# Recitation #10

ENEE 313: Introduction to Device Physics

Fall, 2018

## 1 Week Notes Summary

This week notes include the diode equation, which we've discussed in the last recitation, and MOSFET capacitor. Two important properties of PN junction, the injection of minority carriers with forward bias and a variation of the depletion width, are used in bipolar junction transistor and field-effect transistor, respectively. Again, we're interested in how to derive the drain current versus the gate voltage and drain voltage, how to calculate the MOS capacitance, and how to compute the threshold voltage, etc. Important notes and formulas include

1. The condition of strong inversion: when the surface potential at the interface between the oxide layer and the semiconductor reaches $2\phi_F$, which is twice of the bulk potential of the substrate(here is p-substrate as example)

$$\phi_{s,inv} = 2\phi_F = 2\frac{kT}{q}ln(\frac{N_a}{n_i})$$

2. Width of the depletion region in MOSFET

$$x_{sc} = \sqrt{\frac{2\epsilon_s\phi_s}{qN_a}}$$

   where $\epsilon_s$ is the permitivity of the semiconductor, $\phi_s$ is the work function of the semiconductor.

3. Max. width of the depletion region

$$x_{sc,max} = \sqrt{\frac{2\epsilon_s\phi_{s,inv}}{qN_a}}$$

4. Threshold voltage of an ideal MOS capacitor

$$V_{TH} = -\frac{Q_{sc}}{C_{ox}} + 2\phi_F$$

5. Threshold voltage of an non-ideal MOS capacitor

$$V_{TH} = \Phi_{ms} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{sc}}{C_{ox}} + 2\phi_F$$

where $C_{ox}$ is the oxide layer capacitance and $Q_{sc}$ is the capacitance of the space charge. The difference between ideal case the non-ideal one comes from the work function difference between the metal and the semiconductor.

6. Minimum capacitance is

$$C_m = \frac{C_{ox}C_d}{C_{ox} + C_d}$$

where $C_{ox}$ is the capacitance of the oxide layer/insulator and $C_d$ is the capacitance of the depletion region in the semiconductor side.

7. MOSFET current formula

   (a) Linear region, $V_{GS} - V_{TH} \geq V_{DS}$:

$$I_D = \mu C_{ox}\frac{W}{L}((V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2})$$

   (b) Saturation region, $V_{GS} - V_{TH} \leq V_{DS}$

$$I_D = \frac{\mu C_{ox} W}{2L}(V_{GS} - V_{TH})^2$$

   where $\mu$ is the electron mobility, $W$ is the gate width, $L$ is the gate length.

You may wonder, under saturation, if the channel is pinched off, how does the drain current flow? The reason is since the drain voltage is higher than the source voltage, the electrons are passing not through a narrow channel but through the substrate. Although the channel is pinched off and getting closer to the source end, the electric field between the drain and the channel is very high. So, the conduction continues.

**Exercise 1.** Pinch-off voltage of JFET, problem 6.1 in the textbook
A JEFT consists of $p^+$ regions doped with $10^{18}cm^{-3}$ acceptors and a n region doped with $10^{16}cm^{-3}$ donors. If the width of the n region is $2\mu m$,

1. What is the pinch-off voltage?
2. What does it mean?
3. Calculate the saturation voltage if $V_G = -1V$.

*Solution.* To find the pinch-off voltage, we first need to calculate the built-in potential,

$$\phi_{bt} = \frac{kT}{q} ln(\frac{N_a N_d}{n_i^2}) = 0.81V$$

According to the formula of the width of the depletion region with bias,

$$\begin{aligned} x_{sc} &= \sqrt{\frac{2\epsilon(\phi_{bt} - V)}{q}(\frac{N_a + N_d}{N_a N_d})} \\ &= \sqrt{\frac{2\epsilon(\phi_{bt} - V_{GD})}{qN_d}} \end{aligned}$$

Since the pinch-off voltage is defined as the voltage when $W$ is equal to the half width of the n region, we have

$$\begin{aligned} x_{sc} &= \sqrt{\frac{2\epsilon(\phi_{bt} - V_{GD})}{qN_d}} = a \\ V_{GD} &= \phi_{bt} - \frac{qN_d a^2}{2\epsilon} \end{aligned}$$

,where $a$ is the half width of the n region. Thus, $V_{GD} = 0.81V - 7.6V = -6.79V$. Therefore, the pinch-off voltage is $6.79V$ since it's defined as positive number and it means the the drain current is about to get into the saturation region. Also, since the pinch-off voltage is $-V_{GD} = 6.79V = -V_G + V_D$, $V_{D,sat} = 6.79V - 1V = 5.79V$ □

**Exercise 2.** MOS capacitor
An ideal Si MOSFET capacitor with oxide layer thickness $t_{ox} = 20nm$ and dielectric constant, $\epsilon_r = 11.8$, has dopant concentration $N_a = 10^{18}cm^{-3}$. Let the oxide dielectric constant be $\epsilon_r = 25$,

1. Calculate the maximum depletion region
2. Calculate the minimum capacitance
3. Calculate the threshold voltage

*Solution.* The bulk potential of the n substrate is

$$\phi_F = V_T ln(\frac{N_a}{n_i}) = 0.468V$$

Therefore, the max $x_{sc}$ is

$$\begin{aligned} x_{sc,max} &= \sqrt{\frac{2\epsilon_s \phi_{s,inv}}{qN_a}} \\ &= \sqrt{\frac{2 \times 11.8 \times 8.85 \times 10^{-14} \times 2 \times 0.468}{1.6 \times 10^{-19} \times 10^{18}}} \\ &= 3.5\mu m \end{aligned}$$

To calculate the threshold voltage, since this is an ideal case, we use

$$\begin{aligned} V_{TH} &= 2\phi_F - \frac{Q_{sc}}{C_{ox}} \\ Q_{sc} &= -qN_a x_{sc,max} \\ C_{ox} &= \frac{\epsilon_{ox}}{t_{ox}} \end{aligned}$$

Thus,

$$\begin{aligned} Q_{sc} &= -1.6 \times 10^{-19} \times 10^{16} \times 3.5 \times 10^{-4} = -5.6 \times 10^{-7} C/cm^2 \\ C_{ox} &= \frac{25 \times 8.85 \times 10^{-14}}{20 \times 10^{-7}} = 1.11 \times 10^{-6} F/cm^2 \\ V_{TH} &= 2 \times 0.468 + \frac{5.6 \times 10^{-7}}{1.11 \times 10^{-6}} = 1.44V \end{aligned}$$

Since the depletion capacitance is $C_d = \frac{\epsilon_{Si}}{x_{sc}} = 2.98 \times 10^{-9} F/cm^2$, the minimum capacitance is

$$\begin{aligned} C_{min} &= \frac{C_{ox} C_d}{C_{ox} + C_d} \\ &= 2.97 \times 10^{-9} F/cm^2 \end{aligned}$$

□

**Exercise 3.** MOSFET current
Given a common MOSFET of $W/L = 1$ operating at $V_{GS} = 3V$ and $V_{DS} = 0.5V$, if the drain current is $I_D$, what would you do to double the current?

*Solution.* Assume $V_{TH}$ is $1.5V$, the MOSFET operates at linear region. Thus, the drain current is

$$I_D = \mu C_{ox} \frac{W}{L}((V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}), \text{, for } V_{GS} - V_{TH} \geq V_{DS}$$

Since $V_{GS}$ and $V_{DS}$ remain unchanged, $\mu$ is constant, $V_{TH}$ and $C_{ox}$ won't change unless the doping and the oxide thickness change, to double $I_D$, we can twice the width, $2W$, or half the length, $\frac{L}{2}$. What if it's saturated? □