Recitation 6

ENEE324: Engineering Probability

Spring, 2018

The following problems are from the textbook.

Problem 2.6.33.
A coin that has probability of heads equal to p is tossed successively and independently until a head comes twice or a tail comes twice in a row. Find the expected value of the number of tosses.

Problem 2.6.37.
A transmitter send out either a 1 with probability p, or a 0 with probability $1-p$, independent of earlier transmissions. If the number of transmission within a given time interval has a Poisson PMF with parameter λ, show that the number of 1s transmitted in that same time interval has a Poisson PMF with parameter $p\lambda$

Problem 2.7.41(a)(b)(c).
You drive to work 5 days a week for a full year (50 weeks), and with probability $p = 0.02$ you get a traffic ticket on any given day, independent of other days. Let X be the total number of tickets you get in the year.

1. What is the probability that the number of tickets you get is exactly equal to the expected value of X?

2. Calculate approximately the probability in (a) using a Poisson approximation

3. Any one of the tickets is \$10 or \$20 or \$50 with respective probability 0.5, 0.3, and 0.2, and independent of other tickets. Find the mean and the variance of the amount of money you pay in traffic tickets during the year.

Problem 2.7.42(a)(b).
Here is a probabilistic method for computing the area of a given subset S of the unit square. The method uses a sequence of independent random selection of points in the unit square [0,1] $\times$ [0,1], according to a uniform probability law. If the ith point belongs to the subset S the value of random variable X_i is set to 1, and otherwise it is set to 0. Let $X_1, X_2, ...$ be the sequence of random variables thus defined, and for any n, let

$$S_n = \frac{X_1 + X_2 + X_3 + ... + X_n}{n}$$

1. Show that $\mathbf{E}[S_n]$ is equal to the area of the subset S, and that $\text{var}(S_n)$ diminishes to 0 as n increases

2. Show that to calculate S_n, it is sufficient to know S_{n-1} and X_n, so the past value of X_k, $k = 1, ..., n-1$ do not need to be remembered. Give a formula.