Recitation #11

ENEE 313: Introduction to Device Physics

Fall, 2018

1 Week Notes Summary

Summary for preparation of 2nd mid-term exam.

1. PN junction
   (a) built-in potential
   (b) width of depletion region
   (c) diode capacitance
   (d) diode current equation
2. Generation/Recombination effects: Understand the Shockley-Read-Hall recombination equation.
3. MOS capacitors
   (a) bulk potential of semiconductor of MOSFET
   (b) width of MOSFET depletion region (maximum)
   (c) Minimum capacitance
   (d) threshold voltage for ideal and non-ideal MOSFET
4. MOS transistors Current formulas
   (a) linear region
   (b) saturation region

Exercise 1. Ideal PN junction
Given a Si PN junction, if the bulk resistivity of Si is $1 \Omega m$,

1. find the built-in potential for the Si PN junction at room temperature
2. find the width of depletion region if the applied voltage is $1V$
3. find the maximum electric field in condition of the applied voltage is $1V$

4. What is the capacitance of the junction when the applied voltage $1V$, given the junction cross sectional area is $0.01cm^2$?

5. if the N-side has a resistivity $1\Omega m$, what should be the resistivity of the P-side so that 99% of the total width of the space charge region would be located in the N-side?

6. At room temperature under the forward bias of 0.15 V the current through the PN junction is 1.66 mA. What will be the current through the junction under reverse bias?

7. Now, if we heavily doped the junction with Boron so that it become a $p^+ - n$ junction whose reverse current at room temperature is 0.9 nA/cm^2, calculate the minority-carrier lifetime.

Note that Silicon's $\mu_n = 1500cm^2/Vs$, $\mu_p = 500cm^2/Vs$ and $n_i = 10^{10}cm^{-3}$

Solution. The built-in potential is

$$\phi_{bt} = \frac{kT}{q} ln(\frac{N_a N_d}{n_i^2})$$

To find N_a and N_d, we need to apply the fact that, on N side, $n_i << n \approx N_d$; on P side, $n_i << p \approx N_a$. Therefore, at the quasi-neutral region of both P side and N side,

$$\begin{aligned} \sigma_n &= q\mu_n n \\ \rightarrow N_d \approx n &= \frac{\sigma_n}{q\mu_n} \\ \rightarrow N_d &= 4.17 \times 10^{17} cm^{-3} \end{aligned}$$

$$\begin{aligned} \sigma_p &= q\mu_p p \\ \rightarrow N_a \approx p &= \frac{\sigma_p}{q\mu_p} \\ \rightarrow N_a &= 1.25 \times 10^{18} cm^{-3} \end{aligned}$$

Thus, $\phi_{bt} = 0.88V$.

To calculate the depletion region width,

$$x_{sc} = \sqrt{\frac{\epsilon(\phi_{bt} - V)}{q}(\frac{N_a N_d}{N_a + N_d})} = 1.58\mu m$$

The electric field comes from, on P side,

$$E(x) = \frac{q}{\epsilon} N_a (x_p - x), 0 \leq x \leq x_p$$

So, the maximum electric field is,

$$E_{max} = E(0) = 7.58 \times 10^4 V/cm$$

where $x_p = x_{sc} N_d/(N_d + N_a)$

The capacitance of the junction is

$$C = \frac{\epsilon}{x_s c} = 6.6 \times 10^{-7} Fd/cm^2$$
$$C_{PN} = C \times area = 6.6nF$$

Since $\frac{x_p}{x_n} = \frac{N_d}{N_a}$, $x_n = 0.99x_{sc}$ and $x_p = 0.01x_{sc}$, we have $N_a = 99N_d$. Furthermore, the resistivity of the N-side is $1\Omega m$, we have

$$n = \frac{1}{q\mu_n \rho} = 4.17 \times 10^{13} cm^{-3}$$

Therefore, $Na = 99N_d \approx 99n = 4.12 \times 10^{15} cm^{-3}$

According to the diode current equation,

$$I(V) = I_0 exp(\frac{-qV}{kT} - 1)$$

Since $I(0.15) = 1.66mA$, we have $I_0 = 1.7\mu A$

The reverse current of a $p^+ - n$ junction is

$$I_0 = \frac{qD_p n_i^2}{N_d L_p} = \frac{qn_i^2}{N_d}\sqrt{\frac{D_p}{\tau_p}}$$

Since $\mu = qD/kT$, we can obtain $\tau_p = 0.66ps$ □

Exercise 2. MOS capacitor and transistors

For an ideal $SiO_2 - Si$ MOS capacitor on p substrate at room temperature with $d = 10nm$, $N_a = 5 \times 10^{17} cm^3$, and dielectric permittivities of Si and SiO_2 are 11.8 and 3.9.

1. Find the applied voltage at the interface required to make the silicon surface intrinsic
2. Again, Find it to bring about a strong inversion
3. Find the turn-on(threshold) voltage under high-frequency condition
4. Find the minimum capacitance under high-frequency regime

Solution. The applied voltage appears partially across the insulator, V_i and the semiconductor, ϕ_s is

$$V = V_i + \phi_s$$

The voltage across the insulator is

$$V_i = \frac{-Q_s}{C_i}, \text{ where } C_i = \frac{\epsilon_i}{d}$$

If the voltage across the semiconductor is less than the one needed to bring about a strong inversion, Q_s is

$$Q_s = -qN_aW, \text{ where } W = \sqrt{\tfrac{\epsilon_s \phi_s}{qN_a}}$$

Thus, we have

$$V = \phi_s + \frac{d}{\epsilon_i}\sqrt{q\epsilon_s N_a \phi_s}$$

To make the surface intrinsic, we need

$$\phi_s = \phi_F = \frac{kT}{q} ln(\frac{N_a}{n_i})$$

To make the surface strong inversion, we need

$$\phi_s = 2\phi_F = 2\frac{kT}{q} ln(\frac{N_a}{n_i})$$

where ϕ_F is the bulk potential of the semiconductor. Therefore, the surface becomes intrinsic when $V = 1.03V$; strong inversion when $V = 1.73V$.

To find the minimum capacitance, we already have C_i and need to find C_d, which is $C_d = \frac{\epsilon_s}{W_{max}}$. W is the maximum width of the depletion region.

$$W_{max} = \sqrt{\frac{2\epsilon_s 2\phi_F}{qN_a}} = 4.9\mu m$$

Thus, $C_d = 0.21\mu F$. We know $C_i = \frac{\epsilon_i}{d} = 0.34\mu F$. By the minimum capacitance formula,

$$C_{min} = \frac{C_i C_d}{C_i + C_d} = 0.13\mu F$$

On the other hand, since $V_{TH} = 2\phi_F - \frac{Q_d}{C_i}$ and $Q_d = -qN_aW_{max}$, we have $V_{TH} = 2.05V$ □

Exercise 3. SRH recombination
An N type Silicon substrate with $N_d = 7 \times 10^{15} cm^{-3}$ contains the trap energy level located at the Fermi level and the relaxation time $\tau = 10^{-7}s$.

1. Calculate the recombination rate if n and p are much lower than the equilibrium value
2. if only p is less than n_i, what's the recombination rate?

Solution. According to the SRH recombination equation,

$$R = \frac{1}{\tau}\frac{np - n_i^2}{n + p + 2n_i cosh(\frac{E_t - E_F}{kT})}$$

And the trap energy level locates at the Fermi level. If $n < n_i$ and $p < n_i$, we have

$$R = \frac{1}{\tau} \frac{np - n_i^2}{n + p + 2n_i cosh(\frac{E_t - E_F}{kT})} = -\frac{n_i}{2\tau} = -5.3 \times 10^{16} cm^{-3}/s$$

Since $p < n_i$, $n \approx N_d >> n_i$,

$$R = -\frac{n_i^2}{\tau N_d} = -1.4 \times 10^{11} cm^{-3}$$

$\square$