# Work Energy Power Tutorial

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# 1 Question 1

## 1.1 (a)

Let the net force on the crate be $F_net$, the frictional force on the crate be $f$ and the normal contact force on the crate be $F_N$:

$$\begin{aligned} F_{net} &= mg\sin\theta - f \\ &= mg\sin\theta - F_n\mu_k \\ &= mg\sin\theta - mg\cos\theta\mu_k \\ &= mg(\sin\theta - \mu_k\cos\theta) \end{aligned}$$

By Newton's Second Law:

$$F_{net} = ma$$

$$ma = mg(\sin\theta - \mu_k\cos\theta)$$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$a = 9.81(\sin 25^\circ - 0.19\cos 25^\circ)$$

$$a = 2.456618063$$

$$a = 2.46\,\mathrm{m\,s^{-2}}\ (3\text{ s.f})$$

## 1.2 (b)

Getting the time taken for the crate to travel the distance:

$$s = ut + \frac{1}{2}at^2$$

$$8.15 = 0 + \frac{1}{2}(2.456618063)t^2$$

$$8.15 \times 2 = 2.456618063t^2$$

$$t^2 = 6.635138056$$

$$t = 2.575876173$$

Getting the final speed of the crate:

$$v = v_0 + at$$

$$v = 0 + 2.456618063 \times 2.575876173$$

$$v = 6.327943934$$

$$v = 6.33\,\mathrm{m\,s^{-2}}\ (3\text{ s.f})$$

# 2 Question 2

## 2.1 (a)

First, the force $F$ needs to overcome the frictional force due to the ground ($f_g$):

$$\begin{aligned} f_g &= mg\mu_s \\ &= (5+3)(9.8)(0.6) \\ &= 47.04\,\mathrm{N} \end{aligned}$$

Next, the force $F$ needs to overcome the frictional force on the 3.0 kg $f_{3kg}$:

$$\begin{aligned} f_{3kg} &= mg\mu_s \\ &= 3(9.8)(0.6) \\ &= 17.64\,\mathrm{N} \end{aligned}$$

The force $F$ also needs to overcome the frictional force from the movement of the 5.0 kg causing the 3.0 kg to move backwards ($f_{mov}$):

$$\begin{aligned} f_{mov} = &= mg\mu_s \\ &= 3(9.8)(0.6) \\ &= 17.64\,\mathrm{N} \end{aligned}$$

Thus, the force $F$ is:

$$\begin{aligned} F &= f_g + f_{3kg} + f_{mov} \\ &= 47.4 + 17.64 + 17.64 \\ &= 82.32\,\mathrm{N} \\ &= 82.3\,\mathrm{N}\ (3.\mathrm{s.f}) \end{aligned}$$

## 2.2 (b)

Finding the force when it is 10% greater:

$$\begin{aligned} F &= 1.1(82.32) \\ &= 90.552\,\mathrm{N} \end{aligned}$$

Since the blocks start to move, we will need to use kinetic friction, $\mu_k$. The blocks are also joined together with a rope, so the acceleration of the two blocks will be the same.

From part ($a$):

$$F_{net} = F - (f_g + f_{3kg} + f_{mov})$$

$$F_{net} = 90.552 - ((5+3)(9.8)(0.4) + (3)(9.8)(0.4) + (3)(9.8)(0.4))$$

$$F_{net} = 90.552 - ((5+3)(9.8)(0.4) + (3)(9.8)(0.4) + (3)(9.8)(0.4))$$

$$(5+3)a = 90.552 - ((5+3)(9.8)(0.4) + (3)(9.8)(0.4) + (3)(9.8)(0.4))$$

$$8a = 35.672$$

$$a = 4.459$$

$$a = 4.46\,\mathrm{m\,s^{-2}}\ (3\text{ s.f})$$

# 3 Question 3

## 3.1 (a)

$$F_{net} = F - f$$

$$F_{net} = 650 - 65(9.8)(0.18) - 125(9.8)(0.18)$$

$$ma = 314.84$$

$$190a = 314.84$$

$$a = 1.657052632$$

$$a = 1.66\,\mathrm{m\,s^{-2}}\ (3\text{ s.f})$$

### 3.2 (b)

Let the contact force that acts on the first block be $F_c$:

$$F - F_c - f = ma$$

$$650 - F_c - 65(9.8)(0.18) = 65(1.657052632)$$
$$F_c = 650 - 107.7084211 - 114.66$$
$$F_c = 427.6315789$$
$$F_c = 428\,\mathrm{N}$$

### 3.3 (c)

$$F_{net} = F - f$$
$$F_{net} = 650 - 65(9.8)(0.18) - 125(9.8)(0.18)$$
$$ma = 314.84$$
$$190a = 314.84$$
$$a = 1.657052632$$
$$a = 1.66\,\mathrm{m\,s^{-2}}\ (3\text{ s.f})$$

Let the contact force that acts on the first block be $F_c$:

$$F - F_c - f = ma$$

$$650 - F_c - 125(9.8)(0.18) = 125(1.657052632)$$
$$F_c = 650 - 220.5 - 207.131579$$
$$F_c = 222.368421$$
$$F_c = 222\,\mathrm{N}\ (3\text{ s.f})$$

## 4 Question 4

### 4.1 (a)

Let $l$ be the distance moved by the object with mass $m_1$. When the object with mass $m_1$ moves a distance $l$, $P_2$ moves a distance $l$ towards $P_1$. At the same time, the object with mass $m_2$ also moves a distance $l$ towards $P_2$. As such, the object with mass $m_2$ will effectively be moving $2l$ when the object with mass $m_1$ moves a distance $l$, which would mean that $2a_1 = a_2$.

## 4.2 (b)

The tension in the string for $P_1$ would be:

$$F_{net} = m_1 g - T_1$$

$$m_1 g - T_1 = m_1 a_1$$
$$T_1 = m_1 g - m_1 a_1$$

The tension in the string for $P_2$ would be:

$$F_{net} = T_2$$

$$F_{net} = m_2 a_2$$
$$T_2 = m_2 a_2$$

## 4.3 (c)

The net force on the block with mass $m_1$ is:

$$F_{net} = m_1 g - T_1$$

Since the tension in $P_1$ is the force on $P_2$, which is twice of the tension in the string of $P_2$:

$$m_1 a_1 = m_1 g - 2T_2$$
$$m_1 a_1 = m_1 g - 2m_2 a_2$$

Since $a_1 = 2a_2$:

$$m_1 a_1 = m_1 g - 4m_2 a_1$$
$$a_1(m_1 + 4m_2) = m_1 g$$
$$a_1(m_1 + 4m_2) = m_1 g$$
$$a_1 = \frac{m_1 g}{m_1 + 4m_2}$$

Since $a_1 = 2a_2$:

$$a_2 = \frac{2m_1 g}{m_1 + 4m_2}$$

# 5 Question 5

## 5.1 (a)

Using the conservation of momentum:

$$m_1 v_1 + m_2 v_2 = 0$$

$$\begin{aligned} m_1 v_1 &= -m_2 v_2 \\ 0.500(4.00) &= -3.0v \\ v_2 &= -\frac{2}{3} \\ v_2 &= -0.667\,\mathrm{m\,s^{-1}} \end{aligned}$$

## 5.2 (b)

By the conservation of energy, the potential energy of the block must have been fully converted into the kinetic energy of the block and the wedge:

$$E_p = \mathrm{KE}_{block} + \mathrm{KE}_{wedge}$$

$$\begin{aligned} m_1 g h &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\ (0.500)(9.8)h &= \frac{1}{2}(0.5)(4)^2 + \frac{1}{2}(3)\left(\frac{2}{3}\right)^2 \\ 4.9h &= \frac{14}{3} \\ h &= \frac{20}{21} \\ h &= 0.952\,\mathrm{m} \end{aligned}$$

# 6 Question 6

By the conservation of energy, the potential energy gained by the spring must be equal to the potential energy of the climber, hence:

$$\frac{1}{2}kx^2 = mgh$$

$$\frac{1}{2}kx^2 = mg(2l + x)$$

$$kx^2 = 2mg(2l + x)$$

$$kx^2 = 4mgl + 2mgx$$

$$kx^2 - 2mgx - 4mgl = 0$$

$$x = \frac{2mg \pm \sqrt{(-2mg)^2 - 4(k)(-4mgl)}}{2k}$$

$$x = \frac{2mg \pm \sqrt{4m^2g^2 + 16kmgl}}{2k}$$

$$x = \frac{2mg \pm \sqrt{4m^2g^2\left(1 + \frac{4kl}{mg}\right)}}{2k}$$

$$x = \frac{2mg \pm 2mg\sqrt{1 + \frac{4kl}{mg}}}{2k}$$

$$x = 2mg\left(\frac{1 \pm \sqrt{1 + \frac{4kl}{mg}}}{2k}\right)$$

$$x = mg\left(\frac{1 \pm \sqrt{1 + \frac{4kl}{mg}}}{k}\right)$$

$$x = \frac{mg}{k}\left(1 \pm \sqrt{1 + \frac{4kl}{mg}}\right)$$

Since $x$ is always positive:

$$x = \frac{mg}{k}\left(1 + \sqrt{1 + \frac{4kl}{mg}}\right) \quad \textbf{(Shown)}$$

# 7 Question 7

## 7.1 (a)

### 7.1.1 (i)

The velocity of the air relative to the cyclist would be:

$$v_r = v + w$$

The power developed by the cyclist would be:

$$P = \frac{WD}{t}$$

$$P = F\left(\frac{x}{t}\right)$$

$$P = Fv$$

$$P = (k(v+w)^2)v$$

$$P = kv(v+w)^2$$

### 7.1.2 (ii)

The velocity of the air relative to the cyclist would be:

$$v_r = \sqrt{v^2 + w^2}$$

Let the angle between the velocity of the air and the velocity of the cyclist be $\theta$.

$$\cos\theta = \frac{A}{H}$$
$$\cos\theta = \frac{v}{\sqrt{v^2 + w^2}}$$

Resolving the velocity of the cyclist in the direction of the air resistance:

$$\begin{aligned} v\cos\theta &= v\frac{v}{\sqrt{v^2 + w^2}} \\ &= \frac{v^2}{\sqrt{v^2 + w^2}} \end{aligned}$$

The power developed by the cyclist would be:

$$P = \frac{WD}{t}$$
$$P = F\left(\frac{x}{t}\right)$$
$$P = Fv$$
$$P = (k(\sqrt{v^2 + w^2})^2)\left(\frac{v^2}{\sqrt{v^2 + w^2}}\right)$$
$$P = k(v^2 + w^2)\left(\frac{v^2}{\sqrt{v^2 + w^2}}\right)$$
$$P = kv^2\sqrt{v^2 + w^2}$$

### 7.2 (b)

From **(aii)**, the power required to cycle at speed $v$ in a cross wind of speed $v$ is:

$$\begin{aligned}
P_{cross} &= kv^2\sqrt{v^2+v^2} \\
&= kv^2(\sqrt{2}v) \\
&= \sqrt{2}kv^3
\end{aligned}$$

The power required to cycle at speed $v$ in still air is:

$$\begin{aligned}
P_{still} &= \frac{WD}{t} \\
&= F\left(\frac{x}{t}\right) \\
&= Fv \\
&= kv^2(v) \\
&= kv^3
\end{aligned}$$

$$\begin{aligned}
\frac{P_{cross}}{P_{still}} &= \frac{\sqrt{2}kv^3}{kv^3} \\
\frac{P_{cross}}{P_{still}} &= \sqrt{2}
\end{aligned}$$

Hence, the power required to cycle at speed $v$ in a cross wind speed of speed $v$ is $\sqrt{2}$ greater than the power required to cycle at the same speed $v$ in still air **(shown)**.