Recitation 11

ENEE324: Engineering Probability

Spring, 2018

The following problems are from the textbook.

Problem 5.1.1.
A statistician wants to estimate the mean height h (in meters) of a population, based on n independent samples $X_1, ..., X_n$, chosen uniformly from the entire population. He uses the sample mean $M_n = (X_1 + X_2 + ... + X_n)/n$ as the estimate of h, and a rough guess of 1.0 meters for the standard deviation of the samples X_i.

1. How large should n be so that the standard deviation of M_n is at most 1 centimeter?
2. How large should n be so that Chebyshev's inequality guarantees that the estimate is within 5 centimeters from h, with probability at least 0.99?
3. The statistician realize that all persons in the population have heights between 1.4 and 2.0 meters, and revise the standard deviation figure that he uses based on the bound of Example 5.3. How should the value of n obtained in parts (a) and (b) be revised?

Problem 5.2.4.
In order to estimate f, the true fraction smokers in a large population, Alvin selects n people at random. His estimator M_n is obtained by dividing S_n, the number of smokers in his sample, by n, i.e., $M_n = S_n/n$. Alvin chooses the sample size n to be the smallest possible number for which the Chebyshev inequality yields a guarantee that

$$P(|M_n - f| \geq \epsilon) \leq \delta$$

where ϵ and δ are some prespecified tolerances. Determine how the value of n recommended by the Chebyshev inequality changes in the following cases.

1. The value of ϵ is reduced to half its original value.
2. The probability δ is reduced to half its original value.

Problem 5.3.5.
Let $X_1, X_2, ...$ be independent random variables that are uniformly distributed over [-1, 1]. Show that the sequence $Y_1, Y_2, ...$ converges in probability to some limit, and identify the limit, for each of the following cases:

1. $Y_n = X)n/n$
2. $Y_n = X_1 \times X_2 \times ... \times X_n$

Problem 5.4.10.
A factory produces X_n gadgets on day n, where the X_n are independent and identically distributed random variables, with mean 5 and variance 9.

1. Find an approximation to the probability that the total number of gadgets produced in 100 days is less than 440.
2. Find (approximately) the largest value of n such that

$$P(X_1 + X_2 + ... + X_n \geq 200 + 5n) \leq 0.05$$

3. Let N be the first day on which the total number of gadgets produced exceeds 1000. Calculate an approximation to the probability that $N \geq 220$.