1i) $P_B = P_A + \rho_1 g(1) + \rho_2 g(0.7) - \rho_4 g(0.7)$

$- \rho_6 g(0.8)$

$= 2000 + 9.81(600 + 0.7 \times 700 - 0.7 \times 1000$

$- 700 \times 0.8)$

$= 332.3 Pa$

ii) $\rho_4 \cancel{g}(1 - \Delta h) = \rho_2 \cancel{g}(1)$

$1000(1 - \Delta h) = 700$

$1 - \Delta h = 0.7$

$\Delta h = 0.3$

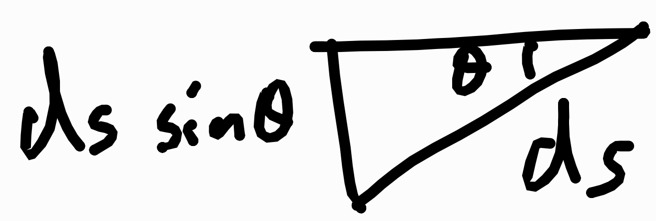
2) $P = P_0 + \rho g h$

$$F = (P_0 + \rho g h)\, ds$$

$$F_h = F \sin\theta$$
$$= (P_0 + \rho g h)\, ds \sin\theta$$

$$F_v = F \cos\theta$$
$$= (P_0 + \rho g h)\, ds \cos\theta$$

Since $ds \sin\theta$ is the height of the infinitesimal segment ds.



The area that the horizontal force is acting on is hence

$$A = \text{width} \times \text{height}$$
$$= 1 \times ds \sin\theta$$
$$= ds \sin\theta,$$

which is the projected area of the gate

2) Since $ds \cos\theta$ is the length of the infinitesimal segment ds, and the vertical force is given

by $F_v = P_0 (ds \cos\theta) \times 1 + \rho g h \, ds \cos\theta \times 1$

$$F_v = P_0 A_{proj} + \rho g h A$$

$$= P_0 A_{proj} + \rho g V$$

$$= P_0 A_{proj} + mg$$

$$= P_0 A_{proj} + w_{fluid}$$

$\therefore$ the vertical force is given by the weight of the volume of fluid above the segment.

3 i) $F_H = \rho_1 g h_1 A + \rho_2 g h_c A$

$= 700 \times 9.81 \times 3 \times 2 \times 2 +$

$1000 \times 9.81 \times \frac{1}{2} \times 2 \times 2 \times 2$

$= 121644 \text{ N}$

$F_{H_{net}} = 121644 - 800 \cos 45° \times 2 \times 2$

$= 119381.2583 \text{ N}$

$\approx 119 \text{ kN}$

ii) Position of the force due to fluid 1 from the pivot

$Y_1 = \frac{1}{2} \times 2$

$= 1 \text{ m}$

Position of the force due to fluid 2 from the pivot

$Y_2 = \frac{2}{3} \times 2$

$= \frac{4}{3} \text{ m}$

Position of the force due to air from the pivot

$Y_{air} = \frac{1}{2} \times 2$

$= 2 \text{ m}$

3ii) Taking moments about the pivot:

$$119381.2583\, y_P = 700 \times 9.81 \times 3 \times 2 \times 2 \times 1$$
$$+ 1000 \times 9.81 \times \tfrac{1}{2} \times 2 \times 2 \times 2 \times \tfrac{4}{3}$$
$$- 800 \cos 45° \times 2 \times 2 \times 1$$

$$y_P = 1.109564937\,m$$

$$Y_R = Y_P + 3$$
$$= 1.109564937 + 3$$
$$= 4.109564937\,m$$
$$\approx 4.1096\,m$$

iii) $F_H = \rho_2 g V + F_{v_{air}}$

$$= 1000 \times 9.81 \times \tfrac{1}{2} \times 2 \times 2 \times 2 + 800 \cos 45° (2 \times 2)$$
$$- 800(2 \times 2)$$

$$= 38302.7417\,N$$
$$\approx 38303\,N$$

3 iv) Distance of the vertical force by fluid 2 from the pivot

$$x_{f_2} = \frac{1}{3} \times 2$$

$$= \frac{2}{3} \text{ m}$$

Distance of the vertical force by air from the pivot

$$x_{air} = \frac{1}{2} \times 2$$

$$= 1 \text{ m}$$

$$38302.7417 x_v = 1000 \times 9.81 \times \frac{1}{2} \times 2 \times 2 \times 2 \left(\frac{2}{3}\right)$$

$$+ (800 \cos 45° - 800)(2 \times 2)(1)$$

$$x_v = 0.658510085 2 \text{ m}$$

$$\approx 0.659 \text{ m}$$

v) Taking moments about the pivot:

$$M_{net} = 38303.7417 (0.6585100852)$$

$$- 119381.2583 (1.109564937)$$

$$= -107237.8581 \text{ Nm}$$

$$\approx -107 \text{ kNm}$$