9.(2a) Taking moments about B

$$R_A L = -2M_0$$

$$R_A = \frac{-2M_0}{L}$$

$$EIy'' = -\frac{2M_0 x}{L} + M_0$$

$$= \frac{M_0}{L}(L - 2x)$$

$$EIy' = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EIy = \frac{M_0}{L}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) + C_1 x + C_2$$

When $x = 0, y = 0$

$$C_2 = 0$$

when $x = L, y = 0$

$$0 = \frac{M_0}{L}\left(\frac{L^3}{2} - \frac{L^3}{3}\right) + C_1 L$$

$$-C_1 L = \frac{M_0 L^2}{6}$$

$$C_1 = -\frac{M_0 L}{6}$$

9.12a) $EI_y = \frac{M_0}{L}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) - \frac{M_0 L}{6}x$

$$EI_y = \frac{M_0}{L}\left(\frac{Lx^2}{2} - \frac{x^3}{3} - \frac{L^2}{6}x\right)$$

$$y = \frac{M_0}{EIL}\left(\frac{Lx^2}{2} - \frac{x^3}{3} - \frac{L^2}{6}x\right)$$

$$EIy' = \frac{M_0}{L}\left(Lx - x^2\right) - \frac{M_0 L}{6}$$

$$y' = \frac{M_0}{EIL}\left(Lx - x^2 - \frac{L^2}{6}\right)$$

when $y' = 0$

$$0 = Lx - x^2 - \frac{L^2}{6}$$

$$x^2 - Lx + \frac{L^2}{6} = 0$$

$$x = \frac{L \pm \sqrt{L^2 - 4(1)\left(\frac{L^2}{6}\right)}}{2(1)}$$

$$= L \pm \frac{\frac{L}{\sqrt{3}}}{2}$$

$$= \frac{1}{2}\left(L \pm \frac{L}{\sqrt{3}}\right)$$

9.12a) Since $0 \leq x \leq \frac{1}{2}L$,

$$x = \frac{1}{2}\left(L - \frac{L}{\sqrt{3}}\right)$$

$$= 0.211324865 4L$$

9.12b) $$y = \frac{M_0}{EIL}\left(\frac{Lx^2}{2} - \frac{x^3}{3} - \frac{L^2}{6}x\right)$$

$$-1.2\times10^{-3} = \frac{224\times10^3}{200\times10^9 \times 556\times10^{-6}L} \times$$

$$\left(\frac{L(0.211324L)^2}{2} - \frac{(0.211324L)^3}{3} - \frac{L^2(0.211324L)}{6}\right)$$

$$-1.2\times10^{-3} = \frac{7}{3475K}\left(-0.0160375074 8L^{\frac{2}{3}}\right)$$

$$L = 6.094675278\,m$$

$$\simeq 6.09\,m$$

9.43 a) Taking moments about A,

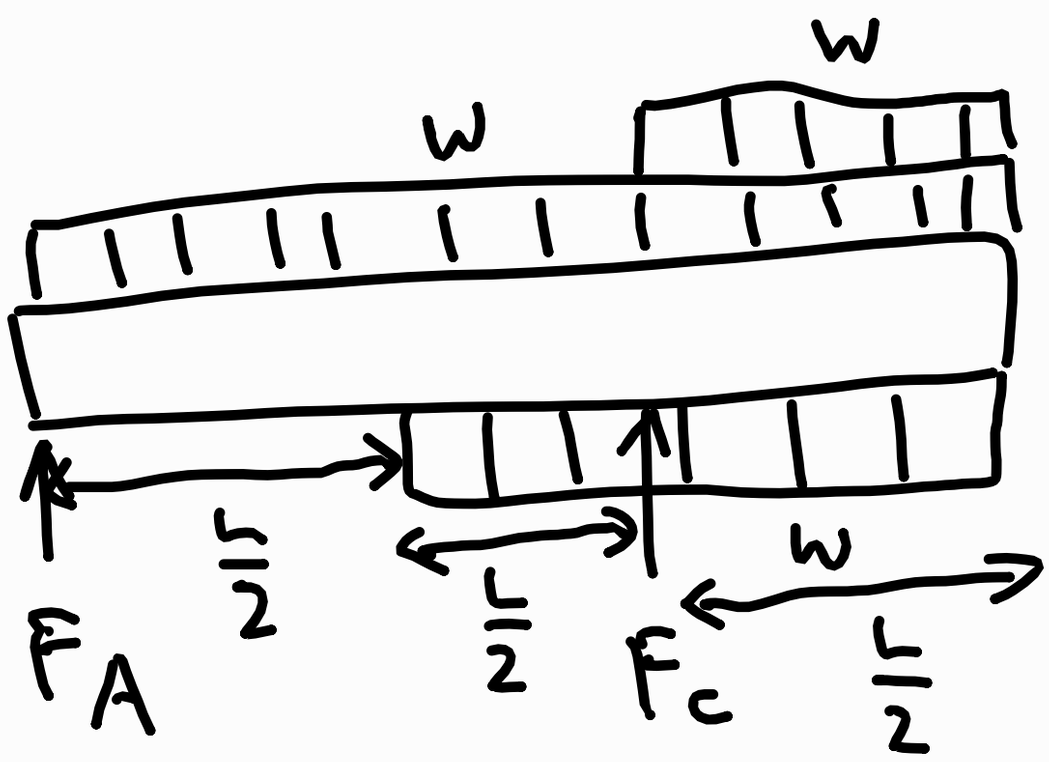
$$\frac{wL}{2} \times \frac{L}{4} + \frac{wL}{2} \times \frac{5}{4} L = F_c L$$

$$F_c = \frac{3wL}{4}$$

By equilibrium:

$$wL = \frac{3wL}{4} + F_A$$

$$F_A = \frac{wL}{4}$$



$$EIy'' = F_A x - \frac{1}{2} w x^2 + \frac{1}{2} w \langle x - \frac{L}{2} \rangle^2$$

$$+ F_c \langle x - L \rangle - \frac{1}{2} w \langle x - L \rangle^2$$

$$= \frac{1}{4} wLx - \frac{1}{2} wx^2 + \frac{1}{2} w \langle x - \frac{L}{2} \rangle^2$$

$$+ \frac{3}{4} wL \langle x - L \rangle - \frac{1}{2} w \langle x - L \rangle^2$$

9.43a) $EIy' = \frac{1}{8}wLx^2 - \frac{1}{6}wx^3 + \frac{1}{6}w\langle x-\frac{L}{2}\rangle^3$

$+ \frac{3}{8}wL\langle x-L\rangle^2 - \frac{1}{6}w\langle x-L\rangle^3 + C_1$

$EIy = \frac{1}{24}wLx^3 - \frac{1}{24}wx^4 + \frac{1}{24}w\langle x-\frac{L}{2}\rangle^4$

$+ \frac{1}{8}wL\langle x-L\rangle^3 - \frac{1}{24}w\langle x-L\rangle^4 + C_1x + C_2$

when $x=0, y=0$

$C_2 = 0$

when $x=L, y=0$

$0 = \frac{1}{24}wL^4 - \frac{1}{24}wL^4 + \frac{1}{384}wL^4 + C_1L$

$C_1 = -\frac{1}{384}wL^3$

$EIy = \frac{1}{24}wLx^3 - \frac{1}{24}wx^4 + \frac{1}{24}w\langle x-\frac{L}{2}\rangle^4$

$+ \frac{1}{8}wL\langle x-L\rangle^3 - \frac{1}{24}w\langle x-L\rangle^4 - \frac{1}{384}wL^3x$

$y = \frac{w}{24EI}\left(Lx^3 - x^4 + \langle x-\frac{L}{2}\rangle^4 + 3L\langle x-L\rangle^3 - \langle x-L\rangle^4 - \frac{1}{16}L^3x\right)$

9.43 b) when $x = \frac{L}{2}$,

$$y = \frac{w}{24EI}\left(\frac{1}{8}L^4 - \frac{1}{16}L^4 - \frac{1}{32}L^4\right)$$

$$= \frac{wL^4}{768EI}$$

9.43 c) when $x = \frac{3L}{2}$,

$$y = \frac{w}{24EI}\left(\frac{27}{8}L^4 - \frac{81}{16}L^4 + L^4 + \frac{3}{8}L^4 - \frac{1}{16}L^4 - \frac{3}{32}L^4\right)$$

$$= \frac{-5wL^4}{256EI}$$

9.47a) Taking moments at B

$$5.4F_A = 1.8(3\times3.6) + 1.8(6.2)$$

$$F_A = \frac{17}{3}\,kN$$

$$EIy'' = \frac{17}{3}x - \frac{3}{2}\langle x-1.8\rangle^2 - 6.2\langle x-3.6\rangle$$

$$EIy' = \frac{17}{6}x^2 - \frac{1}{2}\langle x-1.8\rangle^3 - \frac{31}{10}\langle x-3.6\rangle^2 + C_1$$

$$EIy = \frac{17}{18}x^3 - \frac{1}{8}\langle x-1.8\rangle^4 - \frac{31}{30}\langle x-3.6\rangle^3 + C_1x + C_2$$

when $x=0$, $y=0$

$$C_2 = 0$$

when $x=5.4$, $y=0$

$$0 = \frac{17}{18}(5.4)^3 - \frac{1}{8}(3.6)^4 - \frac{31}{30}(1.8)^3 + 5.4C_1$$

$$C_1 = -22.536$$

$$E = 200\times10^6\,kPa$$

$$I = 129\times10^{-6}\,m^4$$

9.47a) when $x = 0$,

$200 \times 10^6 \times 129 \times 10^{-6}\, y' = -22.536$

$y' = -8.734883721 \times 10^{-4}$ rad

$\simeq 8.73 \times 10^{-4}$ rad

9.47b) when $x = 2.7$,

$200 \times 10^6 \times 129 \times 10^{-6}\, y = \frac{17}{18}(2.7)^3 - \frac{1}{8}(0.9)^4 - 22.536(2.7)$

$y = -1.641074128 \times 10^{-3}$ m

$\simeq -1.64 \times 10^{-3}$ m

9.16(a) Taking moments about B,

$$4.8F_A = 3.6(30\times 2.4) + 50\times 2.4$$

$$F_A = 79\,kN$$

Bending moment at C,

$$M_C = 50\times 1.2$$

$$= 60\,kNm$$

$$EIy'' = 79x - \frac{30}{2}x^2 + \frac{30}{2}\langle x-2.4\rangle^2 - 50\langle x-3.6\rangle$$

$$- 60\langle x-3.6\rangle^0$$

$$= 79x - 15x^2 + 15\langle x-2.4\rangle^2 - 50\langle x-3.6\rangle$$

$$- 60\langle x-3.6\rangle^0$$

$$EIy' = \frac{79}{2}x^2 - 5x^3 + 5\langle x-2.4\rangle^3 - 25\langle x-3.6\rangle^2$$

$$- 60\langle x-3.6\rangle + C_1$$

$$EIy = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x-2.4\rangle^4 - \frac{25}{3}\langle x-3.6\rangle^3$$

$$- 30\langle x-3.6\rangle^2 + C_1x + C_2$$

9.161a) When $x = 0, y = 0$

$$C_2 = 0$$

When $x = 4.8, y = 0$

$$0 = \frac{79}{6}(4.8)^3 - \frac{5}{4}(4.8)^4 + \frac{5}{4}(2.4)^4 - \frac{25}{3}(1.2)^3$$
$$- 30(1.2)^2 + 4.8C_1$$

$$C_1 = -161.76 \text{ kN m}^2$$

$$EI_y' = \frac{79}{2}x^2 - 5x^3 + 5\langle x-2.4\rangle^3 - 25\langle x-3.6\rangle^2$$
$$- 60\langle x-3.6\rangle - 161.76$$

$$EI_y = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x-2.4\rangle^4 - \frac{25}{3}\langle x-3.6\rangle^3$$

$$- 30\langle x-3.6\rangle^2 - 161.76x$$

$$E = 200 \times 10^6 \text{ kPa}$$

when $x = 0$

$$200\times10^6 \times 212\times10^{-6}\, y' = -161.76$$

$$y' = -3.815094434 \times 10^{-3} \text{ rad}$$
$$\approx -3.82 \times 10^{-3} \text{ rad}$$

b) when $x = 2.4$,

$$200\times10^6 \times 212\times10^{-6}\, y = \frac{79}{6}(2.4)^3 - \frac{5}{4}(2.4)^4 - 161.76(2.4)$$

$$y = -5.841509434 \times 10^{-3} \text{ m}$$

$$\approx -5.84 \times 10^{-3} \text{ m}$$