Recitation 9

ENEE324: Engineering Probability

Spring, 2018

The following problems are from the textbook.

Problem 3.5.27.
Let X and Y be continuous random variables with joint PDF $f_{X,Y}$, let A be a subset of the two-dimensional plane, and let $C = \{(X, Y) \in A\}$. Assume that $P(C) > 0$, and define

$$f_{X,Y|C}(x, y) = \begin{cases} \frac{f_{X,Y}(x,y)}{P(C)}, (x, y) \in A \\ 0, o.w. \end{cases}$$

1. Show that $f_{X,Y|C}$ is a legitimate joint PDF
2. Consider a partition of the two-dimensional plan into disjoint subsets $A_i, i = 1, ..., n$, let $C_i = \{(X, Y) \in A_i\}$, and assume that $P(C_i) > 0$ for all i. Derive the following version of the total probability theorem

$$f_{X,Y}(x, y) = \Sigma_{i=1}^{n} P(C_i) f_{X,Y|C_i}(x, y)$$

Problem 3.6.34.
A defective coin minting machine produces coins whose probability of heads is a rand variable P with PDF

$$f_P(p) = \begin{cases} pe^p, p \in [0, 1] 0, o.w. \end{cases}$$

A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent.

1. Find the probability that a coin toss results in heads.
2. Given that a coin toss resulted in heads, find the conditional PDF of P.
3. Given that the first coin toss resulted in heads, find the conditional probability of heads on the next toss.

Problem 4.1.7.
Two points are chosen randomly and independently from the interval [0, 1] according to a uniform distribution. Show that the expected distance between the two points is 1/3.

Problem 4.1.14.
The lifetime of two light bulbs are modeled as independent and exponential random variables X and Y, with parameters λ and μ, respectively. The time at which a light bulb first burns out is $Z = min\{X, Y\}$. Show that Z is an exponential random variable with parameter $\lambda + \mu$.

Problem 4.3.24.
A retired professor comes to the office at a time which is uniformly distributed between 9 a.m. and 1 p.m., performs a single task, and leaves when the task is completed. The duration of the task is exponentially distributed with parameter $\lambda(y) = 1/(5 - y)$, where y is the length of the time interval between 9 a.m. and the time of his arrival.

1. What is the expected amount of time that the professor devotes to the task?

2. What is the expected time at which the task is completed?

3. The professor has a Ph. D. student who on a given day comes to see him at a time that is uniformly distributed between 9 a.m. and 5 p.m. If the student does not find the professor, he leaves and does not return. If he finds the professor, he spends an amount of time that is uniformly distributed between 0 and 1 hour. The professor will spend the same total amount of time on his task regardless of whether he is interrupted by the student. What is the expected amount of time that the professor will spend with the student and what is the expected time at which he will leave his office?