1 Linkages

Degrees of freedom:
$DoF = 3(n_L - 1) - 2n'_J - n''_J$

4-bar linkage condition:
$L_{max} \le L_{min} + L_a + L_b$

Grashof condition:
$L_{max} + L_{min} \le L_a + L_b$

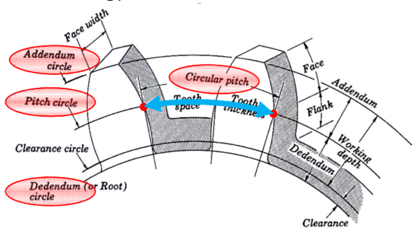
Actual number of joints = Number of links attached to joint −1

1.1 Types of linkages

Crank-rocker: Shortest link next to the fixed link
Drag-link: Shortest link is the fixed link
Double rocker: Shortest link is opposite the fixed link
Change-point or crossover-position: All links are collinear
Triple rocker: Non-Grashof linkage, none of the links makes a $360°$ rotation

2 Gears

Terminology:



Module:
$m = \frac{d_p}{N}$

Circular pitch:
$p_c = \pi \frac{\pi d_p}{N} = \pi m$

Diametral pitch (P_d):
$\frac{1}{P_d} = \frac{m}{25.4}$

Pitch circle radius:
$r = \frac{mN}{2}$

Tooth thickness:
$t = \frac{\pi}{2}$

Addendum:
$a = m$

Velocity ratio:
$r_v = \frac{\omega_2}{\omega_1} = \frac{RPM_2}{RPM_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$

Centre distance:
$c = \frac{d_{p1}+d_{p2}}{2} = \frac{m(N_1+N_2)}{2} = \frac{N_1+N_2}{2P_d}$

Base-circle radius:
$r_b = r\cos\phi$

Base pitch:
$p_b = m\pi\cos\phi = \frac{\pi}{P}\cos\phi$

Contact ratio:
$$C.R. = \frac{\sqrt{(r_2+a_2)^2 - r_2^2\cos^2\phi - r_2\sin\theta}}{p_b} + \frac{\sqrt{(r_1+a_1)^2 - r_1^2\cos^2\phi} - r_1\sin\theta}{p_b}$$

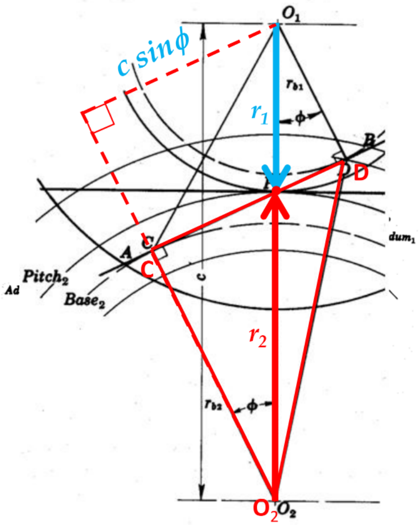
Power:
$P = \omega T$

Tangential force on a gear:
$F_t = \frac{T}{r} = \frac{2T}{mN}$

Radial force on a gear:
$F_r = F_t \tan\phi$

2.1 Avoiding interference

Condition:



$r_1 + a_1 \le \sqrt{r_1^2\cos^2\phi + c^2\sin^2\phi}$

$r_2 + a_2 \le \sqrt{r_2^2\cos^2\phi + c^2\sin^2\phi}$

Pinion and rack condition:
$N \ge \frac{2k}{\sin^2\phi}$

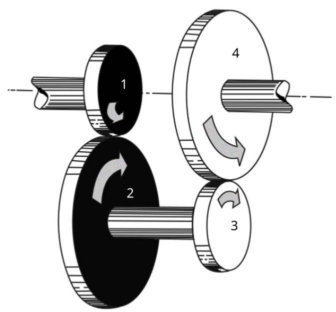
2.2 Gear trains

Formula:
$\frac{\text{Target gear}, n_T}{\text{Driving gear}, n_D} = \frac{\text{Driving gear}}{\text{Driven gear}}$

Planetary gear chains:
$\frac{n_T - n_c}{n_D - n_c} = \left(-\frac{N_1}{N_2}\right)\left(-\frac{N_3}{N_4}\right)\cdots$

Getting the gear ratio:
◇ − for external gear
◇ + for internal gear

2.3 Reverted gear trains



External gears:
$r_1 + r_2 = r_3 + r_4$

External to internal gears:
$r_{internal_1} - r_{external_1} = r_{internal_2} - r_{external_2}$

3 Vector loop equations

◇ Set up a global reference frame.
◇ Assign a direction vector to all links.
◇ Ensure the direction vectors all form a loop and write an equation based on that.
◇ Split the vectors into their sine and cosine components.
◇ Figure out which vectors don't change their length or direction and set them as constants.
◇ Differentiate to find the velocity and acceleration if necessary.

4 Cam motion

Uniform motion:
$s = \frac{L}{\beta}\theta$

$\dot{s} = \frac{L}{\beta}\dot{\theta}$

$\ddot{s} = \frac{L}{\beta}\ddot{\theta}$

$\dddot{s} = \frac{L}{\beta}\dddot{\theta}$

Parabolic motion:
1st parabola: $s = \frac{2L}{\beta^2}\theta^2$

2nd parabola: $s = -L + \frac{4L}{\beta}\theta - \frac{2L}{\beta^2}\theta^2$

Simple harmonic motion:
$s = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$

$\dot{s} = \frac{\pi L\omega}{2\beta}\sin\frac{\pi\theta}{\beta}$

$\ddot{s} = \frac{L}{2}\left(\frac{\pi L\omega}{2\beta}\right)^2\cos\frac{\pi\theta}{\beta}$

$\dddot{s} = \frac{L}{2}\left(\frac{\pi L\omega}{2\beta}\right)^3\sin\frac{\pi\theta}{\beta}$

Cycloidal motion:
$s = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\beta}\right)\right)$

Rise and return:
◇ During rise, replace θ with $\theta - \theta_i$
◇ During return, replace θ with $\theta_e - \theta$

5 Cam profile

Cam profile coordinates:
$x = -(r_b + s)\sin\theta - \frac{ds}{d\theta}\cos\theta$

$y = -(r_b + s)\cos\theta - \frac{ds}{d\theta}\sin\theta$

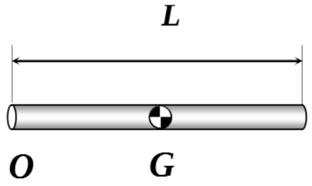
Radius of curvature:
$\rho = r_b + s + \frac{d^2s}{d\theta^2}$

Avoiding any cusps in the offset profile:
$r_b > -s - \frac{d^2s}{d\theta^2}$

6 Moment of inertia

General equation:
$I_G = \int r^2\rho\, dV$

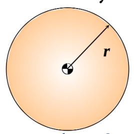
Slender rod:



$I_G = \frac{1}{12}mL^2$

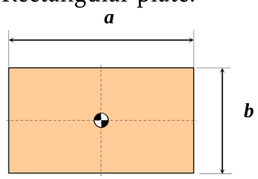
$I_O = \frac{1}{3}mL^2$

Disk or cylinder:



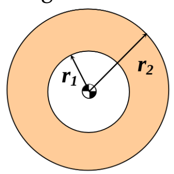
$I_G = \frac{1}{2}mr^2$

Rectangular plate:



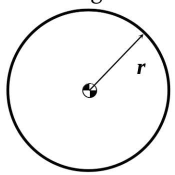
$I_G = \frac{1}{12}m(a^2 + b^2)$

Ring:



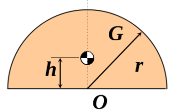
$I_G = \frac{1}{2}m(r_1^2 + r_2^2)$

Thin ring:



$I_G = mr^2$

Semicircular plate:



$I_G = \frac{1}{2}mr^2$

$h = \frac{4r}{3\pi}$

$I_O = \frac{1}{2}mr^2 - mh^2$

Parallel axis theorem:
$I_A = I_G + m|\vec{r}_{AG}|^2$

7 Force analysis

7.1 D'Alembert's Principle

◇ The direction of the inertial force (ma) or moment ($I\alpha$) is **opposite** to the direction of the resultant force or moment.
◇ **Torque** (T) is not an inertial moment, so its direction does not need to be flipped.
◇ The direction of the inertial acceleration (a) or angular velocity (ω) is **opposite** to the direction of the acceleration force or angular velocity.

7.2 Steps

◇ Identify all **two-force members**. Two-force members are members with only two forces and no external force or torque.
◇ For dynamic force analysis, **two-force members** must be **massless**.
◇ For dynamic force analysis, **two-force members with mass** but with **1 net moment** can also be considered as a two force member.
◇ Sliders and pin-in-slot joints always have a normal contact force acting opposite to the wall or slot they are resting on.
◇ Draw applied forces and moments. Forces that have unknown directions are separated into the positive x and y components.
◇ When moving from one link to another, remember to invert the direction of the forces acting on the other link.
◇ **Two-force members** and **normal contact forces** have **known** directions.
◇ Write the equilibrium equations for each free body. There is a total of $3N$ equations for N bodies.
◇ Solve the equations for the unknowns.

8 Maths

8.1 Derivatives

Chain rule:
$\frac{dz}{dx} = \frac{dz}{dy}\cdot\frac{dy}{dx}$

Product rule:
$\frac{d}{dx}(u\cdot v) = \frac{du}{dx}\cdot v + u\cdot\frac{dv}{dx}$

Quotient rule:
$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Standard derivatives:
$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\csc x) = -\csc x\cot x$

$\frac{d}{dx}(\sec x) = \sec x\tan x$

8.2 Integrals

$\int \sin x\, dx = -\cos x$

$\int \cos x\, dx = \sin x$

$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$
$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$
$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|\sqrt{x^2-a^2}+x\right|$
$\int \tan x\,dx = \ln|\sec x|$
$\int \cot x\,dx = \ln|\sin x|$
$\int \csc x\,dx = -\ln|\csc x + \cot x|$
$\int \sec x\,dx = -\ln|\sec x + \tan x|$
Integration by parts:
$\int u\,dv = uv - \int v\,du$

8.3 Trigonometric identities

Quotient identities:
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$
Reciprocal identities:
$\sin\theta = \frac{1}{\csc\theta}$
$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$
$\cot\theta = \frac{1}{\tan\theta}$
Pythagorean identities:
$\sin^2\theta + \cos^2\theta = 1$
$\sec^2\theta - \tan^2\theta = 1$
$\csc^2\theta - \cot^2\theta = 1$
Even/odd identities:
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
$\cot(-\theta) = -\cot\theta$
$\csc(-\theta) = -\csc\theta$
$\sec(-\theta) = \sec\theta$
Co-function identities:
$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$
$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$
$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$
$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$
$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$
$\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$
$\frac{\pi}{2}$ radians $= 90°$
Sum/difference identities:
$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$
$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$
$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$
Double angle identities:
$\sin(2\theta) = 2\sin\theta\cos\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$\cos(2\theta) = 2\cos^2\theta - 1$
$\cos(2\theta) = 1 - 2\sin^2\theta$
$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$
Half angle identities:
$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$
$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$
$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$
Sum to product of 2 angles:
$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$
$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$
$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$
$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$
Product to sum of 2 angles:
$\sin\theta\sin\phi = \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{2}$
$\cos\theta\cos\phi = \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{2}$
$\sin\theta\cos\phi = \frac{\sin(\theta+\phi)+\sin(\theta-\phi)}{2}$
$\cos\theta\sin\phi = \frac{\sin(\theta+\phi)-\sin(\theta-\phi)}{2}$
Law of sines:
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Law of cosines:
$a^2 = b^2 + c^2 - 2bc\cos A$
Area of a triangle:
$A = \frac{1}{2}ab\sin C$