1) $$\cancel{\rho c_p \frac{\partial T}{\partial t}} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}$$

$$\cancel{+\dot{e}_g}$$

$$0 = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)$$

$$0 = k\frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$\int \frac{d^2 T}{dx^2}\, dx = 0$$

$$\frac{dT}{dx} = C_1$$

$$\int \frac{dT}{dx}\, dx = C_1$$

$$T(x) = C_1 x + C_2$$

1a) when $x=0$, $T=T_1$,

$$C_2 = T_1$$

when $x=L$, $T=T_2$

$$T_2 = C_1 L + T_1$$

$$C_1 = \frac{T_2 - T_1}{L}$$

$$\therefore T(x) = \frac{T_2 - T_1}{L} x + T_1$$

b) when $x=0$, $\frac{dT}{dx} = 0$,

$$C_1 = 0$$

when $x=L$, $T=T_S$,

$$C_2 = T_S$$

$$\therefore T(x) = T_S$$

2 a) $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)} + \cancel{\dot{e}_{gen}} = \cancel{\rho c \frac{\partial T}{\partial t}}$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = 0$$

$$\frac{d^2 T}{dx^2} = 0$$

b) $\int \frac{d^2 T}{dx^2} dx = 0$

$$\frac{dT}{dx} = c_1$$

$$\int \frac{dT}{dx} dx = c_1$$

$$T(x) = c_1 x + c_2$$

when $x = 0$,

$$\dot{q} = k\left(-\frac{dT}{dx}\bigg|_{x=0}\right)$$

$$\frac{\dot{Q}}{A} = k(-c_1)$$

$$c_1 = -\frac{\dot{Q}}{kA}$$

2b) $c_1 = \dfrac{-800}{160 \times (10^{-2})^2 \times 60}$

$= -\dfrac{2500}{3}$

when $x = 0.6 \times 10^{-2}$, $T = 112°C$

$112 = -\dfrac{2500}{3}(0.6 \times 10^{-2}) + c_2$

$c_2 = 117$

$\therefore T(x) = -\dfrac{2500}{3}x + 117$

c) When $x = 0$,

$T(x=0) = -\dfrac{2500}{3}(0) + 117$

$= 117°C$

3) $\cancel{\rho c_p \frac{\partial T}{\partial t}} = \frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi}\right)} + \cancel{\frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right)} + \dot{e}_{g}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + \dot{e}_{gen} = 0$$

$$\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) = -\frac{\dot{e}_{gen}}{k} r$$

$$\int \frac{d}{dr}\left(r \frac{dT}{dr}\right) dr = -\int \frac{\dot{e}_{gen}}{k} r \, dr$$

$$r \frac{dT}{dr} = -\frac{\dot{e}_{gen}}{2k} r^2 + D$$

$$\frac{dT}{dr} = -\frac{\dot{e}_{gen}}{2k} r + \frac{D}{r}$$

$$\int \frac{dT}{dr} dr = -\int \frac{\dot{e}_{gen}}{2k} r - \frac{D}{r} \, dr$$

$$T(r) = -\left(\frac{\dot{e}_{gen}}{4k} r^2 - D \ln|r|\right) + c_2$$

$$= -\frac{\dot{e}_{gen}}{4k} r^2 + c_1 \ln|r| + c_2$$

3) when $r=0$,

$$c_1 = 0$$

when $r = r_0$, $T = 105$,

$$105 = -\frac{\dot{e}gen}{4k} r_0^2 + c_2$$

$$c_2 = \frac{\dot{e}gen}{4k} r_0^2 + 105$$

$$T(r) = -\frac{\dot{e}gen}{4k} r^2 + \frac{\dot{e}gen}{4k} r_0^2 + 105$$

$$= \frac{\dot{e}gen}{4k}(r_0^2 - r^2) + 105$$

$$\dot{E}gen = 2kW$$

$$\dot{e}gen = \frac{2}{V}$$

$$= \frac{2}{\pi r^2 h}$$

$$= \frac{2}{\pi(2\times10^{-3})^2 \times 0.5}$$

$$= 318309.8862 \, kWm^{-3}$$

3) When $r=0$,

$$T=\frac{318309.8862\times10^{3}}{4(15)}(2\times10^{-3})^{2}+105$$

$$=126.2206591^{\circ}C$$

$$\approx 126^{\circ}C$$

4) $\cancel{\rho c_p \frac{\partial T}{\partial t}} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)} + \dot{e}_{gen}$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{e}_{gen} = 0$$

$$k\frac{d^2T}{dx^2} + ax^2 = 0$$

$$\frac{d^2T}{dx^2} = -\frac{ax^2}{k}$$

$$\int \frac{d^2T}{dx^2}\,dx = -\int \frac{ax^2}{k}\,dx$$

$$\frac{dT}{dx} = -\frac{ax^3}{3k} + c_1$$

$$\int \frac{dT}{dx} = \int -\frac{ax^3}{3k} + c_1\,dx$$

$$T = -\frac{ax^4}{12k} + c_1x + c_2$$

5b) When $x=0$, $T=T_0$

$$C_2 = T_0$$

When $x=L$, $\frac{dT}{dx}=0$

$$0 = -\frac{aL^3}{3k} + c_1$$

$$C_1 = \frac{aL^3}{3k}$$

$$T = -\frac{ax^4}{12k} + \frac{aL^3}{3k}x + T_0$$

c) Highest temperature point is when $\frac{dT}{dx}=0$

$$0 = -\frac{ax^3}{3k} + \frac{aL^3}{3k}$$

$$\frac{\cancel{a}x^{\cancel{3}}}{\cancel{3k}} = \frac{\cancel{a}L^{\cancel{3}}}{\cancel{3k}}$$

$$x = L$$

$$T = -\frac{135(0.3)^4 \times 10^3}{12(9)} + \frac{135(0.3)^3 \times 10^3}{3(9)} \times 0.3 + 400$$

$$= 430.375\,^\circ C$$

$$\approx 430\,^\circ C$$