Recitation 13

ENEE324: Engineering Probability

Spring, 2018

The following problems are from the textbook except problem 8.1.3 and problem 8.3.2 are from Yates-Goodman's book of probability.

Problem 9.3.22a and .22b.
There are two hypotheses about the probability of heads for a given coin: $\theta = 0.5$ (hypothesis H_0) and $\theta = 0.6$ (hypothesis H_1). Let X be the number of heads obtained in n tosses, where n is large enough so that normal approximations are appropriate. We test H_0 against H_1 by rejecting H_0 if X is greater than some suitably chosen threshold k_n.

1. What should be the value of k_n so that the probability of false rejection is ≤ 0.05?
2. What is the smallest value of n for which both probabilities of false rejection and false acceptance can be made less than or equal to 0.05?

Problem 9.4.25.
Let X be a normal random variable with mean μ and unit variance. We want to test the hypothesis $\mu = 5$ at the 5% level of significance, using n independent samples of X.

1. What is the range of values of the sample mean for which the hypothesis is accepted?
2. Let $n = 10$. Calculate the probability of falsely accepting the hypothesis $\mu = 5$ when the true value of μ is 4.

Problem 8.1.3.
When a chip fabrication facility is operating normally, the lifetime of a microchip operated at temperature T, measure in degree Celsius, is given by ab exponential λ random variable X with expected value $\mathbf{E}[X] = 1/\lambda = (200/T)^2$ years. Occasionally, the chip fabrication plant has contamination problems and the chips tend to fail much more rapidly. To test for contamination problems, each day m chips are subjected to a one-day test at $T = 100C$. Based on the number N of chips that fail in one day, design a significance test for the null hypothesis test H_0 that the plant is operating normally.

1. Suppose the rejection set of the test is $R = \{N > 0\}$. Find the significance level of the test as a function of m, the number of chips tested.
2. How many chips must be tested so that the significance level is $\alpha = 0.01$

3. If we raise the temperature of the test, does the number of chips we need to test increase or decrease?

Problem 8.3.2.
A multilevel QPSK communications system transmits three bits every unit of time. For each possible sequence ijk of three bits, one of eight symbols, $\{s_{000}.s_{001}, ..., s_{111}\}$, is transmitted. When signal s_{ijk} is transmitted, the receiver output is

$$X = s_{ijk} + N$$

where N is a Gaussian(0, $\sigma^2 I$) random vector. The 2-dimensional signal vector $s_{000}, ..., s_{001}$ are shown in figure 1. Let H_{ijk} denote the hypothesis that s_{ijk} was transmitted. The receiver

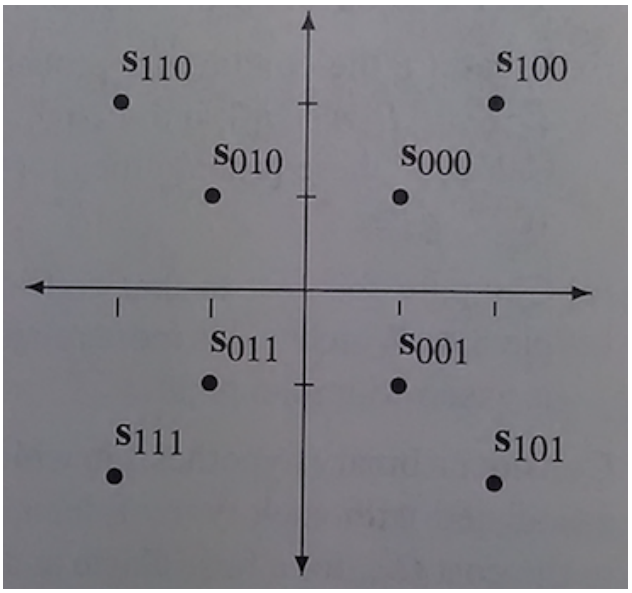

Figure 1: Signal vector

output $X = [X_1 X_2]'$ is used to decide the acceptance sets $\{A_{000}, ..., A_{111}\}$. If all eight symbols are equally likely, sketch the acceptance sets.