

NANYANG TECHNOLOGICAL UNIVERSITY

School of Mechanical and Aerospace Engineering

E3.6 TRANSIENT HEAT CONDUCTION

Thermal & Fluid Lab
N3-B2C-06

Name of Student: _____ Lab Group: _____

Matric No: _____ Date: _____

Name of Supervisor: _____

Grade: _____

NOTE: This title page should be attached to your log sheets before submission.

E3.6 TRANSIENT HEAT CONDUCTION

SUMMARY

I) Area of Study

Heat transfer, Thermodynamics

II) Learning Objectives

To study the unsteady state heat conduction by temperature measurement.

a. Theoretical Models, Principles and Concepts

One-dimensional transient heat transfer at a solid cylinder:

(1) Unsteady heat conduction in the cylinder, and

(2) Heat transfer under forced convection around the cylinder;

Thermal resistance including heat conduction resistance and heat convection resistance;

Biot Number;

Lumped thermal capacity method; and

Graphical solutions of heat transfer at the cylinder by using Heisler charts.

b. Experimental Techniques

Investigate temperature variation with time and convection flow when a stainless steel cylinder is subjected to heat suddenly.

c. Instrumentation

The Unsteady State Heat Transfer Unit H111G, which comprises a stainless steel water bath and integral flow duct with external water circulating pump;

HP data acquisition unit; and

Thermocouples.

d. Data Analysis

Dimensionless analysis;

Order of magnitude calculation;

Applying graphical solutions in heat transfer; and

Statistical and Error analysis

E3.6

TRANSIENT HEAT CONDUCTION

1 INTRODUCTION

One of the most important properties of solid metals is their ability to conduct heat. Some metals have very high thermal conductivities while others conduct heat poorly. The purpose of this experiment will be to explore how heat transfer rates in a stainless steel cylindrical rod heated by a forced convection flow. The heat convection coefficients will be calculated and compared with two different forced convection rates.

Heat transfer is driven by a temperature gradient between an object and its surroundings. The metal rod will thus gain heat to the surrounding medium, which in this experiment is a water bath (the water bath has a built-in pump for forced convection). Since the rod in this experiment will transfer heat without maintaining a constant temperature, the process is known as unsteady state. This means that the rate of heat transfer is not constant with time. The rod will be allowed to heat from room temperature to hot water bath temperature with forced convection with various speeds. The Biot number and heat convection coefficient will be analyzed based on non-dimensional time and temperature in the **Herster charts**.

2 OBJECTIVES

To study unsteady state heat transfer/conduction and investigate of the temperature variation with time and heat flow with a stainless steel cylinder that is subjected to sudden heating.

3. THEORY

In unsteady state heat conduction, temperature is a function of both time and spatial coordinates. In the absence of internal heat generation, the temperature response of a body is governed by Fourier's equation. For a one-dimensional case shown in Figure 1, this equation is reduced to:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad \text{Eq. (1)}$$

where T is temperature, t is time, x is a spatial coordinate. For a cylinder or sphere, x is in the radial direction, R . α is the thermal diffusivity, which is the ratio of the rate at which a solid can transfer heat (characterized by the thermal conductivity k) to the rate at which it can store heat (characterized by ρc), that is, $\alpha = k/(\rho c)$, c is the specific heat capacity of solid, and ρ is the density of solid.

Since analytical solutions of Eq. (1) are complicated and difficult to use, approximate solutions of adequate accuracy for most engineering problems are used.

3.1 Notation

Bi	Biot number
A	area of surface (m^2)
α	thermal diffusivity (m^2/s)
ρ	density of solid (kg/m^3)
c	specific heat of solid ($J/kg \cdot K$)
v	volume of cylinder (m^3)
x	characteristic dimension (m)
q	heat flux (W/m^2)
k	thermal conductivity of cylinder ($W/m \cdot K$)
T_1	temperature of water bath (K)
T_2	temperature close to the cylinder surface (K)

T_3	temperature at the center of cylinder (K)
T_C	temperature at the center of cylinder (K)
T_∞	temperature in far field (K)
t	Time (s)
τ	Time constant (s)
i	initial state
R_{cond}	conduction resistance (K/W)
R_{conv}	convection resistance (K/W)
R	diameter of the stainless steel cylinder (m)
h	heat convection coefficient ($\text{W}/\text{m}^2 \cdot \text{K}$)
Fo	non-dimensional time
θ	non-dimensional temperature difference

3.2 Thermal Resistance

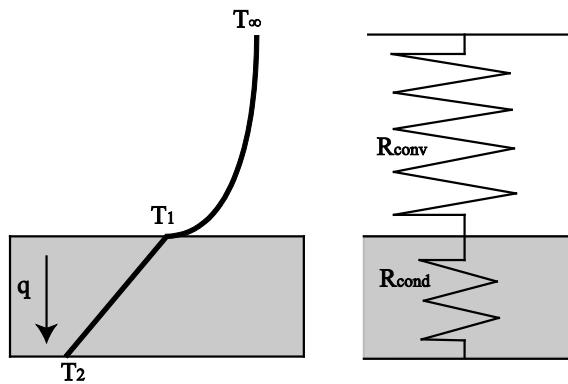


Figure 1 The one-dimensional heat conduction with convective boundary condition

As shown in Figure 1, considering the simple case of one dimensional heat conduction problem with convective boundary condition, the heat flux, q , through the whole system is governed by

$$q = \frac{T_\infty - T_1}{R_{conv}} = \frac{T_1 - T_2}{R_{cond}} = \frac{T_\infty - T_2}{R_{Total}} \quad \text{Eq. (2)}$$

where $R_{conv} = 1/h$, $R_{cond} = x/k$ and $R_{Total} = R_{conv} + R_{cond}$, h is the heat transfer coefficient at the fluid wall interface, and T_∞ is temperature of the surrounding.

3.3 Biot Number

For the typical heat transfer problem shown in Figure 1, the heat flux q depends on the value of the sum of both resistances while the temperature distribution in the slab depends on the relative values of the two resistances, which is quantified by the Biot Number (Bi) defined as:

$$Bi = \frac{R_{cond}}{R_{conv}} = \frac{hx}{k} \quad \text{Eq. (3)}$$

where x is the radius, R for a cylinder.

Low Biot Number ($Bi < 0.1$)

A Biot number less than 0.1 indicates that the conduction thermal resistance (R_{cond}) is practically negligible compared with the convection resistance (R_{conv}). In this case, temperature T_1 and T_2 (Figure 1) are approximately the same, and the solid is assumed to

have a uniform temperature. The transient thermal response of such a system can be obtained by consideration of the changes in the internal energy of the system expressed in terms of changes of the assumed uniform temperature. This approximation is called the **Lumped Thermal Capacity** method.

The governing equation in this case is determined from the energy balance between the heat transfer by convection through the surface and the change in the internal energy of the solid:

$$\rho v c \frac{\partial T}{\partial t} = hA(T - T_{\infty}) \quad \text{Eq. (4)}$$

where A is the surface area of the body for convection. v is the volume of the body. T is the temperature of solid. T_{∞} is the temperature in far field. In this experiment, $T_{\infty} = T_1$.

If the initial condition is $T = T_i$ at $t = 0$, then the solution of equation (4) above is given as:

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = e^{-\left(\frac{hA}{\rho v c}\right)t} = e^{-\left(\frac{t}{\tau}\right)} \quad \text{Eq. (5)}$$

where the quantity $\tau = \rho v c / hA$ is called *Time Constant* of the system.

High Biot Number ($Bi > 0.1$)

In this case, R_{cond} is not negligible and temperature distribution in the solid is not uniform, and the solutions for temperature are available only in graphical form for the following specific cases: the cylinders where the diameter is small compared to the length.

Graphical solutions for the stainless steel cylinder, called **Heisler charts**, are given in Appendix. The graphical solutions give the geometric center or centerline temperature of the solid. The graphical solutions are presented in terms of the non-dimensional temperature difference, θ which is defined as,

$$\theta = \frac{T_c - T_{\infty}}{T_i - T_{\infty}} \quad \text{Eq. (6)}$$

The parameter, T_c , is the centerline or central temperature of solid after time period t from the initial time (when the body started to be heated). In this experiment, $T_c = T_3$.

The non-dimensional time or Fourier Number, Fo is defined as,

$$Fo = \frac{\alpha t}{R^2} \quad \text{Eq. (7)}$$

Three parameters are used in the graphical solution charts (**Heisler charts**): the inverse Biot Number ($1/Bi$), Fourier Number (Fo), and non-dimensional temperature difference (θ). With any two of these parameters computed, the graphs may be used to calculate the third unknown parameter. All the proceeding solutions are for the central or centerline temperature within a body. Off center temperatures may be calculated from knowledge of the centerline temperature at any time using the charts given in Appendix.

4. EQUIPMENT

4.1 Equipment List

- The Unsteady State Heat Transfer Unit H111G, which comprises a stainless steel water bath and integral flow duct with external water circulating pump;
- HP data acquisition unit;
- Thermocouples; and
- Stainless steel cylindrical rod 20mm in diameter, 100mm in length.

4.2 Equipment technical data

Radius of the cylinder	R	10 mm
Thermal conductivity	k	$16.3 \text{ Wm}^{-1}\text{K}^{-1}$
Specific heat capacity	c	460 Jkg^{-1}
Density	ρ	8500 kgm^{-3}
Thermal diffusivity	α	$0.45 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$

4.3 Experimental Setup

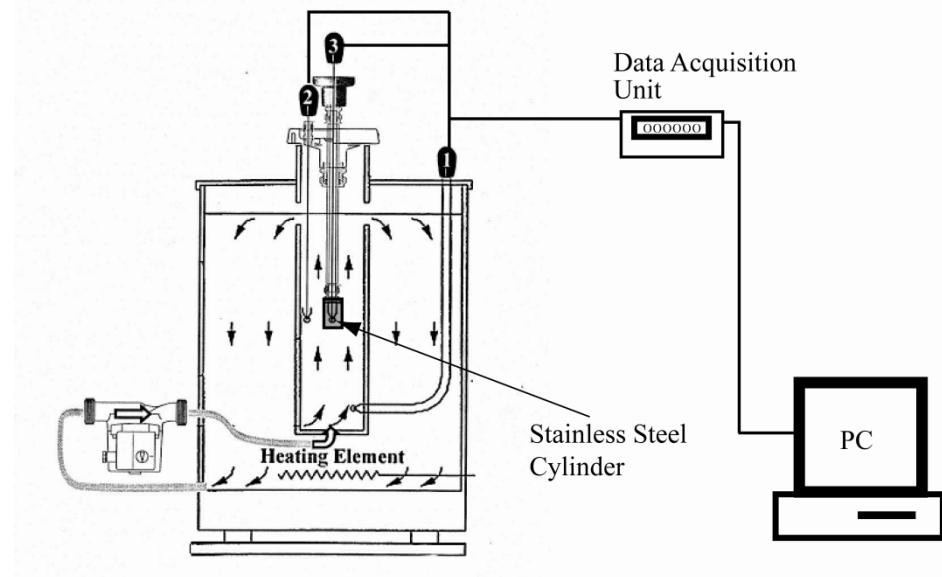


Figure 2 The sketch of the experimental apparatus

The Unsteady State Heat Transfer Unit H111G, shown in Figure 2, is designed specifically for this experiment. The unit comprises a stainless steel water bath and integral flow duct with external water circulating pump. The bath can be heated by a thermostatically controlled 3kW electric heating element in the base of the bath. The adjustment of the thermostat allows the bath to be set to a nominally constant temperature before beginning the experimental procedure. The stainless steel cylinder with 20 mm in diameter and 100 mm in length is fitted with a thermocouple (T_3) well at its geometric center to allow measurement of its core temperature. A thermocouple (T_2) is used to measure water temperature adjacent to the shape. Water from the tank is taken from a combined inlet and discharge fitting and circulated by the pump (variable pump speeds) into the base of the flow duct. The water flows upwards past the cylinder under test and returns to the tank through overflow holes in the duct. Due to the small mass of the shapes relative to the large volume of water contained in the bath the bulk temperature of the bath (T_1) remains essentially constant during the experimental period.

5. EXPERIMENT PROCEDURE

5.1 Operating manual:

E3.6 operating manual

5.2 Experimental method:

- 1) Set the water circulating pump to high speed.
- 2) Turn on the water heater and stabilize the water bath temperature T_1 at approximately 80 to 85°C.
- 3) Install the 20mm-in-diameter stainless steel cylinder in shape carrier of H111G unit.
- 4) Record the starting condition temperature and then plunge the shape in the flow duct. Then record all the temperatures via data acquisition unit and time till the cylinder temperature reaches the water bath temperature.
- 5) Once the stainless steel cylinder reaches the water bath temperature, remove it from the tank.
- 6) Change the pump speed to low speed and repeat the above step 2) – 4).

5.3 Safety:

- 1) Do not open the cover of boiler; and
- 2) Ask the technicians for a help if the experiment is stuck.

6. RESULTS

Draw the curves of T_1 , T_2 and T_3 versus Time. Calculate the values of parameters (θ , Fo , $1/Bi$, and h) using the formulas and **Heisler charts** provided in THEORY section. Attach a set of the sample calculations to the Data Sheet. Some required parameters for calculation can be found in THEORY section.

Reduce the water pump from the high speed to the low speed, and repeat the experiment in 2) to 4). Compare your results with those from experiment with the high speed.

To save the experiment time, each student will use **Heisler charts** to find ONLY one value of $1/Bi$ based on θ and Fo for high speed and low speed experiments respectively (see the dark line in the attached Data Sheet).

All the experimental data and plots may be shared within a group. The sample data calculation, analysis, and discussion should be done individually.

7. DISCUSSION

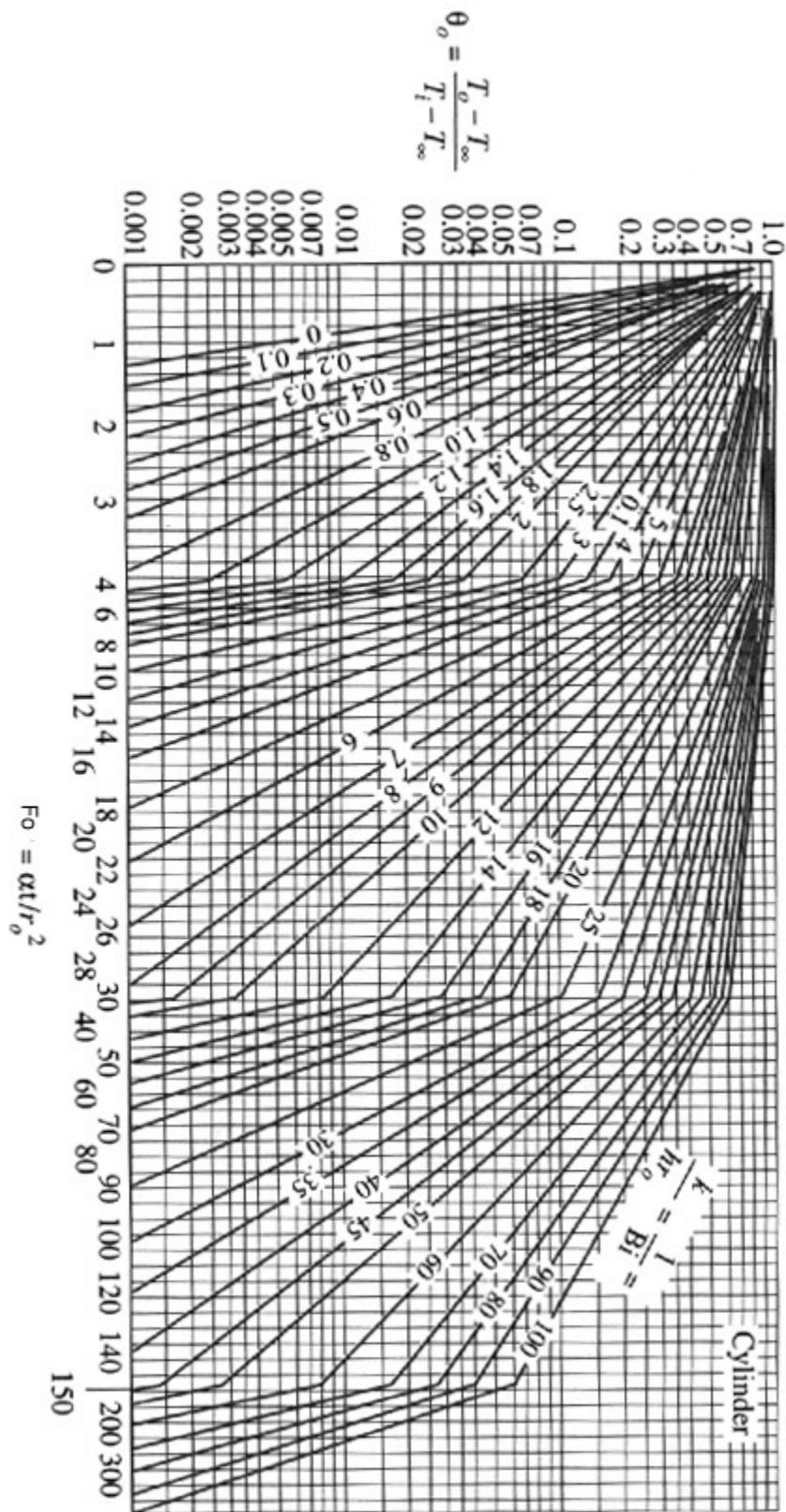
The following four points is required to discuss,

- 1) T_1 , T_2 , and T_3 versus Time at different pump rates;
- 2) The non-dimensional temperature difference, θ versus Time at different pump rates;
- 3) The effect of pump rates on the heat convection coefficient (h); and
- 4) The heat convective the heat convection coefficient (h) versus Time.

8. REFERENCES

Cengal, Y.A. *Introduction to Thermodynamics and Heat Transfer*. McGraw-Hill.

Appendix Heisler charts of cylinder



LOG SHEET**EXPERIMENT E3.6: Transient Heat Conduction****Test Shape: 20mm-in-diameter Stainless Steel Cylinder at different speeds**

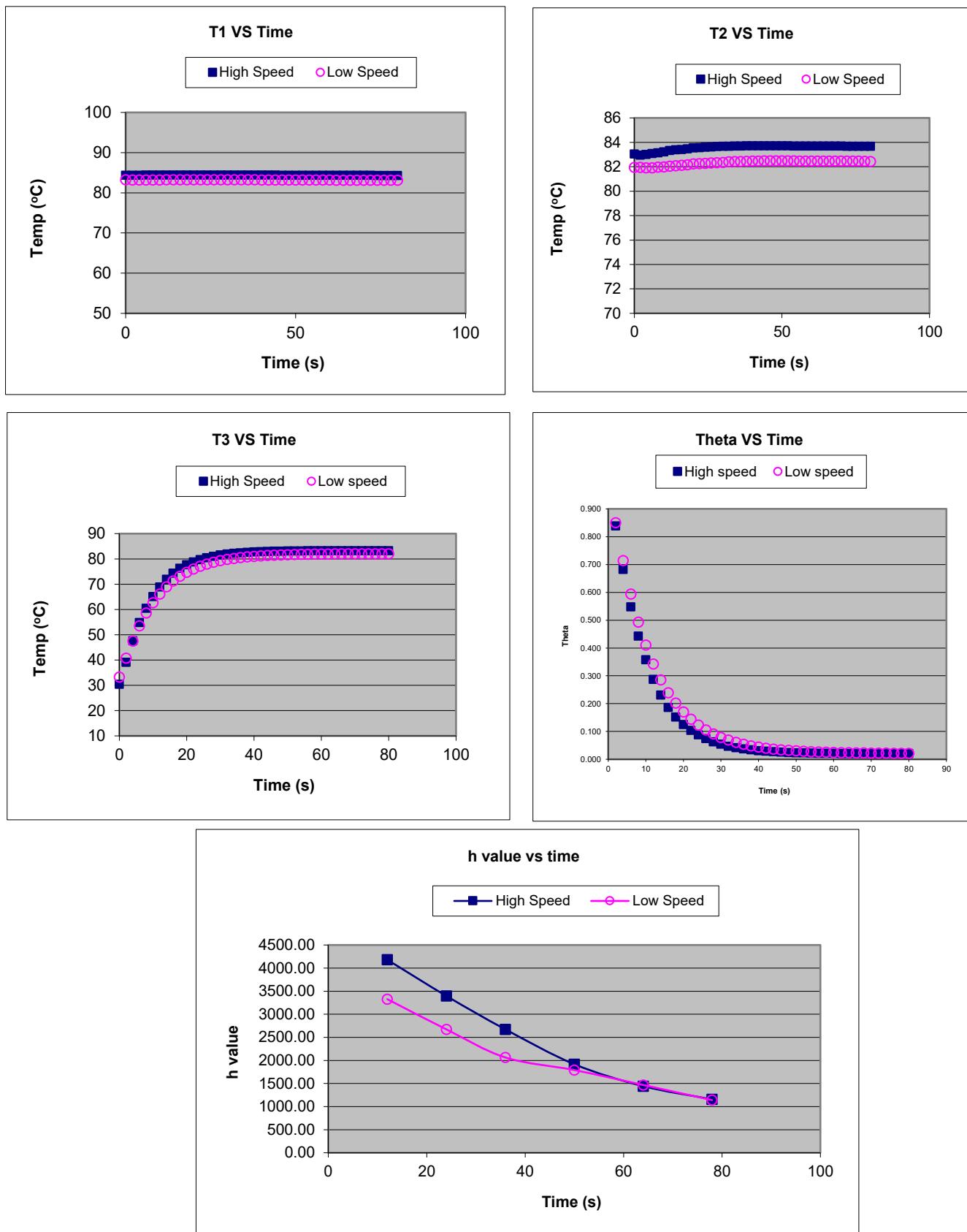
Pump Speed: High or Low							
Time (s)	T1	T2	T3	theta	Fo	1/Bi	h
0							
2							
4							
6							
8							
10							
12							
14							
16							
18							
20							
22							
24							
26							
28							
30							
32							
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60							
62							
64							
66							
68							
70							
72							
74							
76							
78							
80							

A sample of calculation (use a separate page if necessary):

Pump Speed High				Stainless Steel radius = 10 mm			
Time (s)	T1	T2	T3	theta	Fo	1/Bi	h
0	84.36	83.04	30.36	No need to calculate at time=zero			#DIV/0!
2	84.34	82.94	39.08	0.838	0.09		#DIV/0!
4	84.36	83.01	47.55	0.682	0.18		#DIV/0!
6	84.37	83.07	54.8	0.547	0.27		#DIV/0!
8	84.38	83.14	60.47	0.443	0.36		#DIV/0!
10	84.39	83.22	65.08	0.357	0.45		#DIV/0!
12	84.39	83.32	68.88	0.287	0.54	0.39	4179.49
14	84.39	83.38	71.94	0.230	0.63		#DIV/0!
16	84.4	83.41	74.36	0.186	0.72		#DIV/0!
18	84.41	83.47	76.23	0.151	0.81		#DIV/0!
20	84.4	83.53	77.67	0.125	0.9		#DIV/0!
22	84.41	83.55	78.79	0.104	0.99		#DIV/0!
24	84.41	83.59	79.68	0.088	1.08	0.48	3395.83
26	84.41	83.62	80.42	0.074	1.17		#DIV/0!
28	84.41	83.65	81.02	0.063	1.26		#DIV/0!
30	84.4	83.67	81.5	0.054	1.35		#DIV/0!
32	84.39	83.68	81.87	0.047	1.44		#DIV/0!
34	84.39	83.69	82.15	0.041	1.53		#DIV/0!
36	84.39	83.7	82.38	0.037	1.62	0.61	2672.13
38	84.38	83.71	82.56	0.034	1.71		#DIV/0!
40	84.38	83.72	82.71	0.031	1.8		#DIV/0!
42	84.38	83.72	82.83	0.029	1.89		#DIV/0!
44	84.37	83.72	82.92	0.027	1.98		#DIV/0!
46	84.36	83.71	82.99	0.025	2.07		#DIV/0!
48	84.35	83.71	83.05	0.024	2.16		#DIV/0!
50	84.35	83.71	83.09	0.023	2.25	0.85	1917.6
52	84.35	83.71	83.12	0.023	2.34		#DIV/0!
54	84.34	83.7	83.15	0.022	2.43		#DIV/0!
56	84.34	83.7	83.17	0.022	2.52		#DIV/0!
58	84.33	83.7	83.18	0.021	2.61		#DIV/0!
60	84.33	83.7	83.2	0.021	2.7		#DIV/0!
62	84.32	83.69	83.2	0.021	2.79		#DIV/0!
64	84.32	83.69	83.21	0.021	2.88	1.13	1442.5
66	84.32	83.69	83.22	0.020	2.97		#DIV/0!
68	84.33	83.69	83.23	0.020	3.06		#DIV/0!
70	84.32	83.69	83.23	0.020	3.15		#DIV/0!
72	84.32	83.68	83.24	0.020	3.24		#DIV/0!
74	84.31	83.67	83.23	0.020	3.33		#DIV/0!
76	84.31	83.67	83.23	0.020	3.42		#DIV/0!
78	84.31	83.67	83.23	0.0200	3.51	1.41	1156.0
80	84.31	83.67	83.23	0.020	3.6		#DIV/0!

Pump Speed Low				Stainless Steel radius = 10 mm			
Time (s)	T1	T2	T3	theta	Fo	1/Bi	h
0	83.25	81.95	33.23	No need to calculate at time=zero			#DIV/0!
2	83.16	81.94	40.72	0.850	0.09		#DIV/0!
4	83.18	81.91	47.51	0.714	0.18		#DIV/0!
6	83.19	81.91	53.52	0.594	0.27		#DIV/0!
8	83.18	81.96	58.57	0.493	0.36		#DIV/0!
10	83.18	81.98	62.68	0.410	0.45		#DIV/0!
12	83.21	82.05	66.11	0.342	0.54	0.49	3326.53
14	83.21	82.09	68.96	0.285	0.63		#DIV/0!
16	83.21	82.13	71.26	0.239	0.72		#DIV/0!
18	83.21	82.16	73.12	0.202	0.81		#DIV/0!
20	83.21	82.23	74.7	0.170	0.9		#DIV/0!
22	83.22	82.25	76.03	0.144	0.99		#DIV/0!
24	83.21	82.28	77.08	0.123	1.08	0.61	2672.13
26	83.22	82.32	77.95	0.105	1.17		#DIV/0!
28	83.22	82.33	78.68	0.091	1.26		#DIV/0!
30	83.22	82.36	79.28	0.079	1.35		#DIV/0!
32	83.22	82.4	79.77	0.069	1.44		#DIV/0!
34	83.22	82.42	80.19	0.061	1.53		#DIV/0!
36	83.21	82.43	80.52	0.054	1.62	0.79	2063.29
38	83.21	82.45	80.79	0.048	1.71		#DIV/0!
40	83.2	82.46	81.02	0.044	1.8		#DIV/0!
42	83.2	82.47	81.2	0.040	1.89		#DIV/0!
44	83.19	82.48	81.36	0.037	1.98		#DIV/0!
46	83.18	82.48	81.48	0.034	2.07		#DIV/0!
48	83.18	82.48	81.58	0.032	2.16		#DIV/0!
50	83.17	82.48	81.66	0.030	2.25	0.91	1791.21
52	83.18	82.48	81.74	0.029	2.34		#DIV/0!
54	83.17	82.48	81.8	0.027	2.43		#DIV/0!
56	83.15	82.47	81.83	0.026	2.52		#DIV/0!
58	83.15	82.47	81.87	0.026	2.61		#DIV/0!
60	83.15	82.47	81.91	0.025	2.7		#DIV/0!
62	83.14	82.47	81.94	0.024	2.79		#DIV/0!
64	83.13	82.47	81.96	0.023	2.88	1.11	1468.47
66	83.13	82.47	81.98	0.023	2.97		#DIV/0!
68	83.12	82.46	81.99	0.023	3.06		#DIV/0!
70	83.12	82.47	82.02	0.022	3.15		#DIV/0!
72	83.12	82.46	82.02	0.022	3.24		#DIV/0!
74	83.12	82.47	82.03	0.022	3.33		#DIV/0!
76	83.11	82.46	82.03	0.022	3.42		#DIV/0!
78	83.1	82.45	82.03	0.021	3.51	1.43	1139.86
80	83.1	82.43	82.03	0.021	3.6		#DIV/0!

Charts



1 Sample calculations

$$R = 10 \text{ mm}$$

$$k = 16.3 \text{ W m}^{-1} \text{ K}^{-1}$$

$$c = 460 \text{ J kg}^{-1}$$

$$\rho = 8500 \text{ kg m}^{-3}$$

$$\alpha = 0.45 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$$

1.1 High speed

When $t = 36 \text{ s}$:

$$T_\infty = T_1 = 84.39$$

$$T_c = T_3 = 82.38$$

$$T_i = T_{3@t=0 \text{ s}} = 30.36$$

$$\begin{aligned} \theta &= \frac{T_c - T_\infty}{T_i - T_\infty} \\ &= \frac{82.38 - 84.39}{30.36 - 84.39} \\ &= \frac{67}{1801} \\ &\approx 0.037 \text{ (same as computed value)} \end{aligned}$$

$$\begin{aligned} F_o &= \frac{\alpha t}{R^2} \\ &= \frac{(0.45 \times 10^{-5})(36)}{(0.01)^2} \\ &= 1.62 \text{ (same as computed value)} \end{aligned}$$

From the chart, $\frac{1}{\text{Bi}} = 0.61$:

$$\begin{aligned} h &= \frac{\text{Bi } k}{R} \\ &= \frac{\left(\frac{1}{0.61}\right)(16.3)}{0.01} \\ &= \frac{163000}{61} \\ &\approx 2672.13 \text{ (same as computed value)} \end{aligned}$$

1.2 Low speed

When $t = 36$ s:

$$T_\infty = T_1 = 83.21$$

$$T_c = T_3 = 80.52$$

$$T_i = T_{3@t=0\text{ s}} = 33.23$$

$$\begin{aligned}\theta &= \frac{T_c - T_\infty}{T_i - T_\infty} \\ &= \frac{80.52 - 83.21}{33.23 - 83.21} \\ &= \frac{269}{4998} \\ &\approx 0.054 \text{ (same as computed value)}\end{aligned}$$

$$\begin{aligned}F_o &= \frac{\alpha t}{R^2} \\ &= \frac{(0.45 \times 10^{-5})(36)}{(0.01)^2} \\ &= 1.62 \text{ (same as computed value)}\end{aligned}$$

From the chart, $\frac{1}{\text{Bi}} = 0.79$:

$$\begin{aligned}h &= \frac{\text{Bi } k}{R} \\ &= \frac{\left(\frac{1}{0.79}\right)(16.3)}{0.01} \\ &= \frac{163000}{79} \\ &\approx 2063.29 \text{ (same as computed value)}\end{aligned}$$

2 Discussion

The following four points is required to discuss:

- a) T_1 , T_2 , and T_3 versus time at different pump rates;
- b) The non-dimensional temperature difference, θ versus time at different pump rates;
- c) The effect of pump rates on the heat convection coefficient (h);
- d) The heat convection coefficient (h) versus time.

2.1 T_1 , T_2 , and T_3 versus time at different pump rates

Overall, when the pump speed is high, all the temperatures T_1 , T_2 , and T_3 are generally higher than when the pump speed is low.

For the graph of T_1 against time, the temperatures are relatively constant for both low-speed and high-speed pump rates due to the water bath maintaining the temperature, which was kept constant for both setups.

For the graph of T_2 against time, for both the low-speed and high-speed pump rates, the temperature increased at a decreasing rate until it reached a constant value.

For the graph of T_3 against time, both the low-speed and high-speed pump produced a graph that increased at a decreasing rate, but the initial rate of increase is much higher than that of the graph of T_2 against time. This is due to the larger temperature difference initially, which results in a much greater rate of heat transfer.

2.2 The non-dimensional temperature difference, θ versus time at different pump rates

The non-dimensional temperature difference, θ versus time decreased at a decreasing rate until it reached a constant value after $t = 64$ s. The high-speed pump had a higher rate of decrease than the low-speed pump. This suggests that the rate of heat transfer is higher for the high-speed pump than it is for the low-speed pump. This is due to higher flow velocity, increasing the turbulence of the water, which increases the Reynolds number, resulting in a higher rate of heat transfer by convection.

2.3 The effect of pump rates on the heat convection coefficient (h)

The heat convection coefficient was higher for the high-speed pump rate compared to the low-speed pump rate, up until about 55 s. This can be attributed to a higher rate of heat transfer by convection as the flow rate of water in the cylinder is higher, and hence more turbulent.

2.4 The heat convection coefficient (h) versus time

The heat convective coefficient for both the low-speed and high-speed pumps decreases at a decreasing rate until it reaches a constant value. In theory, the heat convective current should be constant since the pump rate, which affects the flow rate, is kept constant throughout the experiment. This decrease in the heat convection coefficient is due to the Biot number not being known, and hence needs to be derived from the Heisler charts using the Fourier number and the temperature difference to obtain the inverse of the Biot number. Since the temperature difference decreases over time, it is expected that the Biot number obtained from this method will also decrease with time, which explains the decreasing h values. If Biot number is calculated using the θ and Fourier number values for the entire duration of the experiment, then the h values would remain constant as expected.