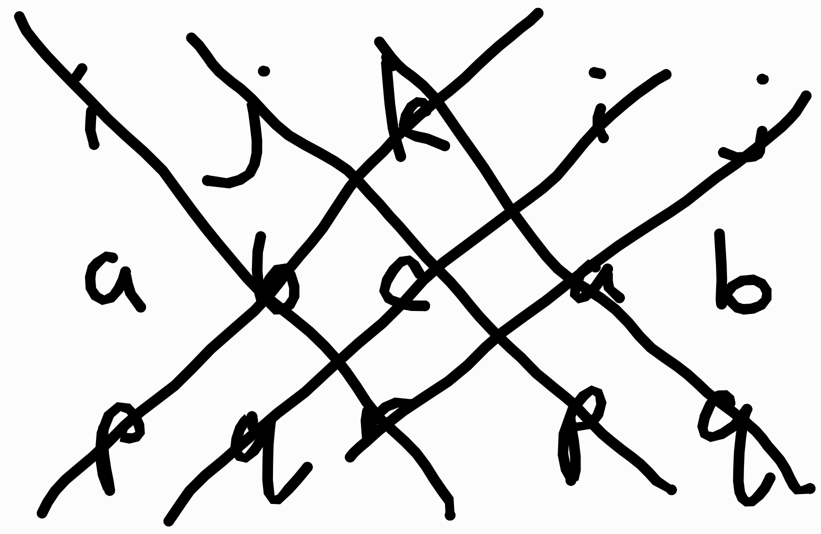
1) Let $\underset{\sim}{u} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$

$\underset{\sim}{v} = p\underset{\sim}{i} + q\underset{\sim}{j} + r\underset{\sim}{k}$

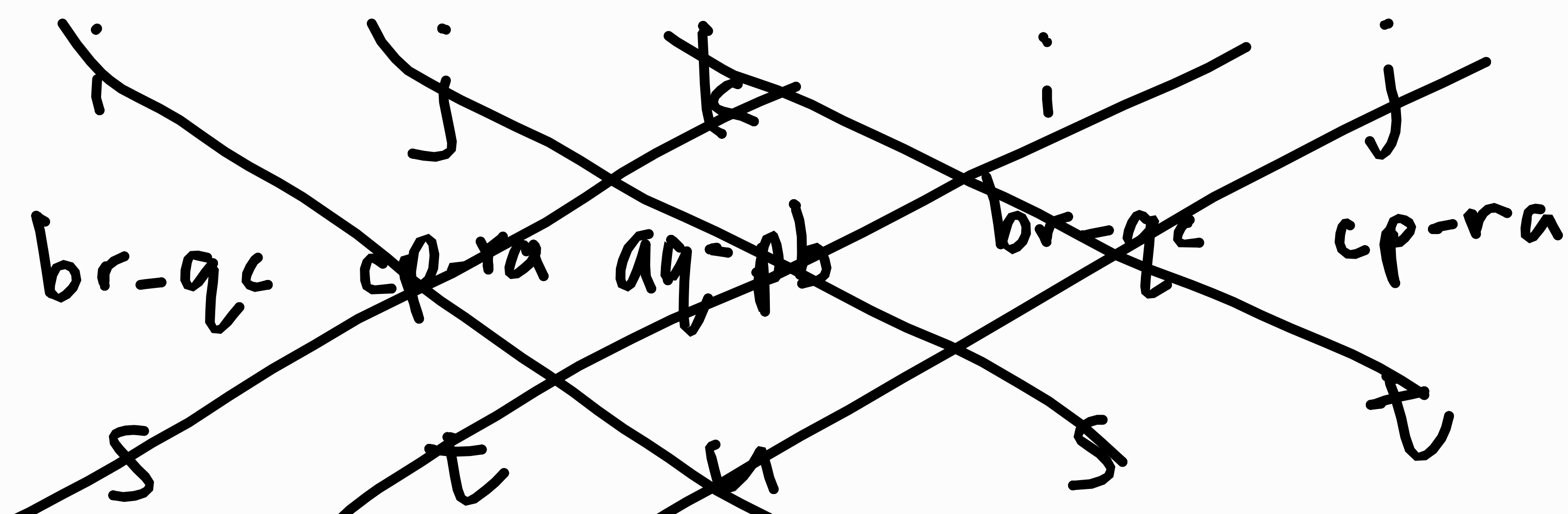
$\underset{\sim}{w} = s\underset{\sim}{i} + t\underset{\sim}{j} + u\underset{\sim}{k}$



$(\underset{\sim}{u} \times \underset{\sim}{v}) \times \underset{\sim}{w}$

$= (br\underset{\sim}{i} + cp\underset{\sim}{j} + aq\underset{\sim}{k} - pb\underset{\sim}{k} - qc\underset{\sim}{i} - ra\underset{\sim}{j}) \times (s\underset{\sim}{i} + t\underset{\sim}{j} + u\underset{\sim}{k})$

$= \left[(br - qc)\underset{\sim}{i} + (cp - ra)\underset{\sim}{j} + (aq - pb)\underset{\sim}{k}\right] \times (s\underset{\sim}{i} + t\underset{\sim}{j} + u\underset{\sim}{k})$



$= u(cp - ra)\underset{\sim}{i} + s(aq - pb)\underset{\sim}{j} + t(br - qc)\underset{\sim}{k}$

$\quad - s(cp - ra)\underset{\sim}{k} - t(aq - pb)\underset{\sim}{i} - u(br - qc)\underset{\sim}{j}$

$= (ucp + pbt - rau + aqt)\underset{\sim}{i} + (saq + uqc - spb - ubr)\underset{\sim}{j}$

$\quad + (tbr + sra - tqc - scp)\underset{\sim}{k}$

2) $(\underset{\sim}{u}\cdot\underset{\sim}{w})\underset{\sim}{v} - (\underset{\sim}{v}\cdot\underset{\sim}{w})\underset{\sim}{u}$

$= (a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k})\cdot(s\underset{\sim}{i} + t\underset{\sim}{j} + u\underset{\sim}{k})(p\underset{\sim}{i} + q\underset{\sim}{j} + r\underset{\sim}{k})$

$\quad - (\underset{\sim}{v}\cdot\underset{\sim}{w})\underset{\sim}{u}$

$= (as + bt + cu)(p\underset{\sim}{i} + q\underset{\sim}{j} + r\underset{\sim}{k})$

$\quad - (p\underset{\sim}{i} + q\underset{\sim}{j} + r\underset{\sim}{k})\cdot(s\underset{\sim}{i} + t\underset{\sim}{j} + u\underset{\sim}{k})(a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k})$

$= (as + bt + cu)(p\underset{\sim}{i} + q\underset{\sim}{j} + r\underset{\sim}{k}) - (ps + qt + ru)(a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k})$

$= (\cancel{asp} + btp + cup \cancel{- psa} - qta - rua)\underset{\sim}{i}$

$\quad + (asq \cancel{+ btq} + cuq - psb \cancel{- qtb} - rub)\underset{\sim}{j}$

$\quad + (asr + btr \cancel{+ cur} - psc - qtc \cancel{- ruc})\underset{\sim}{k}$

$= (ucp + pbt - rau - aqt)\underset{\sim}{i}$

$\quad + (saq + uqc - spb - ubr)\underset{\sim}{j}$

$\quad + (sra + tbr - tqc - scp)\underset{\sim}{k}$

$= (\underset{\sim}{u}\times\underset{\sim}{v})\times\underset{\sim}{w}$ (shown)

2a) Let $\vec{OA} = (2,1,3)$

$\vec{OB} = (3,0,2)$

$\vec{OC} = (-1,1,4)$

$$\begin{aligned}\vec{AB} &= \vec{OB} - \vec{OA} \\ &= (3,0,2) - (2,1,3) \\ &= (1,-1,-1)\end{aligned}$$

$$\begin{aligned}\vec{AC} &= \vec{OC} - \vec{OA} \\ &= (-1,1,4) - (2,1,3) \\ &= (-3,0,1)\end{aligned}$$

Let $\underset{\sim}{n} = \vec{AB} \times \vec{AC}$

$$\begin{array}{ccccc} i & j & k & i & j \\ 1 & -1 & -1 & 1 & -1 \\ -3 & 0 & 1 & -3 & 0 \end{array}$$

$$\begin{aligned}\underset{\sim}{n} &= (1,-1,-1) \times (-3,0,1) \\ &= -\underset{\sim}{i} + 3\underset{\sim}{j} + 0 - 3\underset{\sim}{k} - 0 - \underset{\sim}{j} \\ &= (-1,2,-3)\end{aligned}$$

$\therefore$ the equation of the plane is

$$-x + 2y - 3z = k$$

2a) When $x=2, y=1, z=3$

$$-2+2-9=k$$
$$k=-9$$

$\therefore$ the equation of the plane is

$$-x+2y-3z=-9$$
$$x-2y+3z=9$$

b) The straight line is:

$$(-2,5,-2)+t(1,-2,3),\ t\in\mathbb{R}$$

c) $x-2y+3z=9$

When $x=-2+t,\ y=5-2t,\ z=-2+3t,$

$$(-2+t)-2(5-2t)+3(-2+3t)=9$$
$$-2+t-10+4t-6+9t=9$$
$$14t=27$$
$$t=\frac{27}{14}$$

The point is $\left(-2+\frac{27}{14},\ 5-2\left(\frac{27}{14}\right),\ -2+3\left(\frac{27}{14}\right)\right)$

$$\left(-\frac{1}{14},\ \frac{8}{7},\ \frac{53}{14}\right)$$

3) Let $\underset{\sim}{x} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$

For $\underset{\sim}{x}$ to be perpendicular to $\underset{\sim}{i} - \underset{\sim}{k}$,

$$\underset{\sim}{x} \cdot (\underset{\sim}{i} - \underset{\sim}{k}) = 0$$

$$(a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}) \cdot (\underset{\sim}{i} - \underset{\sim}{k}) = 0$$

$$a - c = 0$$

$$a = c$$

$$\therefore \underset{\sim}{x} = s\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad s, t \in \mathbb{R}$$

$$\hat{\underset{\sim}{x}} = \frac{1}{|\underset{\sim}{x}|}\underset{\sim}{x}$$

$$= \frac{1}{\sqrt{2t^2 + s^2}}\left[s\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right], \quad s, t \in \mathbb{R}$$

$$= \frac{t}{\sqrt{2t^2+s^2}}\underset{\sim}{i} + \frac{s}{\sqrt{2t^2+s^2}}\underset{\sim}{j} + \frac{t}{\sqrt{2t^2+s^2}}\underset{\sim}{k}$$

$\therefore$ The unit vectors perpendicular to $(\underset{\sim}{i} - \underset{\sim}{k})$ are :

$$\frac{t}{\sqrt{2t^2+s^2}}\underset{\sim}{i} + \frac{s}{\sqrt{2t^2+s^2}}\underset{\sim}{j} + \frac{t}{\sqrt{2t^2+s^2}}\underset{\sim}{k}, \quad s, t \in \mathbb{R}, \quad 2t^2 + s^2 \neq 0$$

4a) Let $t = x-1$,

$$36t^2 + 25y^2 + 9(z+2)^2 = 36$$

$$25y^2 + 9(z+2)^2 = 36 - 36t^2$$

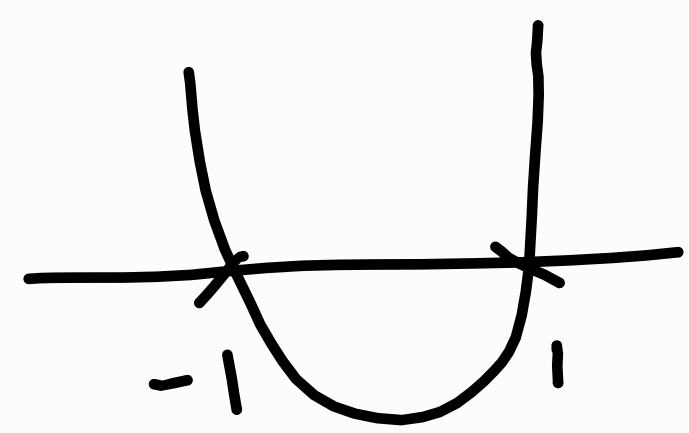
$$(5y)^2 + [3(z+2)]^2 = 36(1-t^2)$$

$$(5y)^2 + [3(z+2)]^2 = \left[6\sqrt{1-t^2}\right]^2$$

Since $(5y)^2 + [3(z+2)]^2 \geq 0$

$$1 - t^2 \geq 0$$

$$t^2 \leq 1$$



$$\therefore -1 \leq t \leq 1$$

Using $\cos^2\theta + \sin^2\theta = 1$, $\theta \in [0, 2\pi]$

Let $5y = 6\sqrt{1-t^2}\cos\theta$,

$$y = \frac{6}{5}\sqrt{1-t^2}\cos\theta$$

Let $3(z+2) = 6\sqrt{1-t^2}\sin\theta$

$$z+2 = 2\sqrt{1-t^2}\sin\theta$$

$$z = 2\sqrt{1-t^2}\sin\theta - 2$$

4a) $\therefore$ A parametric representation is:

$$\left.\begin{aligned} x &= t+1 \\ y &= \frac{6}{5}\sqrt{1-t^2}\cos\theta \\ z &= 2\sqrt{1-t^2}\sin\theta - 2 \end{aligned}\right\} \begin{aligned} & t \in [-1,1] \\ & \theta \in [0,2\pi] \end{aligned}$$

b) $1+x^2+y^2-z=0$

$x^2+y^2 = z-1$

Let $z=t$,

$\therefore 2 \leq t \leq 9$

$x^2+y^2 = t-1$

Using $\cos^2\theta + \sin^2\theta = 1$,

Let $x = \sqrt{t-1}\cos\theta$,

$y = \sqrt{t-1}\sin\theta$, $\theta \in [0,2\pi]$

$\therefore$ A parametric representation is:

$$\left.\begin{aligned} x &= \sqrt{t-1}\cos\theta \\ y &= \sqrt{t-1}\sin\theta \\ z &= t \end{aligned}\right\} \begin{aligned} & t \in [2,9], \\ & \theta \in [0,2\pi] \end{aligned}$$