

Using a body attached frame on A called f,

$$\vec{v}_{B/f} = 0$$

$$\vec{v}_B = \vec{v}_{B/f} + \vec{v}_{B'}$$

$$\vec{v}_B = \vec{v}_{B/f} + \vec{v}_A + \vec{\omega}_f \times \vec{r}_{BA}$$

$$v_B \angle -70^\circ = 0 + 900\hat{i} + \dot{\theta}\hat{k} \times 300 \angle -30^\circ$$

$$v_B (0.342\hat{i} - 0.939692\hat{j}) = 900\hat{i} + 300\dot{\theta}\hat{k} \times (\frac{\sqrt{3}}{2}\hat{i} - 0.5\hat{j})$$

$$v_B (0.342\hat{i} - 0.939692\hat{j}) = 900\hat{i} + 300\dot{\theta}(\frac{\sqrt{3}}{2}\hat{j} + 0.5\hat{i})$$

$$0.342v_B\hat{i} - 150\dot{\theta}\hat{i} - 0.939692v_B\hat{j} - 150\sqrt{3}\hat{j} = 400\hat{i} + 0\hat{j}$$

Solving,

$$v_B = 1017.464287 \text{ mms}^{-1}$$

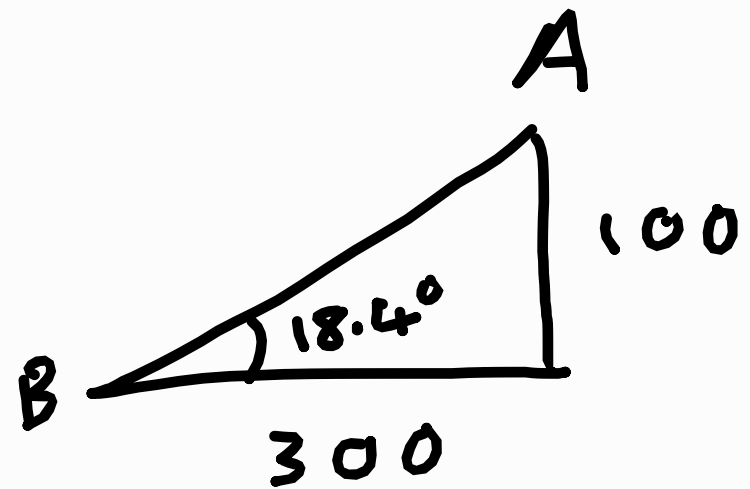
$$\dot{\theta} = -3.68004471 \text{ rads}^{-1}$$

$$\therefore \vec{v}_B \approx 1017 \angle -70^\circ$$

$$\vec{\omega} \approx -3.68 \hat{k}$$

2a) A and E are fixed points, so relative velocity and acceleration with respect to them are equal.

Length of AB is constant  
so B is circular motion about A



$$\vec{v}_B = \vec{\omega} \times \vec{r}_{BA}$$

$$= 3\vec{k} \times (-300\vec{i} - 100\vec{j})$$

$$= 300\vec{j} - 400\vec{i}$$

$$\vec{v}_B = \vec{v}_{B/f} + \vec{v}_{B'}$$

$$= 0 + \vec{v}_D + \vec{\omega}_{f_{BD}} \times \vec{r}_{B'D} \rightarrow$$

$$= \vec{v}_D + \omega_{f_{BD}} \vec{k} \times (200\vec{j})$$

$$= \vec{v}_D - 200\omega_{f_{BD}}\vec{i}$$

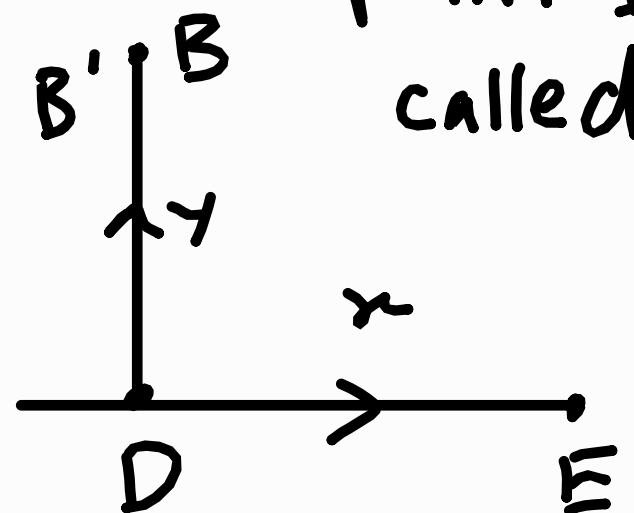
$$= \omega_{f_{DE}} \times \vec{r}_{DE} - 200\omega_{f_{BD}}\vec{i}$$

D is in circular motion about E

$$= \omega_{f_{DE}} \vec{k} \times (-225\vec{j}) - 200\omega_{f_{BD}}\vec{i}$$

$$= -200\omega_{f_{BD}}\vec{i} - 225\omega_{f_{DE}}\vec{j}$$

Attach body-attached frame at point D called f



B is in circular motion about D

ijkij

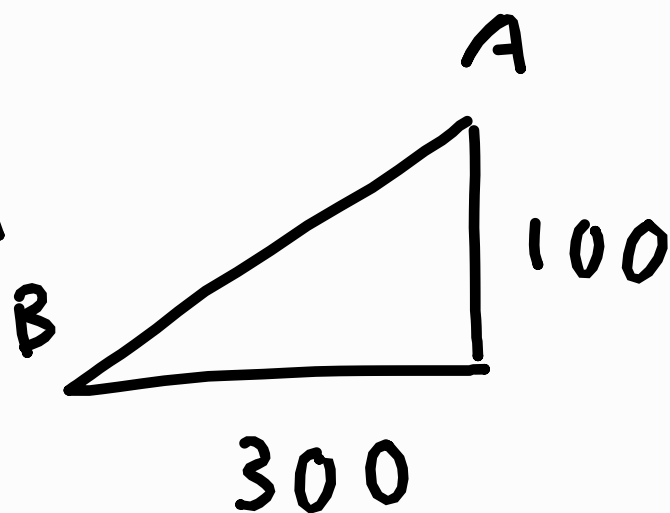
$$2a) \quad 300\hat{i} - 900\hat{j} = -200\omega_{BD}\hat{i} - 225\omega_{DE}\hat{j}$$

Solving,

$$\omega_{BD} = -1.5 \text{ rad/s}$$

$$\omega_{DE} = 4 \text{ rad/s}$$

$$2b) \quad \vec{a}_B = \cancel{\vec{a}_A} + \cancel{\vec{\alpha} \times \vec{r}_{B/A}} - \omega^2 \vec{r}_{B/A}$$



$$= -\omega^2 \vec{r}_{B/A}$$

$$= -3^2 (-300\hat{i} - 100\hat{j})$$

$$= 2700\hat{i} + 900\hat{j}$$

$$\vec{a}_B = \cancel{\vec{a}_{B/f}} + \vec{a}_{B'} + \cancel{2\vec{\omega} \times \vec{r}_{B/f}}$$

$$= \vec{a}_{B'}$$

$$= \cancel{\vec{a}_{BD}} + \vec{\alpha}_{BD} \times \vec{r}_{BD} - \omega_{BD}^2 \vec{r}_{BD}$$



$$+ \cancel{\vec{a}_{DE}} + \vec{\alpha}_{DE} \times \vec{r}_{DE} - \omega_{DE}^2 \vec{r}_{DE}$$

$\hat{i} \hat{j} \hat{k}$

$$= \alpha_{BD} \hat{k} \times (200\hat{j}) - (-1.5)^2 (200\hat{j}) + \alpha_{DE} \hat{k} \times (-225\hat{i})$$

$$- 4^2 (-225\hat{i})$$

$$= -200\alpha_{BD}\hat{i} - 450\hat{j} - 225\alpha_{DE}\hat{j} + 3600\hat{i}$$

$$2b) \quad 2700 = -200\alpha_{BD} + 3600$$

$$\alpha_{BD} = 4.5$$

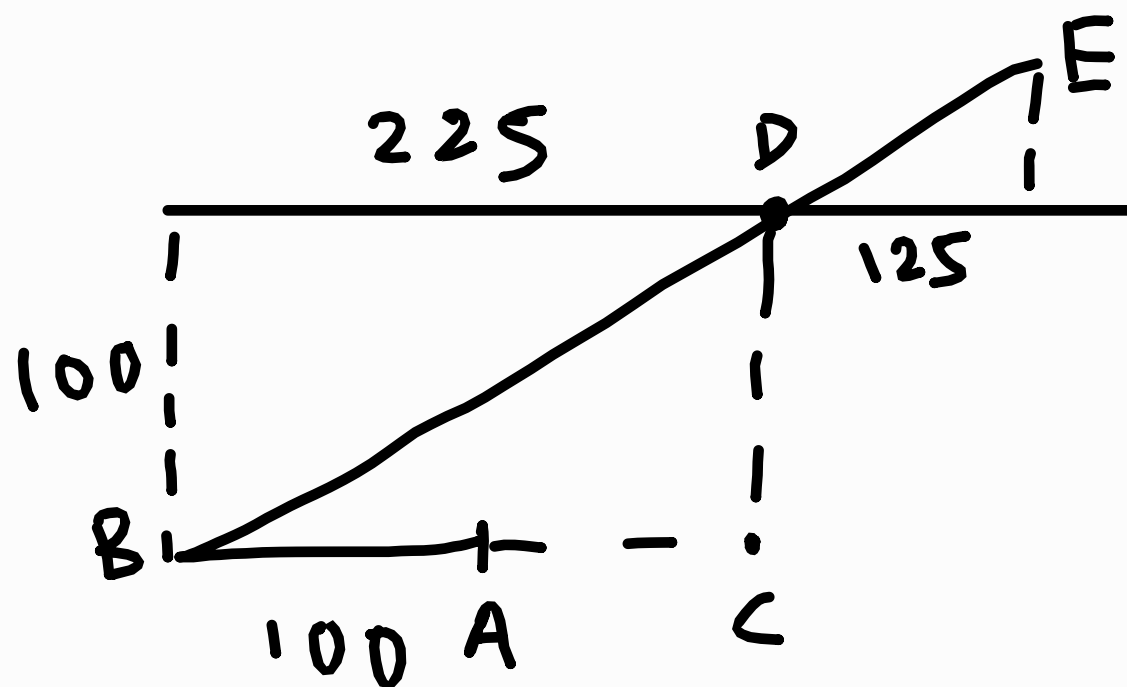
$$\Rightarrow \vec{\alpha}_{BD} = 4.5 \underline{k} \text{ rad s}^{-2}$$

$$900 = -225\alpha_{DE} - 450$$

$$\alpha_{DE} = 6$$

$$\Rightarrow \vec{\alpha}_{DE} = 6 \underline{k} \text{ rad s}^{-2}$$

3a) Let  $C$  be the instant centre.



$$b) \vec{v}_D = -120\hat{j}$$

$$\vec{v}_D = \vec{r}_{DC} \times \vec{\omega}$$

$$-120\hat{j} = 100\hat{j} \times \omega\hat{k}$$

$$-120\hat{j} = 100\omega(\hat{i})$$

$$\omega = 1.2 \rightarrow \vec{\omega}_{BE} = 1.2\hat{k} \text{ rads}^{-1}$$

$$\vec{v}_B = \vec{r}_{BC} \times \vec{\omega}$$

$$\hat{i} \quad \hat{j} \quad \hat{k}$$

$$= -225\hat{i} \times 1.2\hat{k}$$

$$= 270\hat{j}$$

$$\vec{v}_B = \vec{r}_{BA} \times \vec{\omega}_{BA}$$

$$270\hat{j} = -100\hat{i} \times \omega_{BA}\hat{k}$$

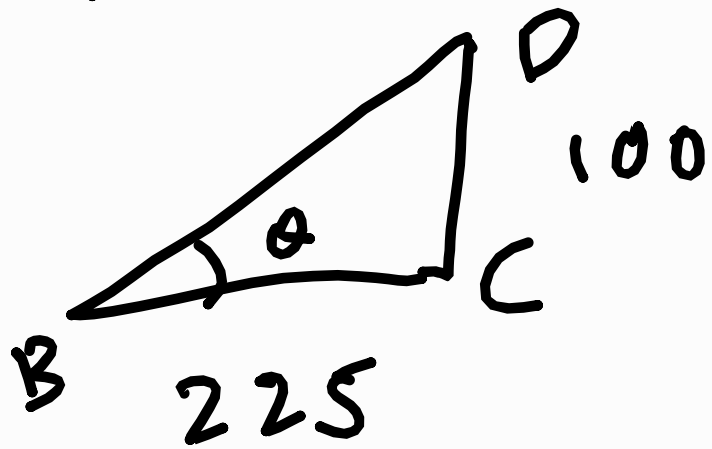
$$270\hat{j} = 100\omega_{BA}\hat{j}$$

$$\omega_{BA} = 2.7 \rightarrow \vec{\omega}_{BA} = 2.7\hat{k} \text{ rads}^{-1}$$

$$3c) \Rightarrow \vec{v}_E = \vec{r}_{EC} \times \vec{\omega} \quad i \ j \ k;$$

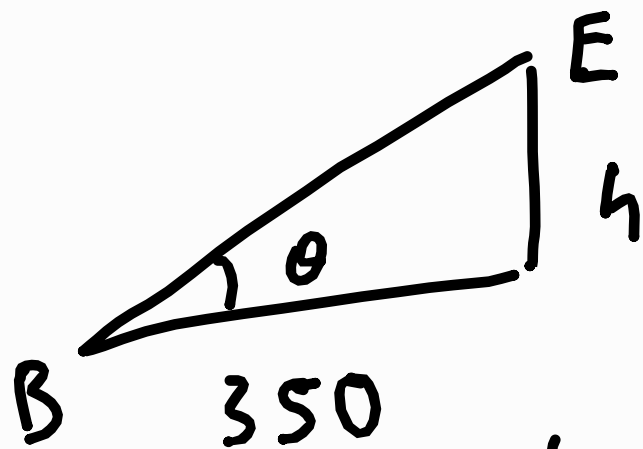
$$= \left( 125 \hat{i} + \frac{1400}{9} \hat{j} \right) \times 1.2 \hat{k}$$

$$= -\frac{560}{3} \hat{i} + 150 \hat{j}$$



$$\theta = \tan^{-1} \left( \frac{100}{225} \right)$$

$$\theta = 23.96^\circ$$



$$\tan \theta = \frac{h}{350}$$

$$\frac{100}{225} = \frac{h}{350}$$

$$h = \frac{1400}{9}$$

$$\begin{aligned}
 4a) \vec{v}_A &= \vec{\omega}_{AO} \times \vec{r}_{AO} && i \quad j \quad k \quad i \quad j \\
 &= \omega_{AO} \underline{\hat{k}} \times (4\underline{\hat{i}} + 3\underline{\hat{j}}) \\
 &= 4\omega_{AO}\underline{\hat{j}} - 3\omega_{AO}\underline{\hat{i}}
 \end{aligned}$$

Attach a frame at E,

$$\begin{aligned}
 \vec{v}_A &= \cancel{\vec{v}_{A|f}} + \vec{v}_E + \vec{\omega}_f \times \vec{r}_{AE} \\
 &= 0.7\underline{\hat{i}} + \omega_f \underline{\hat{k}} \times (-8\underline{\hat{i}} + 8\underline{\hat{j}}) \\
 &= 0.7\underline{\hat{i}} - 8\omega_f \underline{\hat{j}} - 8\omega_f \underline{\hat{j}}
 \end{aligned}$$

$$-3\omega_{AO} = 0.7 - 8\omega_f$$

$$-3\omega_{AO} + 8\omega_f = 0.7 \quad (1)$$

$$4\omega_{AO} = -8\omega_f$$

$$4\omega_{AO} + 8\omega_f = 0 \quad (2)$$

Solving,

$$\omega_{AO} = -0.1 \text{ rads}^{-1}$$

$$\omega_f = 0.05 \text{ rads}^{-1}$$

$$\therefore \omega_f = \omega_{ABDE} = 0.05 \underline{\hat{k}} \text{ rads}^{-1}$$

$$4b) \vec{v}_A = 4\omega_A \vec{j} - 3\omega_A \vec{i} \\ = 0.3\vec{i} - 0.4\vec{j} \text{ rad s}^{-1}$$

$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A \quad \begin{matrix} i & j & k & i & j \end{matrix}$$

$$= 0.3\vec{i} - 0.4\vec{j} + \vec{\omega} \times \vec{r}_{BA}$$

$$= 0.3\vec{i} - 0.4\vec{j} + 0.05\vec{k} \times (-8\vec{j})$$

$$= 0.7\vec{i} - 0.4\vec{j}$$

$$\vec{v}_D = \vec{v}_A + \vec{v}_{D/A}$$

$$= 0.3\vec{i} - 0.4\vec{j} + \vec{\omega} \times \vec{r}_{DA}$$

$$= 0.3\vec{i} - 0.4\vec{j} + 0.05\vec{k} \times (8\vec{j})$$

$$= 0.3\vec{i}$$

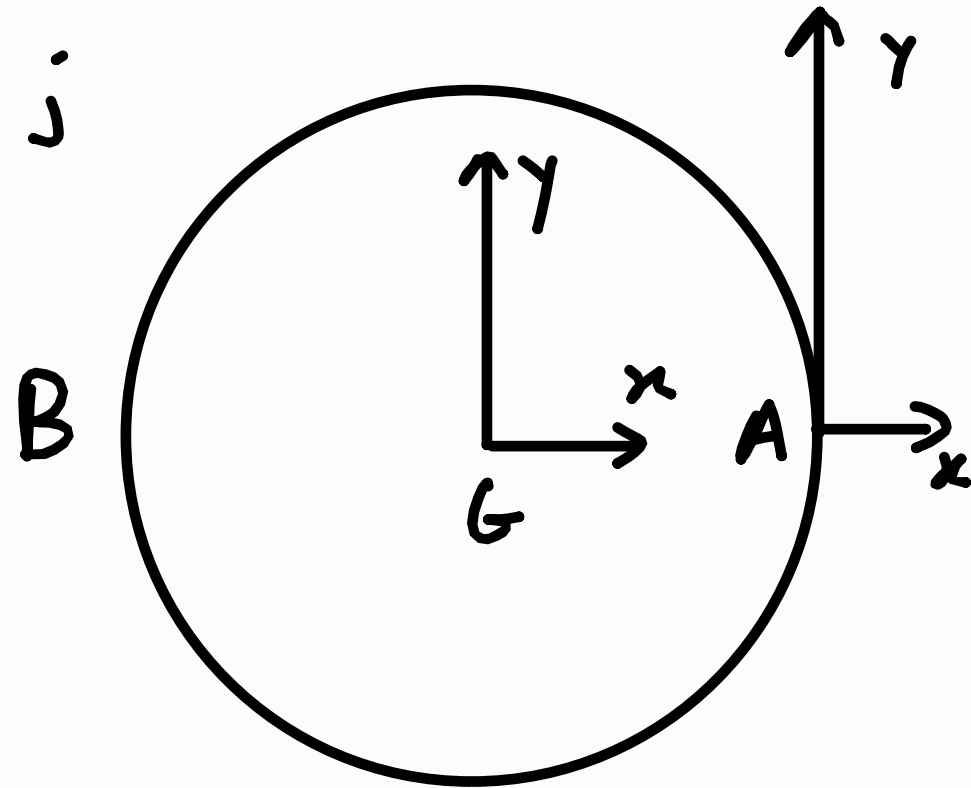


$$5a) \vec{v}_B = \vec{v}_{B/A} + \vec{v}_{B'} \quad i \ j \ k \ i \ j$$

$$= \vec{v}_A + \vec{\omega}_f \times \vec{r}_{BA}$$

$$= -1.8 \hat{j} + \omega_f \hat{k} \times (-1.2 \hat{i})$$

$$= -1.8 \hat{j} - 1.2 \omega_f \hat{j}$$



$$-1.5 = -1.8 - 1.2 \omega_f$$

$$\omega_f = -0.25$$

$$\vec{a}_B = \vec{a}_{B/f} + \vec{a}_{B'} + 2\vec{\omega}_f \times \vec{v}_{B/f}$$

$$= \vec{a}_A + \alpha \times \vec{r}_{BA} - \omega_f^2 \vec{r}_{BA}$$

$$i \ j \ k \ i \ j$$

$$= 0.6 \hat{j} + \alpha \hat{k} \times (-1.2 \hat{i}) - (-0.25)^2 (-1.2 \hat{i}) + \vec{a}_A^n$$

$$= 0.6 \hat{j} - 1.2 \alpha \hat{j} + 0.075 \hat{i} + a_B^n \hat{i}$$

$$-0.45 = 0.6 - 1.2 \alpha$$

$$\alpha = 0.875 \text{ rads}^{-1}$$

$$5) \vec{v}_B = -1.8\hat{j} - 1.2\omega_f\hat{j} \\ = -1.5\hat{j}$$

$$\vec{a}_B = \vec{a}_{B/f_2} + \vec{a}_{B'} + 2\vec{\omega}_f \times \vec{v}_{B/f_2}$$

$$= \vec{a}_G + \vec{\alpha} \times \vec{r}_{BG} - \omega_f^2 \vec{r}_{BG}$$

$$= a_G \hat{j} + 0.875\hat{k} \times (-0.6\hat{i}) - (-0.25)^2 (-0.6\hat{i})$$

$$= a_G \hat{j} - 0.525\hat{j} + \underline{0.0375\hat{i}}$$

$$\therefore \vec{a}_B = 0.0375\hat{i} - 0.45\hat{j}$$