12a) $\begin{vmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{vmatrix}$

$= \cos^2 n\theta - (-\sin^2 n\theta)$

$= \cos^2 n\theta + \sin^2 n\theta$

$= 1$

b) $\left|\begin{array}{ccc|cc} 16 & 22 & 4 & 16 & 22 \\ 4 & -3 & 2 & 4 & -3 \\ 12 & 25 & 2 & 12 & 25 \end{array}\right.$

$= -96 + 528 + 400 - (-144 + 800 + 176)$

$= 0$

12c) $\left|\begin{array}{ccc|cc} a & b & c & a & b \\ c & a & b & c & a \\ b & c & a & b & c \end{array}\right.$

$= a^3 + b^3 + c^3 - (bac + cba + acb)$

$= a^3 + b^3 + c^3 - 3abc$

12d) $\begin{array}{cccc} + & - & + & - \\ \end{array}$
$\begin{vmatrix} -1 & 0 & -1 & -1 \\ 9 & 5 & 10 & 7 \\ 6 & -2 & 7 & 8 \\ 5 & 2 & 5 & 3 \end{vmatrix}$

$= - \left|\begin{array}{ccc|cc} 5 & 10 & 7 & 5 & 10 \\ -2 & 7 & 8 & -2 & 7 \\ 2 & 5 & 3 & 2 & 5 \end{array}\right. + 0 - \left|\begin{array}{ccc|cc} 9 & 5 & 7 & 9 & 5 \\ 6 & -2 & 8 & 6 & -2 \\ 5 & 2 & 3 & 5 & 2 \end{array}\right.$

$+ \left|\begin{array}{ccc|cc} 9 & 5 & 10 & 9 & 5 \\ 6 & -2 & 7 & 6 & -2 \\ 5 & 2 & 5 & 5 & 2 \end{array}\right.$

$= -(195 - 238) - (230 - 164) + (205 - 176)$

$= 6$

(3) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \sim \begin{vmatrix} x-x_1 & y-y_1 & 0 \\ x_1-x_2 & y_1-y_2 & 0 \\ x_2 & y_2 & 1 \end{vmatrix}$

$$= 0 - 0 + \begin{vmatrix} x-x_1 & y-y_1 \\ x_1-x_2 & y_1-y_2 \end{vmatrix}$$

$$= (x-x_1)(y_1-y_2) - (y-y_1)(x_1-x_2)$$

$$(x-x_1)(y_1-y_2) - (y-y_1)(x_1-x_2) = 0$$

$$(x-x_1)(y_1-y_2) = (y-y_1)(x_1-x_2)$$

$$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} \text{ (shown)}$$

14a) $P = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 7 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = PDP^{-1}$$

$$= \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & -3 \\ 0 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -10 & 26 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

14b) $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$

$$\left| \begin{matrix} 3-\lambda & 2 & 2 \\ 2 & 2-\lambda & 0 \\ 2 & 0 & 4-\lambda \end{matrix} \right| \begin{matrix} 3-\lambda & 2 \\ 2 & 2-\lambda \\ 2 & 0 \end{matrix}$$

$$= 4(2-\lambda) + 0 + 4(4-\lambda) - \left[(3-\lambda)(2-\lambda)(4-\lambda) + 0 + 0\right]$$

$$= 8 - 4\lambda + 16 - 4\lambda - (6 - 3\lambda - 2\lambda + \lambda^2)(4-\lambda)$$

$$= 24 - 8\lambda - (\lambda^2 - 5\lambda + 6)(4-\lambda)$$

$$= 24 - 8\lambda - (4\lambda^2 - \lambda^3 - 20\lambda + 5\lambda^2 + 24 - 6\lambda)$$

$$= \cancel{24} - 8\lambda - (-\lambda^3 + 9\lambda^2 - 26\lambda \cancel{+24})$$

$$= \lambda^3 - 9\lambda^2 + 18\lambda$$

$$= \lambda(\lambda^2 - 9\lambda + 18)$$

$$= \lambda(\lambda-3)(\lambda-6)$$

$$\lambda(\lambda-3)(\lambda-6) = 0$$

$$\therefore \lambda = 0, 3, 6$$

14b) When $\lambda = 0$,

$$\begin{bmatrix} 3-0 & 2 & 2 \\ 2 & 2-0 & 0 \\ 2 & 0 & 4-0 \end{bmatrix} \sim \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore x + 2z = 0 \qquad y - 2z = 0$

$x = -2z \qquad y = 2z$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, t \in \mathbb{R}$$

When $\lambda = 3$,

$$\begin{bmatrix} 3-3 & 2 & 2 \\ 2 & 2-3 & 0 \\ 2 & 0 & 4-3 \end{bmatrix} \sim \begin{bmatrix} 0 & 2 & 2 \\ 2 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$2x + z = 0 \qquad y + z = 0$

$x = -\frac{1}{2}z \qquad y = -z$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, t \in \mathbb{R}$$

When $\lambda = 6$,

$$\begin{bmatrix} 3-6 & 2 & 2 \\ 2 & 2-6 & 0 \\ 2 & 0 & 4-6 \end{bmatrix} \sim \begin{bmatrix} -3 & 2 & 2 \\ 2 & -4 & 0 \\ 2 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 0 & 2 & -1 \\ 0 & -4 & 2 \\ 1 & 0 & -1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x - z = 0$ $\quad$ $2y - z = 0$

$x = z$ $\quad$ $2y = z$

$y = \frac{1}{2} z$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \ t \in \mathbb{R}$$

15) $T = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 5 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 3 & 0 & 5 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 3 & 1 & 0 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 3 & 0 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 2 \end{array}\right]$$

$$\therefore T^{-1} = \begin{bmatrix} 5 & 0 & -3 \\ 0 & 1 & 0 \\ -3 & 0 & 2 \end{bmatrix}$$

$$\hat{A} = T^{-1}AT$$

$$= \begin{bmatrix} 5 & 0 & -3 \\ 0 & 1 & 0 \\ -3 & 0 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 & -15 \\ -3 & -4 & 9 \\ 5 & 0 & -15 \end{bmatrix} \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & -30 \\ -3 & -4 & 9 \\ -5 & 0 & 15 \end{bmatrix} \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -70 & 0 & -120 \\ 21 & -4 & 36 \\ 35 & 0 & 60 \end{bmatrix}$$

(5)
$$\left|\begin{array}{ccc|cc} 5-\lambda & 0 & -15 & 5-\lambda & 0 \\ -3 & -4-\lambda & 9 & -3 & -4-\lambda \\ 5 & 0 & -15-\lambda & 5 & 0 \end{array}\right.$$

$$= -75(-4-\lambda)+0+0-\left[(5-\lambda)(-4-\lambda)(-15-\lambda)+0+0\right]$$
$$= 75\lambda+300-(-20-5\lambda+4\lambda+\lambda^2)(-15-\lambda)$$
$$= 75\lambda+300-(\lambda^2-\lambda-20)(-15-\lambda)$$
$$= 75\lambda\cancel{+300}-(-15\lambda^2-\lambda^3+15\lambda+\lambda^2\cancel{+300}+20\lambda)$$
$$= \lambda^3+14\lambda^2+40\lambda$$
$$= \lambda(\lambda^2+14\lambda+40)$$
$$= \lambda(\lambda+4)(\lambda+10)$$

$$\left|\begin{array}{ccc|cc} -70-\lambda & 0 & -120 & -70-\lambda & 0 \\ 21 & -4-\lambda & 36 & 21 & -4-\lambda \\ 35 & 0 & 60-\lambda & 35 & 0 \end{array}\right.$$

$$= -4200(-4-\lambda)+0+0-\left[(-70-\lambda)(-4-\lambda)(60-\lambda)+0+0\right]$$
$$= 16800+4200\lambda-(280+70\lambda+4\lambda+\lambda^2)(60-\lambda)$$
$$= 16800+4200\lambda-(\lambda^2+74\lambda+280)(60-\lambda)$$
$$= \cancel{16800}+4200\lambda-(60\lambda^2-\lambda^3+4440\lambda-74\lambda^2\cancel{+16800}-280\lambda)$$
$$= \lambda^3+14\lambda^2+40\lambda$$
$$= \lambda(\lambda+4)(\lambda+10)$$

$\therefore$ The eigenvectors of both A and $\hat{A}$ are the same.
$\lambda=0$, $\lambda=-4$ and $\lambda=-10$, which are the same.

16a) $\begin{vmatrix} 3-\lambda & 4 \\ 1 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 4$

$= 9 - 6\lambda + \lambda^2 - 4$

$= \lambda^2 - 6\lambda + 5$

$= (\lambda - 1)(\lambda - 5)$

When $\lambda = 1$,

$\begin{bmatrix} 3-1 & 4 \\ 1 & 3-1 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ $\quad x + 2y = 0$

$x = -2y$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \end{bmatrix}, t \in \mathbb{R}$

When $\lambda = 5$,

$\begin{bmatrix} 3-5 & 4 \\ 1 & 3-5 \end{bmatrix} \sim \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$ $\quad x - 2y = 0$

$x = 2y$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \end{bmatrix}, t \in \mathbb{R}$

$T = \begin{bmatrix} -2 & 2 \\ 1 & 1 \end{bmatrix}$

$T^{-1} = \frac{1}{-2-2} \begin{bmatrix} 1 & -2 \\ -1 & -2 \end{bmatrix}$

$= \frac{1}{4} \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

16a) $D = TAT^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$

b) $\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & 2-\lambda \end{vmatrix} \begin{matrix} 1-\lambda & 0 \\ 0 & 3-\lambda \\ 0 & 0 \end{matrix}$

$= 0 + 0 + 0 - (1-\lambda)(3-\lambda)(2-\lambda) - 0 - 0$

$= (\lambda - 1)(3-\lambda)(2-\lambda)$

when $\lambda = 1$,

$$\begin{bmatrix} 1-1 & 0 & 1 \\ 0 & 3-1 & 2 \\ 0 & 0 & 2-1 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$y = 0, \; z = 0$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; t \in \mathbb{R}$

when $\lambda = 2$,

$$\begin{bmatrix} 1-2 & 0 & 1 \\ 0 & 3-2 & 2 \\ 0 & 0 & 2-2 \end{bmatrix} \sim \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x - z = 0 \qquad y + 2z = 0$

$x = z \qquad y = -2z$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \; t \in \mathbb{R}$

16b) When $\lambda = 3$,

$$\begin{bmatrix} 1-3 & 0 & 1 \\ 0 & 3-3 & 2 \\ 0 & 0 & 2-3 \end{bmatrix} \sim \begin{bmatrix} -2 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x = 0, \ z = 0$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ t \in \mathbb{R}$$

$$T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$TAT^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

16c) $\begin{vmatrix} 5-\lambda & 0 & -6 \\ 2 & 1-\lambda & -4 \\ 3 & 0 & -4-\lambda \end{vmatrix} \begin{matrix} 5-\lambda & 0 \\ 2 & 1-\lambda \\ 3 & 0 \end{matrix}$

$= (5-\lambda)(1-\lambda)(-4-\lambda) + 0 + 0 - [-18(1-\lambda)]$

$= (5 - 5\lambda - \lambda + \lambda^2)(-4-\lambda) + 18 - 18\lambda$

$= (\lambda^2 - 6\lambda + 5)(-4-\lambda) + 18 - 18\lambda$

$= -4\lambda^2 - \lambda^3 + 24\lambda + 6\lambda^2 - 20 - 5\lambda + 18 - 18\lambda$

$= -\lambda^3 + 2\lambda^2 + \lambda - 2$

$= (\lambda+1)(\lambda-2)(\lambda-1)$

when $\lambda = -1$,

$$\begin{bmatrix} 5+1 & 0 & -6 \\ 2 & 1+1 & -4 \\ 3 & 0 & -4+1 \end{bmatrix} \sim \begin{bmatrix} 6 & 0 & -6 \\ 2 & 2 & -4 \\ 3 & 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x - z = 0 \qquad y - z = 0$

$x = z \qquad y = z$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, t \in \mathbb{R}$

16c) when $\lambda = 1$,

$$\begin{bmatrix} 5-1 & 0 & -6 \\ 2 & 1-1 & -4 \\ 3 & 0 & -4-1 \end{bmatrix} \sim \begin{bmatrix} 4 & 0 & -6 \\ 2 & 0 & -4 \\ 3 & 0 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 3 & 0 & -5 \\ 0 & 0 & 2 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = 0, z = 0$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, t \in \mathbb{R}$$

$$\begin{bmatrix} 5-2 & 0 & -6 \\ 2 & 1-2 & -4 \\ 3 & 0 & -4-2 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & -6 \\ 2 & -1 & -4 \\ 3 & 0 & -6 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & -4 \\ 1 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x - 2z = 0 \qquad y = 0$

$x = 2z$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, t \in \mathbb{R}$$

16c) $T = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right]$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$TAT^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$