1 Fourier Series

Amplitude linearity:
$V_{out}(t) - V_{out}(0) = \alpha(V_{in}(t) - V_{in}(0))$
Fundamental frequency:
$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$
General form:
$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$
$C_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{A_0}{2}$
$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$
$B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$
Complex form (standard form):
$F(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}\,dt$
$D_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega_0 t}$
$D_n = \frac{A_n - jB_n}{2}$
Cosine form:
$C_n = \sqrt{A_n^2 + B_n^2}$
$\phi_n = -\arctan\left(\frac{B_n}{A_n}\right)$
$F(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi_n)$
Sine form:
$C_n = \sqrt{A_n^2 + B_n^2}$
$\phi_n^* = \arctan\left(\frac{A_n}{B_n}\right)$
$F(t) = C_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \phi_n^*)$
Evenness:
$f(t)g(t)$ and $\frac{f(t)}{g(t)}$ are even only if both $f(t)$ and $g(t)$ are either even or odd
Even function:
$B_n = 0$
$A_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(n\omega_0 t)\,dt$
$C_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt$
Odd function:
$A_n = 0$
$C_0 = 0$
$B_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(n\omega_0 t)\,dt$

2 Linear systems

Linear systems:
$\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^n} = \sum_{m=0}^{M} B_m \frac{d^m X_{in}}{dt^m}$
Homogeneous equation:
$\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^t} = 0$

2.1 Zero-order system

$A_0 X_{out} = B_0 X_{in}$
$X_{out} = K X_{in}$

2.2 First-order system

Differential equations:
$A_1 \frac{dX_{out}}{dt} + A_0 X_{out} = B_0 X_{in}$
$\tau \frac{dX_{out}}{dt} + X_{out} = K X_{in}$
General solution:
$x(t) = (x_0 - x_\infty)e^{-\frac{t}{\tau}} + x_\infty$

2.3 Second-order system

Differential equation:
$A_2 \frac{d^2 X_{out}}{dt^2} + A_1 \frac{dX_{out}}{dt} + A_0 X_{out} = B_0 X_{in}$
Dynamic equation:
$F = m\ddot{x} + b\dot{x} + kx$
Frequency response $\left(\frac{X}{F}\right)$:
$\frac{X}{F} = \frac{k^{-1}}{-\frac{\omega^2}{\omega_0^2} + \frac{j\omega}{Q\omega_0} + 1}$
Resonance:
$\omega_0^2 = \omega_n^2 = \frac{k}{m} = \frac{k}{I}$
Damping ratio:
$\zeta = \frac{b}{b_c} = \frac{b}{2\sqrt{mk}}$
Mechanical Q:
$Q^2 = \frac{km}{b}$, when $Q = 0.5$, the system is critically damped

2.4 Characteristic equation

Equation:
$\sum_{n=0}^{N} A_n s^n = 0$
Primary ($N = 1$):
$A_1 s + A_0 = 0$
$s = \frac{A_0}{A_1}$, if$A_0 \neq 0$
Quadratic ($N = 2$):
$A_2 s^2 + A_1 s + A_0 = 0$
$s = \frac{-A_1 \pm \sqrt{A_1^2 - 4A_0A_2}}{2A_2}$, if$A_2 \neq 0$

3 Solving homogeneous equations

3.1 Primary ($N = 1$)

Single real root: $s_1 = r$
General solution:
$C_0 e^{rt}$

3.2 Secondary ($N = 2$)

Two conjugate roots: $s_1 = a + bi$, $s_2 = a - bi$
General solution:
$(C_1 \sin(bt) + C_2 \cos(bt))e^{at}$
Two different real roots: $s_1 \neq s_2$
General solution:
$C_1 e^{s_1 t} + C_2 e^{s_2 t}$
Double real roots: $s_1 = s_2 = r$
General solution:
$(C_1 + C_2 t)e^{rt}$

3.3 Multiple roots ($N = k$)

Multiple roots: $s_1 = s_2 = \cdots = s_k = r$
General solution:
$(C_0 + C_1 t + C_2 t^2 + \cdots + C_{k-1} t^{k-1})e^{rt}$

4 Dynamic systems

Magnitude ratio:
$M(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$
Dynamic error (1st order system):
$\delta(\omega) = 1 - M(\omega)$

5 Sampling

Nyquist frequency (f_{max}):
$f_s > 2 f_{max}$
Time interval:
$\Delta t = \frac{1}{f_s}$
Aliased signal frequency:
$f_a = |f_s \cdot i - f_n|$

6 Binary

Calculate arbitrary logarithmic base:
$\log_m n = \frac{\ln n}{\ln m}$
$\log_m n = \frac{\log_{10} n}{\log_{10} m}$

6.1 Steps to convert number into binary

◇ Repeatedly divide the number by 2 until the quotient reaches 0.
◇ Keep track of the remainder at the side.
◇ Write the remainder from the bottom to the top.

7 Op-amp

Characteristics:
1. It has infinite impedance on both inputs, so no current is drawn from the input circuit: $I_+ = I_- = 0$
2. It has infinite gain, so the difference between input voltages is zero: $V_+ = V_-$
3. It has zero output impedance, so the output voltage does not depend on the output current.

Inverting amplifier:
$V_{out} = -\frac{R_F}{R} V_{in}$
Non-inverting amplifier:
$V_{out} = \left(1 + \frac{R_F}{R}\right) V_{in}$
Summing amplifier:
$V_{out} = -\left(\frac{R_F}{R_1} V_1 + \frac{R_F}{R_2} V_2 + \cdots + \frac{R_F}{R_N} V_N\right)$
Difference amplifier:
$V_{out} = \frac{R_F}{R}(V_2 - V_1)$
Integrator:
$V_{out} = -\frac{1}{RC}\int_0^t V_{in}(\tau)\,d\tau$
Differentiator:
$V_{out} = -RC\frac{dV_{in}}{dt}$

8 Filters

Cut-off frequency:
$f_c = \frac{1}{2\pi RC}$
Time constant estimate:
$\tau = \frac{1}{f_c}$
Frequency response:
$M(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$

9 Quantisation

Relative quantisation error $= \frac{\text{Quantisation error}}{\text{Gain} \times \text{Amplitude}}$
Sensitivity:
$K = \frac{\Delta V_{out}}{\Delta V_{in}}$
Change in input after amplification:
$\Delta V_{in} = \frac{Q_{res}}{GK}$

9.1 Quantisation procedure

◇ Figure out the amplitude and frequencies in the given signal.
◇ Apply the Shannon sampling theorem ($f_s > 2 f_{max}$).
◇ Find the amplitude range using the signal given.
◇ Obtain the relative quantisation error and pick the best.

10 Circuits

Voltage:
$V = IR$
Current:
$I = \frac{V}{R}$
Resistance:
$R = \frac{V}{I}$
$R = \rho\frac{L}{A}$
Power:
$P = VI = I^2R = \frac{V^2}{R}$
Capacitance:
$C = \frac{Q}{V} = \frac{\epsilon_0\epsilon_r ES}{Ed} = \frac{\epsilon_0\epsilon_r S}{d}$
Current through a capacitor:
$I = C\frac{dV}{dt}$
Impedance of a capacitor:
$Z_c = \frac{1}{j\omega C}$
Energy in a capacitor:
$E = \frac{1}{2}CV^2$
Power output of a motor:
$P_{out} = \omega T$
RC circuits:
$RC = \frac{V}{I} \cdot \frac{Q}{V} = \frac{Q}{\frac{Q}{t}} = t = \tau$
Voltage through the capacitor:
$V_c(t) = (V_0 - V_\infty)e^{\frac{-t}{\tau}} + V_\infty$
Voltage through an inductor:
$V = L\frac{dI}{dt}$
RL circuits:
$\frac{L}{R} = \frac{V}{\frac{I}{t}} \cdot \frac{I}{V} = t$
Thermistor resistance:
$R = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$

10.1 Kirchhoff's voltage law

◇ Pick a current direction to move through the circuit.
◇ When moving from a negative terminal to a positive terminal, add the voltage, as the voltage has increased.
◇ When moving from a positive terminal to a negative terminal, subtract the voltage, as the voltage has decreased.
◇ Essentially, just do the opposite of the signs suggest you to do.

10.2 Kirchhoff's current law

The total current entering a junction is equal to the total current leaving the junction.

11 Circular motion

Torque:
$T = rF$
Velocity:
$v = r\omega$
Centripetal acceleration:
$a_n = \omega^2 r$
Tangential acceleration:
$a_t = r\alpha$

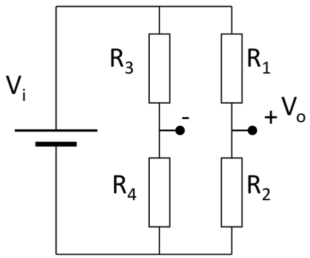
12 Sensors

Gauss law (for capacitive sensors):
$Q = \iint_\Sigma \epsilon_0\epsilon_r \boldsymbol{E}\,dS$
Lorentz force (for proximity sensors):
$\vec{F} = q\vec{v} \times \vec{B}$
Newton's second law:
$F = ma = I\alpha$
Spring force:
$F = kx = k\theta$
Energy of a spring:
$F = \frac{1}{2}kx^2 = \frac{1}{2}k\theta^2$
n-bit encoder resolution:
$\Delta s = \frac{360°}{2^n} = \frac{2\pi}{2^n}$

13 Strain gauges

Poisson's ratio:
$\nu = \frac{\text{lateral strain}}{\text{axial strain}}$
Gauge factor:
$\mathcal{G} = \frac{dR}{R} = \frac{1}{R}\frac{\partial R}{\partial S} = \frac{d\rho}{\rho}\frac{1}{S} + 1 + 2\nu$

13.1 Wheatstone bridge



Wheatstone bridge equations:
$\frac{V^+}{V_i} = \frac{R_2}{R_1 + R_2}$
$\frac{V^-}{V_i} = \frac{R_4}{R_3 + R_4}$
$\frac{V_0}{V_i} = \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}$
Bridge balance condition:
$V_o = 0 \quad \Leftrightarrow \quad R_1R_4 = R_2R_3$
1st order approximation:
$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} - \frac{dR_1}{R_1} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)$
Wheatstone bridge strain equations:
$S_1 = S^a + S^b + S^T$
$S_2 = S^a - S^b + S^T$

13.2 Determine the sum of strains

◇ A strain gauge connected from the positive terminal of the battery (+) to negative terminal of the output (-) connection is added.
◇ A strain gauge connected from the positive terminal of the battery (+) to positive terminal of the output (+) connection is subtracted.

14 Maths

14.1 Derivatives

Chain rule:

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

Product rule:

$\frac{d}{dx}(u \cdot v) = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$

Quotient rule:

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Standard derivatives:

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

14.2 Integrals

$\int \sin x \, dx = -\cos x$

$\int \cos x \, dx = \sin x$

$\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right)$

$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \arcsin\left(\frac{x}{a}\right)$

$\int \frac{1}{x^2-a^2} \, dx = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$

$\int \frac{1}{a^2-x^2} \, dx = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right|$

$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln\left|\sqrt{x^2-a^2} + x\right|$

$\int \tan x \, dx = \ln|\sec x|$

$\int \cot x \, dx = \ln|\sin x|$

$\int \csc x \, dx = -\ln|\csc x + \cot x|$

$\int \sec x \, dx = -\ln|\sec x + \tan x|$

Integration by parts:

$\int u \, dv = uv - \int v \, du$

14.3 Trigonometric identities

Quotient identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal identities:

$\sin\theta = \frac{1}{\csc\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Pythagorean identities:

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

Even/odd identities:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

Co-function identities:

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

$\frac{\pi}{2}$ radians $= 90°$

Sum/difference identities:

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$

Double angle identities:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

Half angle identities:

$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$

$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Sum to product of 2 angles:

$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$

$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$

$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$

$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$

Product to sum of 2 angles:

$\sin\theta\sin\phi = \frac{\cos(\theta-\phi) - \cos(\theta+\phi)}{2}$

$\cos\theta\cos\phi = \frac{\cos(\theta-\phi) + \cos(\theta+\phi)}{2}$

$\sin\theta\cos\phi = \frac{\sin(\theta+\phi) + \sin(\theta-\phi)}{2}$

$\cos\theta\sin\phi = \frac{\sin(\theta+\phi) - \sin(\theta-\phi)}{2}$

Law of sines:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of cosines:

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of a triangle:

$A = \frac{1}{2}ab\sin C$