Recitation #9

ENEE 313: Introduction to Device Physics

Fall, 2018

1 Week Notes Summary

This week notes include continuity equation and the diode equation. Important formulas of the PN junctions include

1. The governing equation for the PN junction diode is

$$I = I_0(e^{V/V_T} - 1) \tag{1}$$

   where I_0 is the saturated current, V is the applied voltage, V_T is the thermal voltage.

2. continuity equation for electrons and holes

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n \tag{2}$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\frac{\partial J_p}{\partial t} + G_p - R_p \tag{3}$$

3. We use the continuity equations to derive the diode equation in

4. When forward bias provided, the diffusion current dominates over the drift current; when reverse bias provided, the diffusion current is usually negligible.

5. The drift current is relatively insensitive to the height of the potential barrier.

Exercise 1. PN junctions
A diode is composed of an abrupt PN junction with donor concentration = N_d and acceptor concentration = N_a and has cross-sectional area A. Assume the diode has infinite length compared to the depletion width. The diode is forward biased with v_A. Given that the excess electron concentration in the P side during v_A is

$$\Delta n(x) = n_{p0}(exp(\frac{v_A}{v_T}) - 1)exp(\frac{x_p - x}{L_n})$$

1. derive an expression for the electron current in the P quasi neutral region in terms of the appropriate parameters including the doping concentration N_d.

2. do it for the N quasi neutral region.

3. If there is no recombination or generation in the depletion region, mathematically show that the electron and hole currents must be constant in this region, and obtain these constant values.

4. What is the total current?

Solution. In equilibrium, $n(x) = n_{p0} + \Delta n(x)$ leads to $\frac{dn(x)}{dx} = \frac{d\Delta n(x)}{dx}$. Since in the quasi neutral region the electric field is 0, we have

$$\begin{aligned} J_n &= qD_n \frac{dn(x)}{dx} = qD_n \frac{d\Delta n(x)}{dx} \\ &= -\frac{qD_n n_{p0}}{L_n}(exp(\frac{v_A}{v_T}) - 1)exp(\frac{x_p - x}{L_n}) \end{aligned}$$

Plus, $n_{p0}N_a = n_i^2$. Thus, the electron current in the p side is

$$\begin{aligned} I_{n_p} &= J_n A \\ &= -\frac{qAD_n n_i^2}{L_n N_a}(exp(\frac{v_A}{v_T}) - 1)exp(\frac{x_p - x}{L_n}) \end{aligned}$$

furthermore, since it's symmetric, we have

$$\begin{aligned} I_{p_n} &= J_p A \\ &= -\frac{qAD_p n_i^2}{L_p N_d}(exp(\frac{v_A}{v_T}) - 1)exp(\frac{x_n + x}{L_p}) \end{aligned}$$

where $x < -x_n$.

By the continuity equation, since we consider the steady state condition and there is no recombination and generation in the depletion region, we have

$$\begin{aligned} \frac{\partial n}{\partial t} &= \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n \\ 0 &= \frac{1}{q}\frac{\partial J_n}{\partial x} \end{aligned}$$

Therefore, $J_n(x) = constant$. Same rule applies to $J_p(x) = constant$.

Total current $J(x) = J_{n_p}(x = x_p) + J_{p_n}(x = -x_n)$. □

Exercise 2. Minority carrier concentrations and currents of PN junction
If a Silicon PN junction is doped with $N_d = 10^{16} cm^{-3}$ and $N_a = 10^{17} cm^{-3}$ and has a cross section area $10^4 \mu m^2$. The diffusion length L_n and L_p are both $10 \mu m$. The mobility of electrons and holes is $1500 cm^2/V - s$ and $500 cm^2/V - s$, respectively. Assume the diode can be considered infinitely long and is applied a forward bias $0.3V$. The following problem is under steady state condition.

1. Calculate the minority carrier concentration as a function of position for this device

2. Calculate the electron and hole currents as a function of position throughout the entire device, as well as the total current

3. without doing any calculations, decide what is larger, x_n or x_p

$n_i = 10^{10} cm^{-3}$ and $\epsilon = 11.8 \times 8.85 \times 10^{-14} Fd/cm$

Solution. Let's seperate the diode into 4 regions, which are

1. Region 1: $-\infty < x \leq -x_n$
2. Region 2: $-x_n \leq x \leq 0$
3. Region 3: $0 \leq x \leq x_p$
4. Region 4: $x_p \leq x < \infty$

To calculate the minor carrier concentration, let's start from the region 1. Since the regions outside the depletion region stay quasi-neutral, where means the electric field is zero. Also, it's steady state condition. We have

1. $\frac{\partial p_n(x)}{\partial t} = 0.$
2. $\frac{\partial \phi}{\partial x} = 0$ in region 1.
3. The Generation / Recombination term can be approximated as $G_p - R_p = -\frac{\Delta p_n(x)}{\tau_p}$

Therefore, this results in two equations to solve

$$0 = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n(x)}{\tau_p}$$
$$J_p = -qD_p\frac{\partial p_n(x)}{\partial x}$$

Since $p_n(x) = p_{n0} + \Delta p_n(x)$ as p_{n0} is constant. We have

$$0 = D_p\frac{\partial^2 \Delta p_n(x)}{\partial x^2} - \frac{\Delta p_n(x)}{\tau_p}$$

Thus, $\Delta p_n(x) = Aexp(\frac{x}{L_p}) + Bexp(-\frac{x}{L_p})$. Apply the boundary conditions,

1. As $x \to -\infty$, $\Delta p_n(x)$ should be bounded, $B = 0$
2. At $x = -x_n$, $\Delta p_n(-x_n) = p_{n0}exp(\frac{V_A}{V_T} - 1)$

Thus, we have, in the region 1,

$$\Delta p_n(x) = p_{n0}exp(\frac{V_A}{V_T} - 1)exp(\frac{x_n + x}{L_p}) \tag{4}$$

Similarly, in the region 4, we have

$$\Delta n_p(x) = n_{p0}exp(\frac{V_A}{V_T} - 1)exp(\frac{x_p - x}{L_n}) \tag{5}$$

Remember the carrier concentration in the depletion region(region 2 and region 3) is ≈ 0.

To calculate current density, since we know the change of minority carriers concentration as function of position on each side, we can simply derive the density as the following,

$$\begin{aligned} J_p(x) &= -qD_p\frac{d\Delta p_n(x)}{dx} \\ &= -\frac{qD_pp_{n0}}{Lp}[exp(\frac{V_A}{V_T}) - 1]exp(\frac{x_n + x}{L_p}),\ -\infty \leq x \leq -x_n \end{aligned}$$

Same rule applies to holes,

$$\begin{aligned} J_n(x) &= -qD_n\frac{d\Delta n_p(x)}{dx} \\ &= -\frac{qD_nn_{p0}}{Ln}[exp(\frac{V_A}{V_T}) - 1]exp(\frac{x_p - x}{L_n}),\ x_p \leq x \leq \infty \end{aligned}$$

Furthermore, since in the depletion region, current density of carriers remains constant, we know

$$\begin{aligned} J_p(-x_n) &= -\frac{qD_pp_{n0}}{Lp}[exp(\frac{V_A}{V_T}) - 1],\ -x_n \leq x \leq x_p \\ J_n(x_p) &= -\frac{qD_nn_{p0}}{Ln}[exp(\frac{V_A}{V_T}) - 1],\ -x_n \leq x \leq x_p \end{aligned}$$

For $J_p(x)$ on the P side, we need to use Kirchhoff's current law implying that total current remains constant throughout the diode. $J_{total} = J_p(-x_n) + J_n(x_p)$.

$$\begin{aligned} J_p(x) &= J_{total} - J_n(x),\ x_p \leq x \leq \infty \\ J_n(x) &= J_{total} - J_p(x),\ -\infty \leq x \leq -x_n \end{aligned}$$

Since $N_a \geq N_d$ and $N_a x_p = N_d x_n$, $x_n \geq x_p$ □

Exercise 3. Diode current

A Si PN junction with cross-sectional area $A = 1mm^2$ is formed with $N_d = 10^{15}cm^{-3}$ and $N_a = 10^{20}cm^{-3}$. $\mu_n = 1500cm^2/Vs$ and $\mu_p = 500cm^2/Vs$. $\tau_n = \tau_p = 2.5ms$.

1. Calculate current with a forward bias $0.7V$, assume that the current diffusion dominated.
2. Which carries most of the current? and why?
3. if want to double the electron current, what to do?

Solution. Recall the intrinsic concentration of Silicon is $n_i = 1.5 \times 10^{10} cm^{-3}$. By Einstein's relationship, $D = \mu \frac{kT}{q}$. Thus, $D_n = 39 \frac{cm^2}{s}$ and $D_p = 13 \frac{cm^2}{s}$. Diffusion length of electrons and holes is $L_p = \sqrt{D_p \tau_p} = 0.18cm$ and $L_n = \sqrt{D_n \tau_n} = 0.31cm$. By the diode current equation,

$$I = qA(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p)(exp(\frac{qV}{kT}) - 1)$$

where $p_n = \frac{n_i^2}{N_d}$ and $n_p = \frac{n_i^2}{N_a}$. Thus, $I = 0.13mA$. Since $N_a >> N_d$, most of the current is carried by holes. Again, since holes dominates electrons in the current, to double the current, halve the donor concentration. □