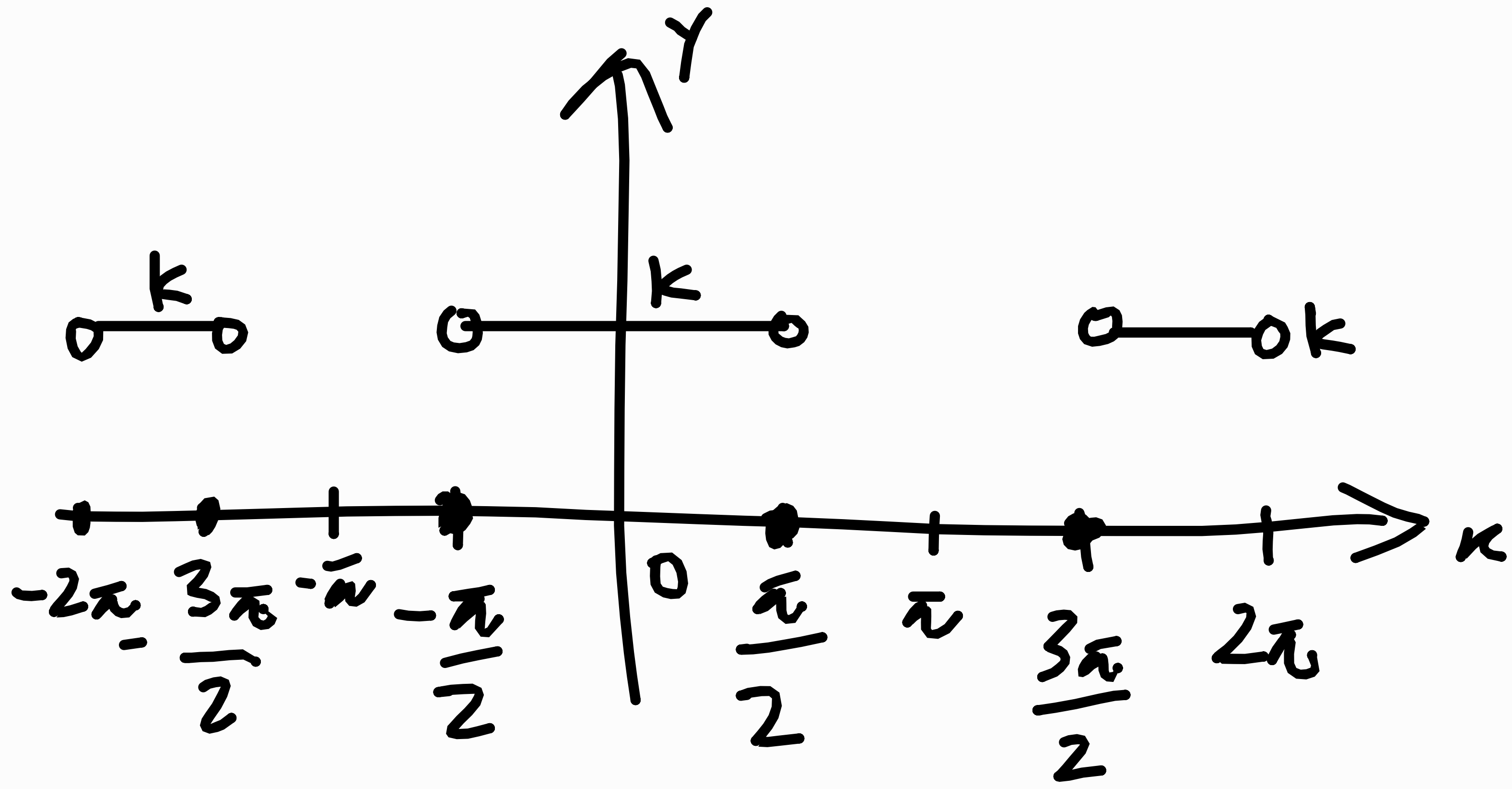
1a)



b) $f(x)$ is even.

c) $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} k\,dx$

$= \frac{k}{2\cancel{\pi}}\left(\cancel{\pi}\right)$

$= \frac{1}{2}k$

1c) $a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} k\cos(nx)\,dx$$

$$= \frac{2k}{\pi}\left[\frac{\sin(nx)}{n}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{2k}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$\therefore$ F.S is $\frac{1}{2}k - \frac{2k}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2}\right)}{n}\cos(nx)$

d) $\text{F.S}_{@x=0} = \frac{1}{2}k + \frac{2k}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2}\right)}{n}$

$$= \frac{1}{2}k + \frac{2k}{\pi}\sin\frac{\pi}{2} + \frac{2k}{2\pi}\sin\left(\frac{2\pi}{2}\right) + \frac{2k}{3\pi}\sin\left(\frac{3\pi}{2}\right) + \frac{2k}{4\pi}\sin\left(\frac{4\pi}{2}\right) + \dots$$

$$= k\left[\frac{1}{2} + \frac{2}{\pi} - \frac{2}{3\pi} + \frac{2}{5\pi} - \frac{2}{7\pi} + \frac{2}{9\pi} - \frac{2}{11\pi} + \dots\right]$$

1d) $F.S_{@\, x=0} = f(0)$

$= k$

$$\therefore \cancel{k}\left[\frac{1}{2}+\frac{2}{\pi}-\frac{2}{3\pi}+\frac{2}{5\pi}-\frac{2}{7\pi}+\frac{2}{9\pi}-\frac{2}{11\pi}+\dots\right]=\cancel{k}$$

$$\frac{1}{2}+\frac{2}{\pi}-\frac{2}{3\pi}+\frac{2}{5\pi}-\frac{2}{7\pi}+\frac{2}{9\pi}-\frac{2}{11\pi}+\dots=1$$

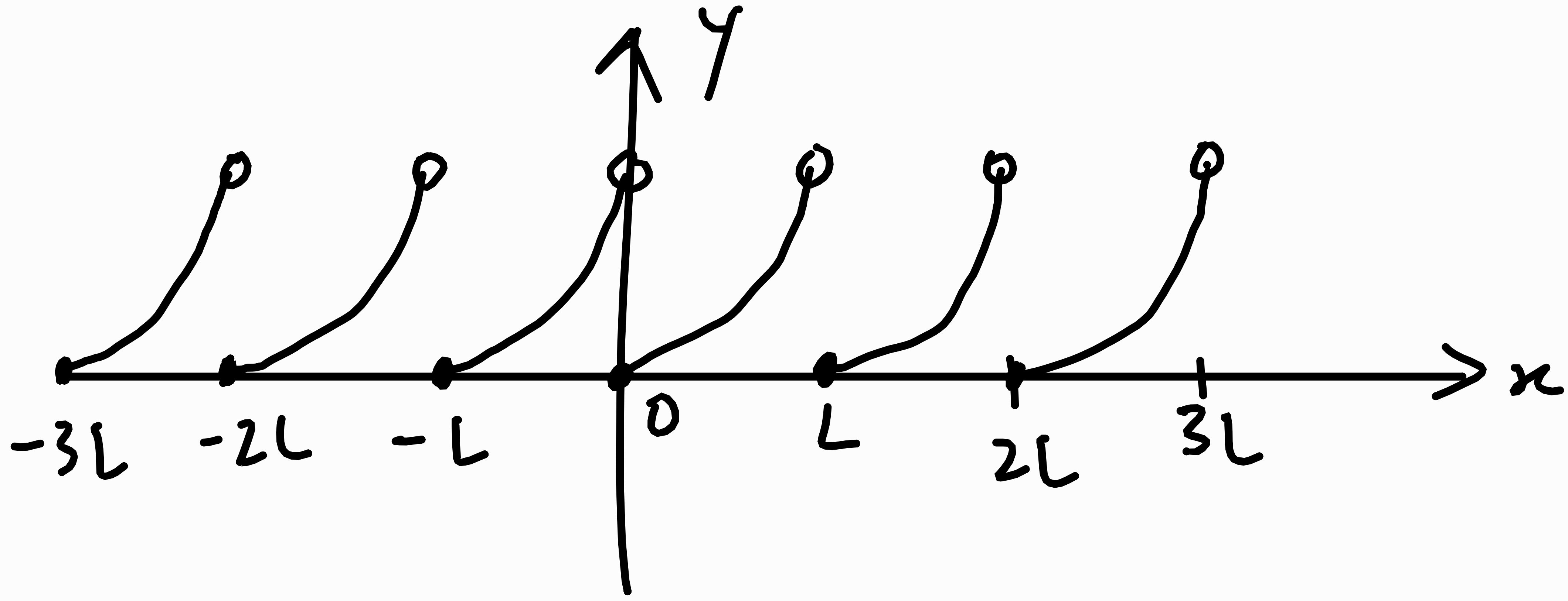
$$\frac{2}{\pi}-\frac{2}{3\pi}+\frac{2}{5\pi}-\frac{2}{7\pi}+\frac{2}{9\pi}-\frac{2}{11\pi}+\dots=\frac{1}{2}$$

$$\frac{2}{\pi}\left[1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\dots\right]=\frac{1}{2}$$

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\dots=\frac{\pi}{4}$$

(shown)

2)



$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{2L} \times 2\int_{0}^{L} x^2\,dx$$

$$= \frac{1}{L}\left[\frac{x^3}{3}\right]_0^L$$

$$= \frac{1}{3}L^2$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{L}\int_0^L x^2\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{L}\left(\frac{1}{\left(\frac{n\pi}{L}\right)^3}\right)\Bigg[\left(\frac{n\pi}{L}\right)^2 x^2 \sin\left(\frac{n\pi x}{L}\right) - 2\sin\left(\frac{n\pi x}{L}\right)$$

$$+ 2\left(\frac{n\pi}{L}\right)x\cos\left(\frac{n\pi x}{L}\right)\Bigg]_0^L$$

2) $$a_n = \frac{2}{L}\left(\frac{1}{\left(\frac{n\pi}{L}\right)^3}\right)\left[\cancel{\left(\frac{n\pi}{L}\right)^2 x^2 \sin\left(\frac{n\pi x}{L}\right)} \cancel{- 2\sin\left(\frac{n\pi x}{L}\right)} + 2\left(\frac{n\pi}{L}\right) x\cos\left(\frac{n\pi x}{L}\right)\right]_0^L$$

$$= \frac{2L^2}{(n\pi)^{\cancel{3}2}}\left[2\cancel{n}\pi\cos(n\pi)\right]$$

$$= \frac{4L^2}{(n\pi)^2}\cos(n\pi)$$

$$\text{F.S} = \frac{1}{3}L^2 + 4L^2\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{(n\pi)^2}\cos\left(\frac{n\pi x}{L}\right)$$

3) $$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\times 2\int_{0}^{L} x^2 \sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{\cancel{L}}\left(\frac{L^{\cancel{3}2}}{(n\pi)^3}\right)\left[2\cos\left(\frac{n\pi x}{L}\right) - \left(\frac{n\pi}{L}\right)^2 x^2 \cos\left(\frac{n\pi x}{L}\right)\right.$$

$$\left.\cancel{+2\left(\frac{n\pi}{L}\right)x\sin\left(\frac{n\pi x}{L}\right)}\right]_0^L$$

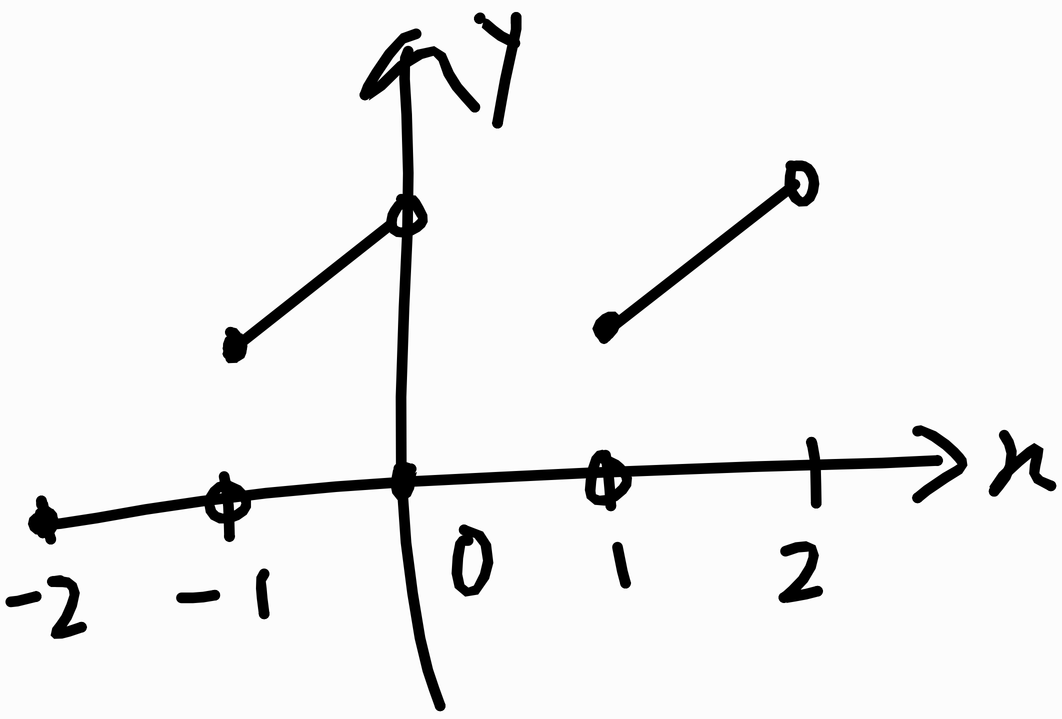
$$= \frac{2L^2}{(n\pi)^3}\left[2\cos(n\pi) - (n\pi)^2\cos(n\pi) - \right.$$

$$\left.(2-0)\right]$$

$$= \frac{2L^2}{(n\pi)^3}\left[2\cos(n\pi) - 2 - (n\pi)^2\cos(n\pi)\right]$$

F.S is $$2L^2\sum_{n=1}^{\infty}\left[\frac{2(\cos(n\pi)-1)}{(n\pi)^3} - \frac{\cos(n\pi)}{n\pi}\right]\sin\left(\frac{n\pi x}{L}\right)$$

4)



$$a_0 = \frac{1}{L}\int_0^L f(x)\,dx$$

$$= \frac{1}{2}\int_0^2 f(x)\,dx$$

$$= \frac{1}{2}\int_1^2 x\,dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{2}\left[2 - \frac{1}{2}\right]$$

$$= \frac{3}{4}$$

4) $a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

$$= \int_1^2 x\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{4}{(n\pi)^2}\left[\cos\left(\frac{n\pi x}{2}\right) + \frac{n\pi}{2}x\sin\left(\frac{n\pi x}{2}\right)\right]_1^2$$

$$= \frac{4}{(n\pi)^2}\left[\cos(n\pi) + \cancel{n\pi\sin(n\pi)} - \left(\cos\left(\frac{n\pi}{2}\right) + \left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)\right)\right]$$

$$= \frac{4}{(n\pi)^2}\left[\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right) - \left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{4}{(n\pi)^2}\left[\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)\right] - \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$\therefore F.S = \frac{3}{4} - \sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{4}{(n\pi)^2}\left[\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)\right]\right)\cos\left(\frac{n\pi x}{L}\right)$$

5) $$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \int_1^2 x\sin\left(\frac{n\pi}{2}\right)dx$$

$$= \frac{4}{(n\pi)^2}\left[\sin\left(\frac{n\pi x}{2}\right) - \frac{n\pi x}{2}\cos\left(\frac{n\pi x}{2}\right)\right]_1^2$$

$$= \frac{4}{(n\pi)^2}\left[\cancel{\sin(n\pi)} - n\pi\cos(n\pi) - \left(\sin\left(\frac{n\pi}{2}\right) - \frac{n\pi}{2}\cos\left(\frac{n\pi}{2}\right)\right)\right]$$

$$= \frac{4}{(n\pi)^2}\left[\frac{n\pi}{2}\cos\left(\frac{n\pi}{2}\right) - \sin\left(\frac{n\pi}{2}\right) - n\pi\cos(n\pi)\right]$$

$$\therefore F.S = \sum_{n=1}^{\infty}\left[\frac{2}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - 2\cos(n\pi)\right] - \frac{4}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)\right]\sin\left(\frac{n\pi x}{L}\right)$$

6a) $c_n = \frac{1}{2L}\int_{\alpha}^{\alpha+2L} f(x) e^{-\frac{in\pi x}{L}} dx$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x e^{-inx} dx$$

$$= -\frac{1}{2n^2\pi}\left[e^{-inx}(-inx-1)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2n^2\pi}\left[e^{-in\pi}(in\pi+1) - e^{in\pi}(1-in\pi)\right]$$

$$= \frac{1}{2n^2\pi}\left[\left(\cos(n\pi) - \cancel{i\sin(n\pi)}\right)(in\pi \cancel{+1}) - \left(\cos(n\pi) + \cancel{i\sin(n\pi)}\right)(\cancel{1} - in\pi)\right]$$

$$= \frac{1}{\cancel{2}n^{\cancel{2}}\cancel{\pi}}(\cancel{2}i\cancel{n\pi})\cos(n\pi)$$

$$= \frac{i}{n}\cos(n\pi)$$

$$c_0 = \frac{1}{2L}\int_{\alpha}^{\alpha+2L} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx = 0$$

$$F.S = i\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{\cos(n\pi)}{n} e^{inx}$$

6b) $c_n = \frac{1}{2L}\int_{\alpha}^{\alpha+2L} f(x)\, e^{-\frac{in\pi x}{L}}\, dx$

$$= \frac{1}{2\pi}\int_{0}^{2\pi} x e^{-inx}\, dx$$

$$= -\frac{1}{2n^2\pi}\left[e^{-inx}(-inx-1)\right]_{0}^{2\pi}$$

$$= \frac{1}{2n^2\pi}\left[e^{-2in\pi}(2in\pi-1)-(0-1)\right]$$

$$= \frac{1}{2n^2\pi}\left[\left(\overset{1}{\cancel{\cos(2n\pi)}} - \overset{0}{\cancel{i\sin(2n\pi)}}\right)(2in\pi-1)+1\right]$$

$$= \frac{1}{\cancel{2}n^{\cancel{2}}\cancel{\pi}}\left[\left(\cancel{2}in\cancel{\pi}\cancel{-1}\right)\cancel{+1}\right]$$

$$= \frac{i}{n}$$

$$c_0 = \frac{1}{2L}\int_{\alpha}^{\alpha+2L} f(x)\, dx = \frac{1}{2\pi}\int_{0}^{2\pi} x\, dx = \frac{1}{\cancel{2\pi}}\left[\frac{\cancel{4}\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$= \pi$$

$$F.S = \pi + i\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1}{n} e^{inx}$$