1a) $\text{curl}\ \underset{\sim}{F} = \nabla \times \underset{\sim}{F}$

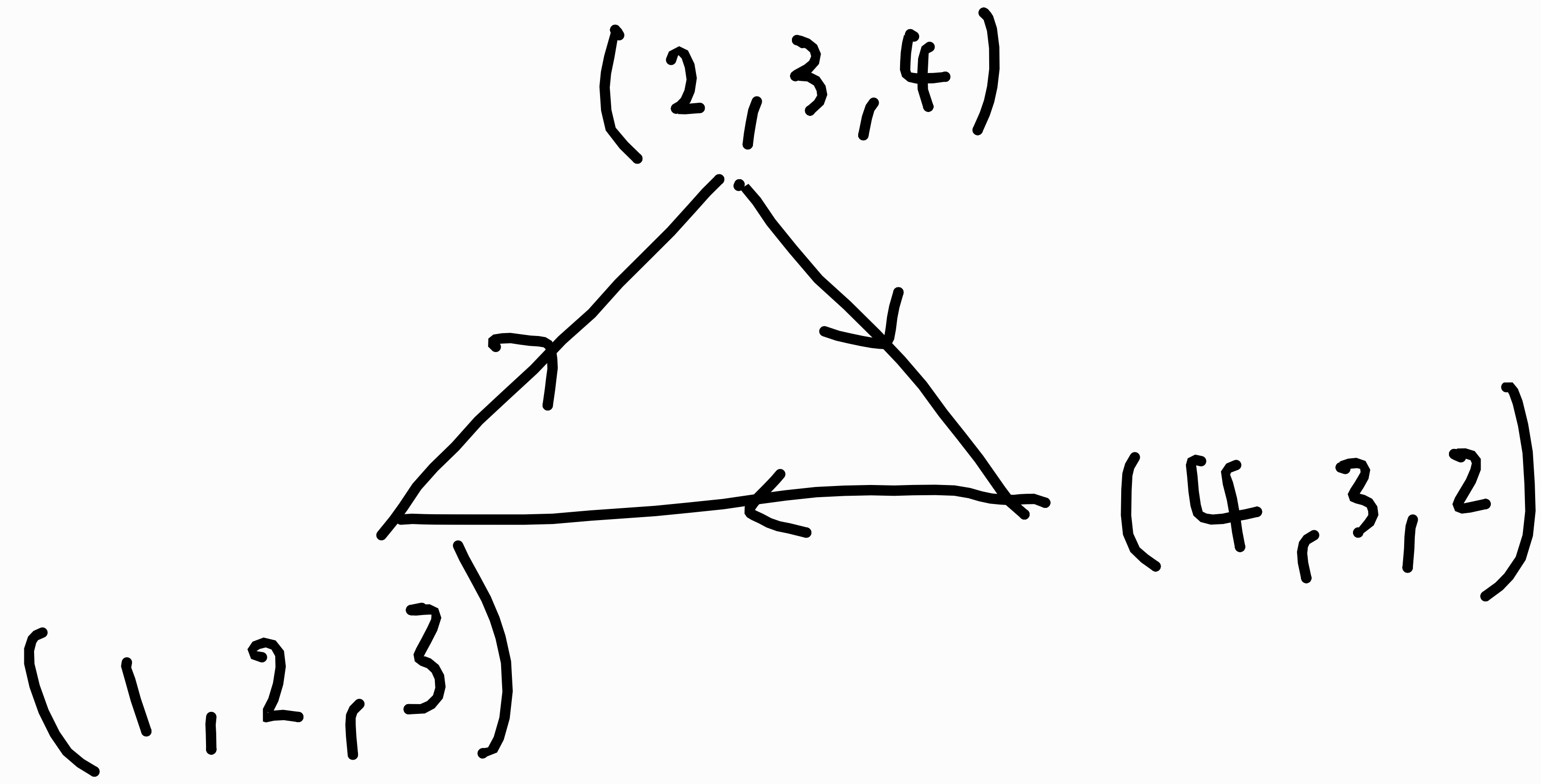
$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (2z, -x, -2y)$$

$$\begin{array}{ccccc} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} & \underset{\sim}{i} & \underset{\sim}{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 2z & -x & -2y & 2z & -x \end{array}$$

$$= -2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k} - 0 - 0 - 0$$

$$= (-2, 2, -1)$$

1b)



Let S be the oriented surface whose oriented boundary is the triangle C. By Stoke's Theorem, the circulation of $\mathbf{F}$ along C is:

$$\oint_c P\,dx + Q\,dy + R\,dz = \iint_S \text{curl}\,\mathbf{F} \cdot \mathbf{u}\,dS$$

Finding the normal vector of the surface,

$$\mathbf{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$$

$$= \left(\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right) \times \left(\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right)$$

$$= (1,1,1) \times (3,1,-1)$$

$$\begin{array}{ccccc} \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{i} & \mathbf{j} \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 3 & 1 \end{array}$$

$$= -\mathbf{i} + 3\mathbf{j} + \mathbf{k} - 3\mathbf{k} - \mathbf{i} + \mathbf{j}$$

$$= (-2, 4, -2)$$

1b) $\underset{\sim}{u} = \dfrac{\underset{\sim}{n}}{\|\underset{\sim}{n}\|}$

Area of $S = \frac{1}{2} ab\sin C$

$= \frac{1}{2} \|\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}\|$

$= \frac{1}{2} \|\underset{\sim}{n}\|$

$$\iint_S \text{curl } F \cdot \underset{\sim}{u} \, dS = \iint_S (-2, 2, -1) \cdot \frac{(-2, 4, -2)}{\|\underset{\sim}{n}\|} dS$$

$$= \frac{14}{\|\underset{\sim}{n}\|} \iint_S dS$$

$$= \frac{14}{\cancel{\|\underset{\sim}{n}\|}} \times \frac{1}{2} \cancel{\|\underset{\sim}{n}\|}$$

$$= 7$$

2a) $\det A = \left|\begin{matrix} 0 & 1 & a \\ 0 & a & 1 \\ 1 & 1 & 1 \end{matrix}\right| \begin{matrix} 0 & 1 \\ 0 & a \\ 1 & 1 \end{matrix}$

$= 1 - a^2$

b) A is invertible if and only if $\det A \neq 0$

When A is invertible, i.e. $a \neq \pm 1$, $A\underset{\sim}{x} = \underset{\sim}{b}$ will have exactly one solution.

For $a = 1$,

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ there are infinitely many solutions when $a = 1$.

For $a = -1$,

$$\begin{bmatrix} 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$\therefore$ the system is inconsistent, which means there are no solutions when $a = -1$.

3a) span S is defined to be the set:

$$\text{span } S = \{k_1\underset{\sim}{x}_1 + k_2\underset{\sim}{x}_2 + \ldots + k_r\underset{\sim}{x}_r : k_1, k_2, \ldots k_r \in \mathbb{R}\}$$

3b) Let $\underset{\sim}{x} = c_1\underset{\sim}{x}_1 + c_2\underset{\sim}{x}_2 + \ldots + c_r\underset{\sim}{x}_r$, $c_1, c_2, \ldots, c_r \in \mathbb{R}$

as $\underset{\sim}{x} \in \text{span } S$

For $\underset{\sim}{u} \in \text{span}(S \cup \{\underset{\sim}{x}\})$

$$\Downarrow$$

$$\underset{\sim}{u} = k_1\underset{\sim}{x}_1 + k_2\underset{\sim}{x}_2 + \ldots + k_r\underset{\sim}{x}_r + k\underset{\sim}{x}, \quad k_1, k_2 \ldots, k_r, k \in \mathbb{R}$$

$$= k_1\underset{\sim}{x}_1 + \ldots + k_r\underset{\sim}{x}_r + k(c_1\underset{\sim}{x}_1 + c_2\underset{\sim}{x}_2 + \ldots + c_r\underset{\sim}{x}_r)$$

$$= (k_1 + kc_1)\underset{\sim}{x}_1 + (k_2 + kc_2)\underset{\sim}{x}_2 + \ldots + (k_r + kc_r)\underset{\sim}{x}_r$$

$$\Downarrow$$

$$\underset{\sim}{u} \in \text{span } S$$

For $\underset{\sim}{u} \in \text{span } S$

$$\Downarrow$$

$$\underset{\sim}{u} = k_1\underset{\sim}{x}_1 + k_2\underset{\sim}{x}_2 + \ldots + k_r\underset{\sim}{x}_r + 0\underset{\sim}{x}, \quad k_1, k_2, \ldots k_r, k \in \mathbb{R}$$

$$\Downarrow$$

$$\underset{\sim}{u} \in \text{span}(S \cup \{\underset{\sim}{x}\}) \quad \therefore \text{span } S = \text{span}(S \cup \{\underset{\sim}{x}\})$$