1a) $f(t) = e^{a-bt}$

$$e^{a-bt} = e^{-bt}(e^{a})$$

$$\mathcal{L}\{e^{a-bt}\} = \frac{e^{a}}{s+b}$$

b) $\sin(wt+\delta) = \sin(wt)\cos\delta + \cos(wt)\sin\delta$

$$\mathcal{L}\{\sin(wt+\delta)\} = \frac{w\cos\delta}{s^2+w^2} + \frac{(\sin\delta)s}{s^2+w^2}$$

$$= \frac{w\cos\delta + s\sin\delta}{s^2+w^2}$$

2) $\mathcal{L}\{f(t)\} = \int_{t=0}^{t\to\infty} f(t)e^{-st}\,dt$

$$= \int_0^1 (1-t)e^{-st}\,dt$$

$$= \int_0^1 e^{-st} - te^{-st}\,dt$$

2) $\mathcal{L}\{f(t)\} = \int_0^1 e^{-st} - te^{-st}\,dt$

$$= -\frac{1}{s}\left[e^{-st}\right]_0^1 + \left[\frac{st+1}{s^2}e^{-st}\right]_0^1$$

$$= -\frac{1}{s}\left[e^{-st} - 1\right] + \left[\frac{s+1}{s^2}e^{-s} - \frac{1}{s^2}\right]$$

$$= \frac{1}{s} - \cancel{\frac{s}{s^2}e^{-st}} + \frac{\cancel{s}+1}{s^2}e^{-s} - \frac{1}{s^2}$$

$$= \frac{1}{s} + \frac{1}{s^2}\left(e^{-s} - 1\right)$$

3a) $G(s) = \dfrac{0.1s + 0.9}{s^2 + 3.24}$

$$= \frac{0.1s}{s^2+3.24} + \frac{0.9}{s^2+3.24}$$

$$g(t) = \frac{1}{10}\cos(1.8t) + \frac{1}{2}\sin(1.8t)$$

b) $G(s) = \dfrac{-s-10}{s^2-s-2}$

$$= \frac{-s-10}{(s-2)(s+1)}$$

$$\frac{-s-10}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1}$$

$$-s-10 = A(s+1) + B(s-2)$$

when $s = -1$,

$$-9 = -3B$$

$$B = 3$$

3b) when $s = 2$,

$$-12 = 3A$$

$$A = -4$$

$$\therefore G(s) = -\frac{4}{s-2} + \frac{3}{s+1}$$

$$g(t) = 3e^{-t} - 4e^{2t}$$

4a) $\frac{d}{dt}\left(\cos^2(t)\right) = -2\cos(t)\sin(t)$

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$$

$$\mathcal{L}\{-2\cos(t)\sin(t)\} = s\mathcal{L}\{f(t)\} - 1$$

$$\mathcal{L}\{-\sin(2t)\} = s\mathcal{L}\{f(t)\} - 1$$

$$1 - \frac{2}{s^2+4} = s\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos^2(t)\} = \frac{1}{s} - \frac{2}{s(s^2+4)}$$

4b) $\cos(2t) = 2\cos^2(t) - 1$

$$\mathcal{L}\{\cos(2t)\} = \mathcal{L}\{2\cos^2(t) - 1\}$$

$$\frac{s}{s^2+4} = 2\mathcal{L}\{\cos^2(t)\} - \frac{1}{s}$$

$$\frac{1}{s} + \frac{s}{s^2+4} = 2\mathcal{L}\{\cos^2(t)\}$$

$$\mathcal{L}\{\cos^2(t)\} = \left(\frac{1}{s} + \frac{s}{s^2+4}\right) \times \frac{1}{2}$$

$$= \frac{s^2+4+s^2}{2s(s^2+4)}$$

$$= \frac{2s^2+4}{2s(s^2+4)}$$

$$= \frac{\cancel{2}(s^2+2)}{\cancel{2}s(s^2+4)}$$

$$= \frac{s^2+2}{s(s^2+4)}$$

5a) Let $f(t) = t^2$

$$F(s) = \mathcal{L}\{t^2\} = \frac{2}{s^3}$$

$$\mathcal{L}\{t^2 e^{-3t}\} = F(s+3)$$

$$= \frac{2}{(s+3)^3}$$

b) Let $f(t) = 5\sinh(2t)$

$$F(s) = \mathcal{L}\{5\sinh(2t)\} = \frac{10}{s^2-4}$$

$$\mathcal{L}\{5e^{2t}\sinh(2t)\} = F(s-2)$$

$$= \frac{10}{(s-2)^2-4}$$

6a) $\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$, where

$$f(t) = \mathcal{L}^{-1}\{F(s)\}$$

$$F(s-a) = \frac{1}{(s+1)^2}$$

$$a = -1$$

$$F(s) = \frac{1}{s^2}$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = t$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = te^{-t}$$

b) $$\frac{3}{s^2+6s+18} = \frac{3}{(s+3)^2+3^2}$$

$$= F(s-a)$$

$$\therefore a = -3$$

$$F(s) = \frac{3}{s^2+3^2}$$

$$f(t) = \sin(3t)$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{3}{s^2+6s+18}\right\} = \sin(3t)e^{-3t}$$