

# ENEE 303 Recitation 09

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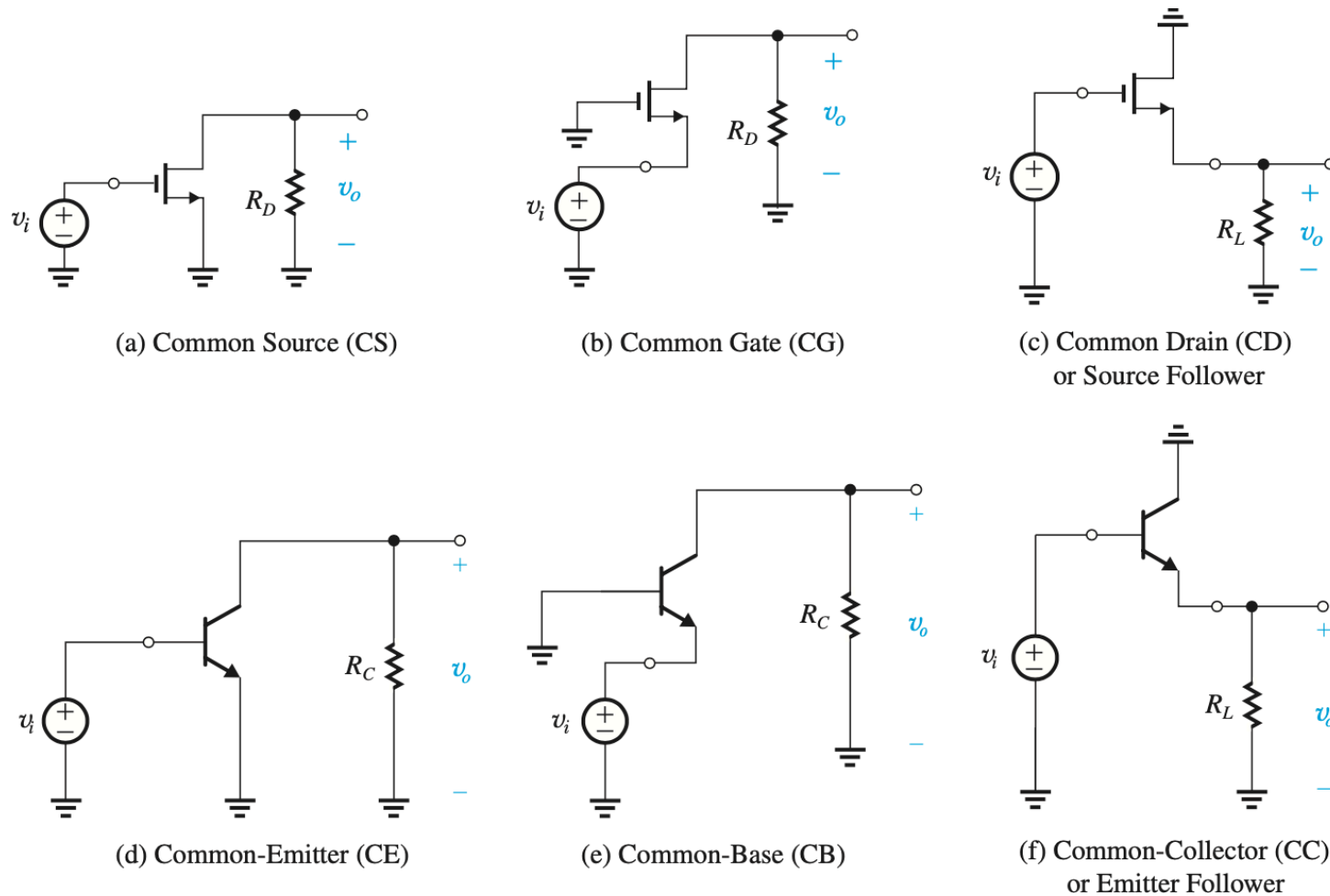
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04/10/2020

# Week Notes

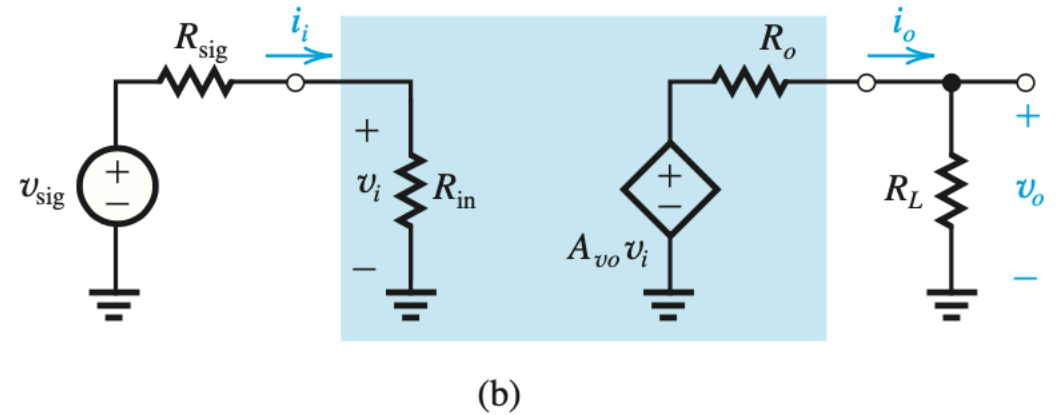
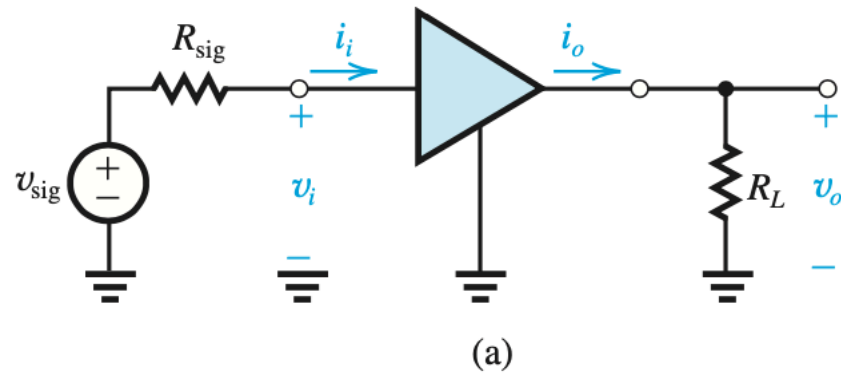
- Three basic configurations of transistor amplifiers and characterizing these amplifiers
  - Common source/emitter
  - Common gate/base
  - Common drain/collector
- 4 example problems + 1 optional example problem

# Basic configurations



**Figure 7.33** The basic configurations of transistor amplifiers. (a)–(c) For the MOSFET; (d)–(f) for the BJT.

# Characterizing transistor amplifiers



Unilateral amplifier: input resistance is independent of load resistance

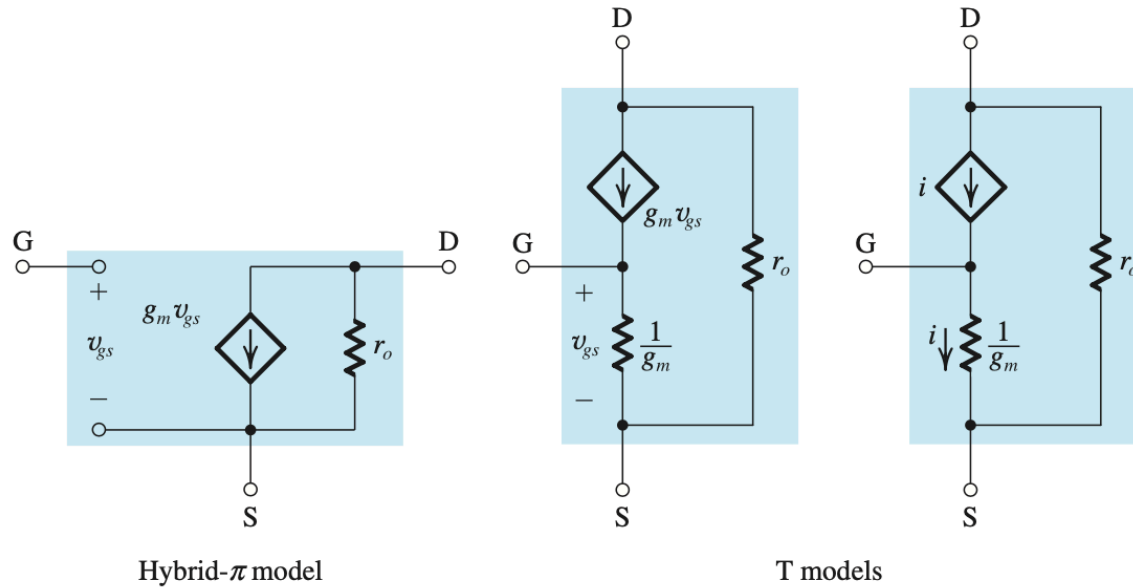
$$R_{in} \equiv \frac{v_i}{i_i} \quad A_{vo} \equiv \left. \frac{v_o}{v_i} \right|_{R_L = \infty} \quad R_o = \frac{v_x}{i_x}$$

$$A_v \equiv \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o} \quad G_v \equiv \frac{v_o}{v_{sig}}$$

# Small signal models

## MOSFET

Small-Signal, Equivalent-Circuit Models



Small signal condition:  $v_{gs} \ll 2(V_{GS} - V_t)$

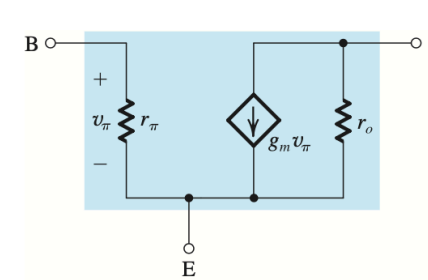
$$g_m \equiv \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{GS}=V_{GS}} = k_n (V_{GS} - V_t)$$

$$A_v \equiv \frac{v_{ds}}{v_{gs}} = -g_m R_D$$

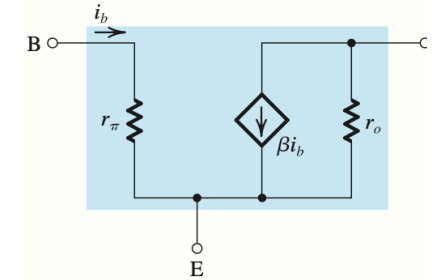
## BJT

Hybrid- $\pi$  Model

■  $(g_m v_\pi)$  Version

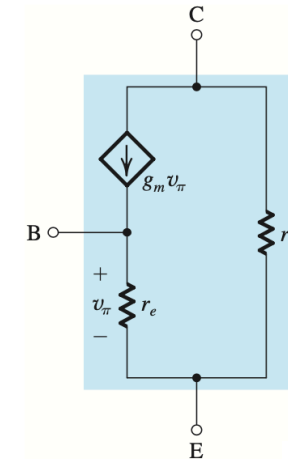


■  $(\beta i_b)$  Version

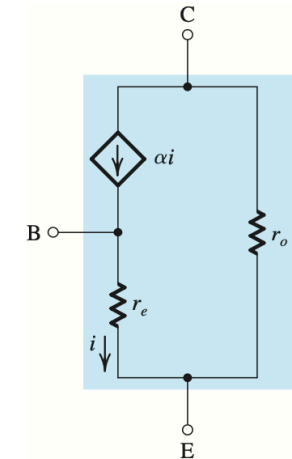


T Model

■  $(g_m v_\pi)$  Version



■  $(\alpha i)$  Version



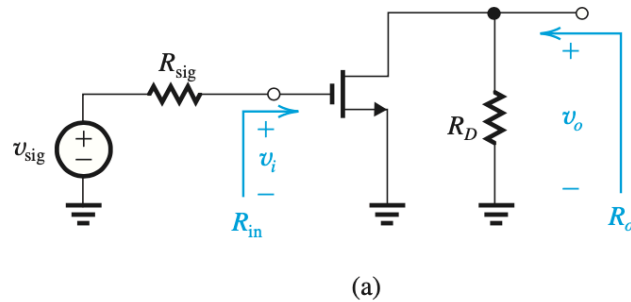
$v_{be} \ll V_T$

$$g_m = \left. \frac{\partial i_C}{\partial v_{BE}} \right|_{i_C=I_C} = \frac{I_C}{V_T} \quad r_\pi = \frac{\beta}{g_m}$$

$$A_v \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C$$

# The common-source/emitter amplifiers

- Characteristics of the CS amplifier



$$R_{in} = \infty$$

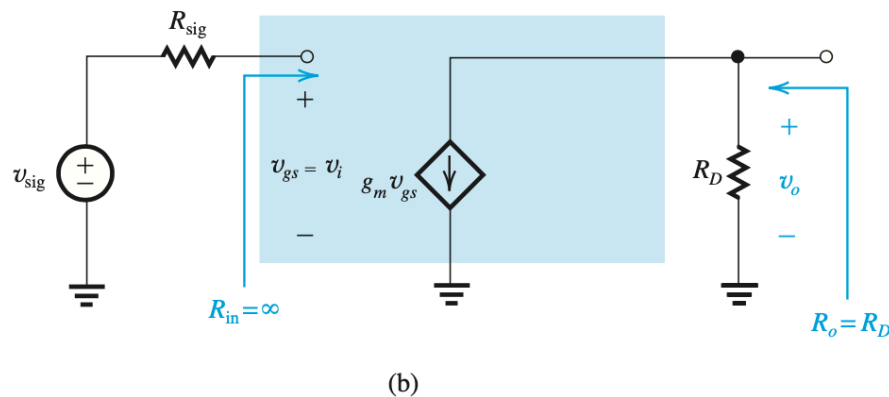
$$v_i = v_{sig}$$

$$v_{gs} = v_i$$

$$v_o = -g_m v_{gs} R_D$$

$$A_{vo} \equiv \frac{v_o}{v_i} = -g_m R_D$$

$$R_o = R_D$$



If a load resistance  $R_L$  is connected across  $R_D$ , the voltage gain  $A_v$  can be obtained

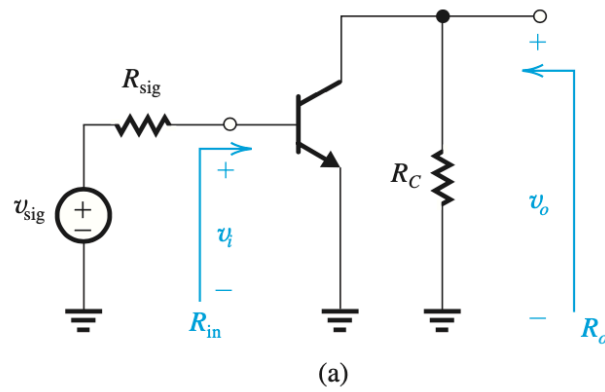
$$A_v = -g_m (R_D \parallel R_L)$$

$$G_v \equiv \frac{v_o}{v_{sig}} = -g_m (R_D \parallel R_L)$$

**Figure 7.35** (a) Common-source amplifier fed with a signal  $v_{sig}$  from a generator with a resistance  $R_{sig}$ . The bias circuit is omitted. (b) The common-source amplifier with the MOSFET replaced with its hybrid- $\pi$  model.

# The common-source/emitter amplifiers

- Characteristics of the CE amplifier



$$R_{in} = r_{\pi} = \beta/g_m$$

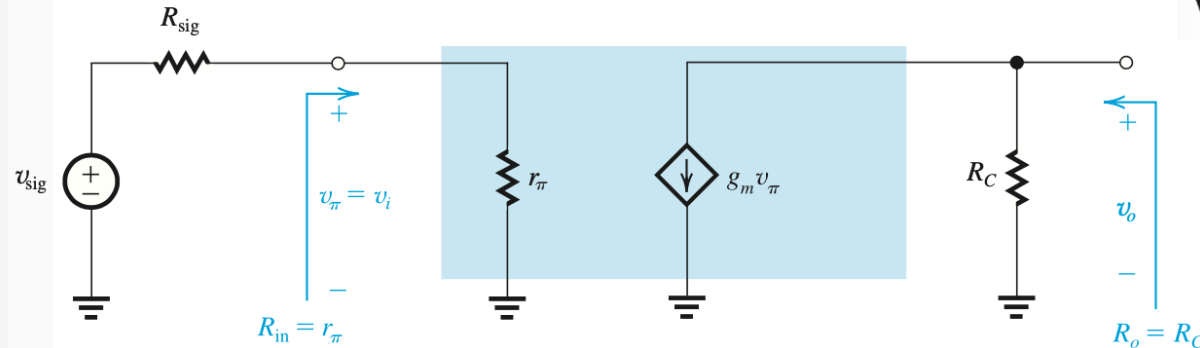
$$v_i = \frac{r_{\pi}}{r_{\pi} + R_{sig}} v_{sig}$$

$$A_{vo} \equiv \frac{v_o}{v_i} = -g_m R_C$$

$$v_{\pi} = v_i$$

$$v_o = -g_m v_{\pi} R_C$$

$$R_o = R_C$$



With a load resistance  $R_L$  connected across  $R_C$ ,

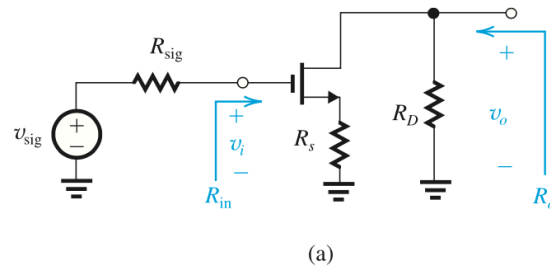
$$A_v = -g_m (R_C \parallel R_L)$$

$$G_v = -\frac{r_{\pi}}{r_{\pi} + R_{sig}} g_m (R_C \parallel R_L)$$

**Figure 7.36** (a) Common-emitter amplifier fed with a signal  $v_{sig}$  from a generator with a resistance  $R_{sig}$ . The bias circuit is omitted. (b) The common-emitter amplifier circuit with the BJT replaced by its hybrid- $\pi$  model.

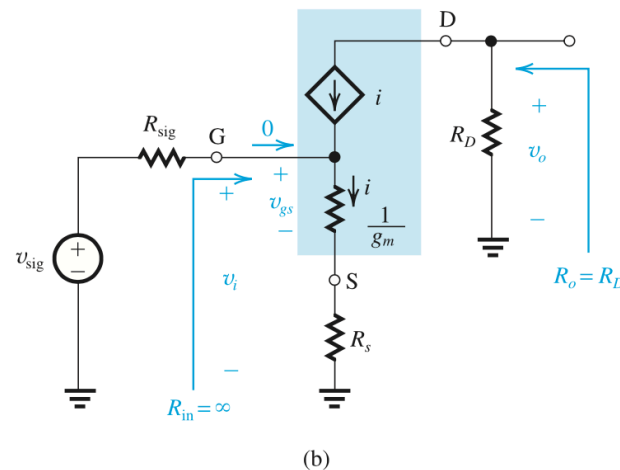
# The common-source/emitter amplifiers with a source/emitter resistance

- Characteristics of the CS amplifier with a source resistance



$$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_s} = \frac{v_i}{1 + g_m R_s}$$

$$v_o = -i R_D \quad i = \frac{v_i}{1/g_m + R_s} = \left( \frac{g_m}{1 + g_m R_s} \right) v_i \quad A_{vo} = -\frac{R_D}{1/g_m + R_s}$$



consider the situation of a load resistance  $R_L$  connected at the output.

$$R_o = R_D$$

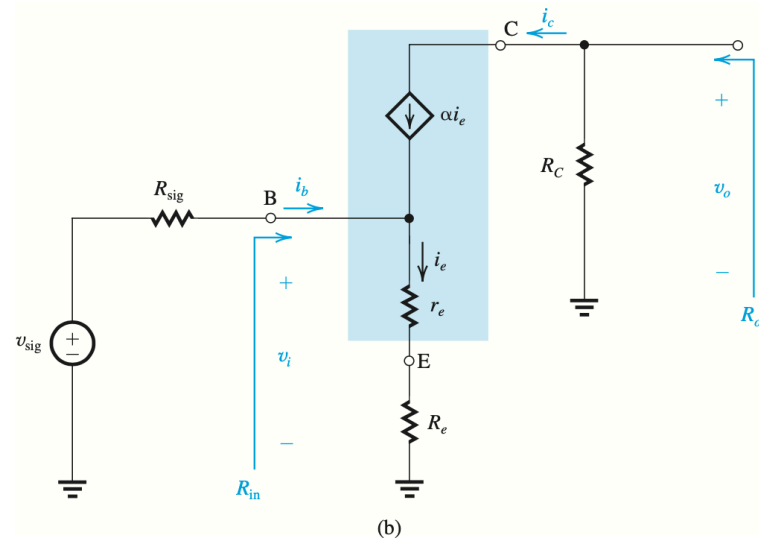
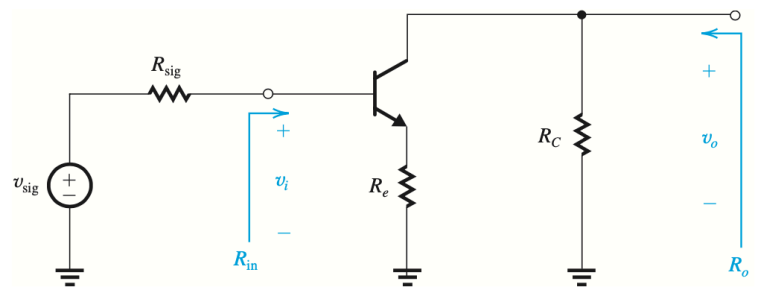
$$A_v = -\frac{g_m (R_D \parallel R_L)}{1 + g_m R_s}$$

**Figure 7.37** The CS amplifier with a source resistance  $R_s$ : (a) circuit without bias details; (b) equivalent circuit with the MOSFET represented by its T model.



# The common-source/emitter amplifiers with a source/emitter resistance

- Characteristics of the CE amplifier with a source resistance



$$R_{in} = (\beta + 1)(r_e + R_e)$$

$$\frac{R_{in}(\text{with } R_e \text{ included})}{R_{in}(\text{without } R_e)} = \frac{(\beta + 1)(r_e + R_e)}{(\beta + 1)r_e} = 1 + \frac{R_e}{r_e} \simeq 1 + g_m R_e$$

$$A_{vo} = -\alpha \frac{R_C}{r_e + R_e} \simeq -\frac{g_m R_C}{1 + g_m R_e}$$

If a load resistance  $R_L$  is connected at the amplifier output,  $A_v$  can be found as

$$A_v = A_{vo} \frac{R_L}{R_L + R_o}$$

$$= -\alpha \frac{R_C}{r_e + R_e} \frac{R_L}{R_L + R_C}$$

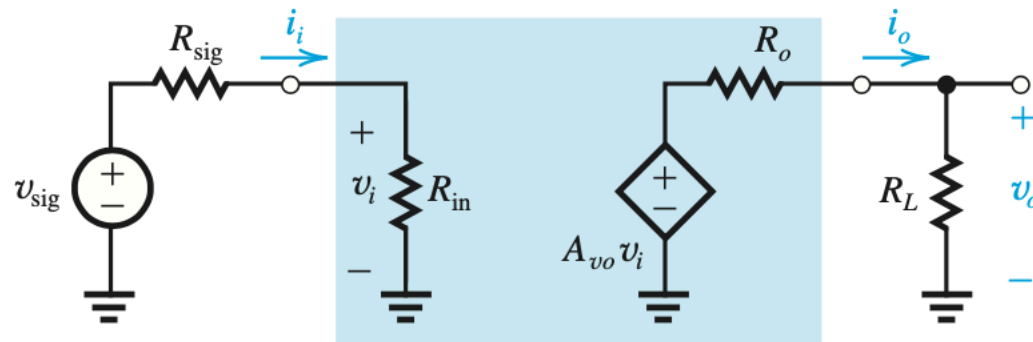
$$= -\alpha \frac{R_C \parallel R_L}{r_e + R_e}$$

$$G_v = -\beta \frac{R_C \parallel R_L}{R_{sig} + (\beta + 1)(r_e + R_e)}$$

**Figure 7.38** The CE amplifier with an emitter resistance  $R_e$ ; (a) circuit without bias details; (b) equivalent circuit with the BJT replaced with its T model.

# Example problem 1

An amplifier with an input resistance of  $100\text{ k}\Omega$ , an open-circuit voltage gain of  $100\text{ V/V}$ , and an output resistance of  $100\text{ }\Omega$  is connected between a  $20\text{ k}\Omega$  signal source and a  $2\text{ k}\Omega$  load. Find the overall voltage gain  $G_v$ . Also find the current gain, defined as the ratio of the load current to the current drawn from the signal source.



(b)

$$R_{in} = 100\text{ k}\Omega \quad R_{sig} = 20\text{ k}\Omega$$

$$R_o = 100\text{ }\Omega \quad R_L = 2\text{ k}\Omega$$

$$A_{vo} = 100\text{ V/V}$$

$$G_v \equiv \frac{v_o}{v_{sig}}$$

$$v_i = \frac{R_{in}}{R_{in} + R_{sig}} v_{sig}$$

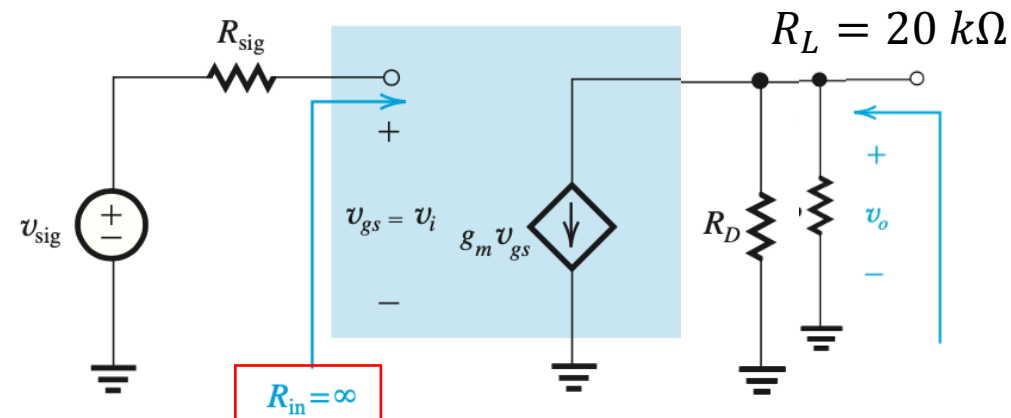
$$v_o = \frac{R_L}{R_o + R_L} A_{vo} v_i = \frac{R_L}{R_o + R_L} A_{vo} \frac{R_{in}}{R_{in} + R_{sig}} v_{sig}$$

$$G_v = \frac{R_L}{R_o + R_L} A_{vo} \frac{R_{in}}{R_{in} + R_{sig}} = 952\text{ V/V}$$

$$A_i = \frac{i_o}{i_i} = \frac{\frac{v_o}{R_L}}{\frac{v_i}{R_{in}}} = \frac{v_o R_{in}}{v_i R_L} = \frac{\frac{R_L}{R_o + R_L} A_{vo} v_i R_{in}}{v_i R_L} = 416.6\text{ A/A}$$

# Example Problem 2

**7.21** A CS amplifier utilizes a MOSFET biased at  $I_D = 0.25$  mA with  $V_{OV} = 0.25$  V and  $R_D = 20$  k $\Omega$ . The amplifier is fed with a signal source having  $R_{sig} = 100$  k $\Omega$ , and a 20-k $\Omega$  load is connected to the output. Find  $R_{in}$ ,  $A_{vo}$ ,  $R_o$ ,  $A_v$ , and  $G_v$ . If, to maintain reasonable linearity, the peak of the input sine-wave signal is limited to 10% of  $2V_{OV}$ , what is the peak of the sine-wave voltage at the output?



$$I_D = \frac{1}{2} k_n V_{OV}^2 \quad g_m = \frac{i_d}{v_{gs}} = k_n V_{OV}$$

$$g_m = \frac{2I_D}{V_{OV}} = 2 \text{ mS}$$

$$R_o = R_D$$

$$A_{vo} = -g_m R_o = -4 \text{ V/V}$$

$$A_v = \frac{v_o}{v_i} = -g_m (R_D || R_L) = -20 \text{ V/V}$$

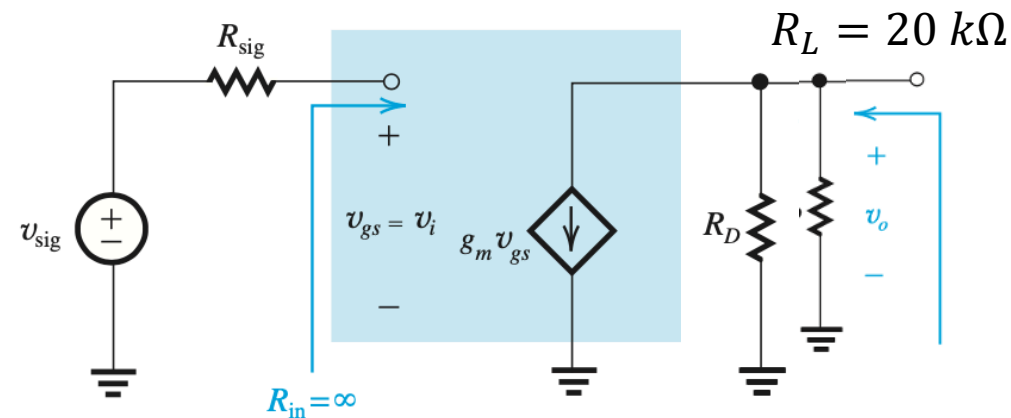
$$G_v = \frac{v_o}{v_{sig}} = \frac{v_o}{v_i} = A_v$$

$$|v_{sig}| = 10\%(2V_{OV}) = 0.05 \text{ V}$$

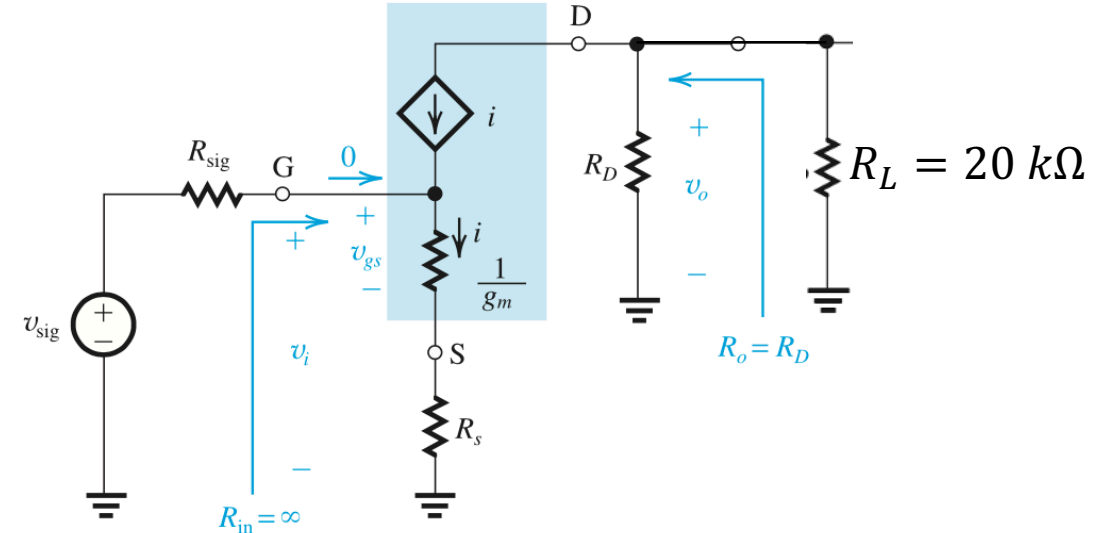
$$|v_o| = G_v |v_{sig}| = 1 \text{ V}$$

# Example problem 3

**7.23** In Exercise 7.21 we applied an input signal  $v_{\text{sig}}$  of 50 mV peak and obtained an output signal of approximately 1 V peak. Assume that for some reason we now have an input signal  $v_{\text{sig}}$  that is 0.2 V peak and that we wish to modify the circuit to keep  $v_{gs}$  unchanged, and thus keep the nonlinear distortion from increasing. What value should we use for  $R_s$ ? What value of  $G_v$  will result? What will the peak signal at the output become? Assume  $r_o = \infty$ .



$$|A_v| = \frac{v_o}{v_i} = 20 \text{ V/V}$$



# Example problem 3 continued

$$v_{gs} = \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_s} v_i = \frac{1}{1 + g_m R_s} v_{sig}$$

$$R_s = \frac{1}{g_m} \left( \frac{v_{sig}}{v_{gs}} - 1 \right) = 1.5 \text{ k}\Omega$$

$$v_{sig} = 200 \text{ mV}$$

$$v_{gs} = 50 \text{ mV}$$

$$G_v = A_v = -\frac{g_m (R_D \parallel R_L)}{1 + g_m R_s} = -5 \text{ V/V}$$

$$|v_o| = G_v |v_{sig}| = 1 \text{ V}$$

# Example problem 4

- Design a CE amplifier with a resistance  $R_e$  in the emitter to meet the following specifications:
  - (i) Input resistance  $R_{in} = 15 \text{ k}\Omega$ .
  - (ii) When fed from a signal source with a peak amplitude of 0.15 V and a source resistance of  $30 \text{ k}\Omega$ , the peak amplitude of  $v_{BE}$  is 5 mV.
- Specify  $R_e$  and the bias current  $I_C$ . The BJT has  $\beta = 74$ . If the total resistance in the collector is  $6 \text{ k}\Omega$ , find the overall voltage gain  $G_v$  and the peak amplitude of the output signal  $v_o$ .

# Example problem 4 continued

$$R_{in} = \frac{v_i}{i_b}$$

$$i_e = i_b + i_c = (1 + \beta)i_b \quad i_b = \frac{i_e}{1 + \beta}$$

$$v_i = i_e(r_e + R_e)$$

$$R_{in} = (1 + \beta)(r_e + R_e) = 15 \text{ k}\Omega$$

$$\frac{v_{BE}}{v_i} = \frac{r_e}{r_e + R_e}$$

$$v_i = i_b R_{in} \quad i_b = \frac{v_{sig} - v_i}{R_{sig}}$$

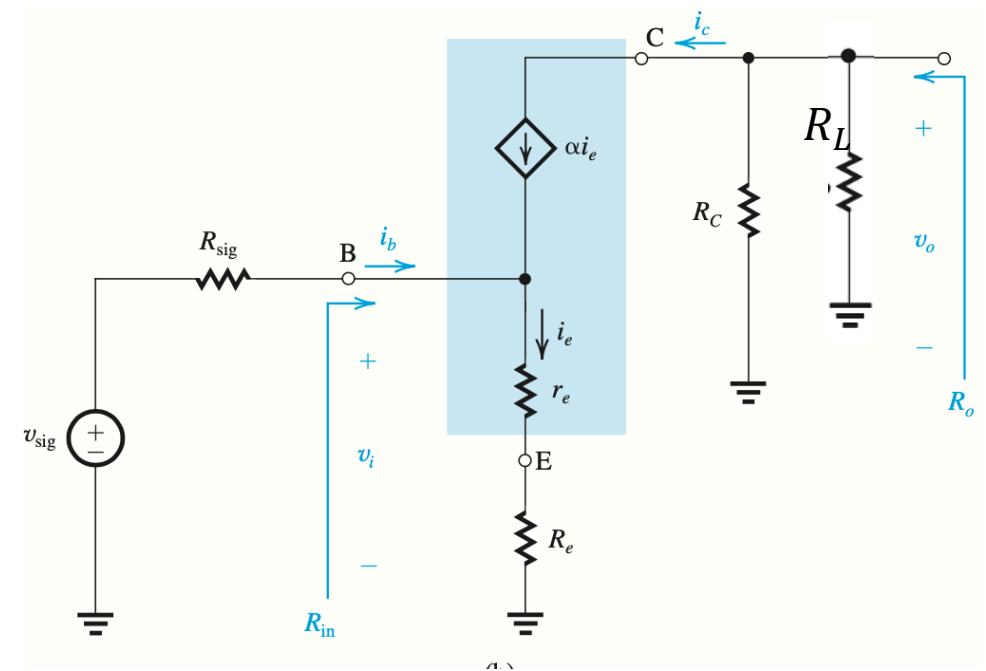
$$v_i = (v_{sig} - v_i) \frac{R_{in}}{R_{sig}}$$

$$v_i = \frac{v_{sig}}{(1 + \frac{R_{in}}{R_{sig}})}$$

$$\frac{v_{BE}}{v_{sig}} = \frac{r_e}{r_e + R_e} \frac{R_{sig}}{R_{sig} + R_{in}}$$

$$r_e = 100 \Omega$$

$$R_e = 200 \Omega$$



Input resistance  $R_{in} = 15 \text{ k}\Omega$

$R_{sig} = 30 \text{ k}\Omega$

$v_{sig} = 0.15 \text{ V}$

$v_{BE} = 5 \text{ mV}$

$\beta = 74$

# Example problem 4 continued

$$v_i = \frac{v_{sig}}{(1 + \frac{R_{in}}{R_{sig}})} = 0.1 \text{ V} \quad i_e = \frac{v_i}{(r_e + R_e)} = 50 \mu\text{A}$$

$$i_c = \alpha i_e \quad \alpha = \frac{\beta}{1 + \beta} \quad i_c = 49.3 \mu\text{A}$$

$$G_v = -\beta \frac{R_C || R_L}{R_{sig} + (\beta + 1)(r_e + R_e)} = -9.86 \text{ V/V}$$

$$A_v = \frac{v_o}{v_i} = -\alpha \frac{R_C || R_L}{r_e + R_e}$$

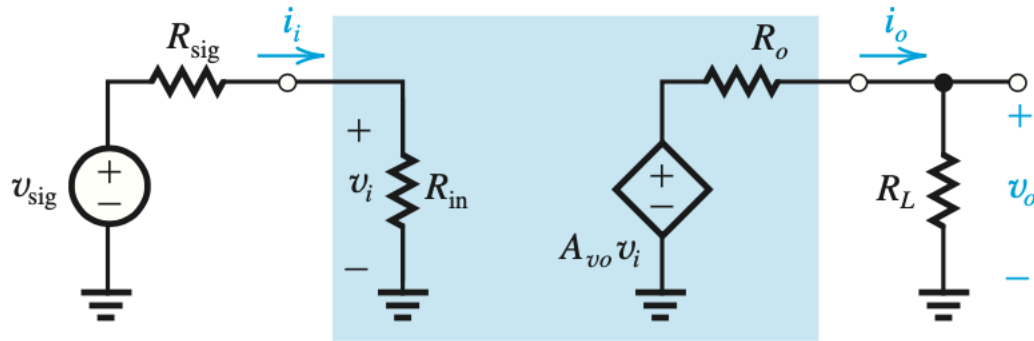
$$v_o = -\alpha \frac{R_C || R_L}{r_e + R_e} v_i = -2.96 \text{ V}$$



## Example problem 5 (optional)

- Specify the parameters  $R_{in}$ ,  $A_{vo}$ , and  $R_o$  of an amplifier that is to be connected between a 100-k $\Omega$  source and a 2-k $\Omega$  load and is required to meet the following specifications:
- (a) No more than 5% of the signal strength is lost in the connection to the amplifier input;
- (b) If the load resistance changes from the nominal value of 2 k $\Omega$  to a low value of 1 k $\Omega$ , the change in output voltage is limited to 5% of nominal value; and
- (c) The nominal overall voltage gain is 10 V/V.

# Example problem 5 continued



(b)

$$R_{sig} = 100 \text{ k}\Omega$$

$$R_L = 2 \text{ k}\Omega$$

$$|v_i| = 0.95|v_{sig}| = \frac{R_{in}}{R_{in} + R_{sig}}|v_{sig}| \quad R_{in} = 1.9 \text{ M}\Omega$$

$$v_{0@2k} = A_{Vo} \frac{R_{2k}}{R_{2k} + R_o} \frac{R_{in}}{R_{in} + R_{sig}} v_{sig}$$

$$v_{0@1k} = A_{Vo} \frac{R_{1k}}{R_{1k} + R_o} \frac{R_{in}}{R_{in} + R_{sig}} v_{sig}$$

$$\frac{v_o}{v_i} = A_{Vo} \frac{R_L}{R_L + R_o}$$

# Example problem 5 continued

$$\frac{v_{0@1k}}{v_{0@2k}} = \frac{\frac{R_{1k}}{R_{1k} + R_o}}{\frac{R_{2k}}{R_{2k} + R_o}}, \text{ where } v_{0@1k} = 0.95v_{0@2k}$$

$$R_o = 111 \, \Omega$$

$$G_v = \frac{v_o}{v_{sig}} = A_{Vo} \frac{R_{1k}}{R_{1k} + R_o} \frac{R_{in}}{R_{in} + R_{sig}} = 10 \, V/V$$

$$A_{Vo} = 11.7 \, V/V$$