Recitation 12

ENEE324: Engineering Probability

Spring, 2018

The following problems are from the textbook except problem 4.11.2 and problem 4.11.5 are from Yates-Goodman.

Problem 8.1.2.
Nefeli, a student in a probability class, takes a multiple-choice test with 10 questions and 3 choices per question. For each question, there are two equally likely possibilities, independent of other questions: either she knows the answer, in which case she answers the question correctly, or else she guesses the answer with probability of success 1/3

1. Given that Nefeli answered correctly the first question, what is the probability that she knew the answer to that question?

2. Given that Nefeli answered correctly 6 out of the 10 questions, what is the posterior PDF of the number of questions of which she knew the answer?

Problem 8.2.7.
We have two boxes, each containing three balls: one black and two white in box 1; two black and one white in box 2. We choose one of the boxes at random, where the probability of choosing box 1 is equal to some given p, and then draw a ball.

1. Describe the MAP rule for deciding the identity of the box based on whether the drawn ball is black or white

2. Assuming that $p = 1/2$ find the probability of an incorrect decision and compare it with the probability of error if no ball had been drawn.

Problem 8.2.4.
The noise voltage in a radar detection system is a Gaussian(0,1) random variable, N. The probability that a target is present is $P(H_1) = 0.01$. In the case of a false alarm, the system issues an unnecessary alert at the cost of $C_{10} = 1$ unit.The cost of a miss is $C_{10} = 10^4$ units because the target could cause a lot of damage. When the target is present, the voltage is $X = 4 + N$, a Gaussian(4,1) random variable. When there is no target present, the voltage is $X = N$, the Gaussian(0,1) random variable. In a binary hypothesis test, the acceptance sets are $A_0 = \{X \leq x_0\}$ and $A_1 = \{X > x_0\}$.

1. What is $x_0 = x_{MAP}$, the decision threshold of the maximum a posteriori probability hypothesis test?

2. What are the error probabilities P_{FA} and P_{MISS} of the MAP test?

Problem 8.2.8.
The probability of heads of a given coin is known to be either q_0 (hypothesis H_0) or q_1 (hypothesis H_1). We toss the coin repeatedly and independently, and record the number of heads before a tail is observed for the first time. We assume that $0 < q_0 < q_1 < 1$, and that we are given prior probabilities $P(H_0)$ and $P(H_1)$. For parts (a) and (b), we also assume that $P(H_1) = P(H_0) = 1/2$

1. Calculate the probability that hypothesis H_1 is true, given that there were exactly k heads before the first tail

2. Consider the decision rule that decides in favor of hypothesis H_1 if $k > k^*$, where k^* is some nonnegative integer, and decides in favor of hypothesis H_0 otherwise. Give a formula for the probability of error in terms of k^*, q_0, and q_1. For what value of k^* is the probability of error minimized? Is there another type of decision rule that would lead to an even lower probability of error?

3. Assume that $q_0 = 0.3, q_1 = 0.7$, and $P(H_1) > 0.7$. How does the optimal choice of k^* (the one that minimizes the probability of error) change as $P(H_1)$ increases from 0.7 to 1.0?