Recitation 10

ENEE324: Engineering Probability

Spring, 2018

The following problems are from the textbook except problem 4.11.2 and problem 4.11.5 are from Yates-Goodman.

Problem 4.11.2.
Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = ce^{-(2x^2-2xy+4y^2)}$$

1. What are $\mathbf{E}[X]$ and $\mathbf{E}[Y]$?
2. Find ρ, the correlation coefficient of X and Y.
3. What are $Var[X]$ and $Var[Y]$?
4. What is the constant c?
5. Are X and Y independent?

Problem 4.11.5.
A person's white blood cell (WBC) count W (measured in thousands of cells per microliter of blood) and body temperature T (in degree Celsius) can be modeled as bivariate Gaussian random variables such that $W \sim Gaussian(7,2)$ and $T \sim Gaussian(37,1)$. To determine whether a person is sick, first the person's temperature T is measured. If $T > 38$, then the person's WBC count is measured. If $W > 10$, the person is declared ill(event I).

1. Suppose W and T are uncorrelated. What is $P(I)$?
2. Now suppose W and T have correlation coefficient $\rho = \frac{1}{\sqrt{2}}$. Find the conditional probability $P(I|T=t)$ that a person is declared ill given that the person's temperature is $T=t$.

Problem 4.4.31.
Find the third, fourth, and fifth moments of an exponential random variable with parameter λ

Problem 4.4.37.
A pizza parlor serves n different types of pizza, and is visited by a number of K of customers in a given period of time, where K is a nonnegative integer random variable with a known associated transform $M_K(s) = \mathbf{E}[e^{sK}]$. Each customer orders a single pizza, with all types of pizza being equally likely, independent of the number of other customers and the types of pizza they order. Give a formula, in terms of $M_K(s)$, for the expected number of different types of pizzas ordered.

Problem 4.5.41.
At a certain time, the number of people that enter an elevator is a Poisson random variable with parameter λ. The weight of each person is independent of every other person's weight, and is uniformly distributed between 100 and 200 lbs. Let X_i be the fraction of 100 by which the *ith* person exceeds 100lbs, e.g., if the 7th person weighs 176lbs., then $X_7 = 0.75$. Let Y be the sum of the X_i.

1. Find the transform associated with Y
2. Use the transform to compute the expected value of Y
3. Verify your answer to part 2 by using the low of iterated expectations.