Recitation #8

ENEE 313: Introduction to Device Physics

Fall, 2018

1 Week Notes Summary

This week transits from metal-semiconductor junctions to p-n semiconductor junctions. Important formulas of the PN junctions include

1. The governing equation for the PN junction diode is

$$I = I_0(e^{qV/V_T} - 1) \tag{1}$$

   where I_0 is the saturated current, V is the applied voltage, V_T is the thermal voltage.

2. $I_0 = (\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p)$, where D is the diffusion coefficient and $L = \sqrt{D\tau}$ is diffusion length.

3. Built-in potential

$$V_0 = V_T ln(\frac{N_a N_d}{n_i^2}) \tag{2}$$

4. Depletion region

$$W = \sqrt{(\frac{2\epsilon\phi_{bt}}{q}(\frac{1}{N_a} + \frac{1}{N_d})} \tag{3}$$

5. Penetration of the transition region into the n and p material,

$$x_{p0} = \frac{WN_d}{N_d + N_a} \tag{4}$$

$$x_{n0} = \frac{WN_a}{N_d + N_a} \tag{5}$$

6. Capacitance per area of diode

$$C = \frac{\epsilon}{W} \text{ for PN junction} \tag{6}$$

Exercise 1. PN junctions
If a Silicon PN junction is doped with $N_d = 10^{16} cm^{-3}$ and $N_a = 10^{17} cm^{-3}$, with zero applied voltage,

1. Determine the length of the depletion region length on the N-side and the P-side
2. Find the electric field as a function of position
3. Give the expression for boundary value potentials
4. Find the built-in potential as a function of position

$n_i = 10^{10} cm^{-3}$ and $\epsilon = 11.8 \times 8.85 \times 10^{-14} Fd/cm$

Solution. To calculate x_p and x_n, we use the formulas,

$$x_p = \sqrt{\frac{2\phi_{bt}\epsilon}{qN_a}\frac{N_d}{N_d + N_a}} = 0.03\mu m \tag{7}$$

$$x_n = \sqrt{\frac{2\phi_{bt}\epsilon}{qN_d}\frac{N_a}{N_d + N_a}} = 0.3\mu m \tag{8}$$

where,

$$\phi_{bt} = 0.026 \times ln(\frac{10^{16} \times 10^{17}}{10^{20}}) = 0.775V \tag{9}$$

According to Gauss's law, $\frac{dE}{dx} = \frac{\rho}{\epsilon}$, where ρ is the charge density and ϵ is the dielectric constant of the material. Let's define the p side of the junction is on the right and $x = 0$ is at the connection between the P-side and the N-side. Therefore, on the P-side ($0 \leq x \leq x_p$),

$$\frac{dE}{dx} = \frac{-qN_a}{\epsilon} \tag{10}$$

$$\tag{11}$$

B.C.s: at x = x_p, $E = 0$. Thus, $E(x) = \frac{qN_a}{\epsilon}(x_p - x)$. Similarly, on the N-side($-x_n \leq x \leq 0$), $E(x) = \frac{qN_d}{\epsilon}(x + x_n)$.

To calculate the built-in potential as a function of position, we know $E(x) = -\frac{d\phi(x)}{dx}$ and need to know the boundary value potentials first.

$$E_F - E_{in} = \frac{kT}{q}ln(\frac{n_0}{n_i}) \tag{12}$$

$$E_{ip} - E_F = \frac{kT}{q}ln(\frac{p_0}{n_i}) \tag{13}$$

Because, on the P-side, $N_a >> n_i$, we can say $p_0 \approx N_a$. Thus, on the P-side,

$$E_{ip} - E_F = \frac{kT}{q}ln(\frac{N_a}{n_i}) = 0.419eV \tag{14}$$

on the N-side,

$$E_F - E_{in} = \frac{kT}{q} ln(\frac{n_0}{n_i}) = 0.359eV \tag{15}$$

We usually define intrinsic potential as reference for convenience. Thus, $\phi(x_p) = -0.419V$ and $\phi(x_n) = 0.359V$. To calculate the bulit-in potential in terms of position, we use $E(x) = -\frac{d\phi(x)}{dx}$. Thus, on the P-side,

$$\phi(x_p) - \phi(x) = -\frac{qN_a}{\epsilon}(x_p - x)^2 \text{for } 0 \leq x \leq x_p \tag{16}$$

Similarly, on the N-side,

$$\phi(x) - \phi(x_n) = -\frac{qN_d}{\epsilon}(x + x_n)^2 \text{for } -x_n \leq x \leq 0 \tag{17}$$

Thus, on the P-side $0 \leq x \leq x_p$,

$$\phi(x) = \frac{qN_a}{\epsilon}(x_p - x)^2 - 0.417V \tag{18}$$

And on the N-side, $-x_n \leq x \leq 0$,

$$\phi(x) = -0.359 + \frac{qN_d}{\epsilon}(x + x_n)^2 \tag{19}$$

□

Exercise 2. PN junctions
A Silicon PN junction is doped with $N_d = 5 \times 10^{17} cm^{-3}$ and $N_a = 10^{17} cm^{-3}$ and has area $0.0001cm^2$. $n_i = 1.5 \times 10^{10} cm^{-3}$

1. Draw and label the band diagram
2. Calculate the difference between Fermi level and the intrinsic Fermi level on both sides
3. Calculate the built in potential
4. Calculate the capacitance

Solution. To calculate the band difference, we know

$$E_F - E_{in} = \frac{kT}{q} ln(\frac{n_0}{n_i}) \tag{20}$$

$$E_{ip} - E_F = \frac{kT}{q} ln(\frac{p_0}{n_i}) \tag{21}$$

Because, on the P-side, $N_a >> n_i$, we can say $p_0 \approx N_a$. Thus, on the P-side,

$$E_{ip} - E_F = \frac{kT}{q} ln(\frac{N_a}{n_i}) = 0.45eV \tag{22}$$

on the N-side,

$$E_F - E_{in} = \frac{kT}{q} ln(\frac{N_d}{n_i}) = 0.408eV \tag{23}$$

Simply use the formula to compute the built-in potential,

$$\phi_{bt} = \frac{kT}{q} ln(\frac{N_a N_d}{n_i^2}) = 0.858V \tag{24}$$

Use the formula to calculate the width of the depletion region,

$$W = \sqrt{(\frac{2\epsilon\phi_{bt}}{q}(\frac{1}{N_a} + \frac{1}{N_d})} = 0.1159\mu m \tag{25}$$

Thus, we can get the capacitance of the diode,

$$C = \frac{\epsilon A}{W} = 9.1 \times 10^{-12} Fd \tag{26}$$

□

Exercise 3. PN junctions
A Silicon PN junctions doped with $N_d = 10^{16} cm^{-3}$ and $N_a = 4 \times 10^{18} cm^{-3}$ has area $2 \times 10^{-3} cm^2$. Calculate ϕ_{bt}, x_n, x_p, Q at the N-side, and $E(x = 0)$ at room temperature.

Solution.

1. Use the formula,

$$\phi_{bt} = \frac{kT}{q} ln(\frac{N_a N_d}{n_i^2}) = 0.85V \tag{27}$$

2. Firstly, calculate W,

$$W = W = \sqrt{(\frac{2\epsilon\phi_{bt}}{q}(\frac{1}{N_a} + \frac{1}{N_d})} = 0.334\mu m \tag{28}$$

Thus,

$$x_n = \frac{W N_a}{N_d + N_a} = 0.333\mu m \tag{29}$$

$$x_p = \frac{W N_d}{N_d + N_a} = 0.83nm \tag{30}$$

3. Q at the N-side is $Q = q \times x_n \times A \times N_d = 0.107nC$

4. $E(x = 0) = \frac{qN_d x_n}{\epsilon} = 5.1 \times 10^4 V/cm$

□