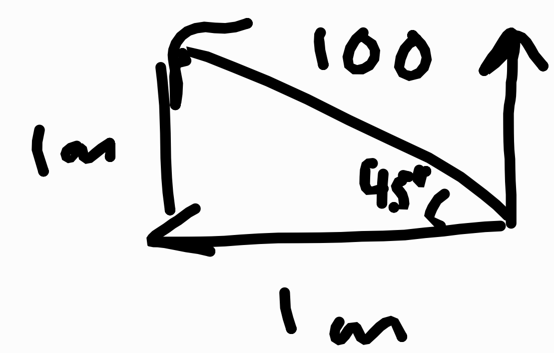
2a) $T_z = T_y = 100 \cos 45°$

$= \frac{100}{\sqrt{2}}$ N



$M_z = T_z (0.5)$

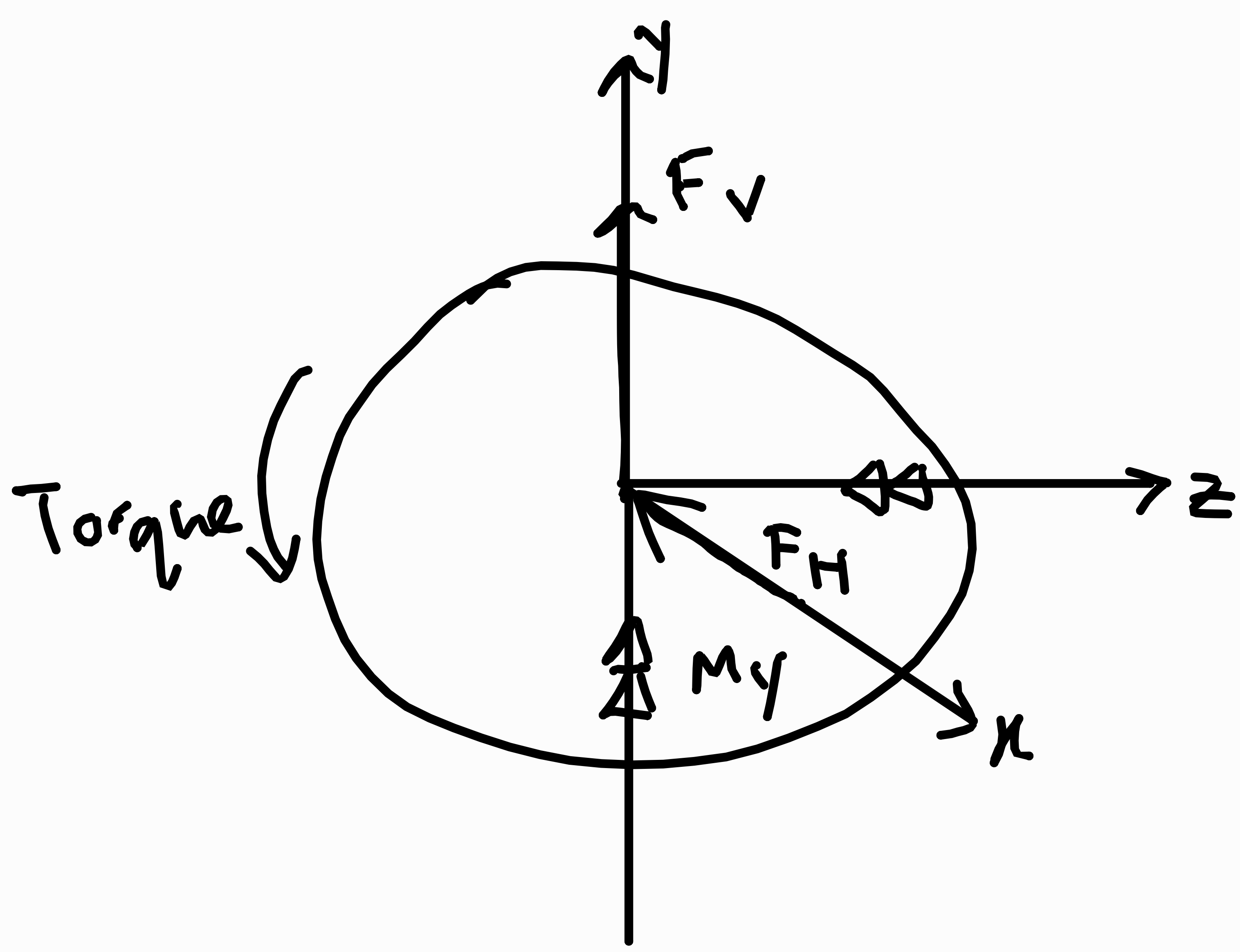
$= \frac{50}{\sqrt{2}}$ Nm

$M_y = T_y (2)$

$= \frac{200}{\sqrt{2}}$ Nm

Torque $= T_z (2)$

$= \frac{200}{\sqrt{2}}$ Nm



2b) $\sigma = \sigma_b + \sigma_{axial}$

$$= \frac{My}{I} + \frac{F_H}{A}$$

$$= \frac{\frac{200}{\sqrt{2}}(20\times10^{-3})}{\frac{1}{4}\pi(20\times10^{-3})^4} + \frac{\frac{100}{\sqrt{2}}}{\pi(20\times10^{-3})^2}$$

$= 22.56417767\,MPa$ (Compression)

$\approx 22.56\,MPa$

$\tau = \tau_T + \tau_b$

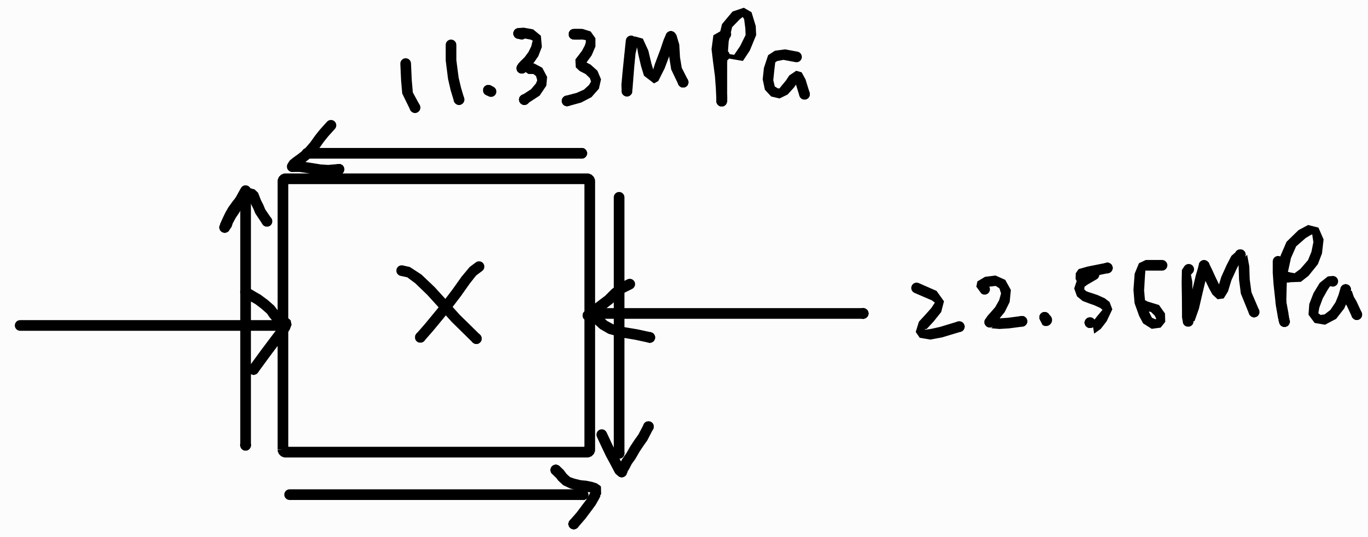
$$= \frac{Tc}{J} + \frac{VQ}{It}$$

$$= \frac{\frac{200}{\sqrt{2}}\cancel{(20\times10^{-3})}}{\frac{1}{2}\pi(20\times10^{-3})^{\cancel{4}3}} + \frac{F_v Q}{It}$$

$$= \frac{\frac{200}{\sqrt{2}}}{\frac{1}{2}\pi(20\times10^{-3})^3} + \frac{\frac{100}{\sqrt{2}}\left(\frac{2}{3}\cancel{(20\times10^{-3})}^3\right)}{\frac{1}{4}\pi(20\times10^{-3})^4(40\times10^{-3})}$$

$= 11.32898031\,MPa$

$\approx 11.33\,MPa$



2c) $$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-22.56}{2} \pm \sqrt{\left(\frac{-22.56}{2}\right)^2 + (-11.33)^2}$$

$$= -11.28 \pm 15.9884747$$

$\therefore \sigma_{max} = 4.706385869 \text{ MPa}$

$\sigma_{min} = -27.27056354 \text{ MPa}$

$\tau_{max} = 15.9884747 \text{ MPa}$

Using the Tresca criterion,

$$\tau_{max} < \frac{\sigma_y}{2}$$

$$15.9884747 < \frac{100}{2}$$

$$15.9884747 < 50$$

Using the Von Mises criterion,

$$\sigma_{max}^2 - \sigma_{max}\sigma_{min} + \sigma_{min}^2 < \sigma_y^2$$

$$4.706385869^2 - 4.706385869(-27.27056354) + (-27.27056354)^2 < 100^2$$

$$894.1794985 < 10000$$

$\therefore$ No failure will occur at X.

1a) Taking moments at C,

$$F_A(3) + 2(1)\left(\tfrac{1}{2}\right) = 5(2)$$

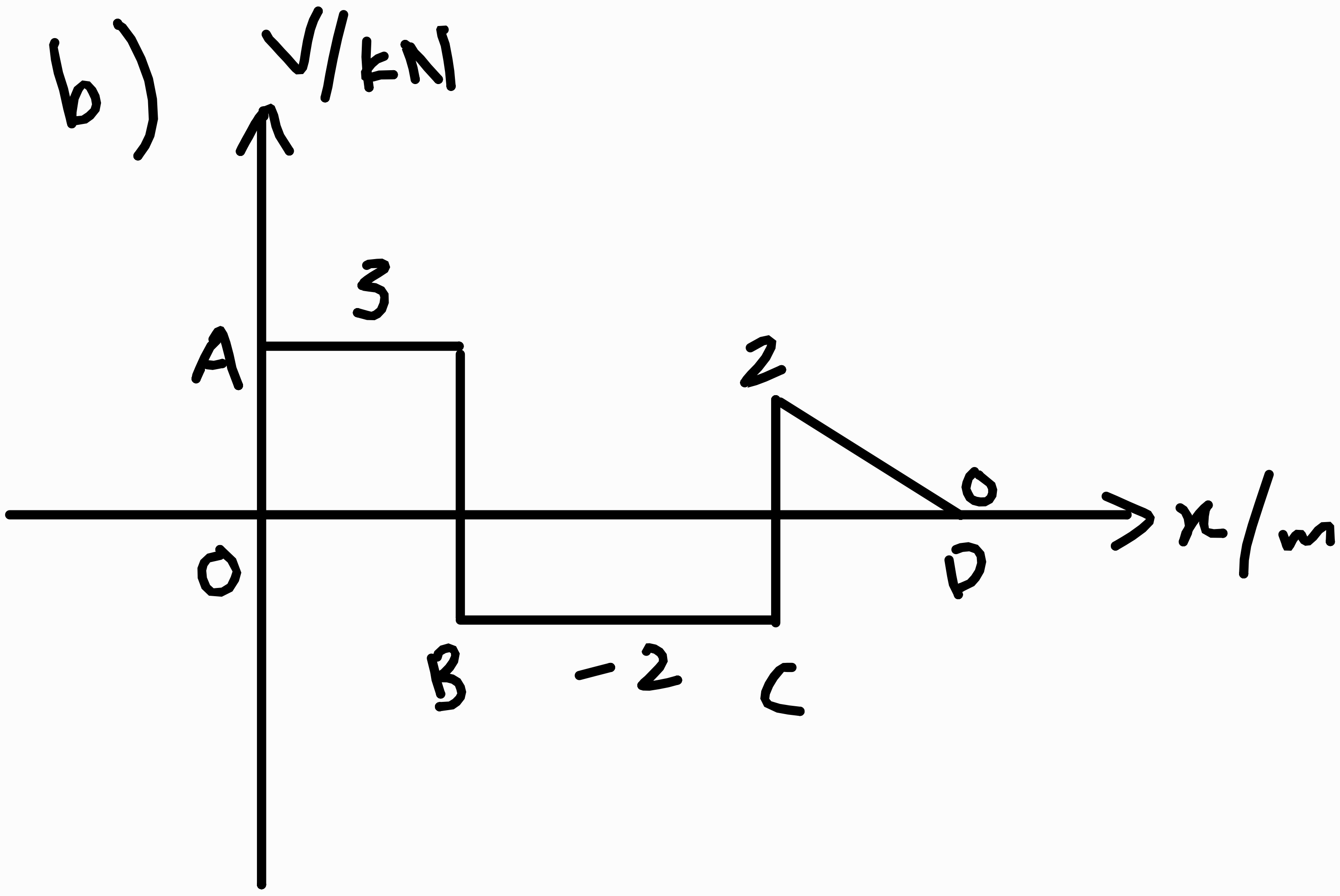
$$F_A = 3\,kN$$

Taking moments at A,

$$F_C(3) = 5(1) + (2)(1)\left(3+\tfrac{1}{2}\right)$$

$$F_C = 4\,kN$$

b)



$V_A = 3\,kN$, $V_B = -2\,kN$, $V_C = 2\,kN$, $V_D = 0\,kN$

1c) $EIy'' = 3x - 5\langle x-1\rangle + 4\langle x-3\rangle - \langle x-3\rangle^2$

$$EIy' = \frac{3x^2}{2} - \frac{5\langle x-1\rangle^2}{2} + \frac{4\langle x-3\rangle^2}{2} - \frac{1}{3}\langle x-3\rangle^3 + C_1$$

$$EIy = \frac{x^3}{2} - \frac{5\langle x-1\rangle^3}{6} + \frac{2\langle x-3\rangle^3}{3} - \frac{1}{12}\langle x-3\rangle^4 + C_1x$$
$$+ C_2$$

When $x=0$, $y=0$,

$$C_2 = 0$$

When $x=3$, $y=0$,

$$0 = \frac{3^3}{2} - \frac{5(2)^3}{6} + 3(C_1)$$

$$C_1 = -\frac{41}{18}$$

1d) When y is max, $y' = 0$,

$$0 = \frac{3x^2}{2} - \frac{5\langle x-1\rangle^2}{2} + 2\langle x-3\rangle^2 - \frac{1}{3}\langle x-3\rangle^3 - \frac{41}{18}$$

Since $1 \leq x \leq 3$,

$$\frac{3x^2}{2} - \frac{5(x-1)^2}{2} = \frac{41}{18}$$

$$3x^2 - 5(x^2 - 2x + 1) = \frac{41}{9}$$

$$-2x^2 + 10x - 5 - \frac{41}{9} = 0$$

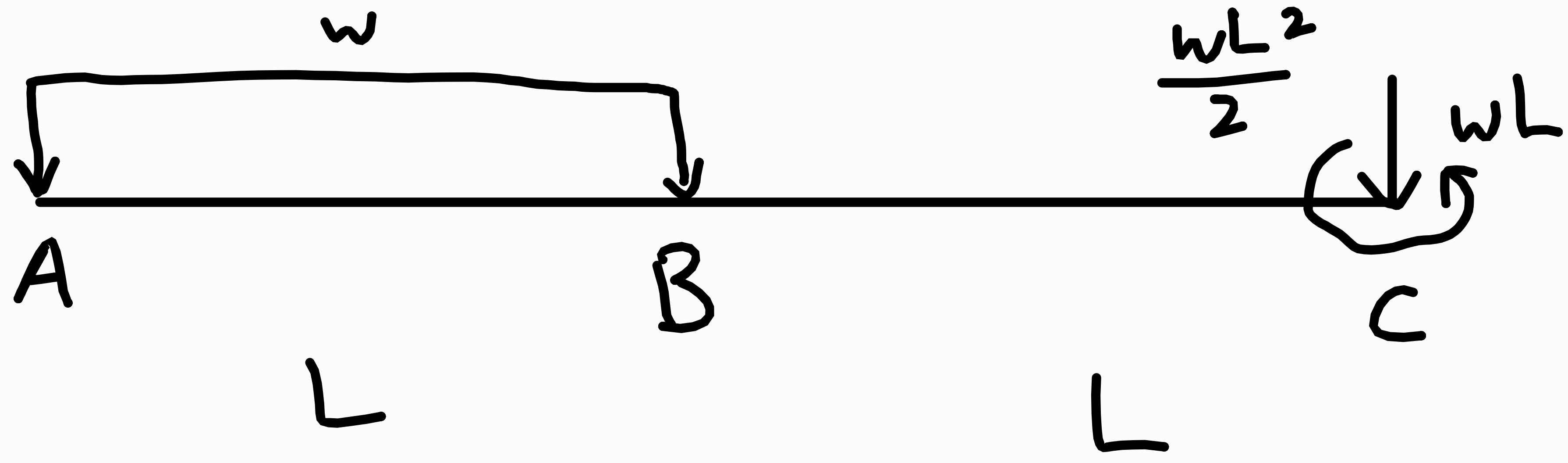
$$x^2 - 5x + \frac{43}{9} = 0$$

$x = 1.28664852$ or $x = 3.713351648$ (reject as $x \leq 3$)

$\therefore x = 1.28664852\,m$

$\approx 1.29\,m$ from A

2a)



$$M_A = \frac{wL^2}{2} + wL(2L) - \frac{wL^2}{2}$$

$$= 2wL^2$$

$$F_A = wL + wL$$
$$= 2wL$$

$$M(x) = 2wLx - 2wL^2x^0 - \frac{w}{2}x^2 + \frac{w}{2}\langle x-L\rangle^2$$

$$EIy'' = 2wLx - 2wL^2x^0 - \frac{w}{2}x^2 + \frac{w}{2}\langle x-L\rangle^2$$

$$EIy' = wLx^2 - 2wL^2x - \frac{w}{6}x^3 + \frac{w}{6}\langle x-L\rangle^3 + c_1$$

$$EIy = \frac{wLx^3}{3} - wL^2x^2 - \frac{w}{24}x^4 + \frac{w}{24}\langle x-L\rangle^4 + c_1x + c_2$$

When $x = 0$, $y = 0$,

$$c_2 = 0$$

When $x = 0$, $y' = 0$,

$$c_1 = 0$$

$$\therefore EIy = \frac{wLx^3}{3} - wL^2x^2 - \frac{w}{24}x^4 + \frac{w}{24}\langle x-L\rangle^4$$

2b) For bending moment to be 0,

$$L \le x \le 2L$$

$$M(x) = 2wLx - 2wL^2 x^0 - \frac{w}{2}x^2 + \frac{w}{2}\langle x-L\rangle^2$$

$$0 = 2wLx - 2wL^2 - \frac{w}{2}x^2 + \frac{w}{2}(x-L)^2$$

$$0 = \cancel{-\frac{w}{2}x^2} + \frac{w}{2}(\cancel{x^2} - 2Lx + L^2) + 2wLx - 2wL^2$$

$$-wLx + \frac{wL^2}{2} + 2wLx - 2wL^2 = 0$$

$$\cancel{w}Lx = \frac{3}{2}\cancel{w}L^{\cancel{2}}$$

$$x = \frac{3}{2}L$$

2c) $EIy = \frac{wLx^3}{3} - wL^2x - \frac{w}{24}x^4 + \frac{w}{24}\langle x-L\rangle^4$

At C, $x = 2L$,

$$EIy = \frac{8wL^4}{3} - 4wL^4 - \frac{2}{3}wL^4 + \frac{w}{24}L^4$$

$$EIy = -\frac{47}{24}wL^4$$

$$y = -\frac{47wL^4}{24EI}$$

2d) When $y = -\frac{2}{180}$, $w = 10\,kN/m$, $L = 1m$,

$E = 200\,GPa$

$$y = -\frac{47wL^4}{24EI}$$

$$-\frac{2}{180} = -\frac{47(10\times10^3)\times1^4}{24\times200\times10^9 I}$$

$$\frac{160}{3}\times10^9 I = 470000$$

$$I = 8.8125\times10^{-6}$$

$$\frac{1}{4}\pi\left(\frac{d}{2}\right)^4 = 8.8125\times10^{-6}$$

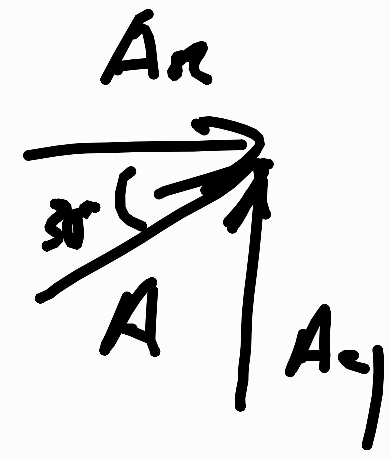
$$\frac{\pi d^4}{64} = 8.8125\times10^{-6}$$

$$d = 0.1157530139\,m$$

$$\approx 115.75\,mm$$

1a) Taking moments about B

$$F_A(120) = 10\sin 30(60)$$



$$F_A = 2.5\,\text{kN}$$

$$F_{Bx} = F_{Ax}$$

$$F_{Bx} = 2.5\cos 30^\circ$$

$$= \frac{5\sqrt{3}}{4}\,\text{kN}$$

$$F_{By} = 10 - F_{Ay}$$

$$= 10 - 2.5\sin 30^\circ$$

$$= 8.75\,\text{kN}$$

1b) At section through H and K,

$$F_{axial} = F_{B_y}\cos 30° + F_{B_x}\cos 60°$$

$$= 8.75\cos 30° + \frac{5\sqrt{3}}{4}\cos 60°$$

$$= \frac{35\sqrt{3}}{8} + \frac{5\sqrt{3}}{8}$$

$$= \frac{40\sqrt{3}}{8}$$

$$= 5\sqrt{3}\ \text{kN}$$

$$V = F_{B_y}\cos 60° - F_{B_x}\cos 30°$$

$$= 8.75\cos 60° - \frac{5\sqrt{3}}{4}\cos 30°$$

$$= \frac{35}{8} - \frac{15}{8}$$

$$= \frac{5}{2}\ \text{kN}$$

$$M = F_{B_y}\cos 60°(40\times10^{-3}) - F_{B_x}\cos 30°(40\times10^{-3})$$

$$= \frac{5}{2}(40\times10^{-3})$$

$$= 0.1\ \text{kNm}$$

1b) $A = 10 \times 24$

$= 240 \text{ mm}^2$

$I = \frac{1}{12}(10)(24)^3$

$= 11520 \text{ mm}^4$

At point H:

$\sigma_x = \frac{F_{axial}}{A}$

$= \frac{5\sqrt{3}}{240}$

$= \frac{\sqrt{3}}{48}$ GPa compression

$\tau_{xy} = \frac{1.5V}{A}$

$= \frac{1.5 \times 2.5}{240}$

$= 15.625 \text{ MPa}$

1b) At point K,

$$\sigma_x = \frac{F_{axial}}{A} + \frac{My}{I}$$

$$= \frac{\sqrt{3}}{48} \times 10^9 + \frac{0.1 \times 10^3 \times 12 \times 10^{-3}}{11520 \times 10^{-12}}$$

$$= 140.2510585 \text{ MPa compression}$$

$$\tau_{xy} = 0$$

1c) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$2\theta_p = \arctan\left(\frac{2(15.625 \times 10^6)}{-\frac{\sqrt{3}}{48} \times 10^9}\right)$$

$$\theta_p = -20.44669732^\circ, \ 69.55330268^\circ$$

1d) $\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \frac{-\frac{\sqrt{3}}{48} \times 10^9}{2} \pm \sqrt{\left(\frac{-\frac{\sqrt{3}}{48} \times 10^9}{2}\right)^2 + (15.625 \times 10^6)^2}$

$= \frac{-\sqrt{3}}{96} \times 10^9 \pm 23867581.74$

$\therefore \sigma_{max} = 5.825385832 \text{MPa}$

$\sigma_{min} = -41.90977766 \text{MPa}$

$\tau_{max} = 23.86758174 \text{MPa}$

1a) $F_V = Q = 1\,kN$

$F_H = P = 2\,kN$

$T = 1(1\times10^{-3})$

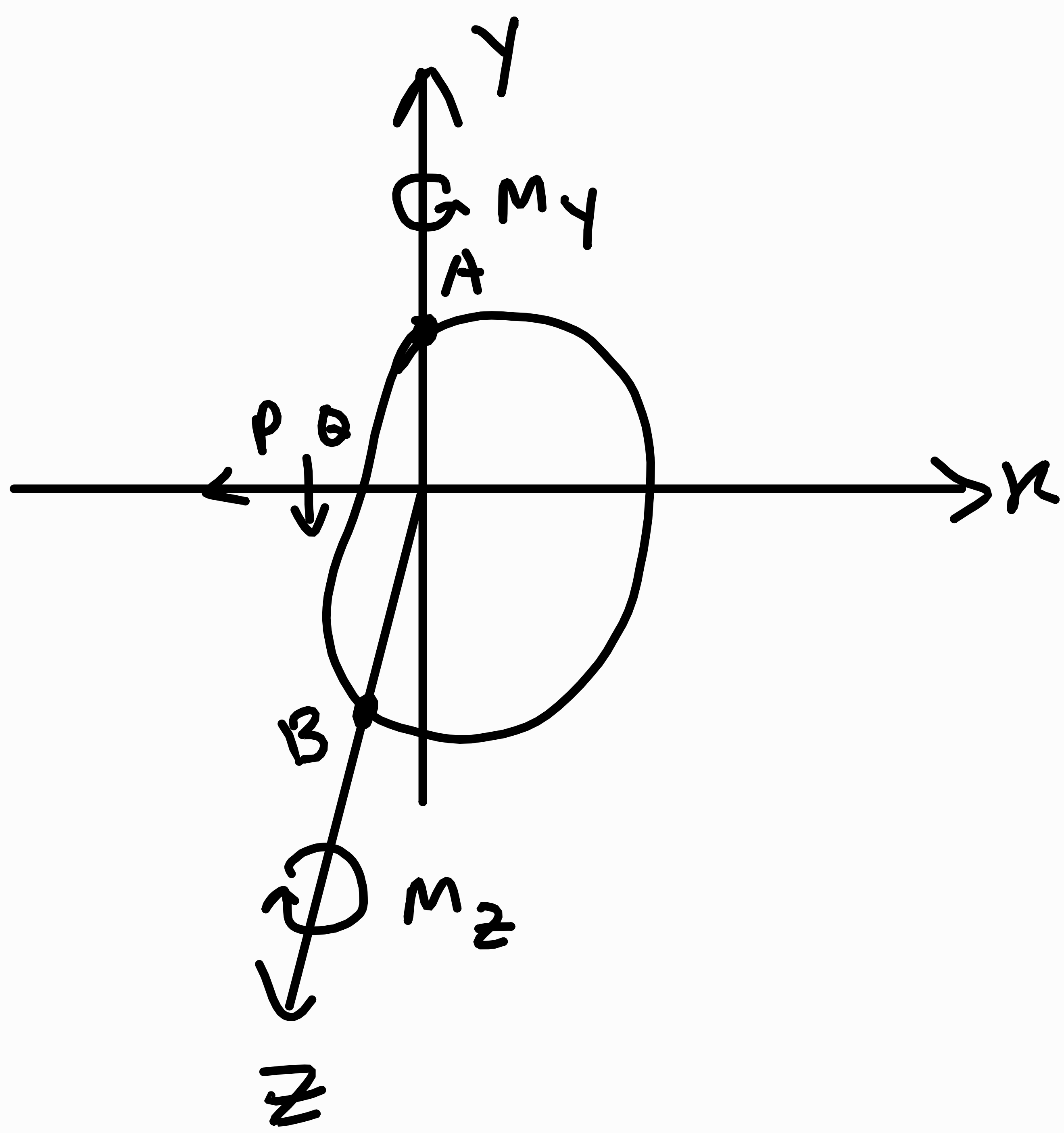
$= 0.1\,kNm$

$M_y = 2(1\times10^{-3})$

$= 0.2\,kNm$

$M_z = 1\times1$

$= 1\,kNm$



1b) $A = (50\times10^{-3})^2\pi$

$= \frac{\pi}{400}$

σ from load

$\sigma_A = \sigma_B = \frac{P}{A}$

$= \frac{2\times10^3}{\frac{\pi}{400}}$

$= 0.2546479089\,\text{MPa}$ tension

σ from M_y

$\sigma_A = 0$

$\sigma_B = \frac{0.2\times10^3(50\times10^{-3})}{\frac{1}{4}\pi(50\times10^{-3})^4}$

$= 2.037183272\,\text{MPa}$ compression

σ from M_z

$\sigma_B = 0$

$\sigma_A = \frac{1\times10^3(50\times10^{-3})}{\frac{1}{4}\pi(50\times10^{-3})^4}$

$= 10.18591636\,\text{MPa}$ compression

1b) $\underline{\tau \text{ from } T}$

$$\tau_A = \tau_B = \frac{0.1\times10^3(50\times10^{-3})}{\frac{1}{2}\pi(50\times10^{-3})^4}$$

$$= 0.5092958179\ MPa$$

$\underline{\tau \text{ from } Q}$

$\tau_A = 0$

$$\tau_B = \frac{VQ}{It}$$

$$= \frac{1\times10^3\left(\frac{2}{3}(50\times10^{-3})^3\right)}{\frac{1}{4}\pi(50\times10^{-3})^4\times100\times10^{-3}}$$

$$= 0.1697652726\ MPa$$

At A,

$\sigma_A = 0.2546479089 - 10.18591636$

$= -9.931268449 MPa$

$\tau_A = 0.5092958179 MPa$

1b) At B,

$$\sigma_B = 0.2546479089 - 2.037183272$$
$$= -1.782535363 \text{ MPa}$$

$$\tau_B = 0.5092958179 - 0.1697652726$$
$$= 0.3395305453 \text{ MPa}$$

1c) $$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-1.782535363}{2} \pm \sqrt{\left(\frac{-1.782535363}{2}\right)^2 + 0.3395305453^2}$$

$$= 0.0624823231 \text{ MPa},$$
$$-1.845017686 \text{ MPa}$$

$\therefore$ $\sigma_{max} = 0.0624823231 \text{ MPa}$

σ_{min} -1.845017686 MPa

$\tau_{max} = 0.9537500044 \text{ MPa}$