

MA3010 Thermodynamics and Heat Transfer

Cheat Sheet (Thermodynamics Section)

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1 Definitions

1.1 Thermodynamics

- Thermodynamics analyses the **amount** of energy (heat or work done) as a system undergoes a process from one equilibrium state to another.
- It only considers the end states in equilibrium and DOES NOT provide information (like time taken) from initial to final state.

1.2 Heat transfer

- Heat transfer analyses the **rate** of energy transfer and **temperature distribution** during a process.
- It is related to time and non-equilibrium states when a temperature gradient exists.

1.3 Change in specific internal energy of an ideal gas

$$\Delta u = c_{v, \text{avg}}(T_2 - T_1)$$

Where:

- Δu is the change in specific internal energy of an ideal gas
- $c_{v, \text{avg}}$ is the specific heat capacity of the ideal gas at constant volume
- T_2 is the final temperature
- T_1 is the initial temperature

1.4 Change in specific enthalpy of an ideal gas

$$\Delta h = c_{p, \text{avg}}(T_2 - T_1)$$

Where:

- Δh is the change in specific enthalpy of an ideal gas
- $c_{p, \text{avg}}$ is the specific heat capacity of the ideal gas at constant pressure
- T_2 is the final temperature
- T_1 is the initial temperature

1.5 Quality (dryness fraction) (x)

Dryness fraction, or quality, x , is the proportion of vapour and liquid by mass in a two-phase mixture. The quality is 0 at the saturated liquid volume (v_f) and 1 at the saturated gas volume (v_g). A two-phase system can be treated as a homogenous mixture for convenience.

$$x = \frac{m_{vapour}}{m_{total}} = \frac{m_g}{m_f + m_g}$$

Where:

- x is the quality or dryness fraction
- m_{vapour} is the mass of vapour
- m_{total} is the total mass
- m_g is the mass of vapour
- m_f is the mass of liquid

1.5.1 Specific entropy in terms of quality (s)

$$s = (1 - x)s_f - xs_g = s_f + xs_{fg}$$

Where:

- s is the specific entropy
- s_f is the specific entropy of the saturated liquid
- s_g is the specific entropy of the saturated vapour
- s_{fg} is the specific entropy of the mixture at evaporation, where $s_{fg} = s_g - s_f$

1.5.2 Quality in terms of specific entropy (x)

$$x = \frac{s - s_f}{s_{fg}}$$

The same formula can also be used for other properties, like specific internal energy (u), specific enthalpy (h) or specific volume (v).

Where:

- x is the quality
- s is the specific entropy of the mixture
- s_f is the specific entropy of the saturated liquid
- s_g is the specific entropy of the saturated vapour
- s_{fg} is the specific entropy of the mixture at evaporation, where $s_{fg} = s_g - s_f$

1.6 Second Law of Thermodynamics

1.6.1 Introduction

1. It identifies the direction of processes.
2. It determines the theoretical limits for the performance of engineering systems, like heat engines and refrigerators.
 - Defines "perfection" for thermodynamic processes
 - Used as a benchmark for real engineering systems
3. It asserts that energy has **quality** as well as quantity. It determines the degree of degradation of energy during a process.
 - Energy at a high temperature has better quality than the same energy at a low temperature.
4. Predicts degree of completion for chemical reactions.
 - A process is completed when entropy stops increasing.

1.7 Thermal energy reservoirs

- A thermal energy reservoir is a hypothetical body with a relatively large **thermal energy capacity (mass × specific heat)** that can absorb or supply finite amounts of heat **without undergoing any change in temperature**.
- In practice, large bodies of water (oceans, lakes and rivers) and the atmospheric air can be modelled as thermal energy reservoirs.

$$Q = mc\Delta T$$

1.7.1 Heat source

A heat source supplies heat energy. Some examples include the Sun, a furnace, etc.

1.7.2 Heat sink

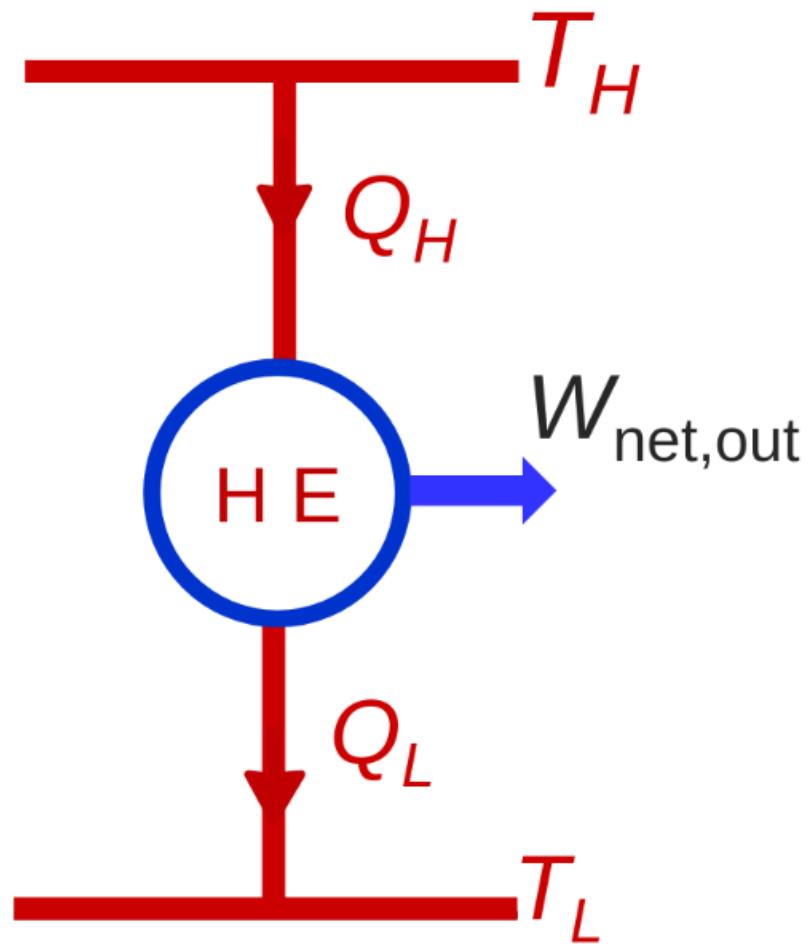
A heat sink absorbs heat energy. Some examples include the river, atmosphere, etc.

1.8 Heat engines

- Work is easily converted into other forms of energy such as heat, but the reverse is more difficult.
- Heat engines convert heat to work.
 1. Receive heat from a high-temperature source.
 2. Convert part of the heat received to work.
 3. Reject the remaining waste heat to a low-temperature sink.
 4. Operate on a **cycle**.
- One example is a steam power plant.
 - Q_{in} is the heat supplied to the steam in the boiler.
 - W_{out} is the work extracted from the steam in the turbine.
 - Q_{out} is the heat rejected by the steam in the condenser.
 - W_{in} is the work required to pump water into the boiler.
- Net output from the heat engine is:

$$W_{\text{net, out}} = W_{out} - W_{in}$$

1.8.1 Diagram



Where:

- T_H is the temperature of the high temperature reservoir
- Q_H is the heat transferred from the high temperature reservoir
- T_L is the temperature of the low temperature reservoir
- Q_L is the heat transferred from the low temperature reservoir
- $W_{\text{net,out}}$ is the net work output of the heat engine

1.8.2 Energy balance

From first law:

$$Q + W = \Delta U$$

For a cycle:

$$Q + W = 0$$

$$Q_H + (-Q_L) + (-W_{\text{net, out}}) = 0$$

$$W_{\text{net, out}} = Q_{in} - Q_{out}$$

Where:

- Q is the heat energy input into the system
- W is the work done on the system
- Q_H, Q_{in} is the heat energy input from the high temperature reservoir (H for high temperature)
- Q_L, Q_{out} is the heat energy output from the low temperature reservoir (L for low temperature)
- $W_{\text{net, out}}$ is the net work output

1.8.3 Thermal efficiency (η)

A measure of how well a heat engine converts heat input into useful work:

$$\text{Thermal Efficiency} = \frac{\text{Net work output}}{\text{Total heat input}}$$

$$\eta_{th} = \frac{W_{\text{net, out}}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$\eta_{th} = \frac{W_{\text{net, out}}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

1.9 Kelvin-Planck statement

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

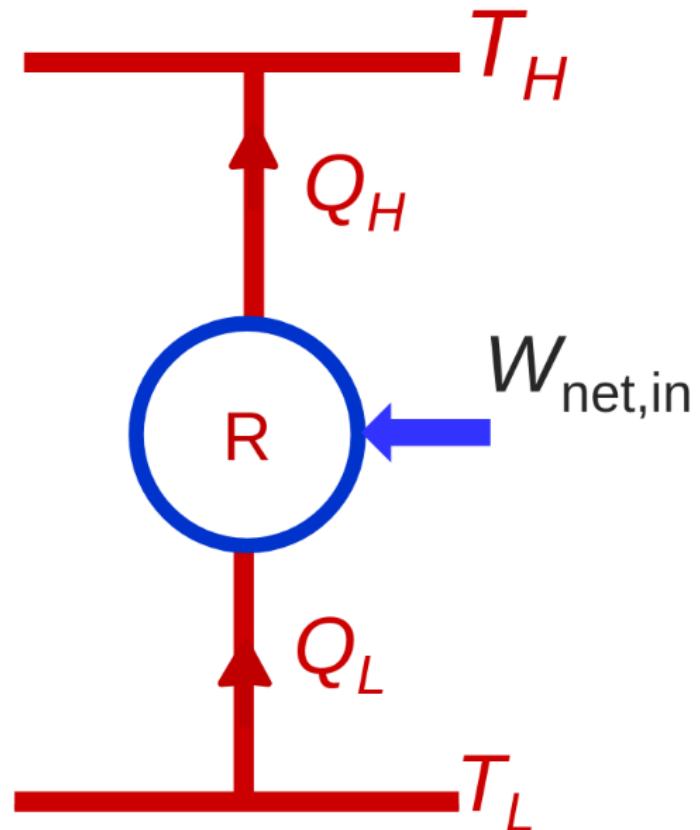
- If $Q_L = 0$, the heat engine will have 100% efficiency.
- However, there is always waste heat produced.
- The cycle cannot be completed without rejecting heat to a low-temperature sink.

Hence, it is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work. Even theoretically perfect heat engines don't have an efficiency of 100%.

This statement is equivalent to the Clausius statement.

1.10 Reverse heat engine (refrigerators and heat pumps)

1.10.1 Diagram



Where:

- T_H is the temperature of the high temperature reservoir
- Q_H is the heat transferred from the high temperature reservoir
- T_L is the temperature of the low temperature reservoir
- Q_L is the heat transferred from the low temperature reservoir
- $W_{\text{net,in}}$ is the net work input of the heat engine

1.10.2 Refrigerators and heat pumps

- Heat transfer from high temperatures to low temperatures by nature
- The opposite can only be achieved using refrigerators and heat pumps
- Refrigerators and heat pumps are examples of "reverse heat engines"
- They operate in a cycle
- The working fluid is called a refrigerant
- Vapour-compression refrigeration system is the most commonly used cycle
- W_{in} is the work input to compressor to compress refrigerant from low to high pressure
- Q_H is the heat rejected by the refrigerant in the condenser
- Q_L is the heat absorbed by refrigerant in evaporator

1.10.3 Energy balance

$$W_{\text{net, in}} = Q_H - Q_L$$

1.11 Coefficient of Performance (*COP*)

- The efficiency of refrigerators and heat pumps is expressed in terms of **coefficient of performance**.
- The formula depends on the function of the machine:

$$\text{Coefficient of Performance} = \frac{\text{Desired output}}{\text{Required input}}$$

1.11.1 Refrigerator

Getting the coefficient of performance:

$$COP_R = \frac{Q_L}{W_{\text{net, in}}}$$

From the energy balance:

$$W_{\text{net, in}} = Q_H - Q_L$$

Hence:

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

1.11.2 Heat pump

Getting the coefficient of performance:

$$COP_{HP} = \frac{Q_H}{W_{\text{net, in}}}$$

From the energy balance:

$$W_{\text{net, in}} = Q_H - Q_L$$

Hence:

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

1.12 Clausius statement

- Heat is never transferred from a cold medium to a warmer one in nature.
- Impossible to have a working refrigerator or heat pump that requires no power input.

Hence, it is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.

This statement is equivalent to the Kelvin-Planck statement.

1.13 Perpetual Motion Machines (PMM)

A device that violates either the first law or second law of thermodynamics.

1.14 Reversible process

A reversible process is defined as one which can be reversed without leaving any trace on the surroundings.

- The state of the system & surroundings can be reverted to initial states at the end of the reverse process.
- It is a theoretical or ideal process, which is usually an ideal version of actual processes.
- Hence, actual devices and system can be approximated as a reversible process at best.
- It serves as the theoretical limit for its corresponding irreversible process and is easy to analyse.
- Thus, actual processes are compared against their corresponding idealised or reversible processes to determine its efficiency.

1.15 Irreversible process

An irreversible process is the opposite of a reversible process.

- It is characteristic of all processes in nature.

1.15.1 Factors

- Friction
- Heat transfer across finite temperature difference
- Mixing of two fluids
- Unrestrained expansion
- Electrical resistance
- Inelastic deformation of solids
- Chemical reaction

The presence of any factors above would cause the process to be irreversible.

1.16 Internally reversible

- Internally reversible means that there is no irreversibility occurring within the system boundaries.
- An example is the boiling of a fluid (constant temperature and pressure process).
- An internally reversible system is the same as a reversible system, the word "internally" is to make it clear that it is the system that is reversible, and not the surroundings.

1.17 Externally reversible

Externally reversible means that there is no irreversibility occurring outside the system boundaries.

1.18 Totally or completely reversible

Totally or completely reversible means that a process is both internally and externally reversible, i.e. there is no irreversibility within the system or surroundings.

1.19 Carnot cycle

- It consists of 4 reversible processes:
 - 2 isothermal processes
 - 2 adiabatic processes
- It is applicable to closed systems or steady flow systems.
- It sets the theoretical limits for heat engines, refrigerators and heat pumps.

1.19.1 The cycle

1. Reversible isothermal expansion (1 - 2, $T_H = \text{constant}$)
 - Gas expands at constant temperature while absorbing heat from energy source.
2. Reversible adiabatic expansion (2 - 3, T_H drops to T_L)
 - Gas does work on surroundings and expands while its temperature drops.
3. Reversible isothermal compression (3 - 4, $T_H = \text{constant}$)
 - Gas compression at constant temperature while losing heat to energy sink.
4. Reversible adiabatic compression (4 - 1, T_L rises to T_H)
 - Work done on gas to compress it and its temperature rises.

1.20 Reverse Carnot cycle (Carnot refrigeration cycle)

- The Carnot cycle is a totally reversible cycle.
- A reversed Carnot cycle becomes the Carnot refrigeration cycle.

1.21 Carnot Principles

1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the two same reservoirs.

$$\eta_{\text{th}, 1, \text{irrev}} < \eta_{\text{th}, 2, \text{irrev}}$$

2. The efficiencies of all reversible heat engines operating between the same two reservoirs are the same.

$$\eta_{\text{th}, 2, \text{rev}} = \eta_{\text{th}, 3, \text{rev}}$$

1.22 Thermodynamic temperature scale (Kelvin scale)

- A temperature scale that is independent of the properties of substances that are used to measure temperature is called a thermodynamic temperature scale.
- It offers great convenience for thermodynamic calculations.
- All reversible heat engines operating between the same two reservoirs have the same efficiency.
- Thus, thermal reservoirs are characterised only by their temperatures.
- Thermal efficiencies of reversible heat engines can be expressed as a function of reservoir temperatures.
- This temperature scale is called the Kelvin scale.
- The magnitudes of temperature units on the Kelvin and Celsius scales are the same, i.e.

$$1\text{ K} \equiv 1\text{ }^{\circ}\text{C}$$

1.23 Absolute temperatures

Absolute temperatures are temperatures on the thermodynamic temperature scale (Kelvin scale).

$$T(\text{K}) = T(\text{ }^{\circ}\text{C} + 273)$$

1.24 Carnot Heat Engines

- Hypothetical heat engine operating on the Carnot cycle
- Most efficient (ideal) heat engine

1.24.1 Thermal efficiency

The thermal efficiency of **any** heat engine is:

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

From the thermodynamic temperature scale:

$$\left(\frac{Q_H}{Q_L} \right) = \frac{T_H}{T_L} \text{ or } \left(\frac{Q_L}{Q_H} \right) = \frac{T_L}{T_H}$$

Hence, the thermal efficiency of a **Carnot heat engine**:

$$\eta_{th, rev} = 1 - \frac{T_L}{T_H}$$

Where:

- η_{th} is the efficiency of a heat engine
- Q_H is the heat transferred from the high temperature reservoir
- Q_L is the heat transferred from the low temperature reservoir
- T_H is the temperature of the high temperature reservoir
- T_L is the temperature of the low temperature reservoir
- $\eta_{th, rev}$ is the efficiency of a Carnot heat engine

So:

- $\eta_{th} < \eta_{th, rev}$ is an irreversible heat engine.
- $\eta_{th} = \eta_{th, rev}$ is a reversible heat engine.
- $\eta_{th} > \eta_{th, rev}$ is an impossible heat engine.
- The efficiency increases with source temperature.
- Energy has higher **quality** at higher temperatures.

1.25 Carnot refrigerators and heat pumps

- A device that operates on the reversed Carnot cycle

The coefficient of performance for any refrigerator is:

$$COP_R = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

Hence, the coefficient of performance for a **Carnot refrigerator**:

$$COP_{R, \text{rev}} = \frac{1}{\frac{T_H}{T_L} - 1}$$

The coefficient of performance for any heat pump is:

$$COP_{HP} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

The coefficient of performance for any heat pump is:

$$COP_{HP, \text{rev}} = \frac{1}{1 - \frac{T_L}{T_H}}$$

Similarly to the efficiency of a Carnot heat engine:

- $COP_R < COP_{R, \text{rev}}$ is an irreversible refrigerator.
- $COP_R = COP_{R, \text{rev}}$ is a reversible refrigerator.
- $COP_R > COP_{R, \text{rev}}$ is an impossible refrigerator.
- $COP_{HP} < COP_{HP, \text{rev}}$ is an irreversible heat pump.
- $COP_{HP} = COP_{HP, \text{rev}}$ is a reversible heat pump.
- $COP_{HP} > COP_{HP, \text{rev}}$ is an impossible heat pump.

1.26 Clausius Inequality

The Clausius inequality states that the cyclic integral of $\frac{\delta Q}{T}$ is always less than or equal to zero for all cycles regardless of the type of cycle.

$$\oint \frac{\delta Q}{T} \leq 0$$

Where:

- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat leaving from the system to the reservoirs.
- T is the common temperature of the reservoirs at a particular instant in time in Kelvin

For reversible cycles:

$$\oint \left(\frac{\delta Q}{T} \right)_{rev} = 0$$

For irreversible cycles:

$$\oint \left(\frac{\delta Q}{T} \right) < 0$$

1.27 Isentropic process

An isentropic process is a process with no change in entropy, or a constant entropy process.

1.28 First Tds equation (Gibbs equation)

- Entropy in differential form:

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$

$$\Delta Q_{\text{int rev}} = T dS$$

- From first law:

$$\Delta Q_{\text{int rev}} - \Delta W_{\text{int rev, out}} = dU$$

- Where:

$$\Delta W_{\text{int rev, out}} = PdV$$

- After substitution:

$$TdS - PdV = dU$$

$$TdS = dU + PdV$$

$$Tds = du + Pdv$$

Where:

- T is the temperature in Kelvin
- dS is the change in entropy
- dU is the change in internal energy
- P is the pressure of the system
- dV is the change in volume
- ds is the change in specific entropy
- du is the change in specific internal energy
- dv is the change in specific volume

1.29 Second Tds equation

- Differentiating the definition of enthalpy ($h = u + Pv$)

$$dh = du + Pdv + vdP$$

$$du = dh - Pdv - vdP$$

- Replace du with the definition of enthalpy ($h = u + Pv$) in the first equation above:

$$Tds = du + Pdv$$

$$Tds = dh - vdP$$

$$ds = \frac{dh}{T} - \frac{vdP}{T}$$

Where:

- T is the temperature in Kelvin
 - ds is the change in specific entropy
 - du is the change in specific internal energy
 - dh is the change in specific enthalpy
 - P is the pressure of the system
 - dv is the change in specific volume
 - dP is the change in pressure of the system
- Entropy change can now be related to changes in other properties as these relations are independent of processes.

1.30 Entropy change for liquid and solids

- Liquids and solids can be approximated as **incompressible substances**, hence $dv \cong 0$

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

$$ds = \frac{du}{T} = \frac{cdT}{T}$$

- For incompressible substances:

$$c_p = c_v = c \quad \rightarrow \quad du = cdT$$

- Integrating:

$$s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{avg} \ln \left(\frac{T_2}{T_1} \right) \quad (\text{kJ kg}^{-1} \text{ K}^{-1})$$

Where:

- s_2 is the final **specific** entropy
- s_1 is the initial **specific** entropy
- c is the specific heat capacity as a function of temperature
- dT is the change in temperature in K
- c_{avg} is the average specific heat capacity
- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin
- Entropy change for liquids and solids are only dependent on temperature and independent of pressure
- For isentropic processes,

$$\Delta s = 0, \quad \Delta T = 0$$

1.31 Entropy change for ideal gases

- For ideal gases:

$$R_{sp} = \frac{R_u}{\text{Molar mass}}$$

$$Pv = R_{sp}T \quad du = c_v dT \quad dh = c_p dT$$

- Substituting into Tds relations:

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

$$ds = c_v \frac{dT}{T} + R_{sp} \frac{dv}{v}$$

$$ds = \frac{dh}{T} - \frac{vdP}{T}$$

$$ds = c_p \frac{dT}{T} - R_{sp} \frac{dP}{p}$$

- Integrating:

$$s_2 - s_1 = \int_1^2 c_v(T) \ln \left(\frac{dT}{T} \right) + R_{sp} \ln \left(\frac{v_2}{v_1} \right)$$

$$s_2 - s_1 = \int_1^2 c_p(T) \ln \left(\frac{dT}{T} \right) - R_{sp} \ln \left(\frac{P_2}{P_1} \right)$$

- Assuming constant (average) specific heats:

$$s_2 - s_1 = \int_1^2 c_{v, \text{ avg}}(T) \ln \left(\frac{dT}{T} \right) + R_{sp} \ln \left(\frac{v_2}{v_1} \right)$$

$$s_2 - s_1 = \int_1^2 c_{p, \text{ avg}}(T) \ln \left(\frac{dT}{T} \right) - R_{sp} \ln \left(\frac{P_2}{P_1} \right)$$

This assumption is sufficiently accurate when the temperature range is less than a few hundred degrees.

1.31.1 Isentropic process for the first Tds equation ($\Delta s = 0$)

- From the first Tds equation:

$$s_2 - s_1 = 0 = c_v \ln \left(\frac{T_2}{T_1} \right) + R_{sp} \ln \left(\frac{v_2}{v_1} \right)$$

$$\ln \left(\frac{T_2}{T_1} \right) = -\frac{R_{sp}}{c_v} \ln \left(\frac{v_2}{v_1} \right) \rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\frac{R_{sp}}{c_v}}$$

- Since:

$$R_{sp} = c_p - c_v \text{ and } k = \frac{c_p}{c_v} \rightarrow \frac{R_{sp}}{c_v} = k - 1$$

- Therefore:

$$\left(\frac{T_2}{T_1} \right)_{isen} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

1.31.2 Isentropic process for the second Tds equation ($\Delta s = 0$)

- From the second Tds equation:

$$s_2 - s_1 = 0 = c_p \ln \left(\frac{T_2}{T_1} \right) - R_{sp} \ln \left(\frac{P_2}{P_1} \right)$$

$$\ln \left(\frac{T_2}{T_1} \right) = \frac{R_{sp}}{c_p} \ln \left(\frac{P_2}{P_1} \right) \rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{R_{sp}}{c_p}}$$

$$\left(\frac{T_2}{T_1} \right)_{isen} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

- Therefore:

$$\left(\frac{P_2}{P_1} \right)_{isen} = \left(\frac{v_1}{v_2} \right)^k \quad \text{or} \quad Pv^k = \text{constant}$$

1.32 Entropy change of a reversible process

- Differential form of entropy:

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \text{ (kJ K}^{-1}\text{)}$$

- Integrate the differential form to give the entropy change for reversible processes:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$

Where:

- dS is the change in entropy for an internally reversible process (int rev)
- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat leaving from the system to the reservoirs.
- T is the common temperature of the reservoirs at a particular instant in time in Kelvin
- ΔS is the change in entropy
- S_2 is the final entropy
- S_1 is the initial entropy

1.32.1 Internally reversible isothermal heat transfer

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0} \right)_{\text{int rev}} = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}}$$

The above reduces to:

$$\Delta S = \frac{Q}{T_0} \quad (\text{kJ K}^{-1})$$

The formula is commonly used to determine the entropy change of thermal energy reservoirs that supply or absorb heat indefinitely at constant temperature.

Where:

- ΔS is the change in total entropy
- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat is leaving from the system to the reservoirs.
- Q is the heat transfer for the internally reversible process
- T_0 is the constant temperature in Kelvin

1.33 Entropy change of an irreversible process

$$\Delta S_{sys} = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$

- The integral will give the value of entropy change only if the integration is carried out along an **internally reversible path** between two states.
- For an **irreversible process**, the entropy change can be determined along an **imaginary internally reversible path**.

Where:

- ΔS_{sys} is the change in entropy of the system
- S_2 is the final entropy
- S_1 is the initial entropy
- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat is leaving from the system to the reservoirs.
- T is the common temperature of the reservoirs at a particular instant in time in Kelvin

1.33.1 Imaginary internally reversible path

- Consider a cycle where:
 - Process 1 to 2 is **arbitrary**
 - Process 2 to 1 is **reversible**
- Clausius inequality:

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

- Integration for the cycle:

$$\int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

- Second term is entropy change:

$$\int_1^2 \frac{\delta Q}{T} + (S_1 - S_2) \leq 0 \quad \rightarrow \quad S_2 - S_1 \geq \frac{\delta Q}{T}$$

- Differential form:

$$dS \geq \frac{\delta Q}{T}$$

Essentially, this means that the entropy change of a **closed system** for an **irreversible process** is **greater than** $\int \frac{\delta Q}{T}$ evaluated for that process.

$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

- The "additional" entropy change for irreversible processes is due to entropy generation due to the presence of irreversibilities.

$$\begin{array}{rclcl} S_2 - S_1 & = & \int_1^2 \frac{\delta Q}{T} & + & S_{gen} \\ \Delta S_{sys} & = & \int_1^2 \frac{\delta Q}{T} & + & S_{gen} \\ \text{Entropy change of a} & = & \text{Entropy transfer due} & + & \text{Entropy generation} \\ \text{closed system} & & \text{to heat transfer} & + & \text{due to irreversibilities} \end{array}$$

1.34 Entropy generation (S_{gen})

- Entropy generation is the entropy generated from an irreversible process.
- This value is always positive for irreversible processes and zero for reversible processes.
- It is not a property, the value is process dependent.

1.35 Systems

1.35.1 Closed system

- Fixed mass
- Energy can be transferred through the boundary by heat or work
- Boundary can be movable

1.35.2 Isolated system

- No mass flow, and no energy transferred through the boundary
- For example, a system of interest and its surroundings constitutes an isolated system.

1.36 Entropy change of a system

Entropy change of a system during a process is equal to the net entropy transfer through the system boundary and entropy generated within the system.

$$\begin{aligned}\Delta S_{system} &= S_{in} - S_{out} + S_{gen} \\ \text{Change in total entropy} &= \text{Total entropy entering} - \text{Total entropy leaving} + \text{Total entropy generated} \\ \text{of the system} &\end{aligned}$$

1.36.1 ΔS_{system}

- Entropy changes when the state of the system changes

$$\Delta S_{system} = S_{final} - S_{initial}$$

- If the entropy at each state is known, the entropy change between the states is simply:

$$\Delta S_{system} = S_{final} - S_{initial} = S_2 - S_1$$

- Entropy change is **zero** in **steady flow** devices during **steady operation** (turbines, compressors, pumps, nozzles, diffusers, heat exchanger, etc.)

$$\Delta S_{system} = S_{final} - S_{initial} = 0$$

1.36.2 $S_{in} - S_{out}$

- Entropy transfer (S_{in} or S_{out}) does not transfer through work

$$S_{work} = 0$$

- For closed systems, entropy is transferred **only by heat**

$$S_{in} - S_{out} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$

Where:

- Q_k is the heat transferred
- T_k is the boundary temperature through which heat is transferred.

1.36.3 Entropy generation (S_{gen})

- A measure of **entropy created by irreversibilities** during a process (friction, heat transfer via finite temperature difference, mixing, etc.)
- Entropy generation is **zero** for **reversible** processes

$$S_{gen} = \begin{cases} < 0 & \text{impossible process} \\ = 0 & \text{reversible process} \\ > 0 & \text{irreversible process} \end{cases}$$

1.36.4 Full equation

$$\Delta S_{system} = S_{in} - S_{out} + S_{gen}$$

For closed systems:

$$\Delta S_{closed} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

Where:

- Q_k is the heat transferred
- T_k is the boundary temperature through which heat is transferred

1.37 Entropy change for closed systems

- Entropy change of a **closed system** may be negative (e.g. heat removal) but **entropy generation can never be negative**.
 - When a system loses heat to the surroundings, entropy change in the surroundings is positive.
- Entropy change of an **isolated system** can never be negative.
 - Heat transfer between the system and its surroundings usually involve a finite temperature difference which is an irreversibility. Hence, there is entropy generation:

$$\Delta S_{isolated} = S_{gen}$$

$$\Delta S_{isolated} = \Delta S_{closed} + \Delta S_{surr} = S_{gen}$$

1.37.1 Adiabatic process (no heat transfer)

For an adiabatic process:

$$\sum \frac{Q_k}{T_k} = 0$$

$$\Delta S_{closed} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

$$\Delta S_{closed} = S_2 - S_1 = 0 + S_{gen}$$

$$\Delta S_{adiabatic\ system} = S_{gen} > 0$$

- In adiabatic closed systems, the change in entropy is only due to entropy generation within the system boundaries
- Since **entropy generation for irreversible processes** can never be less than zero, the entropy in an adiabatic closed system **always increases**

1.37.2 Isentropic process (constant entropy)

- If the process is **adiabatic** (no heat transfer) and **reversible**:

$$\Delta S_{isen} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

$$\Delta S_{isen} = 0 \quad \text{or} \quad S_2 = S_1$$

- A **reversible adiabatic process** is also known as an **isentropic process**.

1.37.3 Isolated systems

- No mass, heat or work transfer for an isolated system

$$\Delta S_{isolated} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

$$\Delta S_{isolated} = S_{gen} > 0$$

- The entropy for an **isolated system** must **always increase**.
- The lower limit of entropy change is only achieved through reversible processes, in which entropy remains constant.
- An isolated system may consist of **any number of subsystems**
- Entropy is an extensive property, so the total entropy is the sum of the entropy of its parts, i.e.

$$\Delta S_{total} = \sum_{i=1} N \Delta S_i > 0$$

- For example, a **closed system** and its surroundings constitutes an **isolated system**.

$$\Delta S_{sys} + \Delta S_{surr} = \Delta S_{isolated} = S_{gen} > 0$$

- For a closed system:

$$\Delta S_{sys} = S_2 - S_1$$

- Consider the surroundings as a thermal energy reservoir (no entropy generation) at temperature T_{surr} receiving heat Q from the system:

$$\Delta S_{surr} = \frac{Q}{T_{surr}}$$

$$S_2 - S_1 + \frac{Q}{T_{surr}} = S_{gen} > 0$$

- Having higher entropy generation means the process is more irreversible.

1.38 Entropy balance for flow control volumes

Entropy transfer by mass flow

- Flow control volumes involve mass inflow, mass outflow, or both.
- Mass contains entropy as well as energy, in proportion to the mass and the state condition.
- Hence:

$$S_{\text{in, mass}} = m_i s_i$$

$$S_{\text{out, mass}} = m_e s_e$$

1.39 Entropy change for flow control volumes

$$\Delta S_{CV} = S_{in} - S_{out} + S_{gen}$$

$$\Delta S_{CV} = \sum m_i s_i - \sum m_e s_e + \sum \frac{Q_k}{T_k} + S_{gen}$$

$$(S_2 - S_1)_{CV} = \sum m_i s_i - \sum m_e s_e + \sum \frac{Q_k}{T_k} + S_{gen}$$

Rate form (a dot above the variable means rate):

$$\Delta \dot{S}_{CV} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

Where:

- ΔS_{CV} is the change in entropy
- $S_{in} - S_{out}$ is the net entropy transfer by **heat and mass flow**
- S_{gen} is the entropy generation

1.39.1 Steady state, steady flow control volume

For a steady state, steady flow process:

$$\Delta \dot{S}_{CV} = 0$$

$$0 = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

1.39.2 Single stream, steady flow process

For a **single stream**, steady flow process:

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

$$\dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

1.39.3 Steady state, single stream and steady flow adiabatic process

For a steady state, single stream and steady flow adiabatic process:

$$\sum \frac{\dot{Q}_k}{T_k} = 0$$

$$\dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$\dot{S}_{gen} \geq 0 \quad \rightarrow \quad s_e \geq s_i$$

The entropy of the fluid will increase as it flows through an adiabatic device.

1.39.4 Adiabatic and reversible process

$$\sum \frac{\dot{Q}_k}{T_k} = 0 \quad \text{and} \quad \dot{S}_{gen} = 0$$

$$\dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$s_e = s_i$$

The entropy doesn't change in such a process, i.e. the process is isentropic, or a constant entropy process.

1.40 Reversible steady flow work

- Consider the differential energy balance equation for a **steady state, steady flow device** undergoing an **internally reversible process**:

$$\delta q_{rev} - \delta w_{rev} = dh + d(KE) + d(PE)$$

- From the Tds equations:

$$\delta q_{rev} = Tds \text{ and } Tds = dh - vdP \quad \therefore \delta q_{rev} = dh - vdP$$

- Substituting:

$$dh - vdP - \delta w_{rev} = dh + d(KE) + d(PE)$$

$$-\delta w_{rev} = vdP + d(KE) + d(PE)$$

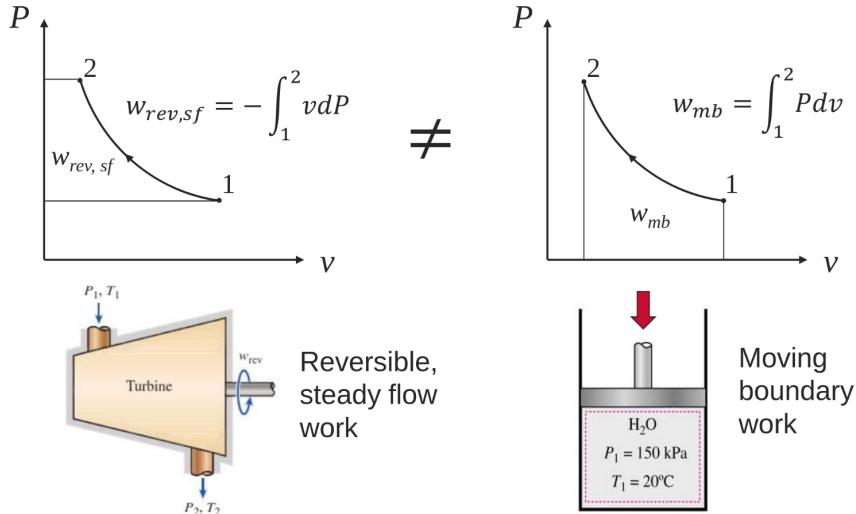
- Integrating:

$$w_{rev} = - \int_i^e v dP - \Delta KE - \Delta PE$$

- When $\Delta KE = 0$ and $\Delta PE = 0$:

$$w_{rev} = - \int_i^e v dP$$

1.40.1 Meaning of reversible steady flow work



1.40.2 Incompressible fluids

- For incompressible fluids (constant specific volume v) and $\Delta KE = 0$ and $\Delta PE = 0$, like pumps:

$$w_{rev} = -v(P_2 - P_1)$$

- In flow devices where it does **no work** and the fluid is **incompressible** (e.g. nozzles or pipes), use the Bernoulli equation

$$0 = -v(P_2 - P_1) - \frac{\nu_2^2 - \nu_1^2}{2} - g(z_2 - z_1)$$

1.40.3 Fluid types

- Examining:

$$w_{rev} = - \int_1^2 v dP$$

- Reversible steady flow work is closely associated to the specific volume v of the fluid flow through the device. The larger the specific volume, the larger the work produced or consumed.
- Pumps handle liquids (small specific volume) and hence consumes less power.
- Compressors handle gases (large specific volume) and tends to consume more power.

1.41 Polytropic work in steady flow devices [formula given]

- For a **polytropic process**:

$$Pv^n = \text{constant}$$

- Work done when **flow** through the device is **polytropic**:

$$w_{poly} = \int_1^2 v \, dP = - \int_1^2 \left(\frac{C}{P} \right)^{\frac{1}{n}} \, dP = -\frac{n}{n-1} (P_2 v_2 - P_1 v_1)$$

- Not to be confused with polytropic work in a **closed system**:

$$w_{poly} = \int_1^2 P \, dv = -\frac{1}{n-1} (P_2 v_2 - P_1 v_1)$$

- For an **ideal gas**:

$$Pv = R_{sp}T$$

$$w_{poly} = -\frac{nR_{sp}(T_2 - T_1)}{n-1} = -\frac{nR_{sp}T_1}{n-1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

1.41.1 Isentropic process for ideal gas

When the process is isentropic:

$$n = k$$

$$w_{isen} = -\frac{kR_{sp}(T_2 - T_1)}{k-1} = \frac{kR_{sp}T_1}{1-k} \left[\left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right]$$

1.41.2 Isothermal process

A gas undergoing unrestrained expansion is an isothermal process.

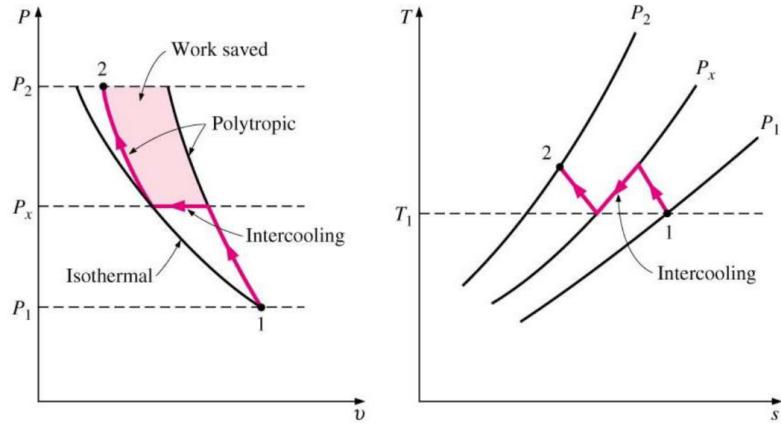
When the process is isothermal:

$$n = 1$$

$$w_{isothermal} = -R_{sp}T \ln \left(\frac{P_2}{P_1} \right)$$

1.41.3 Minimum compressor work

- When compressing an ideal gas between the same pressure ratio, cooling the gas helps reduce the power consumption.
- Multistage compression with inter-cooling:



- Work in two-stage compression with inter-cooling:

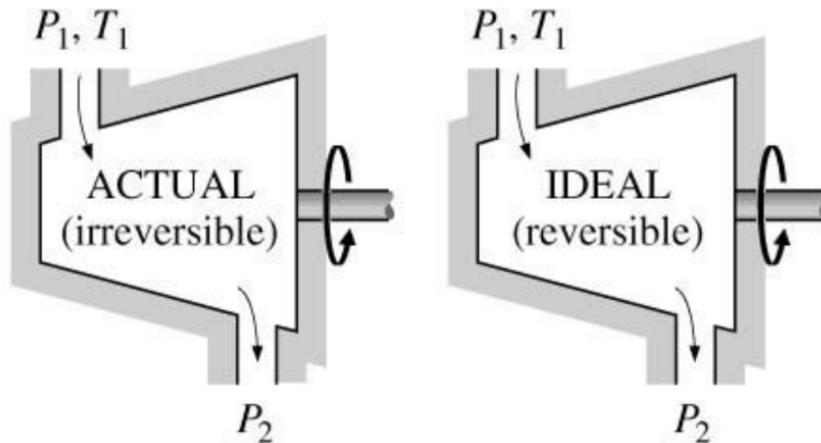
$$w_{comp} = w_{comp1} + w_{comp2} = -\frac{nR_{sp}T_1}{n-1} \left[\left(\frac{P_x}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] - \frac{nR_{sp}T_1}{n-1} \left[\left(\frac{P_2}{P_x} \right)^{\frac{n-1}{n}} - 1 \right]$$

- Minimum total work is when **pressure ratio** across each stage is the **same**:

$$P_x = \sqrt{P_1 P_2} \Rightarrow \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

1.42 Steady state control volume

- Many steady state control volume devices are **adiabatic** or **close to adiabatic** during operation.
- Such devices work **best when irreversibilities are minimised**.
- **Isentropic processes** would serve as the **ideal models** for these devices.
 - Turbines, which extract work from the fluid, decreases the enthalpy of the fluid.
 - Pumps and compressors, which do work on the fluid, increases the enthalpy of the fluid.
 - Nozzles, which accelerate the fluid, decreases the enthalpy but increases the kinetic energy of the fluid. There is no work done for a nozzle.
- **Isentropic processes** would serve as the **ideal process** for such **adiabatic** steady flow devices.
- **Isentropic efficiencies** of turbines, compressors and pumps, and nozzles serve to compare the actual performance of these (**adiabatic**) devices to isentropic conditions at the **same inlet state** and **exit pressure**.



1.43 Isentropic efficiency

Isentropic efficiency is the measure of the **deviation** of the **actual** (adiabatic) process from the **idealised** one.

1.43.1 Figuring out the isentropic efficiency formula

1. Draw a $h - s$ diagram.
2. Draw 2 pressure lines, one representing the initial pressure, and one representing the final pressure.
3. Draw one vertical connecting the two pressure lines to represent the isentropic process.
4. Draw a line that curves to the right connecting the two pressure lines to represent the actual process.
5. To get the efficiency, take the smaller change in enthalpy over the larger change in enthalpy.

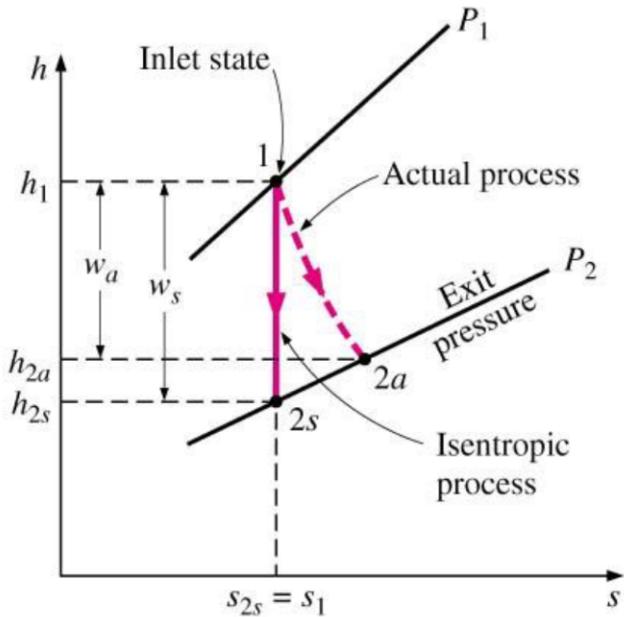
1.43.2 Turbines

$$w_T = h_1 - h_2$$

$$\eta_T = \frac{\text{Actual work}}{\text{Isentropic work}} = \frac{w_a}{w_s}$$

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

- η_T is roughly 90% for large turbines
- η_T is roughly 70% for small turbines



Where:

- w_T is the work done by the turbine
- η_T is the isentropic efficiency of the turbine
- w_s is the isentropic work output of the turbine
- w_a is the actual work output of the turbine
- h_1 is the initial enthalpy of the fluid
- h_{2a} is the final enthalpy of the actual process
- h_{2s} is the final enthalpy of the isentropic process

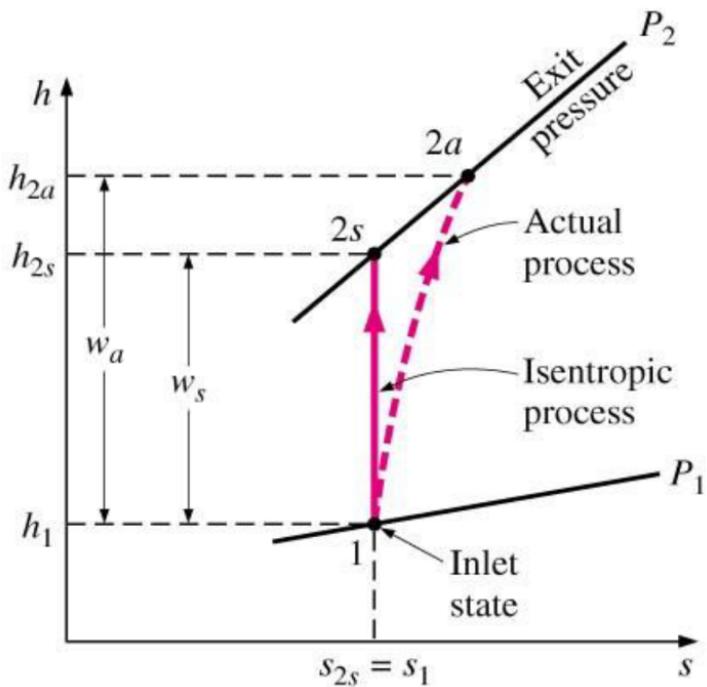
1.43.3 Compressors

$$w_C = -(h_1 - h_2)$$

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{w_s}{w_a}$$

$$\eta_T = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

- η_C is roughly 75 - 85% for well-designed devices



Where:

- w_C is the work required by the compressor
- η_C is the isentropic efficiency of the compressor
- w_s is the isentropic work input into the compressor
- w_a is the actual work input into the compressor
- h_{2a} is the final enthalpy of the actual process
- h_{2s} is the final enthalpy of the isentropic process
- h_1 is the initial enthalpy of the fluid

1.43.4 Pumps

$$w_{rev} = - \int v dP$$
$$w_{on} = - \left(- \int v dP \right)$$
$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

Where:

- w_{rev} is the work done by the fluid in the isentropic process
- w_{on} is the work done on the fluid by the pump in the isentropic process
- η_C is the isentropic efficiency of the pump
- w_s is the isentropic work input of the pump
- w_a is the actual work input of the pump
- v is the specific volume of the fluid
- P_2 is the final pressure of the fluid
- P_1 is the initial pressure of the fluid
- h_{2a} is the final enthalpy of the actual process
- h_1 is the initial enthalpy of the fluid

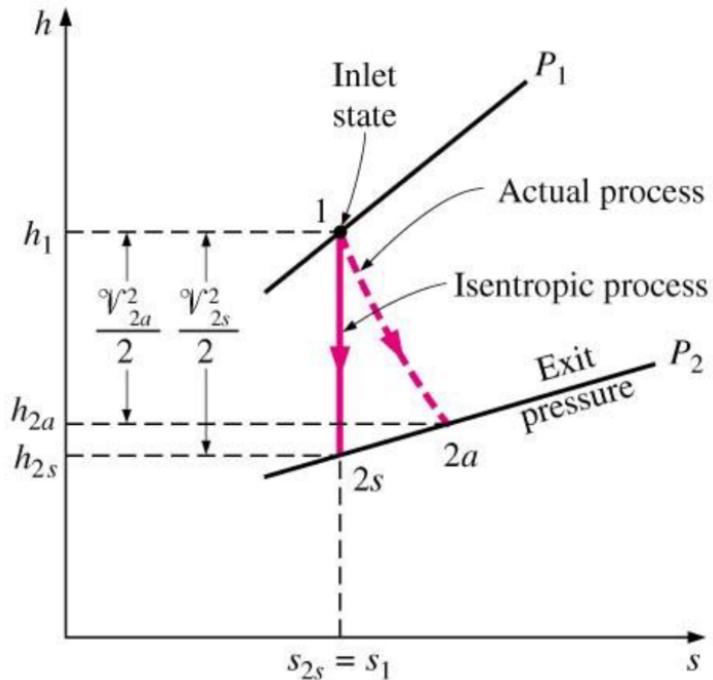
1.43.5 Nozzles

$$\eta_N = \frac{\text{Actual KE at exit}}{\text{Isentropic KE at exit}} = \frac{\nu_{2a}^2}{\nu_{2s}^2}$$

$$h_1 - h_{2a} = \frac{\nu_{2a}^2}{2}$$

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

- Typical η_N is roughly 90 - 95%



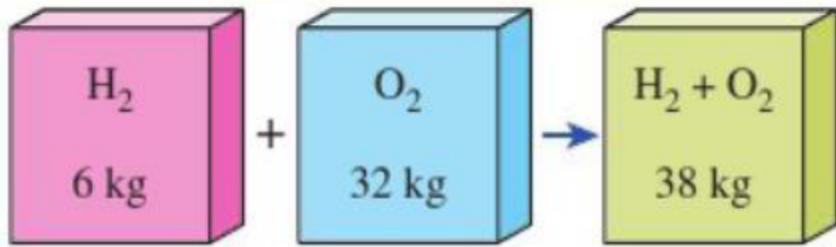
Where:

- η_N is the isentropic efficiency of the nozzle
- ν_{2a} is the final velocity of the actual process
- ν_{2s} is the final velocity of the isentropic process
- h_{2a} is the final enthalpy of the actual process
- h_{2s} is the final enthalpy of the isentropic process
- h_1 is the initial enthalpy of the fluid

1.44 Gravimetric analysis

The total mass of a mixture is equal to the sum of the mass of each component:

$$m_m = \sum_i^k m_i$$



1.44.1 Mass fraction

- Mass fraction is the ratio of mass of a component to the total mass of mixture:

$$mf_i = \frac{m_i}{m_m}$$

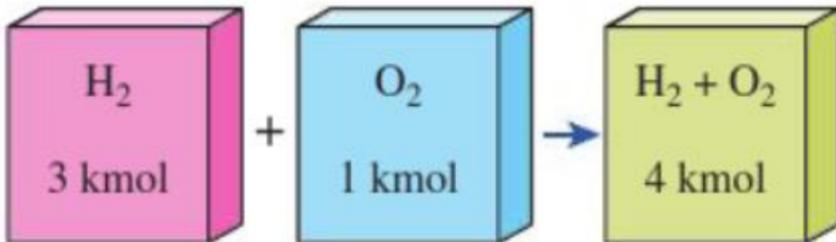
- Sum of all mass fractions in the mixture is equal to 1:

$$\sum_i^k mf_i = 1$$

1.45 Molar analysis

The total number of moles in a **non-reacting mixture** is equal to the sum of the number of moles of each component:

$$N_m = \sum_i^k N_i$$



1.45.1 Mole fraction

- Mole fraction is the ratio of the mole number of a component to the total mole number of the mixture:

$$y_i = \frac{N_i}{N_m}$$

- Sum of all mole fractions in the mixture is equal to 1:

$$\sum_i^k y_i = 1$$

1.46 Composition of gas mixtures

- Mass of substance m is equivalent to the product of its mole number N and molar mass $M(m = NM)$.
- Apparent (or average) molar mass of a mixture can be expressed as:

$$\begin{aligned} M_m &= \frac{m_m}{N_m} \\ &= \frac{\sum m_i}{N_m} \\ &= \frac{\sum N_i M_i}{N_m} \\ &= \sum_i^k y_i M_i \end{aligned}$$

- Or:

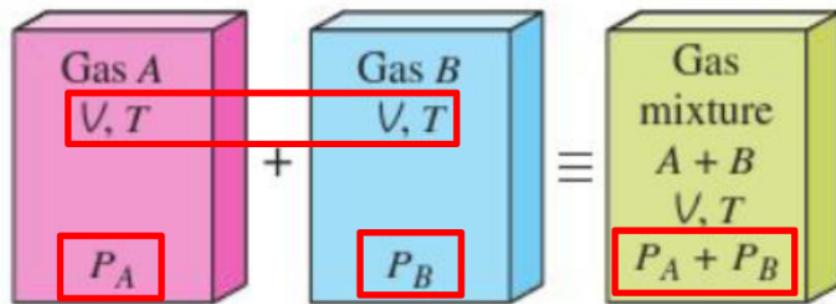
$$\begin{aligned} M_m &= \frac{m_m}{N_m} \\ &= \frac{m_m}{\sum M_i} \\ &= \frac{1}{\frac{\sum m_i}{m_m M_i}} \\ &= \frac{1}{\sum_i^k \frac{m f_i}{M_i}} \end{aligned}$$

- Gas constant for the mixture:

$$R_m = \frac{R_u}{M_m} \quad R_u = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

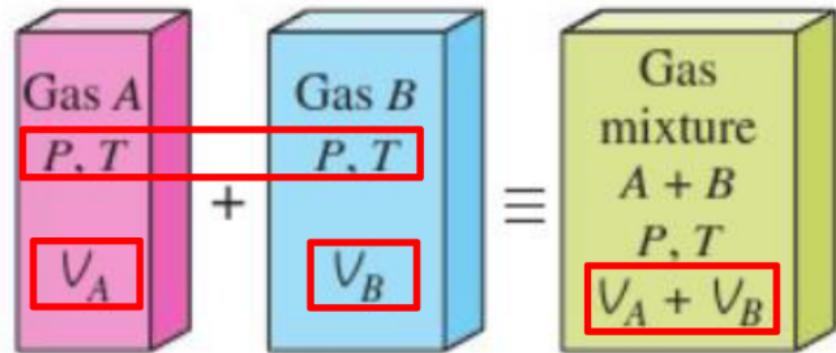
1.47 Dalton's law of additive pressures

The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and volume.



1.48 Amagat's law of additive volumes

The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure.



1.49 $P - v - T$ behaviour of gas mixtures

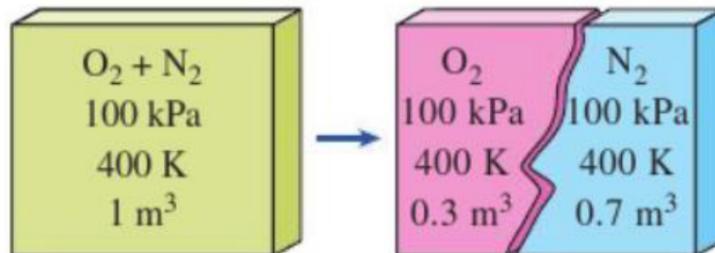
- Dalton's and Amagat's laws hold **exactly** for **ideal-gas mixtures**.
- The laws are **approximate** for **real-gas** mixtures, due to intermolecular forces which may be significant for real gases at high densities.
- Dalton's law:

$$P_m = \sum_i^k P_i(T_m, V_m) \quad P_i = \text{Component pressure}$$

- Amagat's law:

$$V_m = \sum_i^k V_i(T_m, P_m) \quad V_i = \text{Component volume}$$

- The component volume is the volume a component would occupy if it existed alone at T_m and P_m , **not the actual volume** the component occupies **in the mixture**.



1.49.1 Ideal gases

- Pressure fraction:

$$\frac{P_i(T_m, V_m)}{P_m} = \frac{\frac{(N_i R_u T_m)}{V_m}}{\frac{(N_m R_u T_m)}{V_m}} = \frac{N_i}{N_m} = y_i$$

- The quantity $y_i P_m$ is also called the partial pressure (component pressure for ideal gases).

- Volume fraction:

$$\frac{V_i(T_m, P_m)}{V_m} = \frac{\frac{(N_i R_u T_m)}{P_m}}{\frac{(N_m R_u T_m)}{P_m}} = \frac{N_i}{N_m} = y_i$$

- The quantity $y_i V_m$ is also called the partial volume (component volume for ideal gases)
- Dalton's law and Amagat's law are identical for ideal gases:

$$\frac{P_i}{P_m} = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i$$

1.50 Properties of gas mixtures

1.50.1 Extensive properties

Extensive properties of gas mixtures	Changes in extensive properties of gas mixtures
$U_m = \sum_i^k U_i = \sum_i^k m_i u_i = \sum_i^k N_i \bar{u}_i$	$\Delta U_m = \sum_i^k \Delta U_i = \sum_i^k m_i \Delta u_i = \sum_i^k N_i \Delta \bar{u}_i$
$H_m = \sum_i^k H_i = \sum_i^k m_i h_i = \sum_i^k N_i \bar{h}_i$	$\Delta H_m = \sum_i^k \Delta H_i = \sum_i^k m_i \Delta h_i = \sum_i^k N_i \Delta \bar{h}_i$
$S_m = \sum_i^k S_i = \sum_i^k m_i s_i = \sum_i^k N_i \bar{s}_i$	$\Delta S_m = \sum_i^k \Delta S_i = \sum_i^k m_i \Delta s_i = \sum_i^k N_i \Delta \bar{s}_i$

1.50.2 Intensive properties

Intensive properties of a mixture are determined by weighted averages.

$$\begin{array}{l|l} u_m = \sum_i^k m f_i u_i & \bar{u}_m = \sum_i^k y_i \bar{u}_i \\ h_m = \sum_i^k m f_i h_i & \bar{h}_m = \sum_i^k y_i \bar{h}_i \\ s_m = \sum_i^k m f_i s_i & \bar{s}_m = \sum_i^k y_i \bar{s}_i \\ c_{v,m} = \sum_i^k m f_i c_{v,i} & \bar{c}_{v,m} = \sum_i^k y_i \bar{c}_{v,i} \\ c_{p,m} = \sum_i^k m f_i c_{p,i} & \bar{c}_{p,m} = \sum_i^k y_i \bar{c}_{p,i} \end{array}$$

1.51 Dry and atmospheric air

- Air is a mixture of nitrogen, oxygen and small amounts of other gases.
- Atmospheric air contains air and water vapour (moisture)
 - Air and water vapour is atmospheric air
 - Air without water vapour is dry air
- Water vapour (humidity) in the air affects human comfort, and hence is an important consideration for air-conditioning and heating applications
- Water vapour in the air can be treated as an **ideal gas** (the error is less than 0.2%) even when it is a saturated vapour.
 - Atmospheric air can be treated as an ideal-gas mixture of air and water vapour.

1.51.1 Properties

- Dry air:

$$h_{\text{dry air}} = c_p T \quad c_p = 1.005 \text{ kJ kg}^{-1} \text{ }^{\circ}\text{C}^{-1}$$

Use 0 °C as the reference temperature, i.e.

$$\Delta h_{\text{dry air}} = c_p \Delta T$$

- Pressure:

$$P = P_a + P_v$$

The total pressure is the sum of partial pressure of dry air (P_a) and the partial pressure of water vapour P_v .

- Water vapour:

$$h_v(T, \text{low } P) \cong h_g(T)$$

$$h_g(T) \cong 2500.9 + 1.82T \quad (\text{kJ kg}^{-1})$$

- The enthalpy of water vapour in atmospheric air can be taken to be equal to the enthalpy of **saturated vapour** at the **same temperature**.

1.52 Humidity of air

1.52.1 Specific or absolute humidity (ω)

Specific or absolute humidity is the ratio of mass of water vapour to the mass of dry air:

$$\omega = \frac{m_v}{m_a}$$

Alternatively:

$$\begin{aligned}\omega &= \frac{m_v}{m_a} \\ &= \frac{\frac{P_v V}{R_v T}}{\frac{P_a V}{R_a T}} \\ &= \frac{P_v}{R_v} \cdot \frac{R_a}{P_a} \\ &= \frac{R_a}{R_v} \cdot \frac{P_v}{P_a} \\ &= 0.622 \frac{P_v}{P_a} \\ &= \frac{0.622 P_v}{P - P_v} \quad \because P = P_a + P_v\end{aligned}$$

No need to memorise this equation below, as it is given in the exam:

$$\frac{0.622 P_v}{P - P_v}$$

Specific humidity does not change with temperature unless condensation occurs.

1.52.2 Saturated air

- There is a **limit** to the amount of water vapour the air can hold.
- At **maximum**, air is **saturated** with moisture (saturated air):

$$P_v = P_g \quad \text{where } P_g = P_{\text{sat}} @ T$$

- Comfort level depends more on the **amount of moisture present** relative to the **maximum amount** that the air can hold at the same temperature.

1.52.3 Relative humidity (ϕ)

Relative humidity is the ratio of moisture present to maximum amount that the air can hold.

$$\begin{aligned}\phi &= \frac{m_v}{m_g} \\ &= \frac{\frac{P_v V}{R_v T}}{\frac{P_g V}{R_v T}} \\ &= \frac{P_v}{P_g} \quad \text{where } P_g = P_{\text{sat}} @ T\end{aligned}$$

- Relative humidity changes with temperature since maximum moisture the air can hold depends on temperature.
- Relative humidity ranges from 0 (dry air) to 1 (saturated air). The equations below are given in the exam:

$$\begin{aligned}\phi &= \frac{\omega P}{(0.622 + \omega)P_g} \\ \omega &= \frac{0.622\phi P_g}{P - \phi P_g}\end{aligned}$$

- For practical applications, mass of dry air remains constant while the mass of water vapours can change.

1.52.4 Intensive properties

- Total enthalpy of atmospheric air:

$$H = H_a + H_v = m_a h_a + m_v h_v$$

- Divide by mass of dry air (m_a):

$$h = \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$

- Specific enthalpy of atmospheric air is expressed in terms of **per unit mass of dry air**:

$$h = h_a + \omega h_g \quad (\text{kJ kg}^{-1} \text{ dry air}) \quad \therefore h_v \cong h_g$$

- Similarly, **specific volume** of atmospheric air is expressed in terms of per unit mass of dry air as well.

$$\text{Specific volume } (v) : \text{m}^3 \text{ kg}^{-1} \text{ dry air}$$

1.53 Dry-bulb temperature (T_{db})

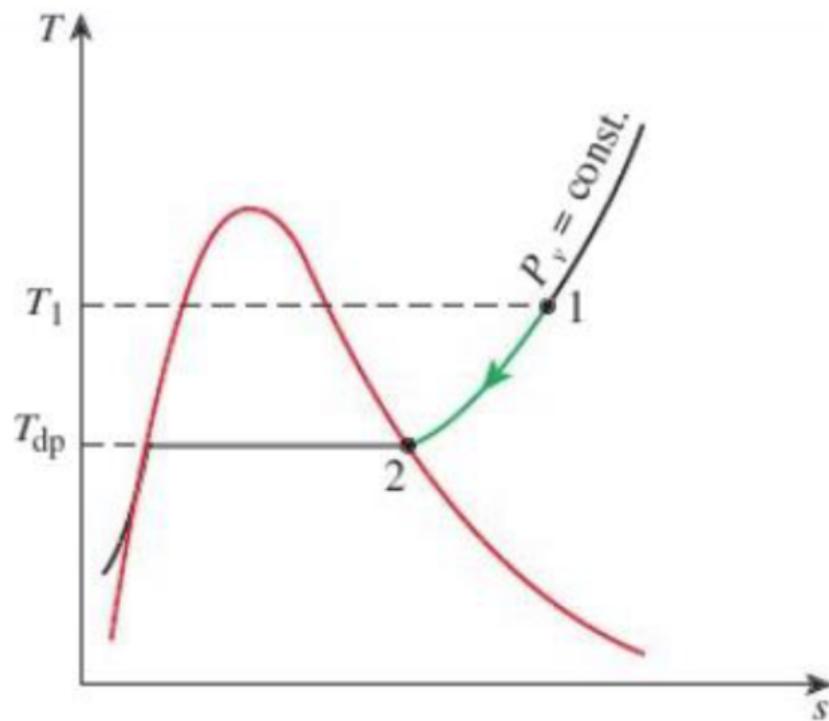
Dry-bulb temperature is the normal temperature of atmospheric air.

1.54 Dew point temperature (T_{dp})

Dew point temperature is the temperature at which condensation begins when air is cooled at constant pressure.

- Saturation temperature corresponding to the saturated vapour pressure.

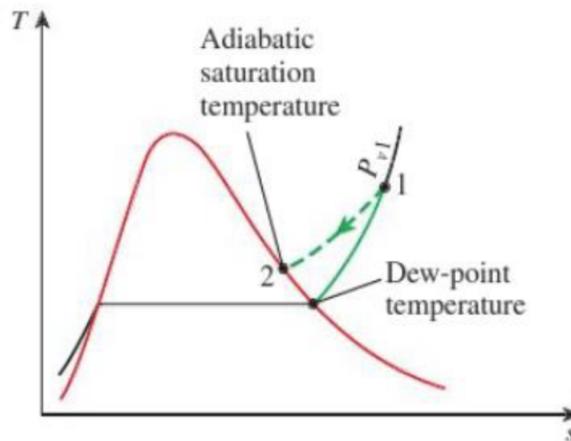
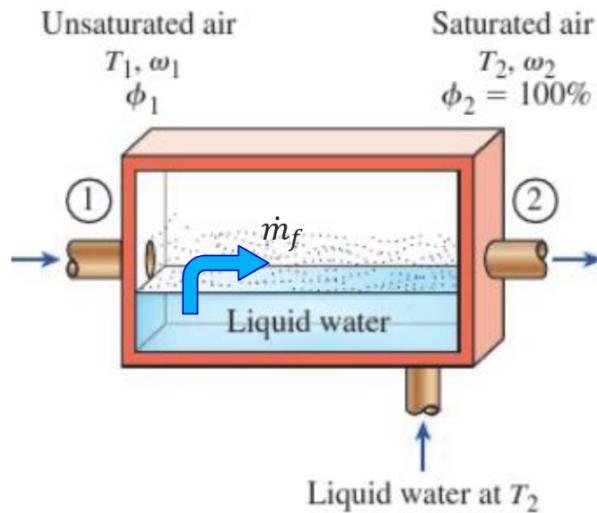
$$T_{dp} = T_{\text{sat}} @ P_v$$



1.55 Adiabatic saturation process

- Relative and specific humidity are important parameters in dealing with atmospheric air.
- Calculation of relative and specific humidity can be achieved by measuring the dew point temperature, but this is not very practical.
- Alternatively, relative and specific humidity can be related to an **adiabatic saturation process**.

Diagrams:



Details:

- Assuming steady flow and negligible kinetic energy and potential energy.

- Mass balance:

$$\begin{aligned} \dot{m}_{a_1} &= \dot{m}_{a_2} = \dot{m}_a \quad (\text{Dry air}) \\ \left. \begin{aligned} \dot{m}_{w_1} + \dot{m}_f &= \dot{m}_{w_2} \\ \omega_1 \dot{m}_{a_1} + \dot{m}_f &= \omega_2 \dot{m}_{a_2} \\ \dot{m}_f &= \dot{m}_a(\omega_2 - \omega_1) \end{aligned} \right\} (\text{Water vapour}) \end{aligned}$$

\dot{m}_f is the evaporation rate.

- Energy balance: $\dot{E}_{in} = \dot{E}_{out}$

$$\begin{aligned} \dot{m}_a h_1 + \dot{m}_f h_{f_2} &= \dot{m}_a h_2 \\ \dot{m}_a h_1 + \dot{m}_a(\omega_2 - \omega_1)h_{f_2} &= \dot{m}_a h_2 \\ c_p T_1 + \omega_1 h_{g_1} + (\omega_2 - \omega_1)h_{f_2} &= c_p T_2 + \omega_2 h_{g_2} \end{aligned}$$

- Rearrange for ω_1 :

$$\omega_1 = \frac{c_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$

$$\begin{aligned} \omega_2 &= \frac{0.622\phi_2 P_{g_2}}{P_2 - \phi_2 P_{g_2}} \quad \text{when } \phi = 100\% \\ &= \frac{0.622P_{g_2}}{P_2 - P_{g_2}} \end{aligned}$$

- T_2 is the adiabatic saturation temperature

- Process for determining T_2 is still very impractical
- Tank needs to be very long depending on the inlet specific humidity and requires perfect insulation and hence a more practical method is required.

1.56 Wet-bulb temperature (T_{wb})

- The wet-bulb temperature is the temperature measured using a thermometer whose bulb is covered with a cotton wick saturated with water and air blowing over the wick.
- Wet-bulb temperature (T_{wb}) is **approximately equal** to adiabatic saturation temperature (T_2) at atmospheric pressures.

$$T_2 \cong T_{wb}$$

- Water evaporating from the wick causes the temperature to drop, so:

$$T_{wb} < T_{dry}$$

- At higher humidity, that means there is lower evaporation rate and hence a higher wet-bulb temperature.
- When the wet-bulb temperature is equal to the temperature of dry air, the air is saturated with water, i.e.

$$T_{wb} = T_{dry} \rightarrow \phi = 100\%$$

1.57 Specific volume of dry air

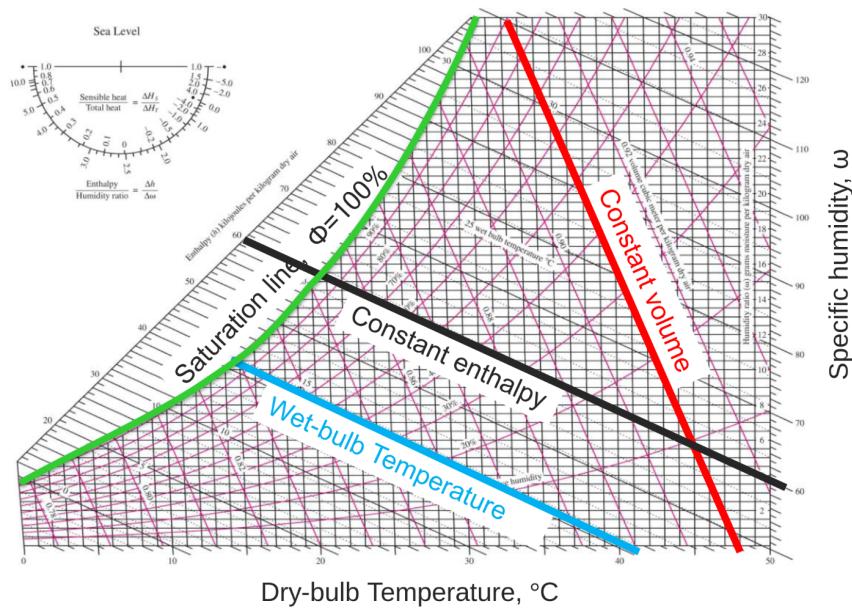
$$v = \frac{P_a T}{R_a}$$

Where:

- v is the specific volume of dry air
- P_a is the partial pressure of dry air
- T is the temperature of the dry air in Kelvin
- R_a is the molar gas constant per unit mass of dry air, which is $0.287 \text{ kJ kg}^{-1} \text{ K}^{-1}$

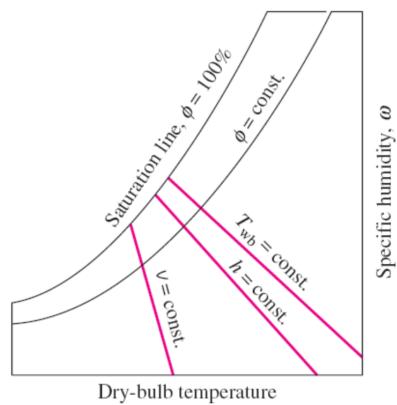
1.58 Psychrometric chart

- The pressure value of the psychrometric chart below is 1 atm or 101.324 kPa, which means it is only applicable this particular pressure value, which is at sea level.
- The relative humidity lines are the lines that curve upwards from left to right. Note that the specific humidity values are on the inside of the y -axis, not outside. The values on the outside to the y -axis are enthalpy values.
- The wet-bulb temperature line is the dotted lines parallel to the constant enthalpy lines, which are solid lines.
- The dew point temperature lines are labelled on the saturation line, but can't be seen as they are blocked by the saturation line label.

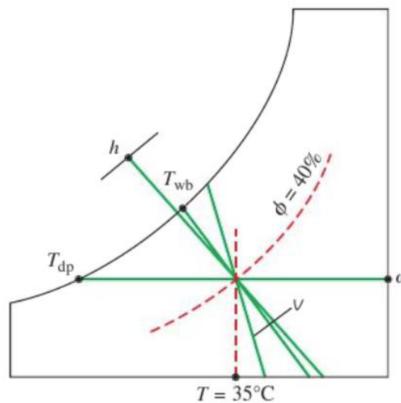


1.58.1 Schematic of the chart

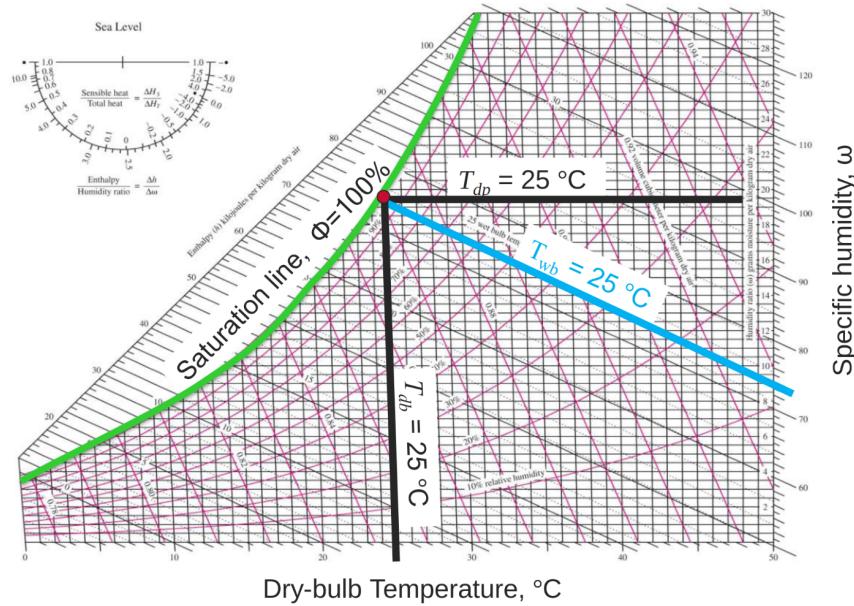
Chart Schematic



E.g. Reading the chart
at $T_{db} = 35^\circ\text{C}$, $\phi = 40\%$



1.58.2 Example

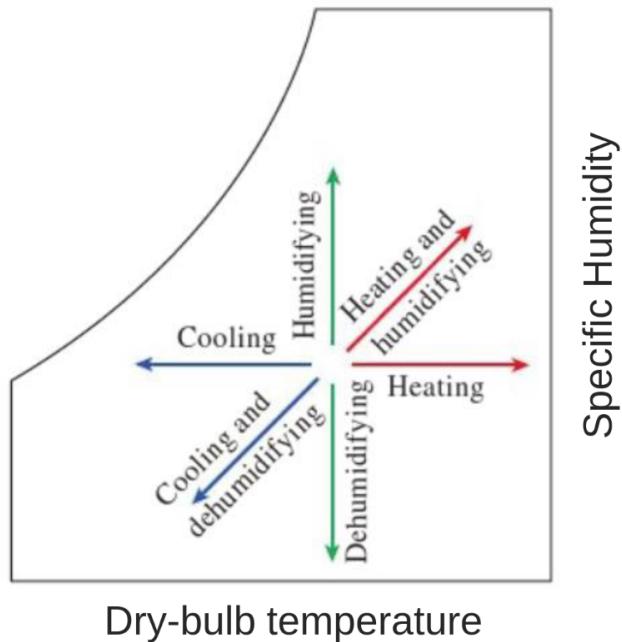


For saturated air:

$$T_{db} = T_{wb} = T_{dp}$$

1.59 Air conditioning processes

- Modern air-conditioning systems can heat, cool, humidify and dehumidify the air.
- Modelling air-conditioning processes on the psychrometric chart:



- These processes are modelled as **steady flow** processes.
- Kinetic energy and potential energy terms are ignored.
- Mass balance: $\sum_{in} \dot{m}_a = \sum_{out} \dot{m}_a$

$$\text{Dry air: } \sum_{in} \dot{m}_a = \sum_{out} \dot{m}_a$$

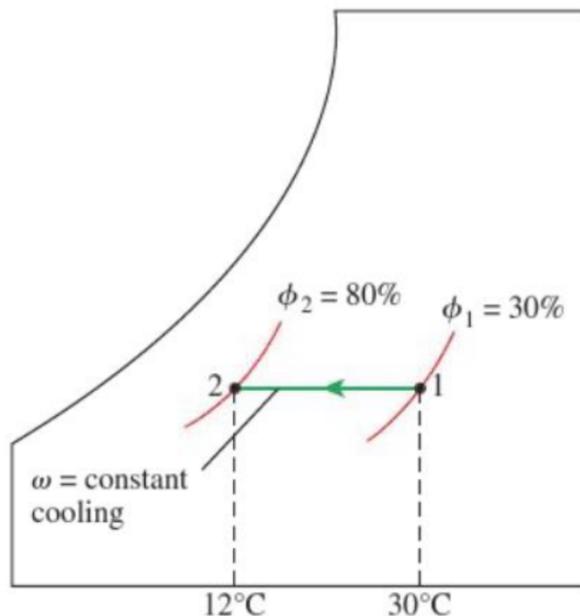
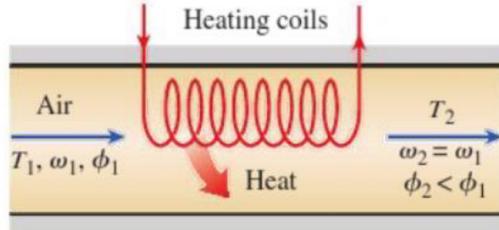
$$\text{Water: } \sum_{in} \dot{m}_w = \sum_{out} \dot{m}_w \rightarrow \sum_{in} \omega \dot{m}_a = \sum_{out} \omega \dot{m}_a$$

- Energy balance: $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}h = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}h$$

- Work term usually consists of **fan work input**, which is small relative to the other terms in the energy equation.

1.59.1 Simple heating and cooling



- Process appears as a horizontal line on the psychrometric chart
- Specific humidity remains constant
- Relative humidity decreases with heating
- Relative humidity increases with cooling
- Mass balance:

$$\begin{aligned}\dot{m}_{a_1} &= \dot{m}_{a_2} = \dot{m}_a \\ \dot{m}_{w_1} &= \dot{m}_{w_2} = \dot{m}_w \quad \rightarrow \quad \omega_1 = \omega_2\end{aligned}$$

- Energy balance:

$$\dot{Q} = \dot{m}_a(h_2 - h_1) \text{ or } q = (h_2 - h_1)$$

1.59.2 Humidification or dehumidification

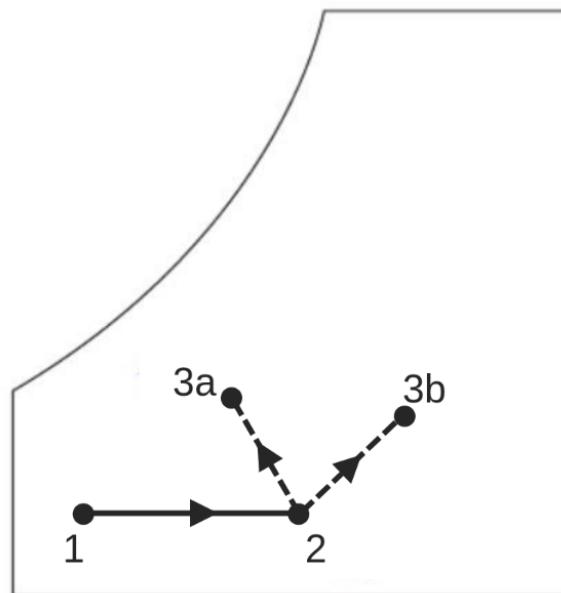
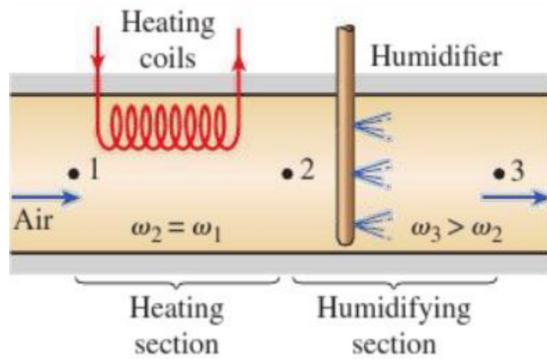
$$\dot{m}_w = \dot{m}_a \Delta \omega$$

Where:

- \dot{m}_w is the mass flow rate of water
- \dot{m}_a is the mass flow rate of dry air
- $\Delta \omega$ is the change in specific humidity

1.59.3 Heating with humidification

Diagrams:



Details:

- 2-step process:
 1. Simple heating
 2. Humidification
- Temperature after humidification depends on the process, for example:
 1. Water spray: temperature decreases after humidification due to evaporation (a).
 2. Steam spray: temperature increases (b).
- Simple heating ($1 \rightarrow 2$):

$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

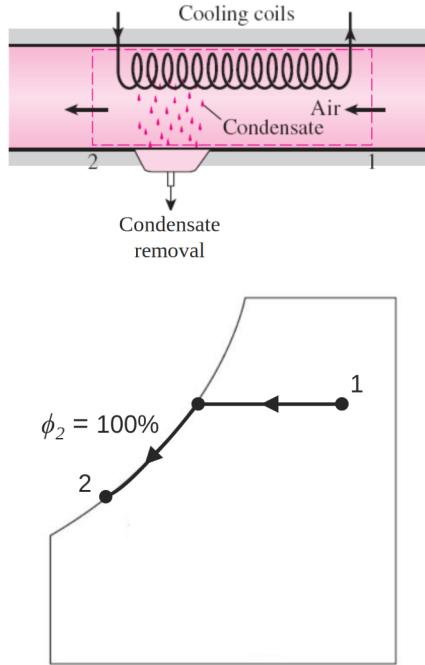
$$\begin{aligned}\dot{m}_{w_1} &= \dot{m}_{w_2} = \dot{m}_w \quad \rightarrow \quad \omega_1 = \omega_2 \\ \dot{Q}_{in} &= \dot{m}_a(h_2 - h_1)\end{aligned}$$

- Humidification ($2 \rightarrow 3$):

$$\omega_2 \dot{m}_{a_2} + \dot{m}_w = \omega_3 \dot{m}_{a_3}$$

$$\dot{m}_w = \dot{m}_a(\omega_3 - \omega_2)$$

1.59.4 Cooling with dehumidification



- Relative humidity increases with cooling, and eventually reaches 100%
- Cooling the air past the dew point temperature causes water vapour to condense. As condensed water (condensate) is removed, the air is dehumidified.
- One-step process.
- Mass balance:

$$\text{Dry air: } \dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$

$$\text{Water: } \dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_w$$

$$\omega_1 \dot{m}_{a1} = \omega_2 \dot{m}_{a2} + \dot{m}_w$$

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$$

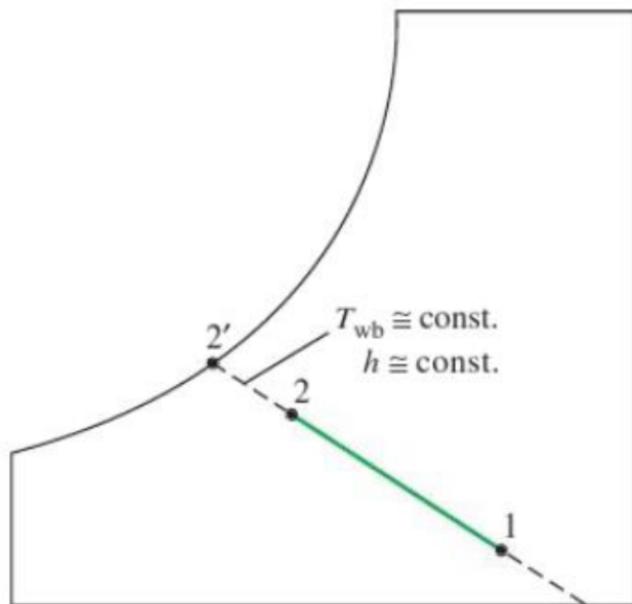
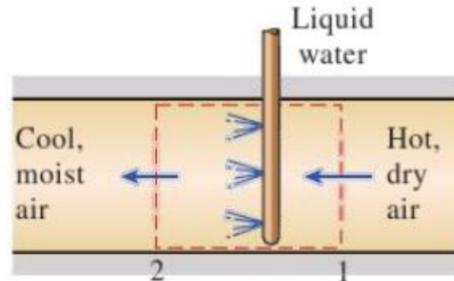
- Energy balance:

$$\dot{Q}_{in} + \sum_{in} \dot{m}h = \dot{Q}_{out} + \sum_{out} \dot{m}h$$

$$\dot{m}_{a1}h_1 = \dot{Q}_{out} + \dot{m}_{a2}h_2 + \dot{m}_w h_w$$

$$\dot{Q}_{out} = \dot{m}_a(h_1 - h_2) - \dot{m}_w h_w$$

1.59.5 Evaporative cooling



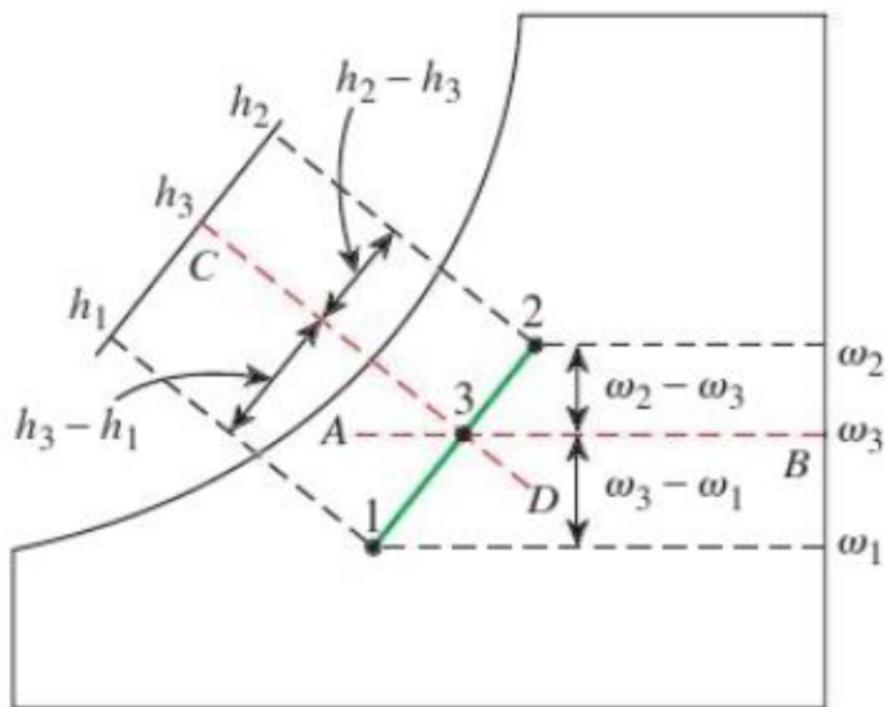
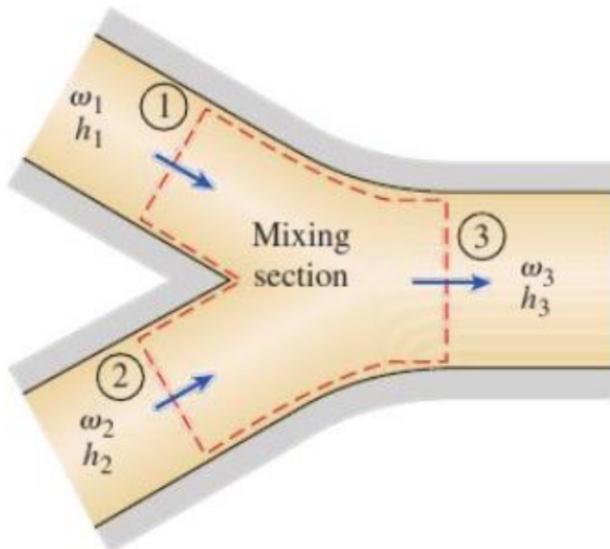
- High cost of cooling is avoided by using evaporative coolers in the desert (hot and dry climates).
- Evaporation of water cools the air and increases the humidity.
- Evaporative cooling is identical to adiabatic saturation process.
- Process follows constant wet-bulb temperature line.
 - Enthalpy is also assumed to remain constant:

$$T_{wb} \approx \text{constant}$$

$$h \approx \text{constant}$$

1.59.6 Adiabatic mixing of airstreams

Diagrams:



Details:

- Mixing of airstreams is commonly used for air-conditioning in large buildings.
- Heat transfer to the surroundings is usually small, so the process is **assumed to be adiabatic**.
- Mass balance:

$$\text{Dry air: } \dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$$

$$\text{Water: } \dot{m}_{w_1} + \dot{m}_{w_2} = \dot{m}_{v_3}$$

$$\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3}$$

- Substituting and eliminating \dot{m}_{a_3} :

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1}$$

- Energy balance:

$$h_1 \dot{m}_{a_1} + h_2 \dot{m}_{a_2} = h_3 \dot{m}_{a_3}$$

- Substituting and eliminating \dot{m}_{a_3} :

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{h_2 - h_3}{h_3 - h_1} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1}$$

- Equation is in the form of a geometric interpolation.
- When two airstreams at states 1 and 2 are mixed adiabatically, the state of the mixture lies in the straight line connecting the two states.

2 Formulas

2.1 Change in specific internal energy of an ideal gas

$$\Delta u = c_{v, \text{ avg}}(T_2 - T_1)$$

Where:

- Δu is the change in specific internal energy of an ideal gas
- $c_{v, \text{ avg}}$ is the specific heat capacity of the ideal gas at constant volume
- T_2 is the final temperature
- T_1 is the initial temperature

2.2 Change in specific enthalpy of an ideal gas

$$\Delta h = c_{p, \text{ avg}}(T_2 - T_1)$$

Where:

- Δu is the change in specific enthalpy of an ideal gas
- $c_{p, \text{ avg}}$ is the specific heat capacity of the ideal gas at constant pressure
- T_2 is the final temperature
- T_1 is the initial temperature

2.3 Quality (dryness fraction) (x)

$$x = \frac{m_{\text{vapour}}}{m_{\text{total}}} = \frac{m_g}{m_f + m_g}$$

Where:

- x is the quality or dryness fraction
- m_{vapour} is the mass of vapour
- m_{total} is the total mass
- m_g is the mass of vapour
- m_f is the mass of liquid

2.3.1 Specific entropy in terms of quality (s)

$$s = s_f + x s_{fg}$$

Where:

- s is the specific entropy
- s_f is the specific entropy of the saturated liquid
- s_g is the specific entropy of the saturated vapour
- s_{fg} is the specific entropy of the mixture at evaporation, where $s_{fg} = s_g - s_f$

2.3.2 Quality in terms of specific entropy (x)

$$x = \frac{s - s_f}{s_{fg}}$$

Where:

- x is the quality
- s is the specific entropy of the mixture
- s_f is the specific entropy of the saturated liquid
- s_g is the specific entropy of the saturated vapour
- s_{fg} is the specific entropy of the mixture at evaporation, where $s_{fg} = s_g - s_f$

2.4 Net work output of a heat engine

$$W_{\text{net, out}} = W_{\text{out}} - W_{\text{in}}$$

Where:

- $W_{\text{net, out}}$ is the net work output of the heat engine
- W_{out} is the work done **by** the heat engine
- W_{in} is the work done **on** the heat engine

2.5 Energy balance of a heat engine

$$W_{\text{net, out}} = Q_{in} - Q_{out}$$

Where:

- $W_{\text{net, out}}$ is the net work output of the heat engine
- Q_{in} is the heat energy input from the high temperature reservoir
- Q_{out} is the heat energy output from the low temperature reservoir

2.6 Thermal efficiency

$$\text{Thermal efficiency} = \frac{\text{Desired output}}{\text{Required input}}$$

2.6.1 Heat engine

$$\eta_{th} = \frac{W_{\text{net, out}}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Where:

- η_{th} is the efficiency of the heat engine
- $W_{\text{net, out}}$ is the net work output of the heat engine
- Q_H is the heat energy transferred from the high temperature reservoir (H for high temperature)
- Q_L is the heat energy transferred from the low temperature reservoir (L for low temperature)

2.6.2 Carnot heat engine

$$\eta_{th, \text{ rev}} = 1 - \frac{T_L}{T_H}$$

Where:

- η_{th} is the efficiency of a Carnot heat engine
- T_H is the temperature of the high temperature reservoir (H for high temperature)
- T_L is the temperature of the low temperature reservoir (L for low temperature)

2.7 Energy balance of a reverse heat engine (refrigerators and heat pumps)

$$W_{\text{net, in}} = Q_H - Q_L$$

Where:

- $W_{\text{net, in}}$ is the net energy input into the reverse heat engine
- Q_H is the heat energy transferred from the high temperature reservoir (H for high temperature)
- Q_L is the heat energy transferred from the low temperature reservoir (L for low temperature)

2.8 Coefficient of performance

$$\text{Coefficient of performance} = \frac{\text{Desired output}}{\text{Required input}}$$

2.8.1 Refrigerator

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

Where:

- COP_R is the coefficient of performance of the refrigerator
- Q_H is the heat energy transferred from the high temperature reservoir (H for high temperature)
- Q_L is the heat energy transferred from the low temperature reservoir (L for low temperature)

2.8.2 Carnot refrigerator

$$COP_{R, \text{rev}} = \frac{T_L}{T_H - T_L} = \frac{1}{\frac{T_H}{T_L} - 1}$$

Where:

- $COP_{R, \text{rev}}$ is the coefficient of performance of a Carnot refrigerator
- T_H is the temperature of the high temperature reservoir (H for high temperature)
- T_L is the temperature of the low temperature reservoir (L for low temperature)

2.8.3 Heat pump

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

Where:

- COP_{HP} is the coefficient of performance of the heat pump
- Q_H is the heat energy transferred from the high temperature reservoir (H for high temperature)
- Q_L is the heat energy transferred from the low temperature reservoir (L for low temperature)

2.8.4 Carnot heat pump

$$COP_{HP, rev} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

Where:

- $COP_{HP, rev}$ is the coefficient of performance of a Carnot heat pump
- T_H is the temperature of the high temperature reservoir (H for high temperature)
- T_L is the temperature of the low temperature reservoir (L for low temperature)

2.9 Clausius Inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

For reversible cycles:

$$\oint \frac{\delta Q}{T} = 0$$

For irreversible cycles:

$$\oint \frac{\delta Q}{T} < 0$$

Where:

- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat is leaving from the system to the reservoirs.
- T is the common temperature of the reservoirs at a particular instant in time in Kelvin

2.10 Entropy

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ K}^{-1})$$

Where:

- dS is the change in entropy for an internally reversible process (int rev)
- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat is leaving from the system to the reservoirs.
- T is the common temperature of the reservoirs at a particular instant in time in Kelvin

2.11 First Tds equation

$$Tds = du + Pdv$$
$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

Where:

- T is the temperature in Kelvin
- ds is the change in **specific** entropy
- du is the change in **specific** internal energy
- P is the pressure of the system
- dv is the change in **specific** volume of the system

2.12 Second Tds equation

$$Tds = dh - vdp$$
$$ds = \frac{dh}{T} - \frac{vdp}{T}$$

Where:

- T is the temperature in Kelvin
- ds is the change in **specific** entropy
- dh is the change in **specific** enthalpy
- v is the **specific** volume of the system
- dP is the change in pressure of the system

2.13 Entropy change for liquids and solids

$$s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{avg} \ln \left(\frac{T_2}{T_1} \right)$$

Where:

- s_2 is the final **specific** entropy
- s_1 is the initial **specific** entropy
- c is the specific heat capacity as a function of temperature
- dT is the change in temperature in K
- c_{avg} is the average specific heat capacity
- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin

2.14 General entropy change for ideal gas

2.14.1 First Tds equation

$$s_2 - s_1 = c_{v, \text{ avg}} \ln \left(\frac{T_2}{T_1} \right) + R_{sp} \ln \left(\frac{v_2}{v_1} \right)$$

Where:

- s_2 is the final **specific** entropy
- s_1 is the initial **specific** entropy
- $c_{v, \text{ avg}}$ is the average specific heat capacity at constant volume
- R_{sp} is the molar gas constant per unit mass, i.e. $R_{sp} = \frac{R}{\text{Molar mass}}$
- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin
- v_2 is the final **specific** volume
- v_1 is the initial **specific** volume

2.14.2 Second Tds equation

$$s_2 - s_1 = c_{p, \text{ avg}} \ln \left(\frac{T_2}{T_1} \right) - R_{sp} \ln \left(\frac{P_2}{P_1} \right)$$

Where:

- s_2 is the final **specific** entropy
- s_1 is the initial **specific** entropy
- $c_{p, \text{ avg}}$ is the average specific heat capacity at constant pressure
- R_{sp} is the molar gas constant per unit mass, i.e. $R_{sp} = \frac{R}{\text{Molar mass}}$
- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin
- P_2 is the final pressure
- P_1 is the initial pressure

2.15 Entropy change for ideal gas for an isentropic process

2.15.1 First Tds equation

$$\left(\frac{T_2}{T_1} \right)_{isen} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

Where:

- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin
- v_2 is the final **specific** volume
- v_1 is the initial **specific** volume
- k is $\frac{c_p}{c_v}$ or the specific heat capacity at constant pressure divided by the specific heat capacity at constant volume

2.15.2 Second Tds equation

$$\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\left(\frac{P_2}{P_1}\right)_{isen} = \left(\frac{v_1}{v_2}\right)^k$$

$$Pv^k = \text{constant}$$

Where:

- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin
- P_2 is the final pressure
- P_1 is the initial pressure
- v_2 is the final **specific** volume
- v_1 is the initial **specific** volume
- k is $\frac{c_p}{c_v}$ or the specific heat capacity at constant pressure divided by the specific heat capacity at constant volume
- P is the pressure of the system
- v is the **specific** volume of the system

2.16 Entropy change of a reversible process

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} (\text{kJ K}^{-1})$$
$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$

Where:

- dS is the change in entropy for an internally reversible process (int rev)
- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat leaving from the system to the reservoirs.
- T is the common temperature of the reservoirs at a particular instant in time in Kelvin
- ΔS is the change in entropy
- S_2 is the final entropy
- S_1 is the initial entropy

2.16.1 Internally reversible isothermal heat transfer

$$\Delta S = \frac{Q}{T_0} (\text{kJ K}^{-1})$$

Where:

- ΔS is the change in total entropy
- Q is the heat transfer for the internally reversible process
- T_0 is the constant temperature in Kelvin

2.17 Entropy change of an irreversible process

$$\Delta S_{sys} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

Where:

- ΔS_{sys} is the entropy change of a closed system for an irreversible process
- S_2 is the final entropy of the closed system
- S_1 is the initial entropy of the closed system
- $\int_1^2 \frac{\delta Q}{T}$ is the entropy change due to heat transfer
- S_{gen} is the entropy generation due to irreversibilities

2.18 Entropy change of a system

$$\Delta S_{system} = S_{in} - S_{out} + S_{gen}$$

Where:

- ΔS_{system} is the total entropy change of the system
- S_{in} is the total entropy entering the system
- S_{out} is the total entropy leaving the system
- S_{gen} is the entropy generated by irreversibilities

2.19 Entropy change of a closed system

$$\Delta S_{closed} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

Where:

- ΔS_{closed} is the total entropy change of a closed system
- S_2 is the final entropy of the system
- S_1 is the initial entropy of the system
- Q_k is the heat transferred
- T_k is the boundary temperature through which heat is transferred
- S_{gen} is the entropy generated by irreversibilities

2.19.1 Adiabatic process (no heat transfer)

$$\sum \frac{Q_k}{T_k} = 0$$

$$\Delta S_{adiabatic} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

$$\Delta S_{adiabatic} = S_2 - S_1 = 0 + S_{gen}$$

$$\Delta S_{adiabatic} = S_{gen} > 0$$

Where:

- Q_k is the heat transferred
- $\Delta S_{adiabatic}$ is the total entropy change of a closed adiabatic system
- T_k is the boundary temperature through which heat is transferred
- S_2 is the final entropy of the system
- S_1 is the initial entropy of the system
- S_{gen} is the entropy generated by irreversibilities

2.19.2 Isentropic process (constant entropy)

$$\begin{aligned}\sum \frac{Q_k}{T_k} &= 0 \quad \text{and} \quad S_{gen} = 0 \\ \Delta S_{isen} &= S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen} \\ \Delta S_{isen} &= 0 \\ S_2 &= S_1\end{aligned}$$

Where:

- Q_k is the heat transferred
- T_k is the boundary temperature through which heat is transferred
- S_{gen} is the entropy generated by irreversibilities
- ΔS_{isen} is the total entropy change of a closed isentropic system
- S_2 is the final entropy of the system
- S_1 is the initial entropy of the system

2.19.3 Isolated system

$$\begin{aligned}\sum \frac{Q_k}{T_k} &= 0 \\ \Delta S_{isolated} &= S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen} \\ \Delta S_{isolated} &= S_2 - S_1 = 0 + S_{gen} \\ \Delta S_{isolated} &= S_{gen} > 0\end{aligned}$$

Where:

- Q_k is the heat transferred
- T_k is the boundary temperature through which heat is transferred
- $\Delta S_{isolated}$ is the total entropy change of an isolated system
- S_{gen} is the entropy generated by irreversibilities
- S_2 is the final entropy of the system
- S_1 is the initial entropy of the system

2.19.4 Closed system with its surroundings (isolated system)

$$\Delta S_{closed} = S_2 - S_1$$

$$\Delta S_{surr} = \frac{Q}{T_{surr}}$$

$$\Delta S_{closed} + \Delta S_{surr} = \Delta S_{isolated} = S_{gen} > 0$$

$$S_2 - S_1 + \frac{Q}{T_{surr}} = S_{gen} > 0$$

Where:

- ΔS_{closed} is the total entropy change of the closed system
- S_2 is the final entropy of the closed system
- S_1 is the initial entropy of the closed system
- ΔS_{surr} is the total entropy change of the closed system
- Q is the heat transferred from the surroundings to the closed system
- T_{surr} is the temperature of the surroundings in K
- $\Delta S_{isolated}$ is the total entropy change of the isolated system consisting of the closed system and its surroundings
- S_{gen} is the entropy generated by irreversibilities

2.20 Entropy balance for flow control volumes

$$S_{\text{in, mass}} = m_i s_i$$

$$S_{\text{out, mass}} = m_e s_e$$

Where:

- $S_{\text{in, mass}}$ is the entropy entering the control volume due to mass entering the control volume
- m_i is the mass entering the control volume
- s_i is the specific entropy of the mass entering the control volume
- $S_{\text{out, mass}}$ is the entropy exiting the control volume due to mass exiting the control volume
- m_e is the mass exiting the control volume
- s_e is the specific entropy of the mass exiting the control volume

2.21 Entropy change for flow control volumes

$$\Delta S_{CV} = \sum m_i s_i - \sum m_e s_e + \sum \frac{Q_k}{T_k} + S_{gen}$$

Rate form (a dot above the variable means rate):

$$\Delta \dot{S}_{CV} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

Where:

- ΔS_{CV} is the total entropy change of the control volume
- m_i is the mass entering the control volume
- s_i is the specific entropy of the mass entering the control volume
- m_e is the mass exiting the control volume
- s_e is the specific entropy of the mass exiting the control volume
- Q_k is the heat transferred
- T_k is the boundary temperature through which heat is transferred
- S_{gen} is the entropy generated by irreversibilities

2.21.1 Steady state, steady flow control volume

$$\Delta\dot{S}_{CV} = 0$$

$$\Delta\dot{S}_{CV} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$0 = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

Where:

- $\Delta\dot{S}_{CV}$ is the rate of total entropy change of the control volume
- \dot{m}_i is the mass rate entering the control volume
- s_i is the specific entropy of the mass entering the control volume
- \dot{m}_e is the mass rate exiting the control volume
- s_e is the specific entropy of the mass exiting the control volume
- \dot{Q}_k is the rate of heat transferred
- T_k is the boundary temperature through which heat is transferred
- \dot{S}_{gen} is the rate of entropy generated by irreversibilities

2.21.2 Single stream, steady flow process

$$\begin{aligned}\Delta\dot{S}_{CV} &= 0 \quad \text{and} \quad \dot{m}_i = \dot{m}_e = \dot{m} \\ \Delta\dot{S}_{CV} &= \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen} \\ 0 &= \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen} \\ \dot{m}(s_e - s_i) &= \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}\end{aligned}$$

Where:

- $\Delta\dot{S}_{CV}$ is the rate of total entropy change of the control volume
- \dot{m} is the mass flow rate through the control volume
- \dot{m}_i is the mass rate entering the control volume
- \dot{m}_e is the mass rate exiting the control volume
- s_i is the specific entropy of the mass entering the control volume
- s_e is the specific entropy of the mass exiting the control volume
- \dot{Q}_k is the rate of heat transferred
- T_k is the boundary temperature through which heat is transferred
- \dot{S}_{gen} is the rate of entropy generated by irreversibilities

2.21.3 Steady state, single stream and steady flow adiabatic process

$$\Delta\dot{S}_{CV} = 0 \quad \text{and} \quad \dot{m}_i = \dot{m}_e = \dot{m} \quad \text{and} \quad \sum \frac{\dot{Q}_k}{T_k} = 0$$

$$\Delta\dot{S}_{CV} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$0 = \dot{m}s_i - \dot{m}_e s_e + 0 + \dot{S}_{gen}$$

$$\dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$\dot{S}_{gen} \geq 0 \quad \rightarrow \quad s_e \geq s_i$$

Where:

- $\Delta\dot{S}_{CV}$ is the rate of total entropy change of the control volume
- \dot{m} is the mass flow rate through the control volume
- \dot{m}_i is the mass rate entering the control volume
- \dot{m}_e is the mass rate exiting the control volume
- \dot{Q}_k is the rate of heat transferred
- T_k is the boundary temperature through which heat is transferred
- s_i is the specific entropy of the mass entering the control volume
- s_e is the specific entropy of the mass exiting the control volume
- \dot{S}_{gen} is the rate of entropy generated by irreversibilities

2.21.4 Adiabatic and reversible process (isentropic process)

$$\begin{aligned}\Delta\dot{S}_{CV} &= 0 \quad \text{and} \quad \dot{m}_i = \dot{m}_e = \dot{m} \quad \text{and} \quad \sum \frac{\dot{Q}_k}{T_k} = 0 \quad \text{and} \quad \dot{S}_{gen} = 0 \\ \Delta\dot{S}_{CV} &= \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen} \\ 0 &= \dot{m}s_i - \dot{m}s_e + 0 + 0 \\ \dot{m}(s_e - s_i) &= \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen} \\ s_e &= s_i\end{aligned}$$

Where:

- $\Delta\dot{S}_{CV}$ is the rate of total entropy change of the control volume
- \dot{m} is the mass flow rate through the control volume
- \dot{m}_i is the mass rate entering the control volume
- \dot{m}_e is the mass rate exiting the control volume
- \dot{Q}_k is the rate of heat transferred
- T_k is the boundary temperature through which heat is transferred
- \dot{S}_{gen} is the rate of entropy generated by irreversibilities
- s_i is the specific entropy of the mass entering the control volume
- s_e is the specific entropy of the mass exiting the control volume

2.22 Reversible steady flow work

$$w_{rev} = - \int_i^e v dP$$

Where:

- w_{rev} is the reversible steady flow work
- v is the specific volume of the fluid
- dP is the infinitesimal change in pressure of the fluid

2.22.1 Incompressible fluids

$$w_{rev} = -v(P_2 - P_1)$$

Where:

- w_{rev} is the reversible steady flow work
- v is the specific volume of the fluid
- P_2 is final pressure of the fluid
- P_1 is initial pressure of the fluid

2.22.2 Incompressible fluids with no work done

$$0 = -v(P_2 - P_1) - \frac{\nu_2^2 - \nu_1^2}{2} - g(z_2 - z_1)$$

Where:

- v is the specific volume of the fluid
- P_2 is final pressure of the fluid
- P_1 is initial pressure of the fluid
- ν_2 is the final velocity of the fluid
- ν_1 is the initial velocity of the fluid
- g is the gravitational constant of 9.81 ms^{-1}
- z_2 is the final height of the fluid
- z_1 is the initial height of the fluid

2.23 Polytropic process

$$Pv^n = \text{constant}$$

Where:

- P is the pressure of the fluid
- v is the specific volume of the fluid
- n is the polytropic index
- constant is an arbitrary constant

2.24 Specific work done when flow through the device is polytropic

$$w_{poly} = -\frac{n}{n-1}(P_2v_2 - P_1v_1)$$

Where:

- w_{poly} is the specific work done, or the work done per unit mass
- n is the polytropic index
- P_2 is the final pressure
- P_1 is the initial pressure
- v_2 is the final specific volume
- v_1 is the initial specific volume

2.24.1 For an ideal gas

$$w_{poly} = -\frac{nR_{sp}(T_2 - T_1)}{n-1} = -\frac{nR_{sp}T_1}{n-1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Where:

- w_{poly} is the specific work done, or the work done per unit mass
- n is the polytropic index
- R_{sp} is the molar gas constant per unit mass, i.e. $R_{sp} = \frac{R}{\text{Molar mass}}$
- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin
- P_2 is the final pressure
- P_1 is the initial pressure

2.24.2 Isentropic process for ideal gas

$$n = k$$

$$w_{isen} = -\frac{kR_{sp}(T_2 - T_1)}{k - 1} = \frac{kR_{sp}T_1}{1 - k} \left[\left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right]$$

Where:

- w_{isen} is the specific work done, or the work done per unit mass
- k is given by $\frac{c_p}{c_v}$
- R_{sp} is the molar gas constant per unit mass, i.e. $R_{sp} = \frac{R}{\text{Molar mass}}$
- T_2 is the final temperature in Kelvin
- T_1 is the initial temperature in Kelvin
- P_2 is the final pressure
- P_1 is the initial pressure

2.24.3 Isothermal process

$$n = 1$$

$$w_{isothermal} = -R_{sp}T \ln \left(\frac{P_2}{P_1} \right)$$

Where:

- $w_{isothermal}$ is the specific work done, or the work done per unit mass
- R_{sp} is the molar gas constant per unit mass, i.e. $R_{sp} = \frac{R}{\text{Molar mass}}$
- T is the temperature of the gas in Kelvin
- P_2 is the final pressure
- P_1 is the initial pressure

2.24.4 Specific work done in two-stage compression with intercooling

$$w_{comp} = w_{comp_1} + w_{comp_2} = -\frac{nR_{sp}T_1}{n-1} \left[\left(\frac{P_x}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] - \frac{nR_{sp}T_1}{n-1} \left[\left(\frac{P_2}{P_x} \right)^{\frac{n-1}{n}} - 1 \right]$$

Where:

- w_{comp} is the specific work done in the two-stage compression with intercooling
- w_{comp_1} is the specific work done in the first stage
- w_{comp_2} is the specific work done in the second stage
- n is the polytropic index
- R_{sp} is the molar gas constant per unit mass, i.e. $R_{sp} = \frac{R}{\text{Molar mass}}$
- T_1 is the initial temperature in Kelvin
- P_x is the pressure of the intermediate stage
- P_2 is the final pressure
- P_1 is the initial pressure

2.24.5 Condition for minimum compressor work

$$P_x = \sqrt{P_1 P_2} \Rightarrow \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

Where:

- P_x is the pressure of the intermediate stage
- P_1 is the initial pressure
- P_2 is the final pressure

2.25 Polytropic specific work done in a closed system

$$w_{poly} = -\frac{1}{n-1}(P_2v_2 - P_1v_1)$$

Where:

- w_{poly} is the specific work done, or the work done per unit mass
- n is the polytropic index
- P_2 is the final pressure
- P_1 is the initial pressure
- v_2 is the final specific volume
- v_1 is the initial specific volume

2.26 Isentropic efficiency

2.26.1 Turbines

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Where:

- η_T is the isentropic efficiency of the turbine
- h_1 is the initial enthalpy of the fluid
- h_{2a} is the final enthalpy of the actual process
- h_{2s} is the final enthalpy of the isentropic process

2.26.2 Compressors

$$\eta_C = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

Where:

- η_C is the isentropic efficiency of the compressor
- h_{2a} is the final enthalpy of the actual process
- h_{2s} is the final enthalpy of the isentropic process
- h_1 is the initial enthalpy of the fluid

2.26.3 Pumps

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

Where:

- η_C is the isentropic efficiency of the pump
- w_s is the isentropic work input of the pump
- w_a is the actual work input of the pump
- v is the specific volume of the fluid
- P_2 is the final pressure of the fluid
- P_1 is the initial pressure of the fluid
- h_{2a} is the final enthalpy of the actual process
- h_1 is the initial enthalpy of the fluid

2.26.4 Nozzles

$$h_1 - h_{2a} = \frac{\nu_{2a}^2}{2}$$

$$h_1 - h_{2s} = \frac{\nu_{2s}^2}{2}$$

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Where:

- h_1 is the initial enthalpy of the fluid
- h_{2a} is the final enthalpy of the actual process
- ν_{2a} is the velocity of the fluid in the actual process
- h_{2s} is the final enthalpy of the isentropic process
- ν_{2s} is the velocity of the fluid in the isentropic process
- η_N is the isentropic efficiency of the nozzle

2.27 Gravimetric analysis

$$m_m = \sum_i^k m_i$$

Where:

- m_m is the mass of a mixture
- m_i is the mass of a component gas in the mixture

2.27.1 Mass fraction (mf)

$$mf_i = \frac{m_i}{m_m}$$

$$\sum_i^k mf_i = 1$$

Where:

- mf_i is the mass fraction of a component gas in the mixture
- m_i is the mass of a component gas in the mixture
- m_m is the mass of the mixture

2.28 Molar analysis

$$N_m = \sum_i^k N_i$$

Where:

- N_m is the number of moles of the mixture
- N_i is the number of moles of a component gas in the mixture

2.28.1 Mole fraction (y)

$$y_i = \frac{N_i}{N_m}$$

$$\sum_i^k y_i = 1$$

Where:

- y_i is the mole fraction of a component gas in the mixture
- N_i is the number of moles of a component gas in the mixture
- N_m is the number of moles of gas in the mixture

2.28.2 Molar mass (M)

$$M_m = \frac{m_m}{N_m}$$

Where:

- M_m is the molar mass of the mixture
- m_m is the mass of the mixture
- N_m is the number of moles of gas in the mixture

2.28.3 Mass specific gas constant (R_m)

$$R_m = \frac{R_u}{M_m}$$

Where:

- R_m is mass specific gas constant
- R_u is the universal gas constant of $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
- M_m is the molar mass of the mixture

2.29 Dalton's law

$$P_m = \sum_i^k P_i(T_m, V_m)$$

Where:

- P_m is the pressure of the mixture
- P_i is the pressure of each component at the temperature and volume of the mixture (T_m, V_m)

2.30 Amagat's law

$$V_m = \sum_i^k V_i(T_m, P_m)$$

Where:

- V_m is the volume of the mixture
- V_i is the volume that each component would occupy if it existed alone at the temperature and pressure of the mixture (T_m, P_m)

2.31 Fractions for ideal gases

2.31.1 Pressure fraction

$$\frac{P_i(T_m, V_m)}{P_m} = \frac{\frac{(N_i R_u T_m)}{V_m}}{\frac{(N_m R_u T_m)}{V_m}} = \frac{N_i}{N_m} = y_i$$

Where:

- P_i is the pressure of the component gas at the temperature and volume of the mixture (T_m, V_m)
- P_m is the pressure of the mixture
- N_i is the number of moles of the component gas in the mixture
- R_u is the universal gas constant of $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
- T_m is the temperature of the mixture
- V_m is the volume of the mixture
- N_m is the number of moles of gas in the mixture
- y_i is the mole fraction of the component gas in the mixture

2.31.2 Partial pressure

$$P_i = y_i P_m$$

Where:

- P_i is the partial pressure (component pressure for ideal gases) of a component of the mixture
- y_i is the mole fraction of the component in the mixture
- P_m is the pressure of the mixture

2.31.3 Volume fraction

$$\frac{V_i(T_m, P_m)}{V_m} = \frac{\frac{(N_i R_u T_m)}{P_m}}{\frac{(N_m R_u T_m)}{P_m}} = \frac{N_i}{N_m} = y_i$$

Where:

- V_i is the volume of the component gas at the temperature and pressure of the mixture (T_m, P_m)
- V_m is the volume of the mixture
- N_i is the number of moles of the component gas in the mixture
- R_u is the universal gas constant of $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
- T_m is the temperature of the mixture
- P_m is the pressure of the mixture
- N_m is the number of moles of gas in the mixture
- y_i is the mole fraction of the component gas in the mixture

2.31.4 Partial volume

$$V_i = y_i V_m$$

Where:

- V_i is the partial volume (component volume for ideal gases) of a component of the mixture
- y_i is the mole fraction of the component in the mixture
- V_m is the volume of the mixture

2.31.5 Dalton's and Amagat's law

$$\frac{P_i}{P_m} = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i$$

Where:

- P_i is the pressure of the component gas in the mixture
- P_m is the pressure of the mixture
- V_i is the volume of the component gas in the mixture
- V_m is the volume of the mixture
- N_i is the number of moles of the component gas in the mixture
- N_m is the number of moles of gas in the mixture

2.32 Properties of gas mixtures

2.32.1 Extensive properties

Extensive properties of gas mixtures	Changes in extensive properties of gas mixtures
$U_m = \sum_i^k U_i = \sum_i^k m_i u_i = \sum_i^k N_i \bar{u}_i$	$\Delta U_m = \sum_i^k \Delta U_i = \sum_i^k m_i \Delta u_i = \sum_i^k N_i \Delta \bar{u}_i$
$H_m = \sum_i^k H_i = \sum_i^k m_i h_i = \sum_i^k N_i \bar{h}_i$	$\Delta H_m = \sum_i^k \Delta H_i = \sum_i^k m_i \Delta h_i = \sum_i^k N_i \Delta \bar{h}_i$
$S_m = \sum_i^k S_i = \sum_i^k m_i s_i = \sum_i^k N_i \bar{s}_i$	$\Delta S_m = \sum_i^k \Delta S_i = \sum_i^k m_i \Delta s_i = \sum_i^k N_i \Delta \bar{s}_i$

2.32.2 Intensive properties

$$\begin{array}{ll} u_m = \sum_i^k m f_i u_i & \bar{u}_m = \sum_i^k y_i \bar{u}_i \\ h_m = \sum_i^k m f_i h_i & \bar{h}_m = \sum_i^k y_i \bar{h}_i \\ s_m = \sum_i^k m f_i s_i & \bar{s}_m = \sum_i^k y_i \bar{s}_i \\ c_{v,m} = \sum_i^k m f_i c_{v,i} & \bar{c}_{v,m} = \sum_i^k y_i \bar{c}_{v,i} \\ c_{p,m} = \sum_i^k m f_i c_{p,i} & \bar{c}_{p,m} = \sum_i^k y_i \bar{c}_{p,i} \end{array}$$

2.33 Properties of air

2.33.1 Specific enthalpy of dry air

$$h_{\text{dry air}} = c_p T \quad c_p = 1.005 \text{ kJ kg}^{-1} \text{ }^{\circ}\text{C}^{-1}$$

$$\Delta h_{\text{dry air}} = c_p \Delta T$$

Where:

- $h_{\text{dry air}}$ is the specific enthalpy of the dry air
- T is the temperature of the dry air
- c_p is the specific heat capacity at constant pressure of the dry air
- $\Delta h_{\text{dry air}}$ is the change in specific enthalpy of the dry air
- ΔT is the change in temperature of the dry air

2.33.2 Pressure

$$P = P_a + P_v$$

Where:

- P is the total pressure of air
- P_a is the partial pressure of dry air
- P_v is the partial pressure of water vapour

2.33.3 Specific enthalpy of water vapour

$$h_v(T, \text{low } P) \cong h_g(T)$$

$$h_g(T) \cong 2500.9 + 1.82T \quad (\text{kJ kg}^{-1})$$

Where:

- h_v is the specific enthalpy of water vapour in atmospheric air (T , low P)
- T is the temperature of the air
- h_g is the enthalpy of saturated water vapour at temperature T

2.34 Humidity of air

2.34.1 Specific or absolute humidity (ω)

$$\omega = \frac{m_v}{m_a} = \frac{0.622P_v}{P - P_v}$$

Where:

- ω is the specific or absolute humidity
- m_v is the mass of water vapour in the air
- m_a is the mass of dry air
- P_v is the pressure of the water vapour
- P is the total pressure of the air

2.34.2 Saturated air

$$P_v = P_g$$

Where:

- P_v is the pressure of the water vapour in the air
- P_g is the pressure of the saturated water vapour at temperature T
- T is the temperature of the air

2.34.3 Relative humidity (ϕ)

$$\phi = \frac{m_v}{m_g} = \frac{P_v}{P_g}$$

Where:

- ϕ is the relative humidity
- m_v is the mass of water vapour in the air
- m_g is the mass of water vapour that the air can hold at saturation temperature, when air becomes a saturated vapour
- P_v is the pressure of the water vapour in the air
- P_g is the pressure of the saturated water vapour at temperature T
- T is the temperature of the air

2.34.4 Converting between absolute and relative humidity [formula given]

$$\phi = \frac{\omega P}{(0.622 + \omega)P_g}$$

$$\omega = \frac{0.622\phi P_g}{P - \phi P_g}$$

Where:

- ϕ is the relative humidity
- ω is the absolute humidity
- P is the total pressure of the mixture
- P_g is the pressure of the saturated water vapour at temperature T
- T is the temperature of the air

2.34.5 Total enthalpy of atmospheric air

$$H = H_a + H_v = m_a h_a + m_v h_v$$

Where:

- H is the total enthalpy of atmospheric air
- H_a is the enthalpy of dry air
- H_v is the enthalpy of water vapour in the air
- m_a is the mass of dry air
- h_a is the specific enthalpy of dry air
- m_v is the mass of water vapour in the air
- h_v is the specific enthalpy of water vapour in the air

2.34.6 Specific enthalpy of atmospheric air

$$h = h_1 + \omega h_g$$

Where:

- h is the specific enthalpy of atmospheric air, expressed in terms of **per unit mass of dry air**
- h_a is the specific enthalpy of dry air
- ω is the absolute humidity
- h_g is the enthalpy of saturated water vapour at the current air temperature

2.35 Dew point temperature (T_{dp})

$$T_{dp} = T_{\text{sat} @ P_v}$$

Where:

- T_{dp} is the dew point temperature
- $T_{\text{sat} @ P_v}$ is the saturation temperature that corresponds to the saturated vapour pressure P_v

2.36 Specific volume of dry air

$$v = \frac{P_a T}{R_a}$$

Where:

- v is the specific volume of dry air
- P_a is the partial pressure of dry air
- T is the temperature of the dry air in Kelvin
- R_a is the molar gas constant per unit mass of dry air, which is $0.287 \text{ kJ kg}^{-1} \text{ K}^{-1}$

2.37 Air conditioning processes

2.37.1 Mass balance in general

$$\begin{aligned} \sum_{in} \dot{m}_a &= \sum_{out} \dot{m}_a \\ \text{Dry air: } \sum_{in} \dot{m}_a &= \sum_{out} \dot{m}_a \\ \text{Water: } \sum_{in} \dot{m}_w &= \sum_{out} \dot{m}_w \quad \rightarrow \quad \sum_{in} \omega \dot{m}_a = \sum_{out} \omega \dot{m}_a \end{aligned}$$

Where:

- \dot{m}_a is the mass flow rate of dry air
- \dot{m}_w is the mass flow rate of water
- ω is the specific or absolute humidity

2.37.2 Energy balance in general

$$\begin{aligned} \dot{E}_{in} &= \dot{E}_{out} \\ \dot{Q}_{in} + \sum_{in} \dot{m}h &= \dot{Q}_{out} + \sum_{out} \dot{m}h \end{aligned}$$

Where:

- \dot{E}_{in} is the rate of energy entering the system
- \dot{E}_{out} is the rate of energy exiting the system
- \dot{Q}_{in} is the heat entering the system
- \dot{m} is the mass flow rate of air
- h is the enthalpy of air
- \dot{Q}_{out} is the heat exiting the system

2.37.3 Simple heating and cooling

- Mass balance:

$$\begin{aligned}\dot{m}_{a_1} &= \dot{m}_{a_2} = \dot{m}_a \\ \dot{m}_{w_1} &= \dot{m}_{w_2} = \dot{m}_w \quad \rightarrow \quad \omega_1 = \omega_2\end{aligned}$$

Where:

- \dot{m}_{a_n} is the mass flow rate of dry air at state n
- \dot{m}_{w_n} is the mass flow rate of water at state n
- ω_n is the absolute humidity of the air at state n

- Energy balance:

$$\dot{Q} = \dot{m}_a(h_2 - h_1) \text{ or } q = (h_2 - h_1)$$

Where:

- \dot{Q} is the rate of heat change of the system
- \dot{m}_a is the mass flow rate of dry air
- h_n is the enthalpy of the air at state n
- q is the change in heat per mass of the system

2.37.4 Humidification or dehumidification

$$\dot{m}_w = \dot{m}_a \Delta \omega$$

Where:

- \dot{m}_w is the mass flow rate of water
- \dot{m}_a is the mass flow rate of dry air
- $\Delta \omega$ is the change in specific humidity

2.37.5 Heating with humidification

- Simple heating ($1 \rightarrow 2$):

$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

$$\dot{m}_{w_1} = \dot{m}_{w_2} = \dot{m}_w \quad \rightarrow \quad \omega_1 = \omega_2$$

$$\dot{Q}_{in} = \dot{m}_a(h_2 - h_1)$$

Where:

- \dot{m}_{a_n} is the mass flow rate of dry air at state n
- \dot{m}_{w_n} is the mass flow rate of water at state n
- ω_n is the absolute humidity at state n
- \dot{m}_a is the mass flow rate of dry air
- \dot{Q}_{in} is the heat input to the air
- h_n is the enthalpy of the air at state n

- Humidification ($2 \rightarrow 3$):

$$\dot{m}_w = \dot{m}_a(\omega_3 - \omega_2)$$

Where:

- \dot{m}_w is the mass flow rate of water
- \dot{m}_a is the mass flow rate of dry air
- ω_n is the absolute humidity at state n

2.37.6 Cooling with dehumidification

- Mass balance:

$$\begin{aligned} \text{Dry air: } \dot{m}_{a_1} &= \dot{m}_{a_2} = \dot{m}_a \\ \text{Water: } \dot{m}_w &= \dot{m}_a(\omega_1 - \omega_2) \end{aligned}$$

Where:

- \dot{m}_{a_n} is the mass flow rate of dry air at state n
- \dot{m}_a is the mass flow rate of dry air
- \dot{m}_w is the mass flow rate of water
- ω_n is the absolute humidity at state n

- Energy balance:

$$\dot{Q}_{out} = \dot{m}_a(h_1 - h_2) - \dot{m}_w h_w$$

Where:

- \dot{Q}_{out} is the rate of heat output from the system
- \dot{m}_a is the mass flow rate of the dry air
- \dot{m}_w is the mass flow rate of water
- h_w is the enthalpy of water

2.37.7 Evaporative cooling

$$T_{wb} \cong \text{constant}$$

$$h \cong \text{constant}$$

Where:

- T_{wb} is the wet-bulb temperature
- h is the enthalpy of the air

2.37.8 Adiabatic mixing of airstreams

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{h_2 - h_3}{h_3 - h_1} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1}$$

Where:

- \dot{m}_{a_n} is the mass flow rate of dry air at state n
- ω_n is the absolute humidity at state n
- h_n is the enthalpy of the air at state n

3 Entropy

- Entropy is a **property**
- It is a measure of the degree of **microscopic disorder** in the system
- Entropy can be **transferred by heat** but **not by work**
- Entropy is **not conserved**
- Entropy can be generated or created but **not destroyed**
- Total entropy for isolated systems (system + surroundings) **increases** for all real processes

3.1 Relation to the Clausius Inequality

- Cyclic integral of **work** and the cyclic integral of **heat** is **not zero**
- However, the cyclic integral of volume (a property) is zero

$$\oint dV = \Delta V_{cyc} = 0$$

- More generally, cyclic integrals of any property are zero

$$\oint d(\text{property}) = 0$$

- Hence, $\left(\frac{\delta Q}{T}\right)_{\text{int rev}}$ represents a property

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \quad (\text{kJ K}^{-1})$$

Where:

- dS is the change in entropy for an internally reversible process (int rev)
- δQ is an infinitesimal amount of heat that is taken from the reservoirs and absorbed by the system. $\delta Q > 0$ if heat from the reservoirs is absorbed by the system, and $\delta Q < 0$ is heat leaving from the system to the reservoirs.
- T is the common temperature of the reservoirs at a particular instant in time in Kelvin

3.2 Molecular disorder

Entropy is a measure of molecular disorder.

- As a system become more disordered, the position of the molecules become less predictable and thus entropy increases.
- Quantity of **energy is always conserved** during an actual process (first law) *but the quality decreases (Second law)
- There is **no entropy transfer** associated with energy transfer in the **form of work**.
- Heat is a form of disorganised energy and some disorganisation (**entropy**) will flow with heat.

3.3 Entropy of pure substances

- Entropy data for **pure substances** (including **real gases**) can be obtained from property tables.
- Specific entropy ($\text{kJ kg}^{-1} \text{K}^{-1}$) values are assigned to reference states for each substance.
 - For water or steam: $s_f = 0 @ 0.01^\circ\text{C}$
 - For R134a refrigerant: $s_f = 0 @ -40^\circ\text{C}$
- Entropy change of a specified mass during a process:

$$\Delta S = S_2 - S_1 = m(s_2 - s_1)$$

Where:

- ΔS is the change in entropy
- S_2 is the final entropy
- S_1 is the initial entropy
- m is the mass of the substance
- s_2 is the final specific entropy
- s_1 is the initial specific entropy

3.4 Determination of entropy from tables

- Superheated vapour region: Tabulated versus pressure (P) and temperature (T).
- Saturated liquid-vapour region: s_f, s_g, s_{fg} are given at saturation pressure (P_{sat}) and temperature (T_{sat}).

$$s = s_f + x s_{fg}$$

Where:

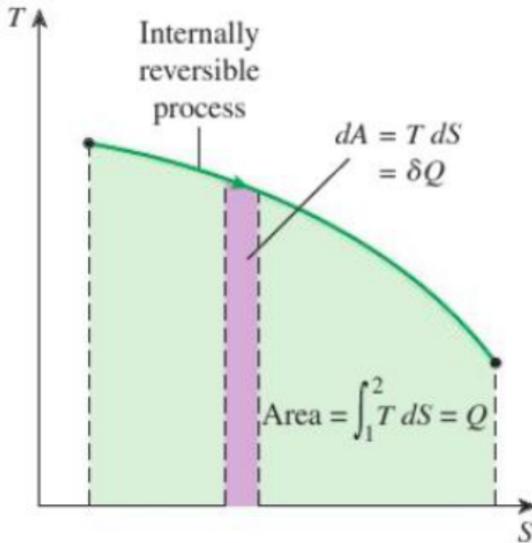
- s is the specific entropy of the substance
- s_f is the specific entropy of the saturated liquid
- s_{fg} is the specific entropy of the saturated vapour minus the specific entropy of the saturated liquid, i.e. $s_{fg} = s_g - s_f$
- Compressed liquid region: Tabulated versus pressure (P) and temperature (T)
 - Fluid is almost incompressible, so the entropy considered is dependent only on temperature, i.e.

$$s \cong s_f @ T$$

3.5 Property diagrams

- First Law analysis: $P - v$ and $T - v$ diagrams
- Second Law analysis: $T - s$ and $h - s$ diagrams

3.5.1 $T - s$ diagram



- Differential form:

$$dS = \left(\frac{\delta Q}{T} \right)_{intrev} \quad (\text{kJ K}^{-1})$$

- Integrating:

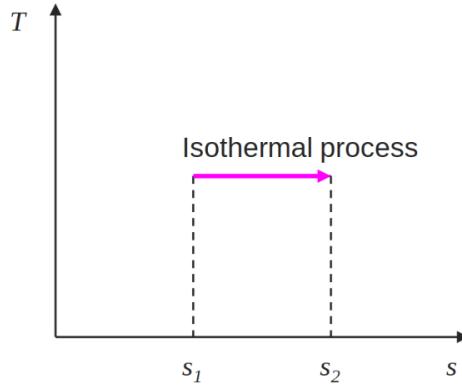
$$Q_{intrev} = \int_1^2 T dS \quad (\text{kJ})$$

$$q_{intrev} = \int_1^2 T ds \quad (\text{kJ})$$

Where:

- Q_{intrev} is the heat transferred in an internally reversible process
- T is the temperature in Kelvin
- dS is the infinitesimal change in entropy
- q_{intrev} is the specific heat transferred in an internally reversible process
- ds is the infinitesimal change in specific entropy
- **Area under the process curve in the T-s diagram represents the heat transfer in an internally reversible process.**

- Internally reversible isothermal process:



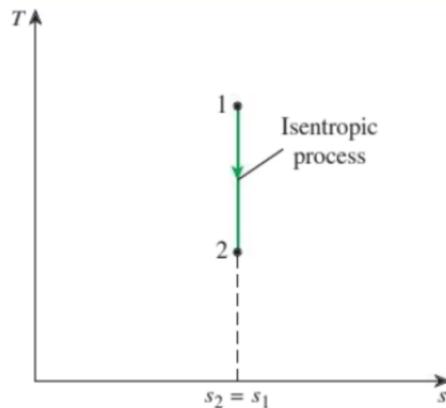
$$Q_{intrev} = T_0 \Delta S$$

$$q_{intrev} = T_0 \Delta s$$

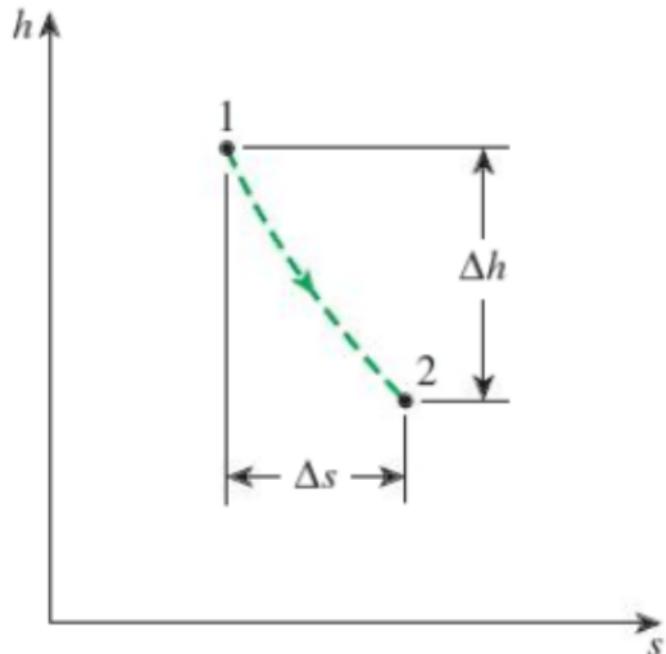
Where:

- Q_{intrev} is the heat transferred in an internally reversible process
- T is the temperature in Kelvin
- ΔS is the change in entropy
- q_{intrev} is the specific heat transferred in an internally reversible process
- Δs is the change in specific entropy

- Isentropic process is a constant entropy process:

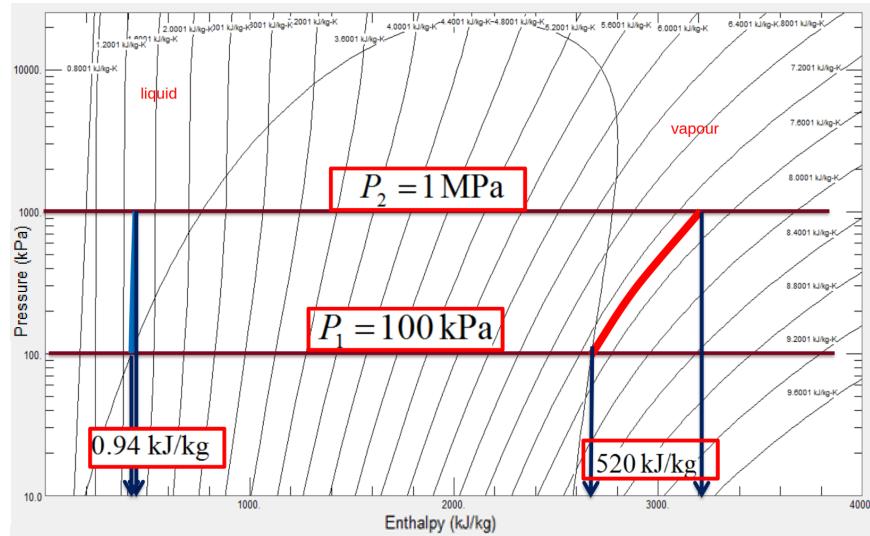


3.5.2 $h - s$ diagram (Mollier diagram)



- Useful in analysis of steady flow devices
- Enthalpy h is the primary property in First Law analysis.
- Entropy s is the property that accounts for irreversibilities in adiabatic processes.

3.5.3 $P - h$ diagram for water



4 Human comfort and air-conditioning

- Humans prefer comfortable environments that are neither too hot nor too cold, and neither too humid nor too dry.
- The weather cannot be changed, but the climate in a confined space can be controlled.
- Modern air-conditioning systems can do more than simply cooling or heating the air.
 - Other functions include humification, dehumidification, cleaning and deodorisation, i.e. these systems condition the air.
- A human body is like a heat engine:
 - Rate of heat generation depends on the level of activity.
 - Waste heat is dissipated to the surroundings.
 - A body feels comfortable when it can dissipate its waste heat freely, and not more.
 - A body at rest dissipates about 100 W when at rest.
- Comfort of the human body depends primarily on three factors:
 1. Dry-bulb temperature
 2. Relative humidity
 3. Air motion
- Relative humidity affects the amount of heat a body can dissipate through sweat evaporation. $40\% < \phi < 60\%$ is usually preferred
- Air motion removes the build up of warm moist air around the body and replaces it with fresh air.
- Air motion should be strong enough for such removal but gentle enough to be unnoticed.
- If air motion is too strong, a chilling effect is produced (wind-chill factor).