty:
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         if c is less than w.dist:
            set w.prev = v and w.dist = c
            enqueue {w, c} into the PQ
```

$\boxed{O(|V|)}$

whole loop

$\boxed{O(|E| \log |E|)}$

Overall:
$O(|E| \log |E| + |V|)$

Walkthrough

Dijkstra(S, G):

```
Initialize: Priority queue (PQ), dist fields to infinity,
            prev fields to -1, done fields to false
Enqueue {S, 0} onto the PQ
while PQ is not empty:
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    for each of v's neighbors, w:
      distance to w through v, c = v.dist + edgeWeight(v, w)
      if c is less than w.dist:
        set w.prev = v and w.dist = c
        enqueue {w, c} into the PQ
```

$\boxed{O(|V|)}$

whole loop $\boxed{O(|E| \log |E|)}$

Overall:
$O(|E| \log |E| + |V|)$

Because $|E| <= |V|^2$ and $\log(|V|^2)$ is just $O(\log(|V|))$ we could tighten to:
$O(|E| \log |V| + |V|)$

Unweighted Shortest Path: Running Time

```
BFS(S):
   Initialize queue, set dist to INFINITY and prev to null for all nodes
   Add S to queue and set S.dist to 0
   while queue is not empty:
      dequeue node curr from head of queue
      set n.visited = true
      for each of curr's neighbors, n:
         if n.dist > curr.dist+1:
            set n.dist to curr.dist+1
            set n's prev to curr
            enqueue n to the queue
// When we get here then we're done exploring from S
```

What is the time complexity (in terms of |V| and |E|) of this algorithm?

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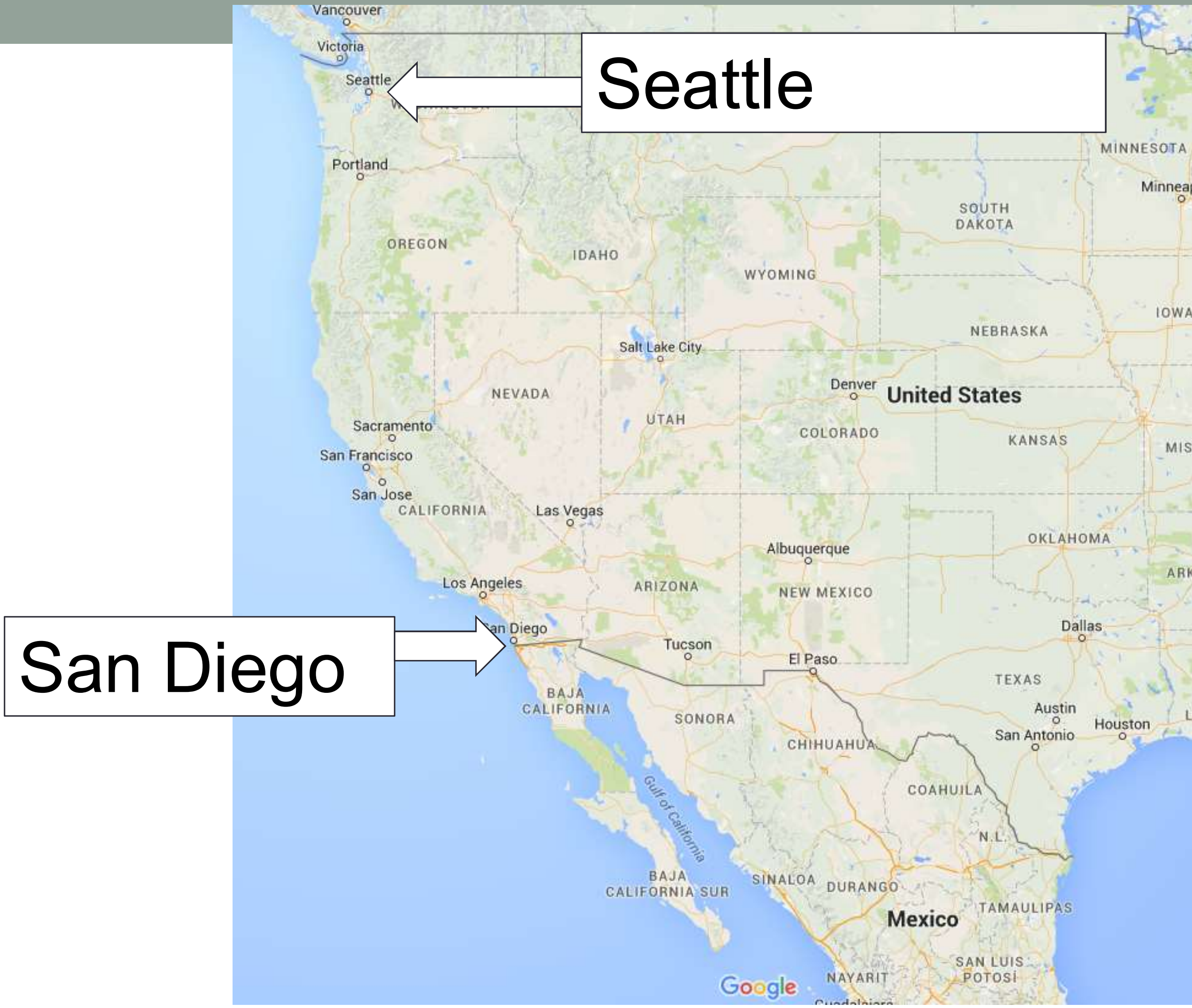
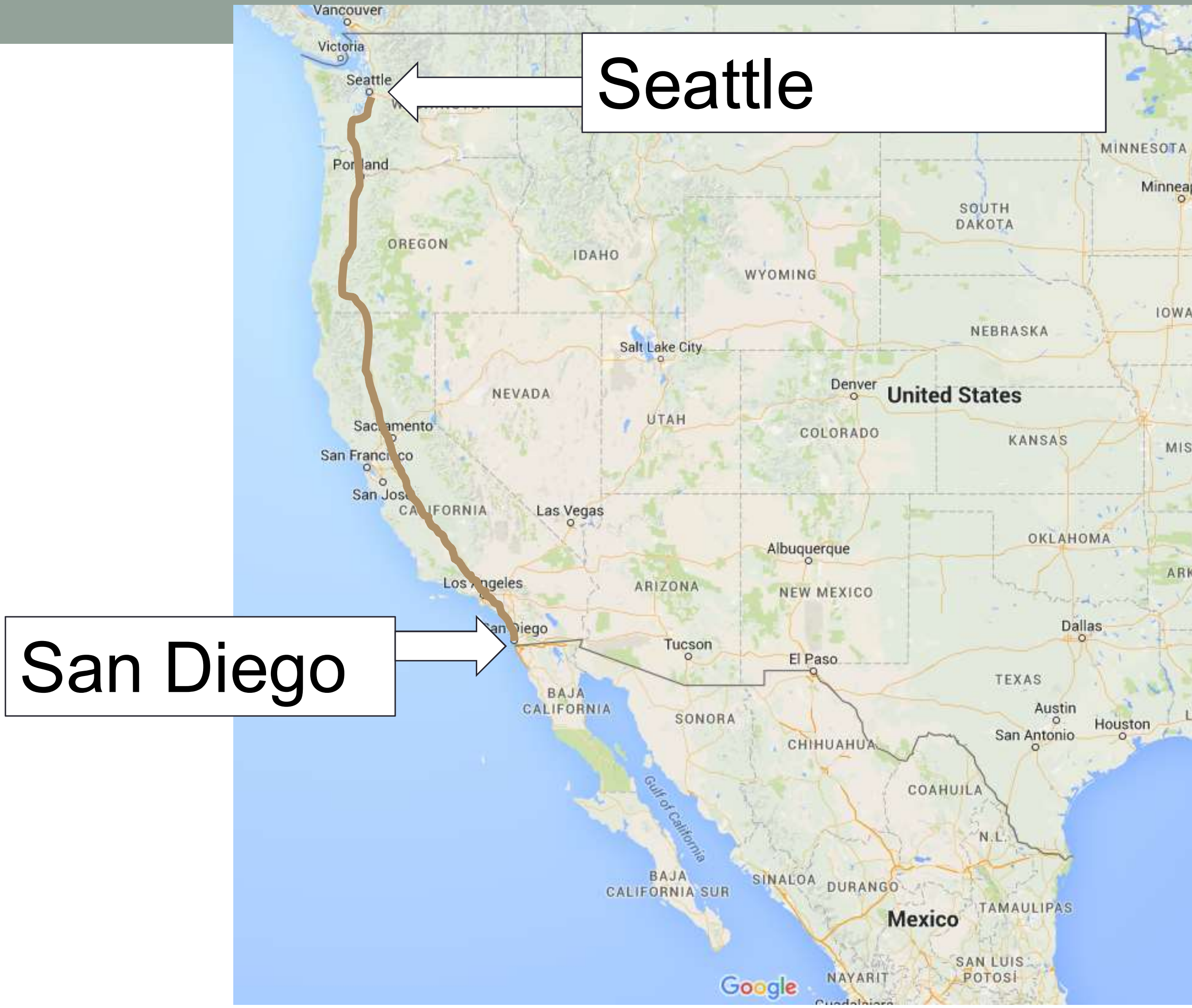


Driving directions from San Diego to Seattle?



Driving directions from San Diego to Seattle?

maps.google.com

Driving directions from San Diego to Seattle?

1255 miles

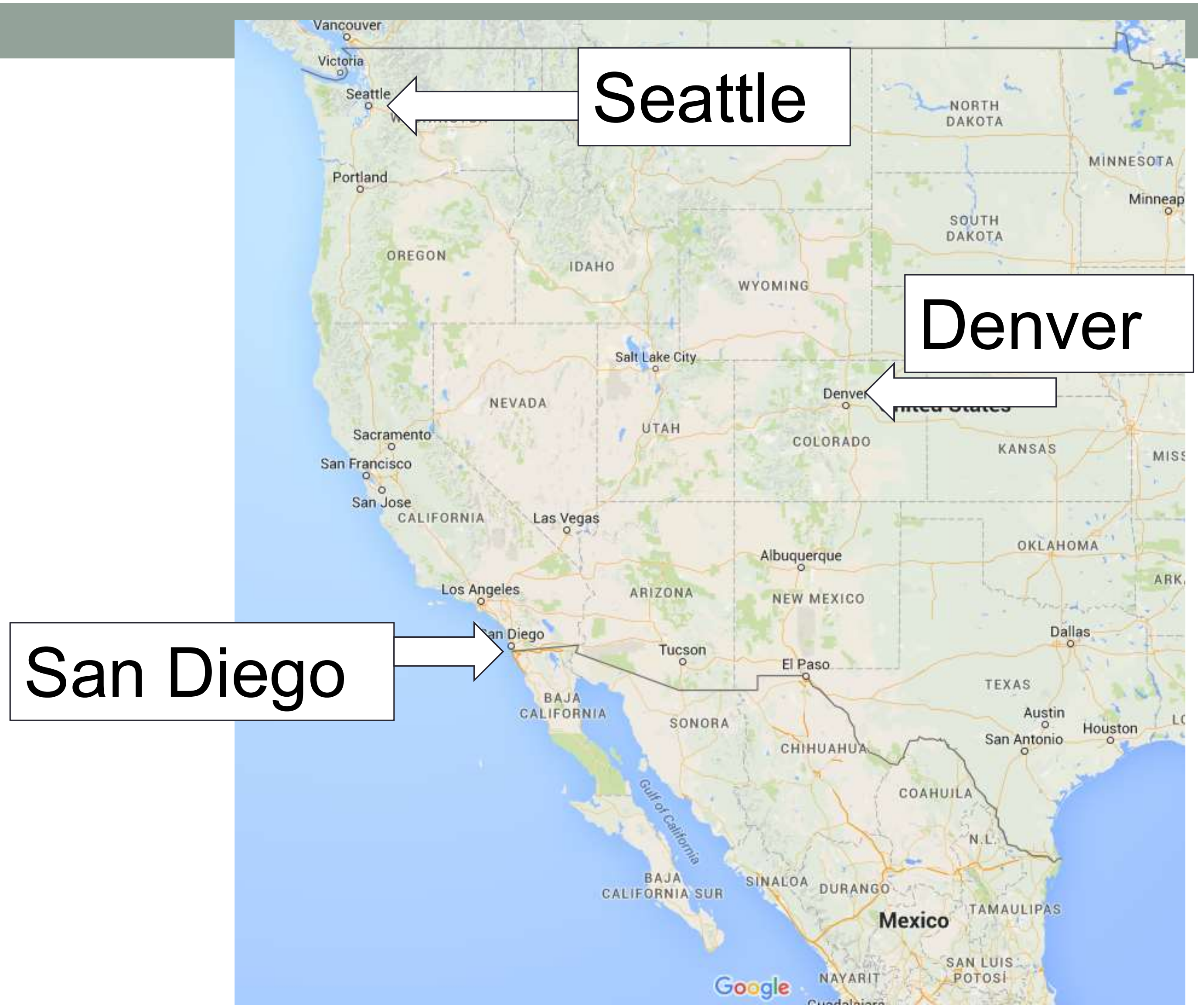
maps.google.com

Dijkstra will find the shortest route. But how?

1255 miles

maps.google.com

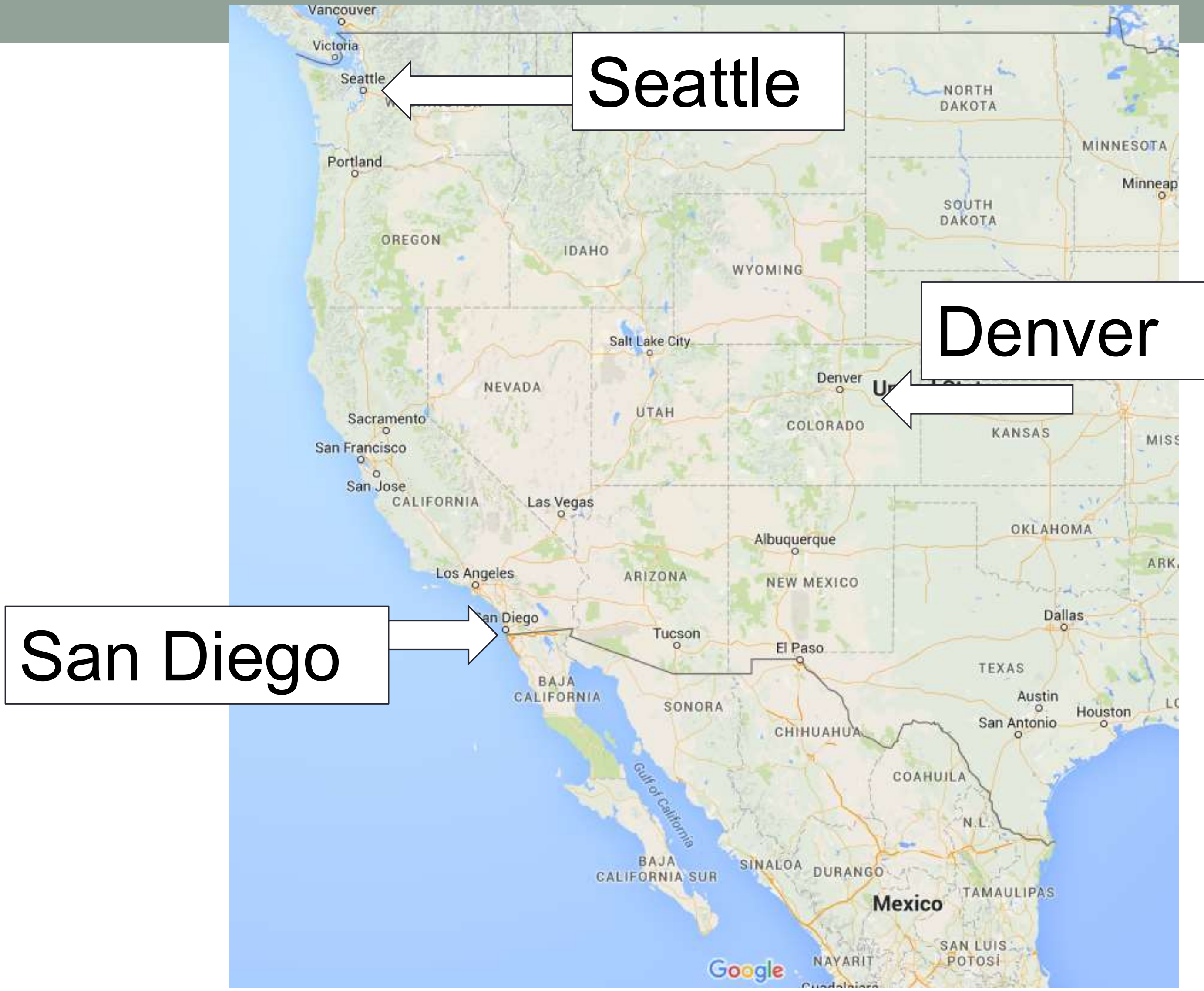
Dijkstra will find the shortest route. But how?



Would Dijkstra have you consider Denver in finding the path to Seattle?

A. Yes
B. No
C. Maybe

maps.google.com



Why would YOU have never considered Denver?

maps.google.com

Going East is the wrong direction!

maps.google.com

Seattle

We should consider distance from target too!

Dijkstra only considers distance from source

Denver

San Diego

Dijkstra's Algorithm

- Priority Queue ordering is based on:

g(n): the distance (cost) from start vertex to vertex n

A* Algorithm

- Priority Queue ordering is based on:

g(n): the distance (cost) from start vertex to vertex n

AND

h(n): the **heuristic estimated cost** from vertex n to goal vertex

A* Algorithm

- Priority Queue ordering is based on:

g(n): the distance (cost) from start vertex to vertex n

AND

h(n): the **heuristic estimated cost** from vertex n to goal vertex

f(n) = g(n) + h(n)

A* Algorithm

- Priority Queue ordering is based on:

g(n): the distance (cost) from start vertex to vertex n

AND

h(n): the **heuristic estimated cost** from vertex n to goal vertex

f(n) = g(n) + h(n)

Dijkstra can be seen as a special case where h(n)=0

A* Algorithm

- Priority Queue ordering is based on:

g(n): the distance (cost) from start vertex to vertex n

AND

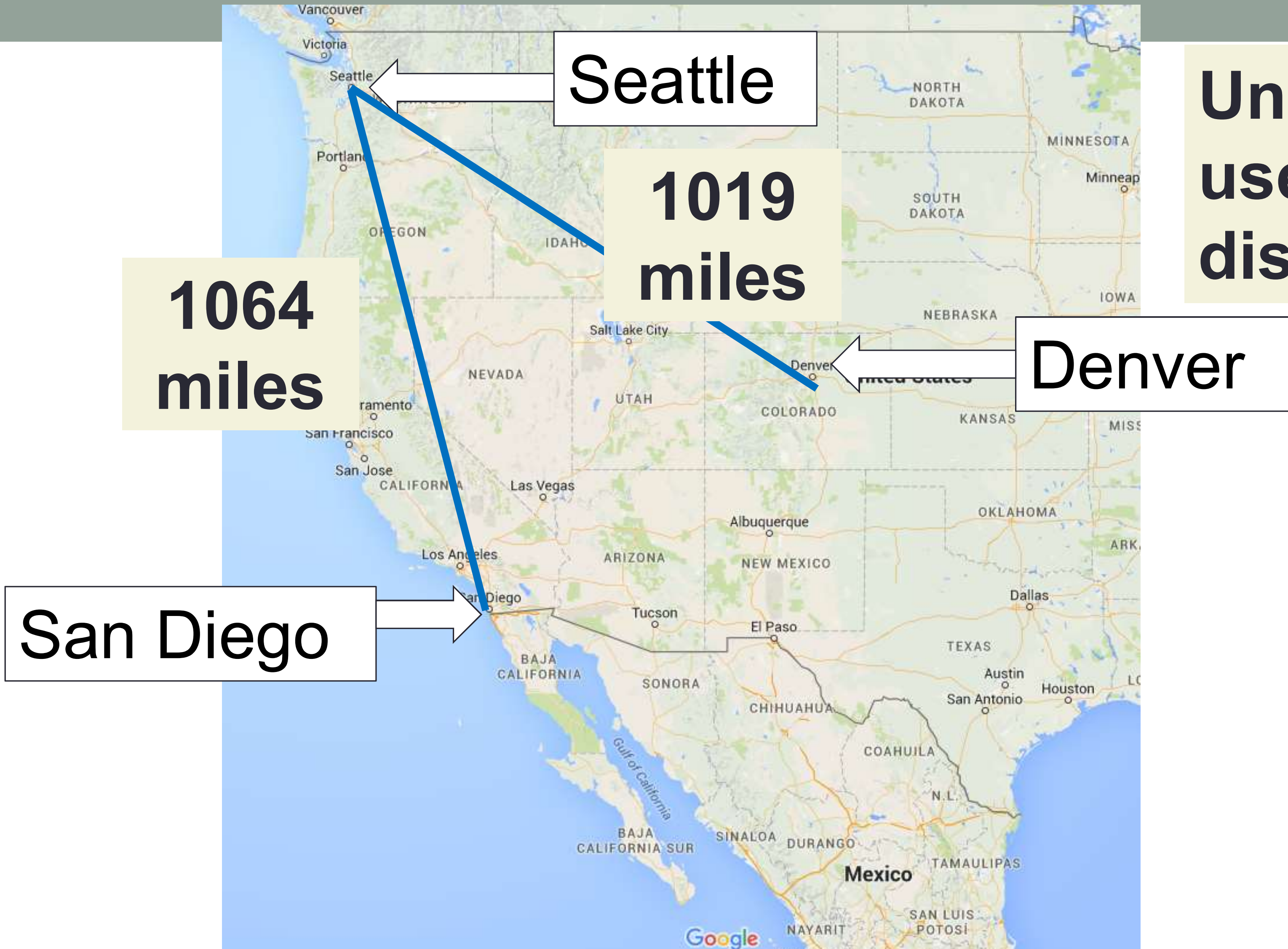
h(n): the **heuristic estimated cost** from vertex n to goal vertex

f(n) = g(n) + h(n)

Guaranteed to find shortest path IF estimate is never an overestimate

maps.google.com

Underestimate: use the exact distance.



http://distancecalculator.globefeed.com/World_Distance_Calculator.asp

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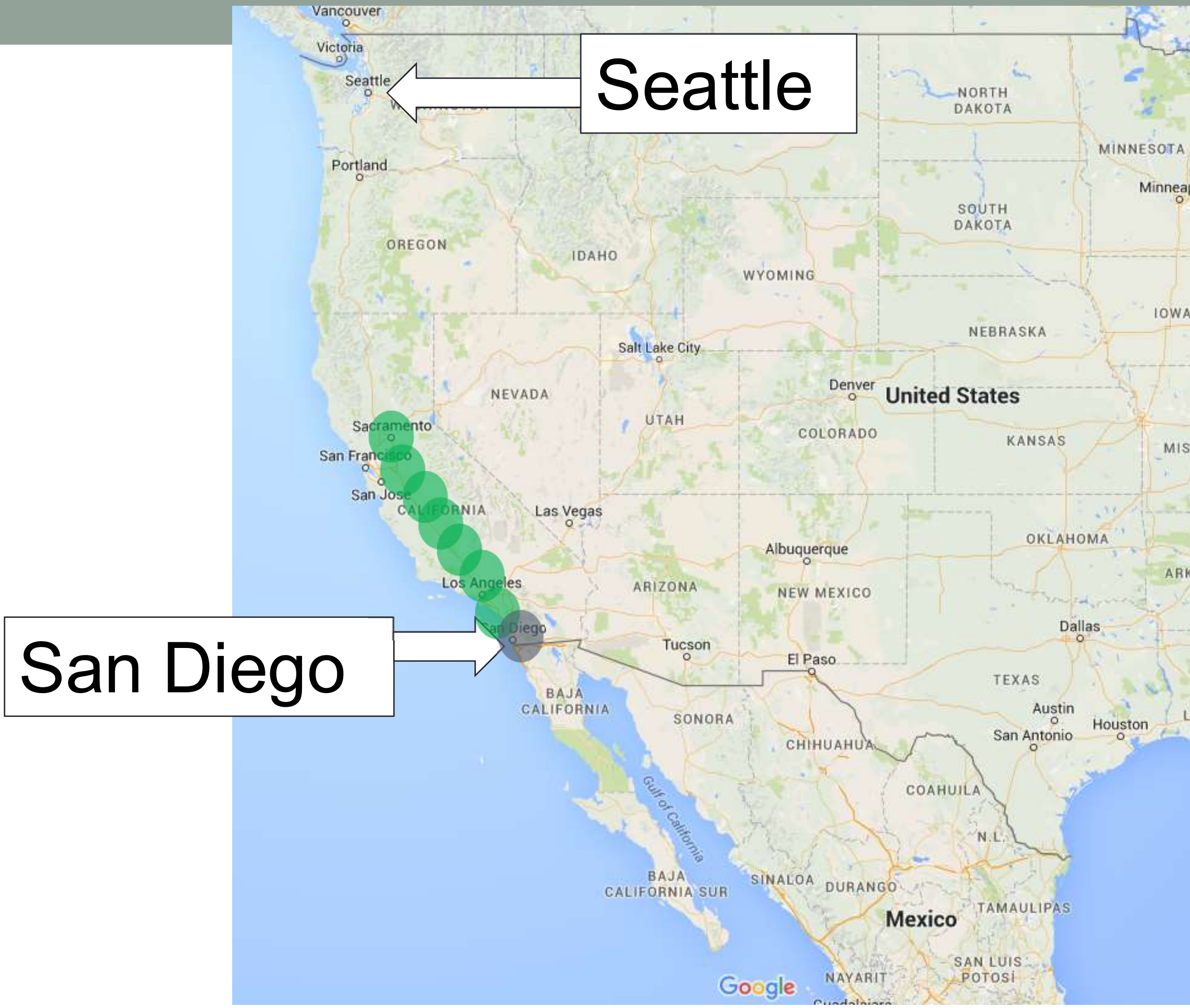
maps.google.com

A*



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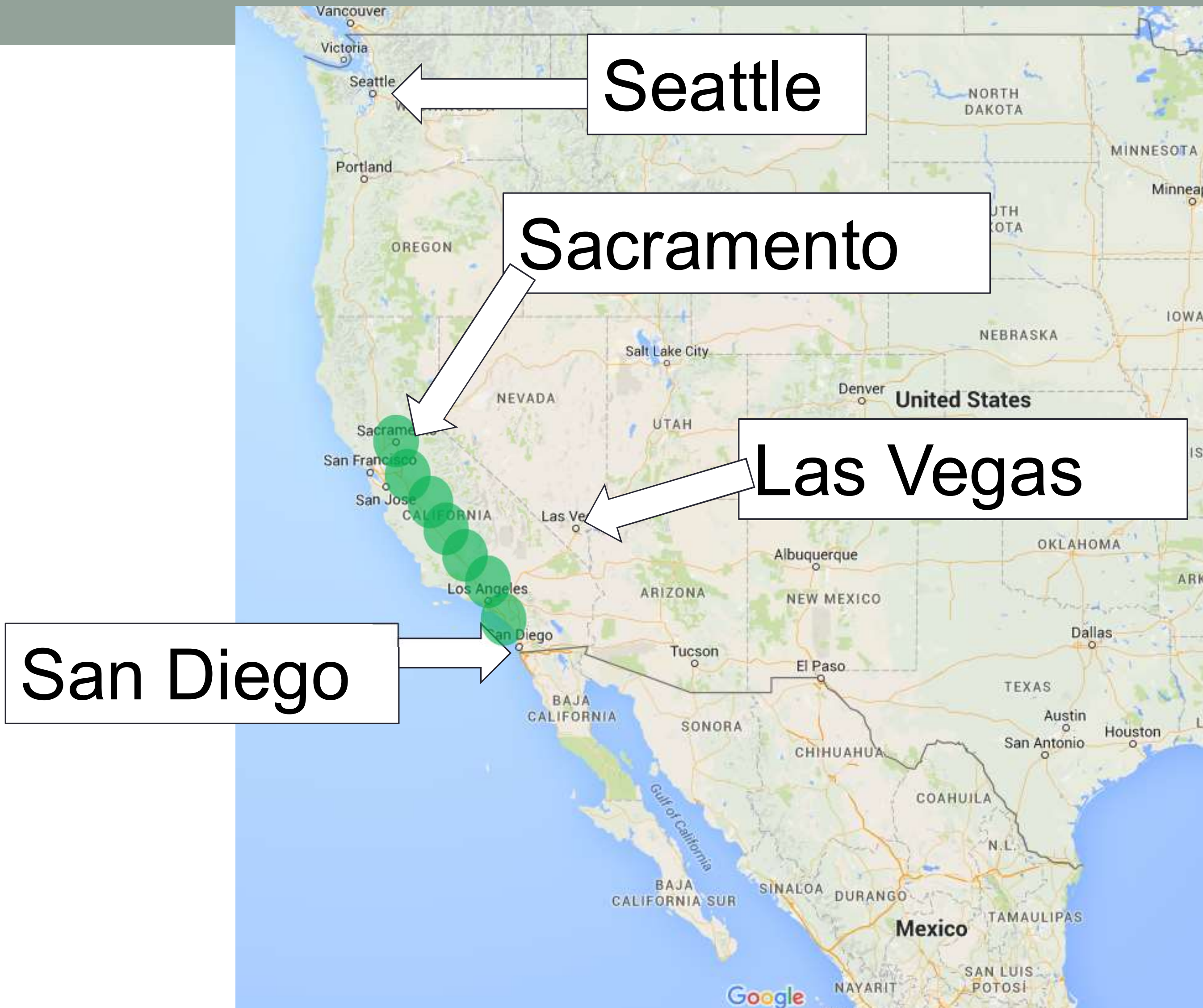
A*



http://distancecalculator.globefeed.com/World_Distance_Calculator.asp

maps.google.com

A*

Seattle

Sacramento

f(n) = 504 + 625 = 1129

Las Vegas

f(n) = 331 + 871 = 1202

San Diego

http://distancecalculator.globefeed.com/World_Distance_Calculator.asp

A* Algorithm

- Priority Queue ordering is based on:

g(n) the distance (cost) from start vertex to vertex n

AND

h(n) the heuristic estimated cost from vertex n to goal vertex

f(n) = g(n) + h(n)

Just change the priority function!