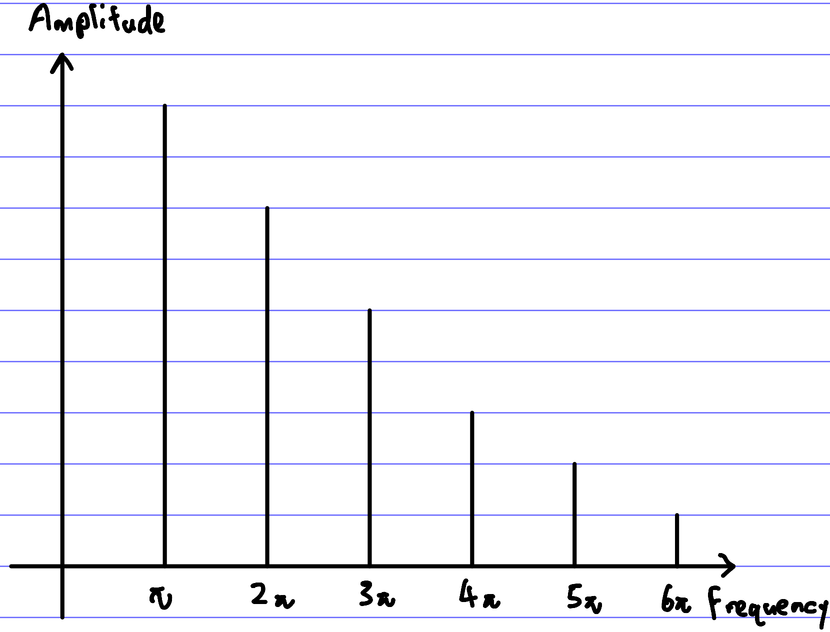
1(a) $F(t) = \sum_{n=1}^{\infty} \frac{4\sqrt{2}}{(2n-1)} e^{-\frac{2n-1}{2}} \{\cos((2n-1)\pi t) + \sin((2n-1)\pi t)\}$

When $n=1$, $F(t) = 4\sqrt{2} e^{-\frac{1}{2}} (\cos \pi t + \sin \pi t)$

When $n=2$, $F(t) = \frac{4\sqrt{2}}{3} e^{-\frac{1}{3}} (\cos 3\pi t + \sin 3\pi t)$

When $n=3$, $F(t) = \frac{4\sqrt{2}}{5} e^{-\frac{1}{5}} (\cos 5\pi t + \sin 5\pi t)$

$\vdots$



1b) $D_n = \frac{A_n - jB_n}{2}$

$= \frac{4\sqrt{2}}{4n-2} e^{-\frac{2n-1}{2}} (1-j)$

let $w_0 = \pi$,

$\therefore F(t) = \sum_{n=-\infty}^{\infty} \frac{4}{4n-2} (1-j) e^{-\frac{2n-1}{2}} e^{j(2n-1)w_0 t}$

$= \sum_{n=-\infty}^{\infty} \frac{4}{4n-2} (1-j) e^{(2n-1)(j-\frac{1}{2})w_0 t}$

c) $f_s > 2f_{max}$

$f_s > 2(5)$

$\therefore f_s > 10\,Hz$

d) $C_n = \sqrt{A_n^2 + B_n^2}$

$\phi_n^* = \arctan\left(\frac{A_n}{B_n}\right)$

$C_n = \sqrt{\left(\frac{4\sqrt{2}}{2n-1} e^{-\frac{2n-1}{2}}\right)^2 + \left(\frac{4\sqrt{2}}{2n-1} e^{-\frac{2n-1}{2}}\right)^2}$

$= \sqrt{2}\left(\frac{4\sqrt{2}}{2n-1} e^{-\frac{2n-1}{2}}\right)$

$= \frac{8}{2n-1} e^{-\frac{2n-1}{2}}$

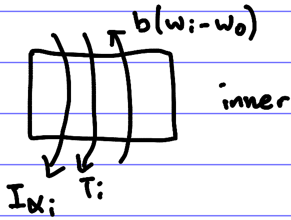
1d) $\phi_n^* = \arctan\left(\frac{A_n}{B_n}\right)$

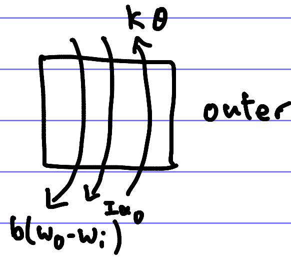
$= \arctan(1) \quad \because A_n = B_n$

$= \frac{\pi}{4}$

$$\therefore F(t) = \sum_{n=1}^{\infty} \frac{8}{2n-1} e^{-\frac{2n-1}{2}} \sin\left((2n-1)\pi t + \frac{\pi}{4}\right)$$

3a)





$T_i = b(w_i - w_o)$

$k\theta = b(w_o - w_i)$

$k\ddot{\theta}$ = rotational force - damper force

$I\alpha$ = rotational force + damper force

b) Let $\theta_o = Ae^{jwt}$, $w_i = Be^{jwt}$

$w_o = jwAe^{jwt}$

$k(Ae^{jwt}) = b(jwAe^{jwt} - Be^{jwt})$

$A\cancel{e^{jwt}}(k - bjw) = -bB\cancel{e^{jwt}}$

$bB = A(bjw - k)$

$\frac{A}{B} = \frac{b}{bjw - k}$

$\frac{\theta_o}{w_i} = \frac{A\cancel{e^{jwt}}}{B\cancel{e^{jwt}}}$

$= \frac{A}{B}$

$= \frac{b}{bjw - k}$

3ci) $H = \frac{b}{bjw - k}$

$$|H| = \frac{b}{\sqrt{b^2w^2 + k^2}}$$

$$\frac{|H|}{b} = \frac{1}{\sqrt{b^2w^2+k^2}}$$

$$\frac{b}{|H|} = \sqrt{b^2w^2+k^2}$$

$$\frac{b^2}{H^2} = b^2w^2 + k^2$$

$$\frac{b^2}{H^2} - b^2w^2 = k^2$$

$$b^2\left(\frac{1}{H^2} - w^2\right) = k^2$$

$$\frac{b^2}{k^2} = \frac{1}{\frac{1}{H^2} - w^2}$$

$$\frac{b}{k} = \frac{1}{\sqrt{\frac{1}{H^2} - w^2}}$$

At $H = 5$, $w = 0.01$,

$$\frac{b}{k} = \frac{1}{\sqrt{\frac{1}{5^2} - 0.01^2}}$$

$$= 5.006261743$$

$$\approx 5.006$$

3c ii) $H = \frac{b}{j\omega b - k}$

$$\lim_{\omega \to 0} \frac{b}{j\omega b - k} = -\frac{b}{k}$$

$$\lim_{\omega \to \infty} \frac{b}{j\omega b - k} = 0$$

At $\frac{1}{\sqrt{2}}$ of the initial value:

$$|H| = \frac{b}{\sqrt{\omega^2 b^2 + k^2}} = \frac{\frac{b}{k}}{\sqrt{2}}$$

$$\frac{\cancel{b}}{\sqrt{\omega^2 b^2 + k^2}} = \frac{b}{\sqrt{2}k}$$

$$\sqrt{2}k = \sqrt{\omega^2 b^2 + k^2}$$

$$2k^2 = \omega^2 b^2 + k^2$$

$$2k^2 = \omega^2 b^2 + k^2$$

$$k^2 = \omega^2 b^2$$

$$\omega^2 = \frac{k^2}{b^2}$$

$$\omega = \frac{k}{b}$$

$$= \frac{1}{5.006}$$

$$= 0.1997498436$$

$$\approx 0.200$$

3d) $w^2 = \frac{k}{I}$

$$I = \frac{k}{w^2}$$

$$= \frac{10}{0.200^2}$$

$$= 250.6265664$$

$$\approx 250.63$$

4a) $E(t) - iR - L\frac{di}{dt} - e = 0$

$$E(t) = iR + L\frac{di}{dt} + e$$

b) $f = m\ddot{x} + c\dot{x} + kx$

$$f = m\ddot{x} + cv + kx$$

c) $E(t) = iR + L\frac{di}{dt} + e$

$$\frac{di}{dt} = \frac{E(t) - iR - e}{L}$$

$$\dot{i} = \frac{E(t) - iR - T_v}{L}$$

4c) $m\ddot{x} + cv + kx = f$

Let $v = \dot{x}$, $\dot{v} = \ddot{x}$

$f = m\dot{v} + cv + kx$

$\dot{v} = -\frac{c}{m}v - \frac{k}{m}x - \frac{1}{m}Ti$

$$\dot{i} = \frac{E(t) - iR - Tv}{L}$$

$$= -\frac{T}{L}v - \frac{R}{L}i + \frac{E(t)}{L}$$

$$\frac{d}{dt}\begin{bmatrix} x \\ v \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{c}{m} & -\frac{1}{m}T \\ 0 & -\frac{T}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x \\ v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} E(t)$$

d) $f = m\ddot{x} + cv + kx$

$v = \dot{x}e^{jw}$

$\dot{v} = \ddot{x}e^{jw}$

$$\ddot{x} = \frac{\dot{v}}{e^{jw}}$$