1) For the projectile:

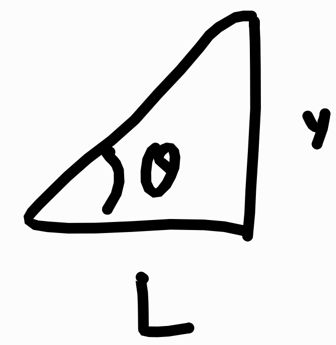
$$x_p = v\cos\theta\, t \qquad -(1)$$

$$y_p = v\sin\theta\, t - \frac{1}{2}gt^2 \qquad -(2)$$

For the target:

$$x_T = L \qquad -(3)$$

$$y_T = L\tan\theta - \frac{1}{2}gt^2 \qquad -(4)$$



$$\tan\theta = \frac{y}{L}$$

$$y = L\tan\theta$$

When $x_p = x_T = L$,

$$v\cos\theta\, t = L \qquad -(5)$$

$$y_p = \frac{v\sin\theta\cos\theta\, t}{\cos\theta} - \frac{1}{2}gt^2 \qquad -(6)$$

Sub (5) into (6)

$$y_p = \frac{L\sin\theta}{\cos\theta} - \frac{1}{2}gt^2$$

$$= L\tan\theta - \frac{1}{2}gt^2$$

$$= y_T$$

$\therefore$ Since $x_p = x_T$ and $y_p = y_T$, the projectile will hit the target.

2) Rate of increase in the plane's speed is the tangential acceleration.

$$a_t = a\cos 60^\circ$$
$$= 21\cos 60^\circ$$
$$= 10.5\,ms^{-2}$$

$$a_n = \frac{v^2}{\rho}$$

$$21\sin 60^\circ = \frac{120^2}{\rho}$$

$$\rho = 791.7946549\,m$$
$$\approx 792\,m$$

3) Speed $= 72 \, km \, h^{-1} = 20 \, ms^{-1}$

a)

$$a_t = -1.25 \, ms^{-2}$$

$$a_n = \frac{v^2}{r}$$

$$= \frac{20^2}{350}$$

$$= \frac{8}{7} \, ms^{-2}$$

$$a = a_t + a_n$$

$$|a| = \sqrt{(-1.25)^2 + \left(\frac{8}{7}\right)^2}$$

$$= 1.693700814 \, ms^{-2}$$

$$\approx 1.694 \, ms^{-2}$$

3b) When $t=4$,

$$v = u + at$$
$$= 20 - 1.25(4)$$
$$= 15\,ms^{-1}$$

$$a_t = -1.25\,ms^{-1}$$
$$a_n = \frac{v^2}{r}$$
$$= \frac{15^2}{350}$$
$$= \frac{9}{14}$$

$$a = a_t + a_n$$
$$|a| = \sqrt{(-1.25)^2 + \left(\frac{9}{14}\right)^2}$$
$$= 1.40561919\,ms^{-2}$$
$$\approx 1.406\,ms^{-2}$$

4) $\theta = 2t^2 \quad r = 60t^2 - 20t^3$

$\dot{\theta} = 4t \quad \dot{r} = 120t - 60t^2$

$\ddot{\theta} = 4 \quad \ddot{r} = 120 - 120t$

$v = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$a = \ddot{r}\hat{e}_r + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r + 2\dot{r}\dot{\theta}\hat{e}_\theta$

a) When $t=1$,

$v = 60\hat{e}_r + 40(4)\hat{e}_\theta$

$= 60\hat{e}_r + 160\hat{e}_\theta$

$= 60(\cos 2\,\underset{\sim}{i} + \sin 2\,\underset{\sim}{j}) + 160(-\sin 2\,\underset{\sim}{i} + \cos 2\,\underset{\sim}{j})$

$= -170.4563985\,\underset{\sim}{i} - 12.02564824\,\underset{\sim}{j}$

$= 170.8800749 \angle 184.035513807^\circ$

$\approx 170.9 \angle 184.04^\circ$ mms-1

4b) $a = 0 + 40(4)\hat{e}_\theta - 640\hat{e}_r + 2(60)(4)\hat{e}_\theta$

$= 640\hat{e}_\theta - 640\hat{e}_r$

$= 640(\hat{e}_\theta - \hat{e}_r)$

$= 640(-\sin 2\underset{\sim}{i} + \cos 2\underset{\sim}{j} - \cos 2\underset{\sim}{i} - \sin 2\underset{\sim}{j})$

$= 640(-0.4931505903\underset{\sim}{i} - 1.325444263\underset{\sim}{j})$

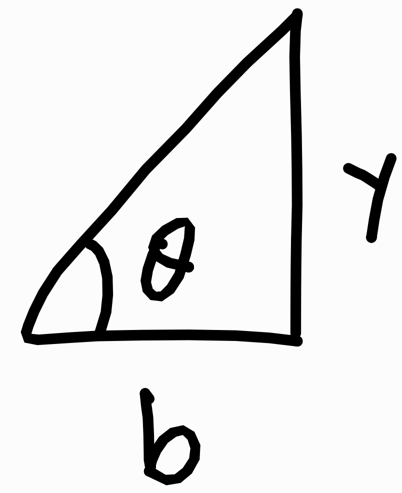
$= 905.0966799\angle 249.591559°$

$\approx 905.1\angle 249.6°$

c) 0

5) $x = b\tan\theta$

$$v = \frac{dx}{dt} = b(\sec^2\theta)\dot{\theta}$$

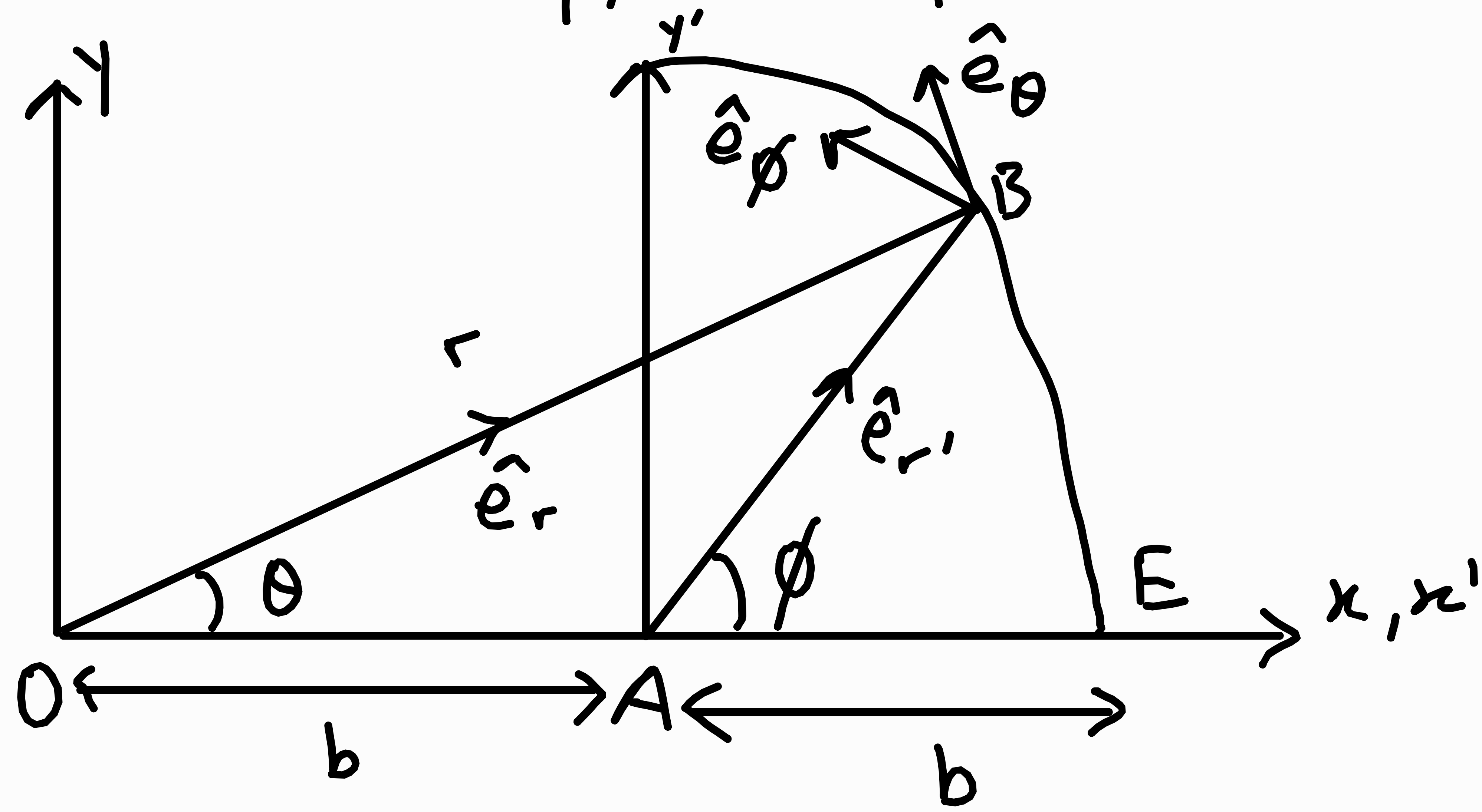


$$a = \frac{dv}{dt} = b\left[\ddot{\theta}\sec^2\theta + 2\dot{\theta}^2\sec^2\theta\tan\theta\right]$$

$\tan\theta = \frac{y}{b}$

$y = b\tan\theta$

6a) Set up two fixed reference frams $O-xy$, $A-x'y'$:



we have $\phi = 2\theta$

$\therefore \dot{\phi} = 2\dot{\theta}$

Since $\dot{\theta}$ is constant,

$\ddot{\phi} = 2\ddot{\theta} = 0$

6a)

Measuring the motion of B in A - x'y':

$$\vec{a}_B = b\ddot{\phi}\hat{e}_\phi - b\dot{\phi}^2\hat{e}_{r'}$$

$$= -b\dot{\phi}^2\hat{e}_{r'}$$

$$|\vec{a}_B| = b\dot{\phi}^2$$

make the angle point towards A instead of away from A

$$\vec{a}_B = b\dot{\phi}^2\angle(\pi + \phi)$$

$$= b4\dot{\theta}^2\angle(\pi + 2\theta)$$

6b) Observed from the frame (f) fixed on rod OC, the relative motion of B is described by:

$$\vec{v}_{B/f} = \dot{r}\hat{e}_r$$

$$\vec{a}_{B/f} = \ddot{r}\hat{e}_r$$

6b) Expressing r with $r = 2b\cos\theta$,

$$r = 2b\cos\theta$$

$$\dot{r} = -2b\dot{\theta}\sin\theta$$

$$\ddot{r} = -2b(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta)$$

$$= -2b\dot{\theta}^2\cos\theta$$

$$\therefore \vec{v}_{B|f} = -2b\dot{\theta}\sin\theta\,\hat{e}_r$$

$$\vec{a}_{B|f} = -2b\dot{\theta}^2\cos\theta\,\hat{e}_r$$

where $\hat{e}_r = \cos\theta\,\underset{\sim}{i} + \sin\theta\,\underset{\sim}{j}$