1) $\underset{\sim}{F} = \left(f(x,y), g(x,y) \right) = \left(2x(y-1), x^2+2y \right)$

$$\frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2x$$

Since $\frac{\partial g}{\partial x} = \frac{\partial f}{\partial y}$, $\underset{\sim}{F}$ is conservative.

$$\frac{\partial \phi}{\partial x} = 2x(y-1)$$

$$= 2xy - 2x$$

$$\phi = \int 2xy - 2x \, dx$$

$$= x^2 y - x^2 + r(y)$$

1) $\frac{\partial \phi}{\partial y} = x^2 + 2y$

$$\phi = \int x^2 + 2y \, dy$$

$$= x^2 y + y^2 + t(x)$$

$$\phi = x^2 y + y^2 - x^2$$

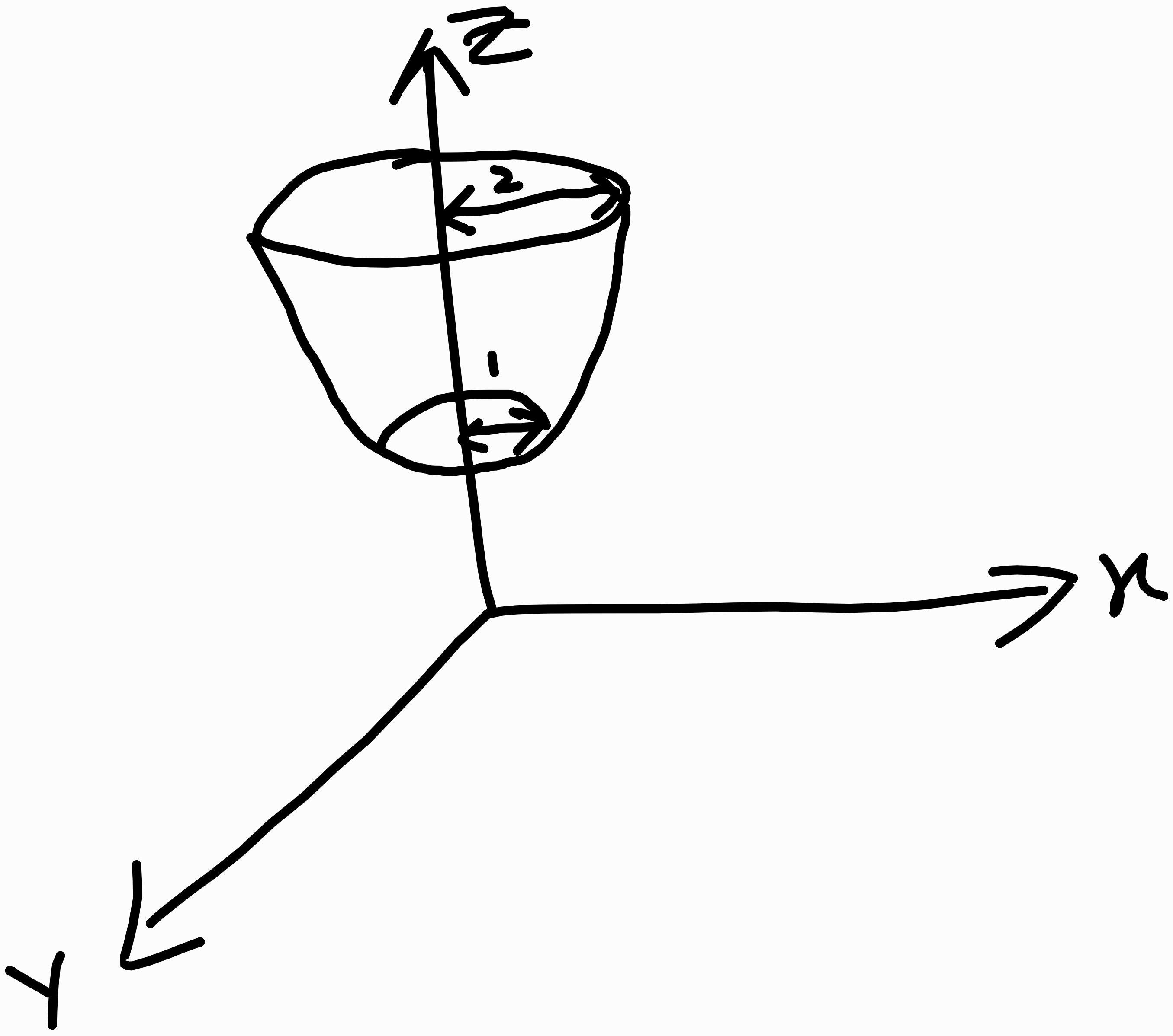
$$\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_C 2x(y-1)\,dx + (x^2+2y)\,dy$$

$$= \phi(1,1) - \phi(2,0)$$

$$= 1 \cancel{+1-1} - (-2^2)$$

$$= 5$$

2)



Surface area $= \iint_S dS$

$$= \iint_S \sqrt{1+4x^2+4y^2}\, dA$$

$$= \int_{r=1}^{r=2} \int_{\theta=0}^{\theta=2\pi} \sqrt{1+4r^2}\, r\, dr\, d\theta$$

$$= \int_{r=1}^{r=2} \int_{\theta=0}^{\theta=2\pi} \frac{1}{8}(8r)\sqrt{1+4r^2}\, dr\, d\theta$$

$$= \frac{1}{8}\int_0^{2\pi} \left[(1+4r^2)^{\frac{3}{2}}\right]_1^2 d\theta$$

2) Surface area $= \frac{1}{8}\int_0^{2\pi} \left[\frac{(1+4r^2)^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^2 d\theta$

$= \frac{1}{12}\int_0^{2\pi} \left(17\sqrt{17} - 5\sqrt{5}\right) d\theta$

$= \frac{1}{12}\left(17\sqrt{17} - 5\sqrt{5}\right)\left[\theta\right]_0^{2\pi}$

$= \frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)$

3) Equation of the plane:

$z = 8 - x - 4y$

$= 8 - x - 4y$

$$\iint_{H} yz\,d\sigma = \iint_{H} y(8-x-4y)\sqrt{1+(-1)^2+(-4)^2}\,dA$$

$$= \sqrt{18}\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=1} r\sin\theta(8-r\cos\theta-4r\sin\theta)\,r\,dr\,d\theta$$

$$= \sqrt{18}\int_{\theta=0}^{2\pi}\int_{r=0}^{1} 8r^2\sin\theta - r^3\sin\theta\cos\theta - 4r^3\sin^2\theta\,dr\,d\theta$$

$$= \sqrt{18}\int_{0}^{2\pi}\left[\frac{8}{3}r^3\sin\theta - \frac{r^4}{4}\sin\theta\cos\theta - r^4\sin^2\theta\right]_0^1 d\theta$$

$$= \sqrt{18}\int_{0}^{2\pi}\frac{8}{3}\sin\theta - \frac{1}{4}\sin\theta\cos\theta - \sin^2\theta\,d\theta$$

$$= \sqrt{18}\int_{0}^{2\pi}\frac{8}{3}\sin\theta - \frac{1}{8}\sin 2\theta - 1 + \cos^2\theta\,d\theta$$

$$= \sqrt{18}\int_{0}^{2\pi}\frac{8}{3}\sin\theta - \frac{1}{8}\sin 2\theta + \frac{1}{2}(2\cos^2\theta - 1) - \frac{1}{2}\,d\theta$$

$$= \sqrt{18}\int_{0}^{2\pi}\frac{8}{3}\sin\theta - \frac{1}{8}\sin 2\theta + \frac{1}{2}\cos 2\theta - \frac{1}{2}\,d\theta$$

$$= \sqrt{18}\left[-\frac{8}{3}\cos\theta + \frac{1}{16}\cos 2\theta + \frac{1}{4}\sin 2\theta - \frac{1}{2}\theta\right]_0^{2\pi}$$

$$= \sqrt{18}(-\pi) = -3\sqrt{2}\pi$$

3b) $x^2+y^2=1$

$$y=\sqrt{1-x^2} \qquad \frac{\partial f}{\partial x}=\frac{x}{\sqrt{1-x^2}}$$

$$=f(x,z) \qquad \frac{\partial f}{\partial y}=0$$

$$\text{Surface area} = \iint_S y\, dS$$

$$=\iint_S \sqrt{1-x^2}\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\, dA$$

$$=\iint_S \sqrt{1-x^2}\sqrt{1+\frac{x^2}{1-x^2}+0^2}\, dA$$

$$=\iint_S \sqrt{1-x^2}\sqrt{\frac{1\cancel{-x^2}\cancel{+x^2}}{1-x^2}}\, dA$$

$$=\iint_S \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\, dA$$

$$=\iint_S dA$$

$$=\text{Area of } R$$

$$=\frac{1}{2}\times 4$$

$$=2$$

4a) $\underset{\sim}{u} = (x^2+y, xyz, z)$

$$\iint_S \underset{\sim}{u} \cdot \hat{n} \, dS = \iiint_T \nabla \cdot \underset{\sim}{u} \, dx\,dy\,dz$$

$$= \iiint_T 2x + xz + 1 \, dx\,dy\,dz$$

$$= \int_{x=0}^{1} \int_{y=-1}^{1} \int_{z=1}^{2} 2x + xz + 1 \, dx\,dy\,dz$$

$$= \int_{x=0}^{1} \int_{z=1}^{2} \left[2xy + xzy + y\right]_{-1}^{1} dx\,dz$$

$$= \int_{x=0}^{1} \int_{z=1}^{2} 4x + 2xz + 2 \, dx\,dz$$

$$= \int_{0}^{1} \left[4xz + xz^2 + 2z\right]_{1}^{2} dx$$

$$= \int_0^1 4x + 3x + 2 \, dx$$

$$= \left[\frac{7x^2}{2} + 2x\right]_0^1$$

$$= \frac{11}{2}$$

4b) $\underset{\sim}{u} = (2x, 0, 3z + xy)$

$$\iint_S \underset{\sim}{u} \cdot \hat{n}\, dS = \iiint_T \nabla \cdot \underset{\sim}{u}\, dx\,dy\,dz$$

$$= \iiint_T 2 + 3\, dx\,dy\,dz$$

$$= 5 \int_{r=0}^{3} \int_{\theta=0}^{2\pi} \int_{z=0}^{5} r\,dr\,d\theta\,dz$$

$$= 5 \int_{r=0}^{3} \int_{\theta=0}^{2\pi} 5r\,dr\,d\theta$$

$$= 25 \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^3 d\theta$$

$$= 25 \int_0^{2\pi} \frac{9}{2}\, d\theta$$

$$= 225\pi$$

5) $\underset{\sim}{F} = (ay^2+z,\ xy,\ x)$

$$\nabla \times \underset{\sim}{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (ay^2+z,\ xy,\ x)$$

$$\begin{array}{ccccc} i & j & k & i & j \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ ay^2+z & xy & x & ay^2+z & xy \end{array}$$

$$= (0-0,\ 1-1,\ y-2ay)$$

$$= (0,\ 0,\ y-2ay)$$

$\therefore a = \frac{1}{2}$ for $\underset{\sim}{F}$ to be conservative

$\therefore \underset{\sim}{F} = \left(\frac{1}{2}y^2+z,\ xy,\ x\right)$

$$\frac{\partial \phi}{\partial x} = \frac{1}{2}y^2 + z$$

$$\phi = \int \frac{1}{2}y^2 + z \, dx$$

$$= \frac{1}{2}y^2 x + zx + g(y) + h(z)$$

5) $$\frac{\partial \phi}{\partial y} = xy$$

$$\phi = \int xy \, dy$$

$$= \frac{1}{2} x y^2 + f(x) + h(z)$$

$$\frac{\partial \phi}{\partial z} = x$$

$$\phi = \int x \, dz$$

$$= xz + f(x) + g(y)$$

$$\therefore \phi = xz + \frac{1}{2} x y^2$$

$$\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r} = \phi(3,0,6) - \phi(2,1,2)$$

$$= 3(6) + \frac{1}{2}(3)(0)^2 - \left[2(2) + \frac{1}{2}(2)(1)^2\right]$$

$$= 13$$