1 Kinematics

Equations of motion:
$s = ut + \frac{1}{2}at^2$
$v = u + at$
$v^2 = u^2 + 2as$
Relative velocity:
$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$
Maximum height (no air resistance):
$h_{max} = \frac{u^2 \sin^2\theta}{2g}$, where θ is the angle to the ground.
Range (no elevation change & no drag):
$R = \frac{u^2 \sin 2\theta}{g}$, where θ is the angle to the ground.

1.1 Vector resolution

$\cos\theta$ stands for closing the angle θ, so the vector that closes the angle is $\cos\theta$ and the vector that opens the angle is $\sin\theta$.

2 Newton's laws

Newton's second law:
$F_{net} = ma$
$F_{net} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$
Newton's second law for variable mass:
$F_{net} = ma + \vec{v}_{rel}\frac{dm}{dt}$
Laminar flow drag:
$F_D = bv$
Turbulent flow drag:
$F_D = kv^2$
Terminal velocity:
$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right)$

3 Circular motion

Arc length:
$s = r\theta$, where θ is in radians.
Angular displacement:
$\theta = \frac{s}{r}$
Frequency:
$\frac{1}{T}$, where T is the period.
Angular velocity:
$\omega = \frac{d\theta}{dt} = 2\pi f = \frac{2\pi}{T} = \frac{v_{tan}}{r}$
Angular acceleration:
$\alpha = \frac{d\omega}{dt}$
Centripetal acceleration:
$a = \frac{v^2}{r} = r\omega^2$
Total linear acceleration:
$\vec{a} = \vec{a}_{tan} - \frac{(\vec{v}_{tan})^2}{R}\hat{r} = \vec{a}_{tan} - R\omega^2\hat{r}$
Angular quantities:
$s = r\theta$
$v_{tan} = r\omega$
$a_{tan} = r\alpha$
Equations of motion:
$\omega_f = \omega_i + \alpha t$
$\theta - \theta_0 = \omega_i t + \frac{1}{2}\alpha t^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta - \theta_0)$
Non-uniform circular motion:
$\vec{a} = \vec{a}_r + \vec{a}_{tan}$

4 Work, Energy, Power

Work done:
$W = \vec{F}\cdot\vec{s} = Fs\cos\theta$
$W = \int_a^b \vec{F}\cdot d\vec{l}$
Kinetic energy:
$KE = \frac{1}{2}mv^2$
Potential energy:
$\Delta U = -\int \vec{F}\cdot d\vec{l}$
Work-energy theorem:
$W = KE_f - KE_i$
$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
Gravity:
$PE_{grav} = mgh = -\int \vec{F}_{grav}\cdot d\vec{l}$
$PE_{grav} = -\frac{GMm}{r}$
$F_{grav} = \frac{GMm}{r^2}$
Force:
$F = -\frac{dU}{dx}$
Springs:
$\vec{F}_{spring} = -k\vec{x}$
$PE_{spring} = \frac{1}{2}kx^2$
Power:
$P_{avg} = \frac{\Delta W}{\Delta t}$
$P = \frac{dW}{dt}$
$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$

5 Momentum

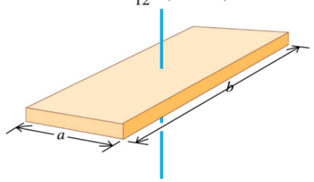
Momentum:
$\vec{p} = m\vec{v}, \quad p = mv$
Impulse:
$\vec{J} = \vec{F}_{net}\Delta t, \quad J = F_{net}\Delta t$
$\vec{J} = \int_{t_1}^{t_2}\vec{F}_{net}\,dt$
Elastic collisions:
$v_A - v_B = v_B' - v_A'$
Coefficient of restitution e:
$v_B' - v_A' = -e(v_B - v_A)$
Centre of mass (CM):
$x_{CM} = \frac{m_1x_1 + \ldots + m_nx_n}{m_1 + \ldots + m_n} = \frac{1}{M}\sum_{i=1}^n m_ix_i$
$x_{CM} = \frac{1}{M}\int x\,dm$

6 Rotation of rigid bodies

Torque:
$\vec{\tau} = \vec{r}\times\vec{F}$
$\tau = rF\sin\theta$
Net torque:
$\vec{\tau}_{net} = I\vec{\alpha}, \quad \tau_{net} = I\alpha$
Moment of inertia (MOI):
$I = m_1r_1^2 + m_2r_2^2 + \cdots = \sum_i m_ir_i^2$
$I = \int r^2\,dm$
Parallel axis theorem:
$I_P = I_{CM} + Md^2$
Perpendicular axis theorem (only for flat objects):
$I_z = I_x + I_y$
Rotational kinetic energy:
$KE_{rot} = \frac{1}{2}I\omega^2$
Work-energy theorem:
$W = \int \tau\,d\theta$
$W = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$
Power:
$P = \tau\omega$
Rolling without slipping:
$v_{CM} = R\omega$
$KE_{total} = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$

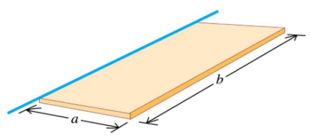
Rectangular plate, axis through center

$I = \frac{1}{12}M(a^2 + b^2)$



Thin rectangular plate, axis along edge

$I = \frac{1}{3}Ma^2$

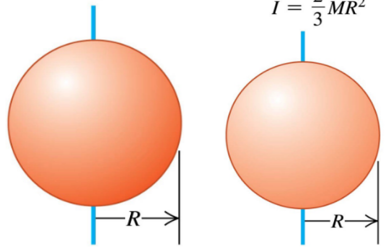


Solid sphere

$I = \frac{2}{5}MR^2$

Thin-walled hollow sphere

$I = \frac{2}{3}MR^2$



Solid cylinder

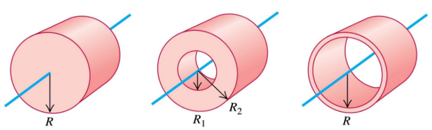
$I = \frac{1}{2}MR^2$

Hollow cylinder

$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Thin-walled hollow cylinder

$I = MR^2$

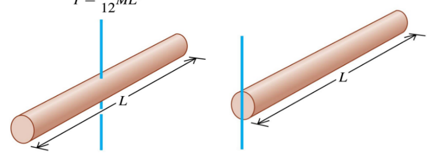


Slender rod, axis through center

$I = \frac{1}{12}ML^2$

Slender rod, axis through one end

$I = \frac{1}{3}ML^2$



7 Angular momentum

Angular momentum:
$\vec{L} = \vec{r}\times\vec{p}, \quad L = rp\sin\theta$
$\vec{L} = \vec{r}\times m\vec{v}, \quad L = rmv\sin\theta$
$\vec{L} = I\vec{\omega}, \quad L = I\omega$
Net torque:
$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$
$\vec{\tau}_{net} = I\vec{\alpha}, \quad \tau_{net} = I\alpha$
Angular velocity of precession:
$\Omega = \frac{\tau_{ext}}{L\sin\theta}$

8 Electric Fields

Coulomb's law:
$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$
Electric force:
$\vec{F} = q\vec{E}$
Electric field:
$\vec{E}_{net} = \int \frac{1}{4\pi\varepsilon_0 r^2}\,dq$
$\vec{E}_{net} = \sum_i \frac{q_i}{4\pi\varepsilon_0 r^2}$
Electric field of a ring of charge:
$E = \frac{Q}{4\pi\varepsilon_0}\frac{x}{(r^2+x^2)^{\frac{3}{2}}}$
Electric field of a cylinder:
$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}$
Electric field of a thin plane of charge:
$E = \frac{\sigma}{2\varepsilon_0}$
Electric field at the surface of a conductor:
$E_\perp = \frac{\sigma}{\varepsilon_0}$
Electric field between two uniformly charged plates:
$E = \frac{V}{d}$
Dipole moment ($-$ to $+$):
$p = qd$
Electric dipole torque:
$\vec{\tau} = \vec{p}\times\vec{E}, \quad \tau = pE\sin\theta = qdE\sin\theta$
Electric potential:
$V = \frac{U}{q}$
$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r}$
$V = -\int \vec{E}\,d\vec{r}$
Electric potential energy of 2 point charges:
$U = \frac{q_1q_2}{4\pi\varepsilon_0 r}$
Electric potential energy of a system of discrete charges:
$U = \frac{1}{2}\sum_i\sum_{j\neq i}\frac{q_iq_j}{4\pi\varepsilon_0 r_{ij}}$
Electric potential energy of a dipole:
$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$
Electric flux:
$\Phi_E = \int \vec{E}\cdot d\vec{A} = E\cos\theta\,dA = EA\cos\theta$
Gauss' Law:
$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

9 DC Circuits

Resistance:
$R = \frac{V}{I}$
Resistors in series:
$R_{eq} = R_1 + R_2 + \cdots$
Resistors in parallel:
$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\right)^{-1}$
Electromotive force (e.m.f):
$\mathcal{E} = \frac{W}{Q}$
Internal resistance r:
$V_{terminal} = \mathcal{E} - Ir$
Power (replace R with X_C or X_L for AC):
$P = VI = I^2R = \frac{V^2}{R}$
Potential divider: (replace R with R, X_C or X_L and R_{total} with Z for AC)
$V = \frac{R}{R_{total}}V_{total}$
Capacitance:
$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$
Capacitance of a conducting sphere:
$C = 4\pi\varepsilon_0 r$
Capacitance of a co-axial cylindrical conductor:
$C = \frac{2\pi\varepsilon_0 L}{\ln\left|\frac{R_{out}}{R_{in}}\right|}$
Capacitors in series:
$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right)^{-1}$
Capacitors in parallel:
$C_{eq} = C_1 + C_2 + \cdots$
Potential energy stored in a capacitor:
$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$
Electric energy density in a vacuum:
$u = \frac{1}{2}\varepsilon_0 E^2$
Electric energy density in the presence of a dielectric:
$u = \frac{1}{2}\varepsilon_r\varepsilon_0E^2 = \frac{1}{2}K\varepsilon_0E^2 = \frac{1}{2}\varepsilon E^2$
Capacitance with a dielectric:
$C = KC_0 = K\frac{\varepsilon_0 A}{d} = \varepsilon\frac{A}{d}$
Gauss' Law in dielectrics:
$\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{encl-free}}{\varepsilon_0}$
Charging a capacitor:
$q = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$
$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}} = I_0e^{-\frac{t}{RC}}$
Discharging a capacitor:
$q = Q_0e^{-\frac{t}{RC}}$
$i = -\frac{dq}{dt} = -\frac{Q_0}{RC}e^{-\frac{t}{RC}} = I_0e^{-\frac{t}{RC}}$
Applying Kirchhoff's laws:
$- \rightarrow +$ (Increasing V) $\longrightarrow$ Add V
$+ \rightarrow -$ (Decreasing V) $\longrightarrow$ Subtract V

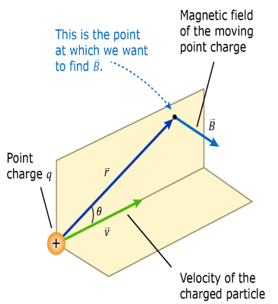
9.1 Capacitors in circuits

◊ When a capacitor is uncharged, it acts like a wire.
◊ When a capacitor is fully charged, it acts like a break in the circuit.

◊ When two charged capacitors are connected, the charges will transfer until the potential difference is the same for both capacitors.
◊ When the capacitors' **same** polarity plates are connected together, the total charge is $Q_1 + Q_2$, so the potential difference is $\frac{Q_1+Q_2}{C_1+C_2}$
◊ When the capacitors' **opposite** polarity plates are connected together, the total charge is $Q_1 - Q_2$, so the potential difference is $\frac{Q_1-Q_2}{C_1+C_2}$

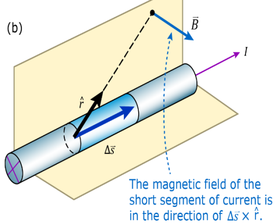
10 Magnetic fields

Biot-Savart law for moving point charge:



$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$

Biot-Savart law for a current:



$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$

Ampere's law:
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$
Use the right-hand grip rule to determine the direction of the current I_{encl}.
Magnetic flux:
$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\cos\phi\, dA = BA\cos\phi$
Gauss' law:
$\oint \vec{B}\cdot d\vec{A} = 0$
Force on a moving charge in a magnetic field:
$\vec{F} = q\vec{v}\times\vec{B}, \quad F = Bqv\sin\theta$
Lorentz force:
$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$
Force on a current in a magnetic field:
$\vec{F} = I\vec{l}\times\vec{B}$
$F = BIl\sin\theta$
Magnetic field of a solenoid:
$B = \mu_0 nI$
Magnetic dipole moment:
$\vec{\mu} = NI\vec{A}$
Magnetic dipole torque:
$\vec{\tau} = \vec{\mu}\times\vec{B}$
$\tau = \mu B\sin\theta = NIAB\sin\theta$
Velocity selector:
$v = \frac{E}{B_{in}}$
Mass spectrometer:
$m = \frac{B_{in}B_1 Rq}{E}$
Hall voltage:
$\Delta V_H = E_H d = v_d Bd$

11 Electromagnetic induction

Magnetic flux linkage:
$N\Phi_B = N\int \vec{B}\cdot d\vec{A} = NBA\cos\theta$
Faraday's law:
$\mathcal{E} = -N\frac{d\Phi_B}{dt}$
E.m.f induced in a moving conductor:
$\mathcal{E} = Blv\sin\theta$
Maxwell-Faraday law:
$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
$\oint \vec{E}\cdot d\vec{l} \neq 0$
Ampere-Maxwell law:
$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{encl} + I_{disp}\right)$
$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{encl} + \varepsilon_0\frac{d\Phi_E}{dt}\right)$
$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{encl} + \varepsilon_0 A\frac{dE}{dt}\right)$
Mutual inductance:
$M = \frac{N_2\Phi_{B2}}{I_1} = \frac{N_1\Phi_{B1}}{I_2}$
Self inductance:
$L = \frac{N\Phi_B}{I}$
Energy stored in the magnetic field:
$U_B = \frac{1}{2}LI^2$
AC generators:
$\mathcal{E} = NBA\omega\sin\omega t$
Transformer:
$\frac{V_s}{V_p} = \frac{N_s}{N_p}$
$V_pI_p = V_sI_s$

12 Inductors

Potential difference across an inductor:
$V = L\frac{dI}{dt}$
Inductors in series:
$L_{eq} = L_1 + L_2 + \cdots$
Inductive reactance:
$X_L = \omega L = 2\pi fL$
Capacitive reactance:
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$
Resonance in AC circuits:
Condition for resonance is: $X_C = X_L$
$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

12.1 RL series circuit

Current:
$I = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

12.2 LC circuit (no voltage source)

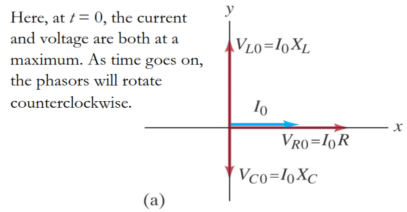
Charge:
$Q = Q_0\cos(\omega t + \phi)$
Angular frequency:
$\omega = 2\pi f = \sqrt{\frac{1}{LC}}$
Total energy:
$U = \frac{Q_0^2}{2C}$

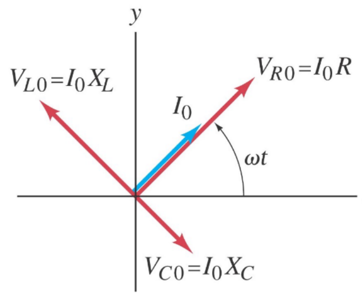
12.3 RCL series circuit (no V source)

Angular frequency (under-damped oscillations):
$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
Charge:
$Q = Q_0 e^{-\frac{Rt}{2L}}\cos(\omega' t + \phi)$

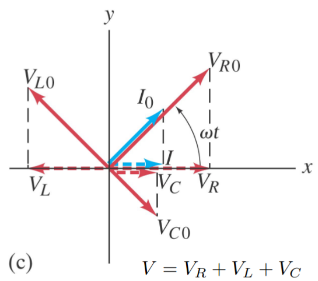
12.4 RCL series circuit

Phasor diagram

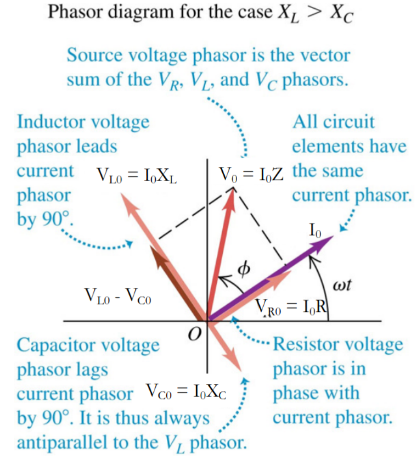




(b) Some time t later, the phasors rotated.



The algebraic sum of voltages across each device at any point in time is equal to the source voltage V. **The algebraic sum is the sum of the projections of each phasor on the x-axis**. The current is the same throughout the series circuit.
Impedance:
$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
Current:
$I = I_0\cos\omega t$
Voltage:
$V = I_0Z\cos(\omega t + \phi)$



Phase angle between voltage and current:
$\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$

12.5 Resistors in AC circuits

Root-mean-square current:
$I_{rms} = \frac{I_0}{\sqrt{2}}$
The current through a resistor is **in phase** with the voltage.

12.6 Inductors in AC circuits

Voltage:
$V = \omega LI_0\cos\left(\omega t + \frac{\pi}{2}\right) = V_0\cos\left(\omega t + \frac{\pi}{2}\right)$
Current:
$I_0 = \frac{V_0}{\omega L}$
The current through an inductor **lags** the voltage by 90°.

12.7 Capacitors in AC circuits

Voltage:
$V = \frac{I_0}{\omega C}\cos\left(\omega t - \frac{\pi}{2}\right) = V_0\cos\left(\omega t - \frac{\pi}{2}\right)$
Current:
$I_0 = V_0\omega C$
The current through a capacitor **leads** the voltage by 90°.

12.8 Filter circuits

Use potential divider equation (under DC circuits) for all filter circuits.

13 Trigonometric identities

Pythagorean identities:
$\sin^2\theta + \cos^2\theta = 1$
$\sec^2\theta - \tan^2\theta = 1$
$\csc^2\theta - \cot^2\theta = 1$
Double angle identities:
$\sin(2\theta) = 2\sin\theta\cos\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$\cos(2\theta) = 2\cos^2\theta - 1$
$\cos(2\theta) = 1 - 2\sin^2\theta$
$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$
Law of sines:
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Law of cosines:
$a^2 = b^2 + c^2 - 2bc\cos A$
Area of a triangle:
$A = \frac{1}{2}ab\sin C$

14 Coordinates

Polar coordinates:
$x = r\cos\theta, \quad y = r\sin\theta$
Cylindrical coordinates:
$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$
Spherical coordinates:
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

15 Steps

15.1 General steps

◊ Always quote formula first.
◊ Draw diagrams.
◊ Change the formula's quantity, dm, dq, etc, into something × the given quantity.
◊ Use symmetry to cancel things out.
◊ Integration too hard? Substitute polar, cylindrical or spherical coordinates.
◊ Infinity as a limit? Substitute with a $\tan\theta$ as $\tan\left(\pm\frac{\pi}{2}\right) \to \pm\infty$.

15.2 Steps to find centre of mass

◊ For discrete masses, treat holes as a mass but subtract them.
◊ For continuous masses, change dm into something × the given quantity, like $\rho\,dV, \rho h\,dA, 2\pi r\rho h\,dr$ for a cylinder.

15.3 Steps to find moment of inertia

◊ Find a symmetrical axis of rotation.
◊ For continuous masses, change dm into something × the given quantity, like $\rho\,dV, \rho h\,dA, 2\pi r\rho h\,dr$ for a cylinder.
◊ Use parallel or perpendicular axis theorem to find the MOI about the actual axis of rotation if necessary.

15.4 Steps to find E-field (continuous)

◊ Convert dq into the something × the given quantity, like $\lambda\,dx, \sigma\,dA, \rho\,dV$.
◊ Note that x and r are different, r is the distance away from the charged object.
◊ A lot of times, E_y will cancel out, so you only need to consider E_x.
◊ If the shape is difficult to integrate, identify a shape that you can stack to get the shape you want, like rings for a disk and disks for a sphere.

15.5 Using Gauss' law

◊ Try to use a symmetrical surface.
◊ Split the surface into sections, like the ends and the curved portion of a cylinder should be considered separately.

15.6 Using Ampere's law

◊ Generally, you want to use either a circle or a rectangle for the loop.
◊ A circle is the easiest, as the B-field at every point on the circle is the same.
◊ Using a rectangle, you will have to add the B-field of every side of the rectangle together to get the overall B-field.