Choosing Managers and Funds: How to Maximize Your Alpha Without Sacrificing Your Target

Gary T. Baierl
and
Peng Chen[1]

April 1999

This article has been submitted to the *Journal of Portfolio Management*.

Abstract

In this paper we describe a method for selecting portfolios of managers or mutual funds to implement a target asset allocation. Our goal is to maximize alpha for each level of tracking error. Furthermore, the routine is designed to meet manager imposed minimum investment requirements by utilizing discrete optimization techniques. A step-by-step example illustrates the practical use of the methods we have developed.

IbbotsonAssociates

1. Introduction

The primary concern of long-term investors should be matching asset allocation targets, as shown in Ibbotson and Kaplan [1998] and Brinson, Singer and Beebower [1991]. The secondary concern is then maximizing alpha. These concerns apply to either the institutional investor who selects managers or the individual investor who selects mutual funds. Each faces minimum investment constraints while selecting from a set of managers in active return-risk space. In this paper, we present an approach to the selection of optimal portfolios in this setting. The output answers both the questions "How many managers (funds) do I need?" and "How much should I allocate to each manager (fund)?" Additionally, the output can be used to answer the question "Should I choose index managers or active managers or both?"

These questions have been independently addressed in similar frameworks by Waring and Castille [1998], and by Demakis [1997]. Their approaches differ somewhat from ours; for example, in the assumptions describing future active returns, the method for controlling deviation from the asset class target, and our explicit accounting for manager imposed minimum investments. Furthermore, we present an example of our procedure with an emphasis on its practicality.

As we have stressed, the method we present is identically applied in the selection of managers or in the selection of mutual funds. Henceforth we will use the terms "funds" and "managers" interchangeably.

The next section presents the optimization framework in which we will operate. The third section demonstrates our methods and output in the form of an example. The final section concludes.

2. The problem

Our starting point for the present analysis is to take the target asset allocation as given. This portion of the implementation may or may not be taken directly off of a mean/variance efficient frontier, and it often is not.

Once the target is assigned, an appropriate set of managers must be selected to determine the manager level portfolio that will best implement the target asset allocation. What we mean by being best is based on the three major themes of this paper. First, using the style weights, the combination of managers we select invests in the same proportion across asset classes as the target. Second, as we choose a portfolio with greater alpha expectations, the tracking error against the target is kept as low as possible. And third, the combination has to be realistic in terms of its dollar recommendations. That is, we cannot invest less than what the manager requires as a minimum.

In order to formally discuss our procedures we will define some terminology. We assume that

$m =$ number of asset classes,

$n =$ number of managers available.

In general, we will let the subscript i range from 1 to m, and j will range from 1 to n.

Thus, unless otherwise noted, i denotes asset classes and j denotes managers. Furthermore

$A_{i,j} =$ style weight of fund j in asset class i,

$b_i =$ target allocation to asset class i,

$a_j =$ alpha for the manager of fund j,

$V_{i,j} =$ covariance of the active returns of managers i and j,

$M_j =$ minimum required (by the manager) investment in fund j in dollars,

$K =$ dollar amount investor is allocating over managers,

$w_j =$ allocation (in percent) to manager j,

$l =$ parameter measuring investor's degree of active risk tolerance,

$\mathbf{1} =$ the $n \times 1$ vector of ones.

The inputs A, a, and V may be obtained through either fundamental analysis or from returns-based style analysis.[2] At this point in the discussion we take these values as given.

Once the parameters describing manager performance are accepted, the problem for the investor is to choose an allocation across managers that achieves the asset class level target, invests at least the minimum required by each manager that is selected, and is mean/variance efficient in terms of the active return. Formally, the problem is to choose w to solve:

(P1) $\min w'Vw - la'w$ subject to

$$Aw = b,$$

$$\text{Either } w_j = 0 \text{ or } w_j \geq \frac{M_j}{K}, \text{ for } j = 1,2,\ldots,n,$$

$$\mathbf{1}'w = 1,$$

$$w \geq 0.$$

The form of the problem in (P1) is the natural extension of mean/variance efficiency to the active portion of the total portfolio return. Furthermore, problem (P1) is a standard quadratic program (QP), except for the second constraint that generally renders the feasible set non-convex. Verbally, this constraint says that if a manager is selected to be in the portfolio, then the level of investment in that manager must reach the imposed minimum level M_j. The procedure used to solve this technicality is to implement a branch and bound routine, solving a sequence of QPs derived from (P1) that will either demonstrate that the constraint set in (P1) is infeasible or determine the solution to (P1). We discuss the details of this aspect of our solution mechanism in the appendix.

3. An example

Now we illustrate the procedure of the second section in a realistic manager choice problem. We selected a target portfolio across five asset classes. Table 1 shows this target along with the data series we have chosen to associate with each asset class. Since the focus of this paper is on the manager level implementation problem, we take the target as given without need for justification.

The first step in the practical implementation of manager selection is to filter out the managers that are not going to be considered in the optimization problem in (P1). This filtration is necessary for several reasons. First, we need to exclude style inconsistent managers, as style drift significantly increases the risk of not hitting the target asset allocation. Second, the filtration process can help to select managers with relatively good performance in terms of alpha and tracking error. Third, the filtration process is necessary to ensure an appropriate number of managers are fed into the optimization. The optimization procedure that we have designed solves a discrete problem due to the consideration of managers 'choice of minimum levels of investment. Unfortunately, this aspect of the procedure can significantly increase the time to find a solution as the number of managers increases. On the other hand, too few managers may render hitting the target to be infeasible. When possible, the inclusion of a set of index managers along with active and blended managers will eliminate this problem. The portion of the manager portfolio allocated to index managers in this setting has popularly been called the completion portfolio, since it complements the blended managers such that the combination hits the target asset allocation.

We proceed to select a set of managers to be input into problem (P1). We begin with a sufficient set of data (sufficient in that the funds have enough return data to produce style weights and active returns for the period from January 1994 to December 1998) on 2,769 mutual funds.[3] Then a returns-based cluster analysis is used to create 8 peer groups of these funds. Grouping funds this way is important because when we select the funds to be fed into the (P1) optimization we will select at least a few funds from each peer group. This ensures that the selected funds cover a large enough variety of styles that the implementation will be able to attain the chosen target.

The cluster analysis was managed so that our resulting peer groups are described by the categories in Table 2. Returns-based style analysis was run on all funds, delivering style weights and active returns.[4] The mean of the active returns is the raw alpha, and the standard deviation of the active returns is the tracking error. Because of the estimation error contained in raw alphas we would normally adjust the alphas in some manner. However, since the topic of this paper is the solution process itself we will simply use the raw alphas as inputs. We assumed that the active returns are independent across managers. Therefore, the covariance matrix of active returns, V , is a diagonal matrix with the squares of the tracking errors on the diagonal.

Now we have all the data necessary for discrimination among funds. We selected 23 funds, including approximately three from each peer group. The selections were based on the following criteria:[5]

1. Include funds with style regression R-squared of at least 0.85.
2. Include funds that fit their peer groups well. This is also measured by an R-squared, which we require to be at least 0.65.
3. Include funds with minimum investment requirements of no more than $10,000.

The goal of the first requirement is to exclude highly style inconsistent funds. The second requirement helps to avoid funds that have been placed into an inappropriate peer group. The third requirement is made under the assumption that the total investment amount in the experiment is $100,000.

With these requirements in mind, for each peer group we included the fund with the lowest tracking error, the fund with the highest information ratio, and the fund with the highest alpha.[6] In cases where the best fund in terms of one of these criteria had already been included, for instance the fund with the highest information ratio may also have the highest alpha, we included the best fund not yet included.

This process produced the set of funds listed in Table 3. The data used in both the selection process and the (P1) optimization on these funds is also listed in Table 3.

The data from Table 3 on our 23 selected funds was input into the optimizer that solves problem (P1), and we obtained five fund allocations that simultaneously hit the asset class target, are efficient in alpha-tracking error space, and satisfy the minimum fund investment requirements. These manager level portfolios, presented in Table 4, are ordered in terms of active risk, so that Portfolio 1 is the minimum tracking error portfolio while Portfolio 5 is the maximum alpha portfolio.

The manager level portfolios make intuitive sense. For instance none of the funds selected on the basis of high alpha made it into the minimum tracking error portfolio, and none of the funds selected on the basis of low tracking errors were included in the maximum alpha portfolio. To analyze the allocations to passive versus active managers, we aggregated the allocations to index managers and active managers for each portfolio.[7] The aggregated allocation is presented in Table 4. Portfolios with high active risk allocate more to active managers than portfolios with low active risk. For instance, the highest active risk portfolio (Portfolio 5) allocates more than 80% to active managers, while the lowest active risk portfolio (Portfolio 1) allocates only 3% to active managers. This shows that the use of index versus active managers depends on the active risk tolerance level of the investor.

Another observation is that our filtration process chose few unnecessary funds in terms of the (P1) optimization results; only two funds failed to show up with some positive exposure for at least one of the five listed portfolios. Finally, notice that the minimum investment requirements were binding in at least a few instances. For example, in Portfolio 1, both the Strong Corporate Bond Fund, and the Fidelity Diversified International Fund had allocations equal to their minimum investment requirements.

In Figure 1, we have presented the performance of these manager level portfolios in alpha-tracking error space. That is, the performance is being measured net of the performance of the asset class target. The frontier displays an optimal trade off of alpha for active risk.

To close this section, we want to mention the fact that the example we have just presented was constructed using mutual funds and investment levels appropriate for an individual investor. This was merely because the data to which we had access provides information on these funds, and this made, we believe, for a good expository presentation. The nature of this process should carry over to the selection of private managers in an institutional setting in a straightforward manner.

4. Conclusion

In this paper, we have presented a method for choosing managers or mutual funds to implement a target asset allocation. The portfolios selected are optimized in terms of alpha for each level of tracking error, with the additional constraint that minimum investment levels are attained. We showed that the selection of active versus index managers depends, as it should, on the investor's active risk tolerance. In general, the higher the risk tolerance the higher the allocation to active management.

The value of the method discussed here relies heavily on the quality of the inputs used, i.e. expected alphas, tracking errors, and correlations among managers' active returns, along with style weights representing management policy. A set of poor quality inputs could lead to unacceptable portfolios. Furthermore, the performance of managers needs to be monitored and evaluated on a continuous basis.

Appendix: Solution method

In this appendix the detailed description of the solution mechanism designed to solve (P1) is given. The best description of the procedure involves the use of a binomial tree diagram.

Associate each node of the tree with two nonintersecting sets of indices J_1 and J_2, both being subsets of $\{1,2,\ldots,n\}$. At each node, the following QP is solved:

$$\min w'Vw - la'w \text{ subject to}$$

$$\begin{aligned}
&Aw = b,\\
&w_j = 0 \text{ for } j \in J_1,\\
&w_j \geq \frac{M_j}{K} \text{ for } j \in J_2,\\
&\mathbf{1}'w = 1,\\
&w \geq 0.
\end{aligned}$$

As in Figure 2, the top node, denoted node 0, of the tree begins with J_1 and J_2 both equal to the empty set and the QP is solved. If a solution is found such that all the constraints in (P1) are satisfied, we have found a solution and are done. In general, there will be a set of managers such that the solution to the problem in the current node tells us to invest a positive amount, but lower than the required minimum. Suppose that manager number 1 has a minimum investment in percentage terms of the total dollar amount of the portfolio of 5%, but the top node optimization solution is 2.5% for this manager. Then the left node immediately below will solve the problem with $J_1 = \{1\}$ and $J_2 = \varnothing$, while the right node solves the problem with $J_1 = \varnothing$ and $J_2 = \{1\}$. In other words, the node on the left branch sets the first manager's investment amount to zero and the node on the right branch requires an investment of at least 5%. Now there are a few possibilities. If both nodes deliver a solution such that all constraints in (P1) are satisfied, then the solution to the overall problem is the solution to the QP in the node with the lowest value of the objective function. That is, if it is better to invest zero with the first manager than it is to invest the minimum or more and both portfolios are feasible for all other constraints, then the general solution is to not invest with the first manager.

The next possibility is that one node satisfies all of the constraints in problem (P1), but the other does not. Suppose that node 2 satisfies all the constraints, and the objective function value at the optimum is zero. Now, the process repeats itself for the left node. That is, we treat it the same way that we treated the top node in the first step. For instance, suppose that when we solve the problem in node 1, manager number 5 receives an investment of 8% while the minimum is 10%. Then we solve the next left node (node 3) with $J_1 = \{1,5\}$ and $J_2 = \varnothing$, while the next right node (node 4) is solved with $J_1 = \{1\}$ and $J_2 = \{5\}$. At this point, regardless of whether or not all of the constraints in (P1) are satisfied, we can make an evaluation of whether or not a particular branch of the tree requires further exploration. Suppose that some of the (P1) constraints remain unsatisfied for node 4, but the solution to the problem in this node delivers an objective value of 1.0. Since we already know that we can achieve a value of 0 with the feasible portfolio in node 2, and further exploration below node 4 can never improve on the value of 1.0, we need not pursue doing so. This is the "bound" in "branch and bound."

The program proceeds as such. Each branch will terminate in one of three ways:

1. The objective value fails to improve on the best known so far for a feasible solution to (P1).

2. A solution that satisfies the constraints in (P1) is found exceeding the best known so far. Save the solution and objective value for future comparison.
3. A set of constraints has been added such that the feasible set has become empty.

Once all possible branches have been explored, the saved solution from possibility 2 will be the solution to (P1), assuming that one exists. Further descriptions of methods to solve discrete optimization problems may be found, for example in Rardin [1998].

References

Brinson, Gary P., Brain D. Singer, and Gilbert L. Beebower, "Determinants of Portfolio Performance II: An Update," *Financial Analysts Journal*, May/June 1991, pp. 40-48.

Demakis, Drew W., "Optimization of Active Risk Across Asset Classes," Research Insights – RogersCasey, 1997.

Ibbotson, Roger G. and Paul D. Kaplan, "Does Asset Allocation Policy Explain 40%, 90%, or 100% of Performance?" working paper, December 1998.

Rardin, Ronald L., *Optimization in Operations Research,* New Jersey: Prentice-Hall, 1998.

Sharpe, William F., "Asset Allocation: Management Style and Performance Measurement," *Journal of Portfolio Management*, Winter 1992, pp. 7-19.

Waring, M. Barton and Chip Castille, "A Framework for Optimal Manager Structure," Investment Insights – Barclays Global Investors, June 1998, pp. 1-20.

Table 1
Target Portfolio

Asset Class	Benchmark Proxy	Weight
Cash	Salomon Brothers 30 Day T-Bill	12.7%
Gov ŧ/Corp Bond	Lehman Brothers Gov ŧ/Corp Bond	27.3%
Large-Cap Domestic Equity	S&P 500	36.3%
Small-Cap Domestic Equity	Russell 2000	16.5%
International Equity	MSCI EAFE	7.2%

Table 2
Peer Group Assignments for Fund Universe

Peer Group	Description	Number of Funds
1	Pure Cash	213
2	Pure Gov t/Corp Bond	482
3	Pure LC Domestic Equity	296
4	Pure SC Domestic Equity	272
5	Pure International Equity	256
6	Domestic Equity Blend	311
7	Domestic Blend	511
8	Domestic/International Blend	428

Table 3
Funds Satisfying Inclusion Requirements

	Fund	Peer Group	Alpha	Tracking Error	Information Ratio	U.S. 30 Day TBill	LB Gvt/Corp	S&P 500	Russell 2000	MSCI EAFE	R Squared	Minimum
1	American National Primary Fund	1	-0.26%	0.03%	-7.78	99.79	0.00	0.00	0.00	0.21	71.68	$1,000
2	Marshall Short-Term Income	1	-0.01%	0.18%	-0.04	78.12	21.60	0.00	0.00	0.28	73.54	$1,000
3	SEI Index Fds-Bond Index	2	-0.01%	0.11%	-0.07	10.40	89.60	0.00	0.00	0.00	99.09	$0
4	Strong Corporate Bond	2	0.14%	0.54%	0.26	0.00	94.49	0.60	3.19	1.72	87.35	$2,500
5	Vanguard Fixed-Long Term UST	2	0.15%	1.13%	0.13	0.00	100.00	0.00	0.00	0.00	95.27	$3,000
6	Vanguard 500 Index Fund	3	-0.01%	0.03%	-0.22	0.00	0.00	99.88	0.00	0.12	100.00	$3,000
7	Vanguard Growth Index Fund	3	0.26%	0.89%	0.29	0.00	0.00	100.00	0.00	0.00	95.43	$3,000
8	Wilshire Targ-Large Co Growth Inv	3	0.20%	1.10%	0.18	0.00	0.00	100.00	0.00	0.00	93.88	$2,500
9	Vanguard Small-Cap Index Fund	4	0.07%	0.23%	0.28	0.00	0.00	1.11	97.72	1.17	99.77	$3,000
10	Galaxy Small Cap Value Inst	4	0.38%	1.27%	0.30	12.47	8.95	0.00	78.58	0.00	89.98	$0
11	USAA Aggressive Growth Fund	4	0.59%	3.29%	0.18	0.00	0.00	0.00	100.00	0.00	86.52	$3,000
12	Schwab International Index Inv	5	-0.05%	0.59%	-0.09	0.00	3.55	8.62	0.50	87.34	97.69	$1,000
13	Fidelity Diversified Intl Fund	5	0.10%	1.14%	0.09	0.79	5.32	15.12	12.64	66.14	90.59	$2,500
14	Managers International Equity	5	0.03%	1.28%	0.03	10.89	3.87	10.80	7.42	67.02	86.60	$2,000
15	Vanguard Extended Market Idx Fund	6	0.11%	0.67%	0.16	0.00	0.00	25.13	74.87	0.00	98.03	$3,000
16	Fidelity Low-Priced Stock	6	0.39%	0.96%	0.41	21.00	1.61	4.90	59.08	13.41	92.52	$2,500
17	Price (T. Rowe) Mid-Cap Growth	6	0.48%	1.59%	0.30	0.00	0.00	20.56	70.41	9.03	89.20	$2,500
18	Vanguard Total Stock Mkt Index	7	0.01%	0.23%	0.02	0.00	0.00	78.43	21.57	0.00	99.70	$3,000
19	Fidelity Dividend Growth Fund	7	0.51%	1.24%	0.42	0.00	4.22	62.80	25.42	7.56	90.47	$2,500
20	Mairs & Power Growth Fund	7	0.36%	1.27%	0.29	0.00	7.72	52.17	36.60	3.51	89.88	$2,500
21	Vanguard Index-Balanced Ptfl	8	-0.01%	0.16%	-0.05	5.85	33.95	48.43	11.77	0.00	99.61	$3,000
22	Price (T. Rowe) Dividend Growth	8	0.30%	0.75%	0.41	10.79	21.79	50.30	14.31	2.80	93.21	$2,500
23	Royce Fund-Premier	8	0.20%	1.17%	0.17	36.41	0.00	4.75	52.27	6.56	85.56	$2,000

Table 4
Alpha-Tracking Error Efficient Portfolios

Fund	Portfolio 1	Portfolio2	Portfolio 3	Portfolio 4	Portfolio 5
American National Primary Fund	7.8%	-	-	-	-
Marshall Short-Term Income	2.4%	10.0%	10.1%	12.8%	15.4%
SEI Index Fds-Bond Index	23.7%	4.2%	-	-	-
Strong Corporate Bond	2.5%	14.0%	11.8%	6.9%	-
Vanguard Fixed-Long Term UST	-	3.6%	6.4%	12.3%	21.3%
Vanguard 500 Index Fund	28.6%	4.9%	-	-	-
Vanguard Growth Index Fund	-	9.7%	4.8%	-	-
Wilshire Targ-Large Co Growth Inv	-	4.9%	-	-	-
Vanguard Small-Cap Index Fund	14.4%	-	-	-	-
Galaxy Small Cap Value Inst	0.4%	2.6%	-	-	-
USAA Aggressive Growth Fund	-	-	-	-	-
Schwab International Index Inv	6.1%	1.4%	-	-	-
Fidelity Diversified Intl Fund	2.5%	2.6%	4.7%	4.2%	3.8%
Managers International Equity	-	2.0%	-	-	-
Vanguard Extended Market Idx Fund	-	-	-	-	-
Fidelity Low-Priced Stock	-	7.4%	8.4%	4.1%	2.8%
Price (T. Rowe) Mid-Cap Growth	-	3.2%	-	-	-
Vanguard Total Stock Mkt Index	3.5%	-	-	-	-
Fidelity Dividend Growth Fund	-	8.5%	26.1%	43.5%	56.7%
Mairs & Power Growth Fund	-	4.7%	-	-	-
Vanguard Index-Balanced Ptfl	8.1%	-	-	-	-
Price (T. Rowe) Dividend Growth	-	14.2%	27.8%	16.2%	-
Royce Fund-Premier	-	2.0%	-	-	-
Allocation to Active Managers	3%	34%	37%	60%	82%
Allocation to Passive Managers	97%	66%	63%	40%	18%

Figure 1

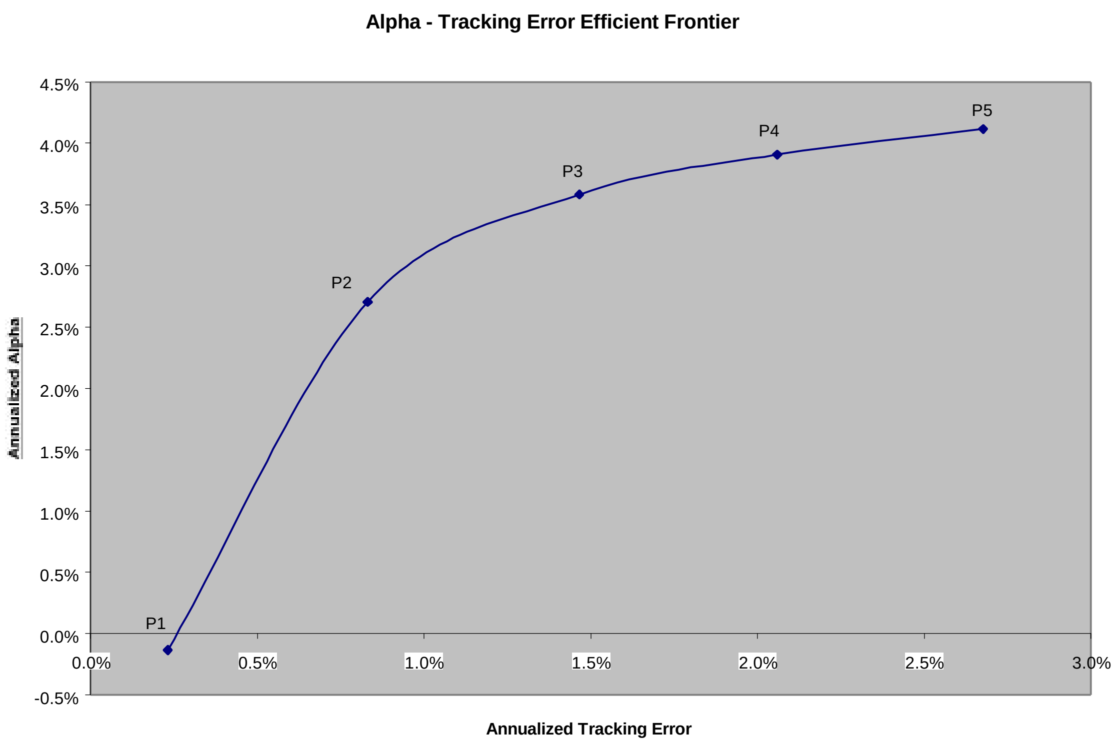
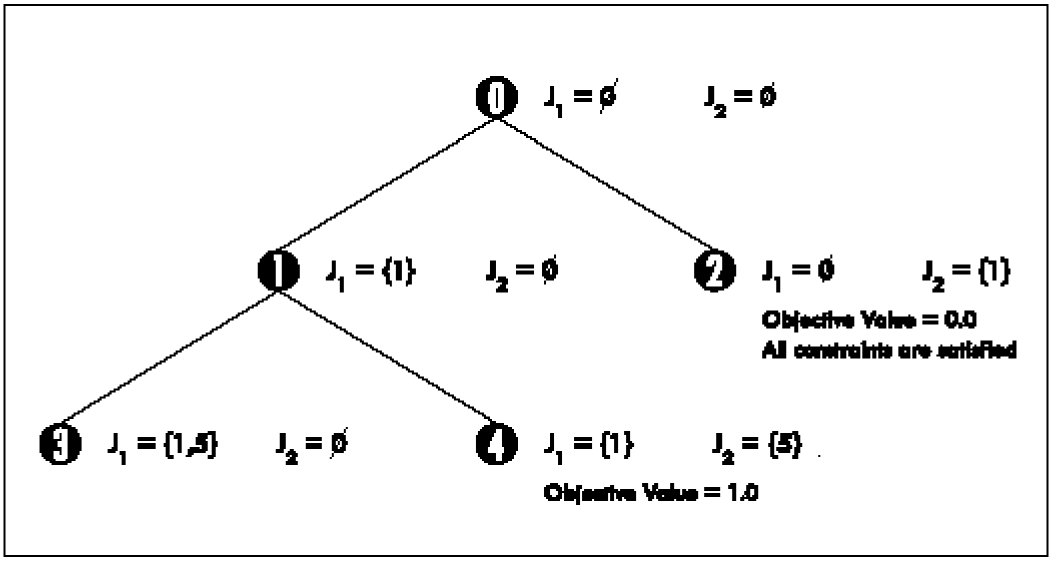

Alpha - Tracking Error Efficient Frontier
4.5%
4.0%
3.5%
3.0%
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
-0.5%
Annualized Alpha
P1
P2
P3
P4
P5
0.0%
0.5%
1.0%
1.5%
2.0%
2.5%
3.0%
Annualized Tracking Error

Figure 2
Discrete Optimization Sequence



[1] Both authors are research consultants at Ibbotson Associates. Gary T. Baierl is the corresponding author and may be reached at Ibbotson Associates, 225 N. Michigan Ave., Suite 700, Chicago, IL 60601, voice: (312) 616-9213, fax: (312) 616-0404, e-mail: gbaierl@ibbotson.com. We thank Roger Ibbotson, Paul Kaplan, and Matt Terrien for helpful suggestions and Adam Cole for help with data management. The authors are responsible for any errors.
[2] Returns-based style analysis was developed by Sharpe [1992].
[3] All fund data was obtained from CDA Wiesenberger.
[4] Fees were subtracted out of the active returns in order to calculate alphas.
[5] Requirements 1 and 2 applied to all selections except the pure cash peer group, where none of the funds had R-squared higher than 0.85. In this group we dropped the style regression R-squared requirement to 0.65. Requirement 3 applied to all selections.
[6] The pure cash peer group was an exception in that we did not seek its highest alpha fund.
[7] Managers with estimated tracking error higher than 1% are defined to be active managers. Managers with estimated tracking error less than 1% are defined to be passive or index managers.