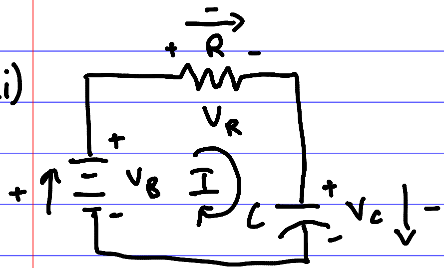
1i)



Using Kirchhoff's voltage law,

$$+V_B - V_R - V_C = 0$$

$$V_B = V_R + V_C \quad -(1)$$

$$V_R = IR \quad -(2)$$

$$C = \frac{Q}{V}$$

$$VC = Q$$

$$\frac{dQ}{dt} = \frac{dV}{dt}C$$

$$I = C\frac{dV}{dt}$$

$$\therefore I = C\frac{dV_c}{dt} \quad -(3)$$

Sub (3) into (2)

$$V_R = C\frac{dV_c}{dt}R \quad -(4)$$

Sub (4) into (1)

$$V_B = C\frac{dV_c}{dt}R + V_c$$

$$\frac{V_B}{RC} = \frac{dV_c}{dt} + \frac{V_c}{RC}$$

$$\frac{dV_c}{dt} = \frac{1}{RC}(V_B - V_c) \quad -(5)$$

i) When $t=0$, $V_c = 0\,V$

When $t=\infty$, $V_\infty = ?$,

$$\cancel{\frac{dV_\infty}{dt}} = \cancel{\frac{1}{RC}}(V_B - V_\infty)$$

$$V_\infty = V_B$$

$$\therefore V_\infty = 6V$$

$$\therefore \frac{dV_c}{dt} = \frac{1}{RC}(6 - V_c)$$

ii) $\tau = RC$

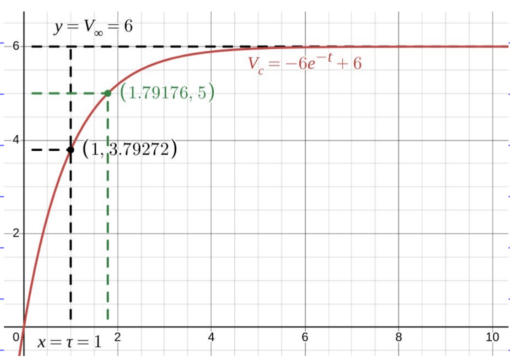
$= 1\times10^{-3} \times 1\times10^{3}$

$= 1s$

iii) $V_c(t) = (V_0 - V_\infty)e^{-\frac{t}{\tau}} + V_\infty$

$$= -6e^{-t} + 6$$

iv) Desmos:



iv) $E = \frac{1}{2}CV^2$

$\therefore E_{max} = \frac{1}{2}CV_{\infty}^2$

$= \frac{1}{2}(1\times10^{-3})(6)^2$

$= 18mJ$

vi) $E_{90\%} = \frac{1}{2}C(V_{90\%})^2$

$0.9(18) = \frac{1}{2}(1)(V_{90\%})^2$

$V_{90\%} = \sqrt{\frac{2(0.9)(18)}{1}}$

$= 5.692099788V$

$\approx 5.69V$

$V_{90\%} = -6e^{-t_{90\%}} + 6$

$5.69 = -6e^{-t_{90\%}} + 6$

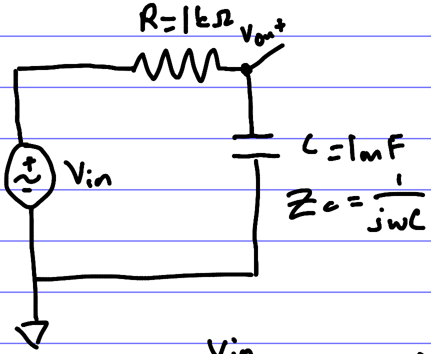
$-0.31 = -6e^{-t_{90\%}}$

$\ln(0.051) = -t_{90\%}$

$t_{90\%} = 2.969739006$

$\approx 2.97s$

2i)



$$I = \frac{V_{in}}{R + Z_c} \quad -(1)$$

$$= \frac{V_{in}}{R + \frac{1}{j\omega C}}$$

$$V_{out} = I \cdot Z_c \quad -(2)$$

$$= I \cdot \frac{1}{j\omega C}$$

Sub (1) into (2),

$$V_{out} = \frac{V_{in}}{R + \frac{1}{j\omega C}} \left(\frac{1}{j\omega C}\right)$$

$$= \frac{V_{in}}{j\omega CR + 1}$$

$$= \frac{V_{in}}{j\omega(1\times10^{-3})(1\times10^{3}) + 1}$$

$$= \frac{V_{in}}{1 + j\omega}$$

ii) $H = \frac{V_{out}}{V_{in}}$

$$= \frac{\cancel{V_{in}}}{1 + j\omega} \times \frac{1}{\cancel{V_{in}}}$$

$$= \frac{1}{1 + j\omega}$$

2iii) $|V_{out}| = \frac{|V_{in}|}{\sqrt{2}}$

$$\frac{|V_{out}|}{|V_{in}|} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{|1+jw|} = \frac{1}{\sqrt{2}}$$

$$|1+jw| = \sqrt{2}$$

$$\sqrt{1^2+w^2} = \sqrt{2}$$

$$1+w^2 = 2$$

$$w^2 = 1$$

$\therefore w = 1$ or $w = -1$ (rejected as $w>0$)

$\therefore w = 1 \, rads^{-1}$

The cutoff frequency is the same as the time constant calculated in the previous question.

iv) $w_0 = 2\pi f_0$

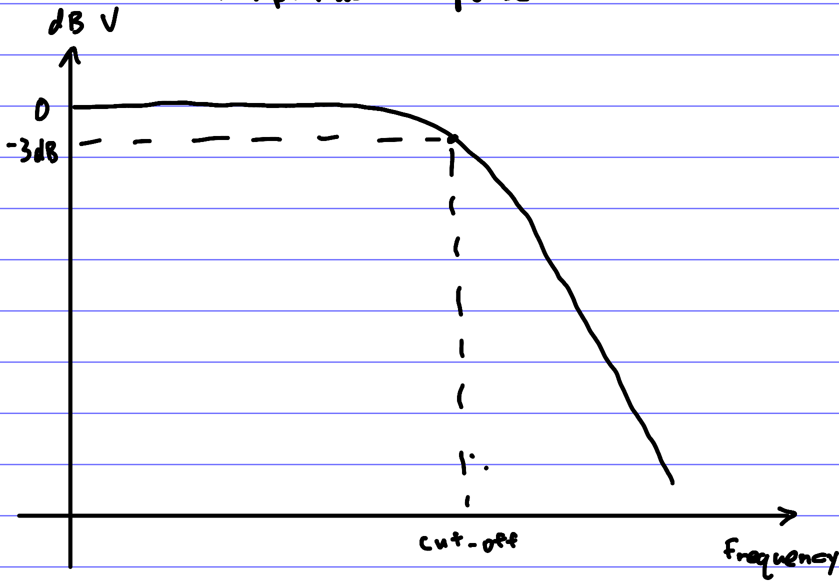
$$1 = 2\pi f_0$$

$$f_0 = \frac{1}{2\pi}$$

$$\approx 0.159 Hz$$

2iv)

Amplitude Response



Phase response