1) $\underset{\sim}{G}(t) = -3\cos(2t)\,\underset{\sim}{i} + 3\sin(2t)\,\underset{\sim}{j} + \underset{\sim}{k}$

$= (-3\cos(2t), 3\sin(2t), 1)$

a) $\underset{\sim}{G} \cdot \underset{\sim}{G} = (-3\cos(2t), 3\sin(2t), 1) \cdot$

$(-3\cos(2t), 3\sin(2t), 1)$

$= 9\cos^2(2t) + 9\sin^2(2t) + 1$

$= 10$

b) $\frac{d\underset{\sim}{G}}{dt} = (6\sin(2t), 6\cos(2t), 0)$

$\frac{d\underset{\sim}{G}}{dt} \cdot \frac{d\underset{\sim}{G}}{dt} = (6\sin(2t), 6\cos(2t), 0) \cdot (6\sin(2t), 6\cos[2t), 0)$

$= 36\sin^2(2t) + 36\cos^2(2t) + 0$

$= 36$

c) $\underset{\sim}{G} \cdot \frac{d\underset{\sim}{G}}{dt} = (-3\cos(2t), 3\sin(2t), 1) \cdot (6\sin(2t), 6\cos(2t), 0)$

$= 18\left[-\cos(2t)\sin(2t) + \cos(2t)\sin(2t)\right]$

$= 18(0)$

$= 0$

1d) Since $\underset{\sim}{G} \cdot \frac{d\underset{\sim}{G}}{dt} = 0$, $\underset{\sim}{G}$ is perpendicular to $\frac{d\underset{\sim}{G}}{dt}$.

Hence,

$$\left| \underset{\sim}{G} \times \frac{d\underset{\sim}{G}}{dt} \right| = |\underset{\sim}{G}| \left| \frac{d\underset{\sim}{G}}{dt} \right| |\sin 90°|$$

$$= \sqrt{10}\sqrt{36}$$

$$= 6\sqrt{10}$$

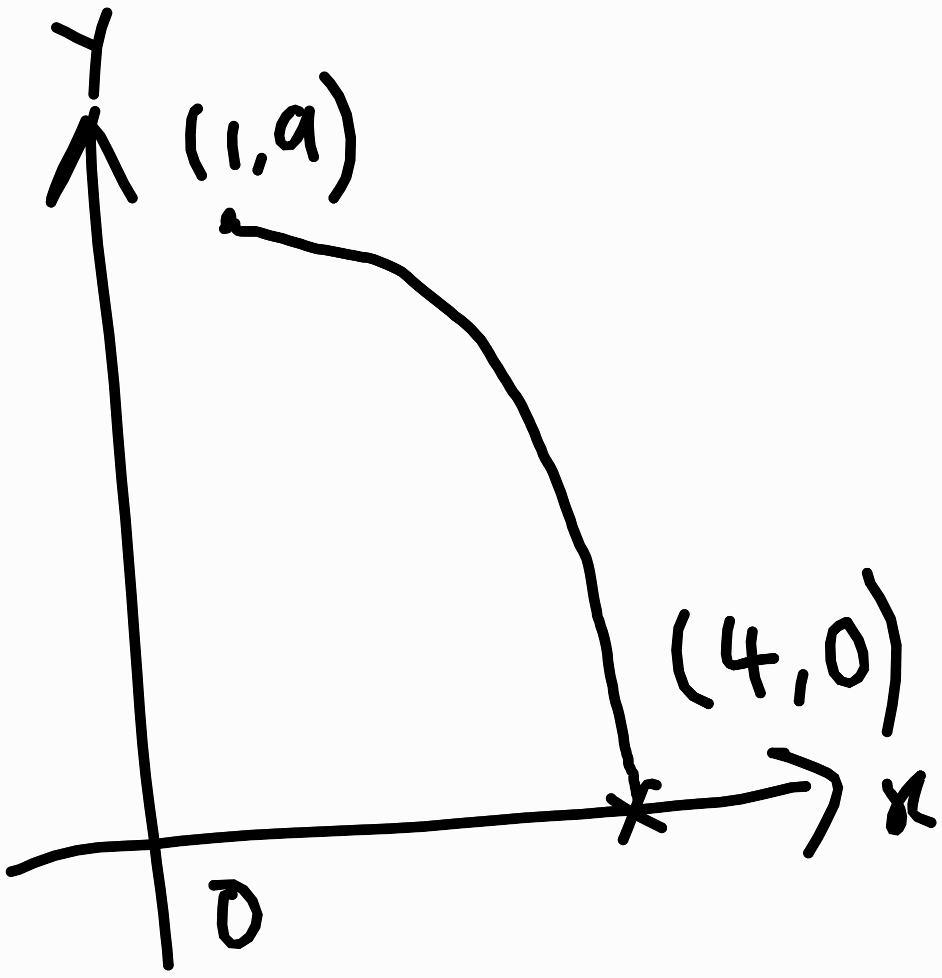
2a) $x = 1 + t$, $y = 9 - t^2$

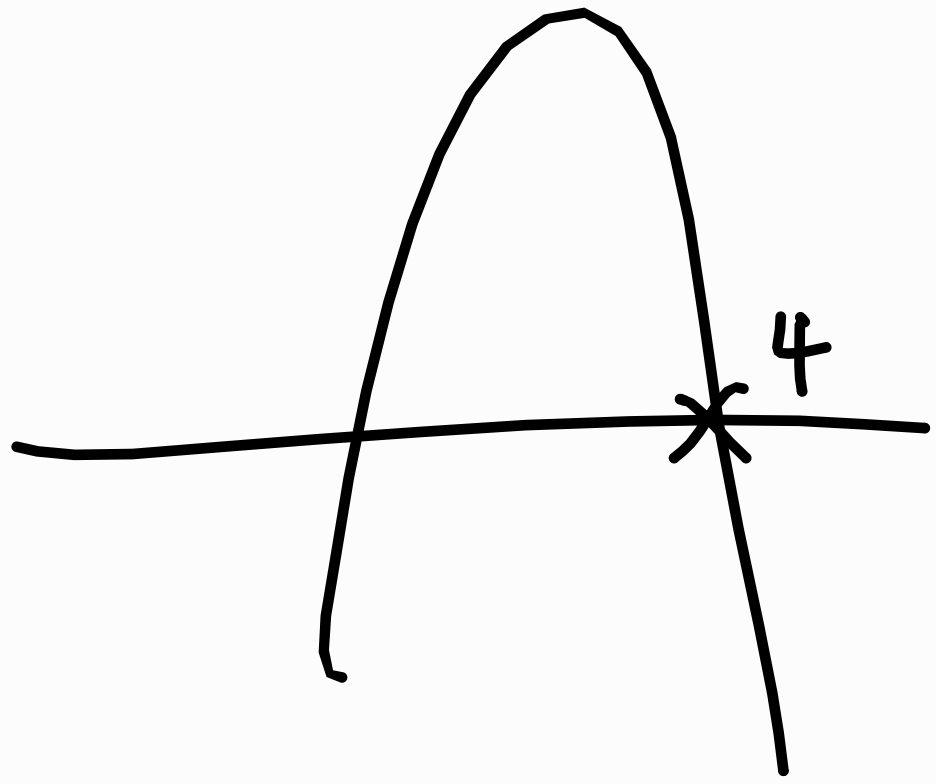
$t = x - 1$, $y = (3+t)(3-t)$

$$\therefore y = (3 + x - 1)(3 - (x-1))$$

$$= (x+2)(4-x)$$

2a)





b) $x = 1 + t$, $y = 9 - t^2$

$\frac{dx}{dt} = 1$ $\quad \frac{dy}{dt} = -2t$

when $t = 5$,

$\frac{dx}{dt} = 1$, $\frac{dy}{dt} = -2(5)$
$= 10$

$\vec{v} = 1\underset{\sim}{i} - 10\underset{\sim}{j}$

c) $\frac{d^2x}{dt^2} = 0$, $\frac{d^2y}{dt^2} = -2$

$\vec{F} = m\vec{a}$
$= 5(-2\underset{\sim}{j})$
$= -10\underset{\sim}{j}$ N

3) $\nabla \psi = \left(\dfrac{\partial \psi}{\partial x}, \dfrac{\partial \psi}{\partial y} \right)$

$$\frac{\partial \psi}{\partial x} = -2y\sin(2xy) + 4y\cos(2xy) + 4x^3 - 2$$

$$\psi = \int -2y\sin(2xy) + 4y\cos(2xy) + 4x^3 - 2 \, dx$$

$$= \frac{-2y\cos(2xy)}{-2y} + \frac{4y\sin(2xy)}{2y} + \frac{4}{4}x^4 - 2x + F(y)$$

$$= \cos(2xy) + 2\sin(2xy) + x^4 - 2x + F(y)$$

$$\frac{\partial \psi}{\partial y} = -2x\sin(2xy) + 4x\cos(2xy) + 5y^4 - 12y$$

$$= \int -2x\sin(2xy) + 4x\cos(2xy) + 5y^4 - 12y \, dy$$

$$= \frac{-2x\cos(2xy)}{-2x} + \frac{4x\sin(2xy)}{2x} + \frac{5}{5}y^5 - \frac{12}{2}y^2 + G(x)$$

$$= \cos(2xy) + 2\sin(2xy) + y^5 - 6y^2 + G(x)$$

$$\therefore \psi = \cos(2xy) + 2\sin(2xy) + x^4 - 2x + y^5 - 6y^2 + c, \; c \in \mathbb{R}$$

4) $f(x,y,z) = x^2+y^2+3z^2+xy+3xz+3yz+x+y+2z+10$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$\frac{\partial f}{\partial x} = 2x+y+3z+1$$

$$\frac{\partial f}{\partial y} = 2y+x+3z+1$$

$$\frac{\partial f}{\partial z} = 6z+3x+3y+2$$

When $\nabla f = \underset{\sim}{0}$,

$$2x+y+3z+11=0$$
$$2y+x+3z+11=0$$
$$6z+3x+3y+12=0$$

Solving,

$$x = -\frac{1}{3}-t, \quad y = -\frac{1}{3}-t, \quad z = t$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore$ These points lie on a straight line parallel to the line $-\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$.

5) $\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}\right)$

$$\frac{\partial \phi}{\partial x} = 3x^2 + 4y$$

$$\phi = \int 3x^2 + 4y \, dx$$

$$= x^3 + 4xy + F(y)$$

$$\frac{\partial \phi}{\partial y} = 4y + 4x + 3$$

$$\phi = \int 4y + 4x + 3 \, dy$$

$$= 2y^2 + 4xy + 3y + G(x)$$

$$\therefore \phi = x^3 + 4xy + 2y^2 + 3y$$

$$\phi(1,3) = 5$$

$$1 + 12 + 18 + 9 + c = 5$$

$$c = -35$$

$$\therefore \phi = x^3 + 4xy + 2y^2 + 3y - 35$$