4.16) $\bar{Y} = \dfrac{\sum \bar{y} A}{\sum A}$

$$= \frac{1.5(15) \times 40 \times 15 + \frac{1}{2}(15)(15)(20)}{40 \times 15 + 20 \times 15}$$

$$= 17.5 \text{ mm}$$

$$I = \frac{1}{12} \times 40 \times 15^3 + 40 \times 15 (1.5 \times 15 - 17.5)^2 + \frac{1}{12} \times 20 \times 15^3 + 20 \times 15 (17.5 - 7.5)^2$$

$$= 61875 \text{ mm}^4$$

$$= 6.1875 \times 10^{-8} \text{ m}^4$$

4.16)

$$\sigma = \frac{My}{I}$$

$$M = \frac{\sigma I}{y}$$

Taking the tension at the top part of the beam:

$$M = \frac{24 \times 10^{6} (6.1875 \times 10^{-8})}{(30 - 17.5) \times 10^{-3}}$$

$$= 118.8 \text{ Nm in tension}$$

Taking the compression at the bottom part of the beam:

$$M = \frac{30 \times 10^{6} (6.1875 \times 10^{-8})}{17.5 \times 10^{-3}}$$

$$= 106.0714286$$

$$\approx 106.1 \text{Nm}$$

$\therefore$ M_{max} is 106.1 Nm.

4.99a) Loading is centric:

$$F = 60 \times 3 = 180\,kN$$

$$A = (240 \times 10^{-3}) \times 90 \times 10^{-3}$$

$$= 0.0216\,m^3$$

$$\sigma_A = \sigma_B = \frac{-180 \times 10^3}{0.0216}$$

$$= -\frac{25}{3} \times 10^6$$

$$\approx -8.33\,MPa \text{ (Compression)}$$

b) Transferring the forces to the centroid

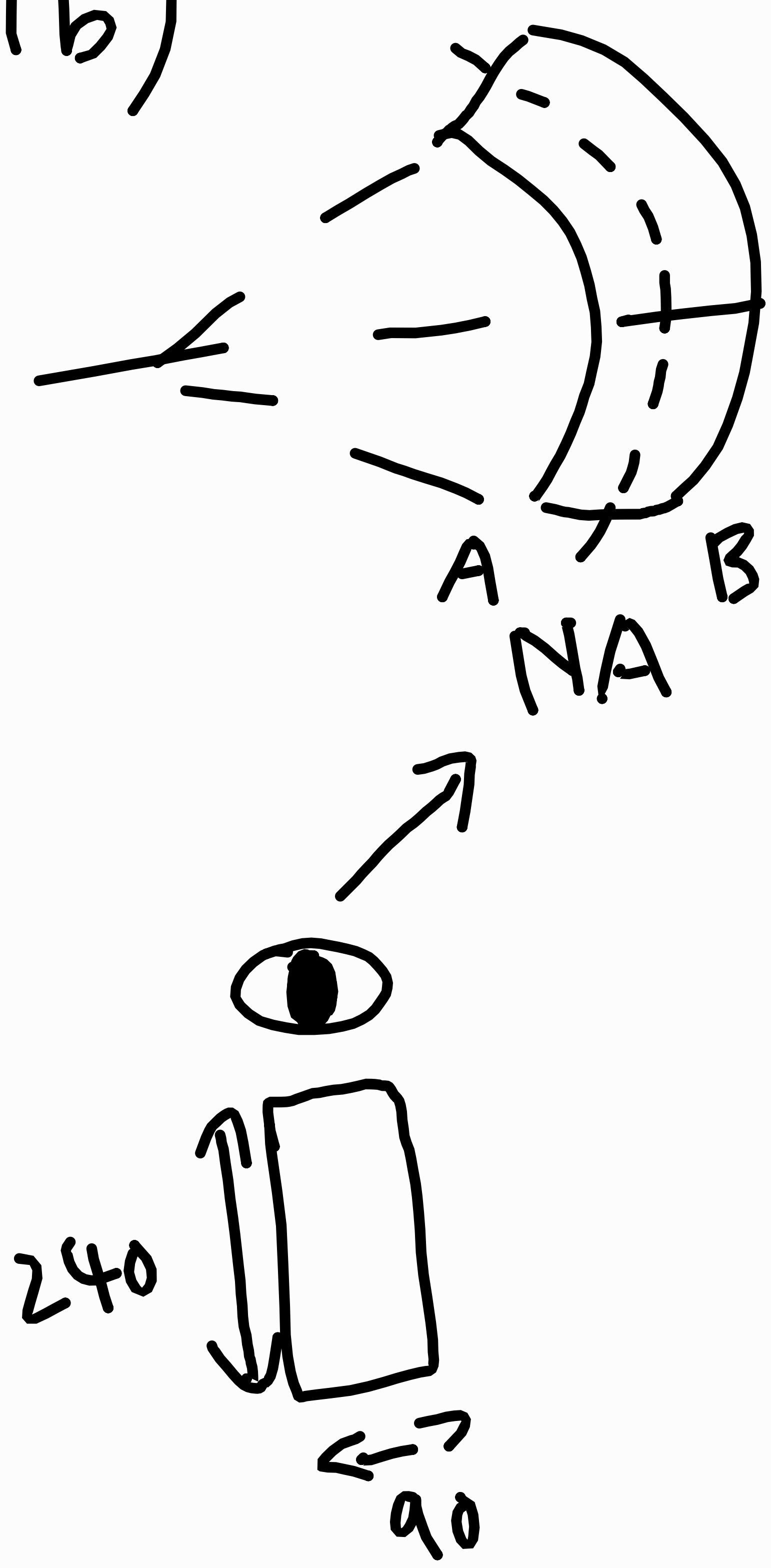
$$F = 2 \times 60 = 120\,kN$$

$$A = 240 \times 90 = 21600\,mm^2$$

$$\sigma = \frac{-F}{A} = -\frac{120 \times 10^3}{21600 \times 10^{-6}}$$

$$= -\frac{50}{9}\,MPa \text{ (Compression)}$$

4.99b)

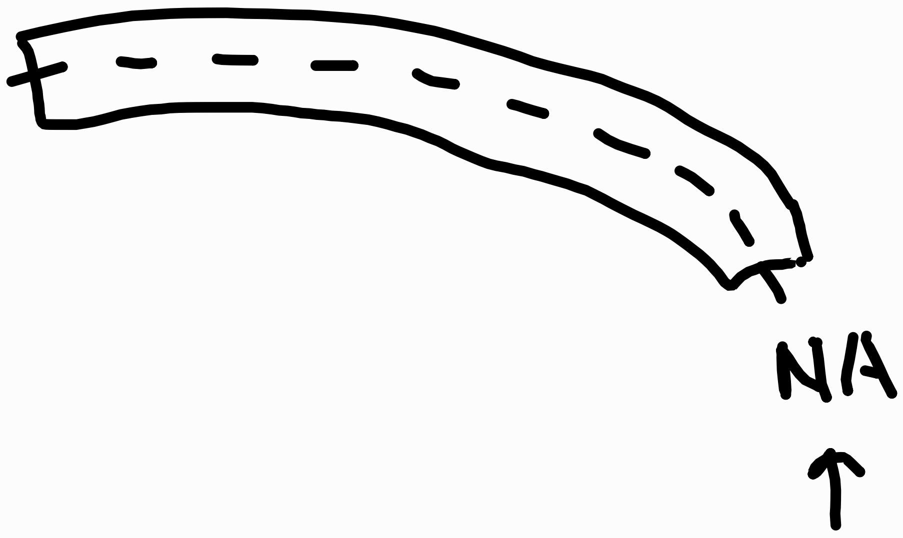


$$I = \frac{1}{12} b h^3$$
$$= \frac{1}{12}(90\times10^{-3})(240\times10^{-3})^3$$
$$= \frac{81}{781250} m^4$$

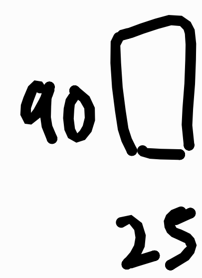
$$\sigma_A = \sigma - \frac{My}{I}$$
$$= -\frac{50}{9}\times10^6 - \frac{150\times60(120\times10^{-3})}{\frac{81}{781250}}$$
$$= -15972222.22 Pa$$
$$\approx -15.97 MPa$$

$$\sigma_B = \sigma + \frac{My}{I}$$
$$= -\frac{50}{9}\times10^6 + \frac{150\times60(120\times10^{-3})}{\frac{81}{781250}}$$
$$= 4861111.111 Pa$$
$$\approx 4.86 MPa$$

4.122)



$$\sigma_A = E\varepsilon_A$$
$$= 200\times10^{9}(350\times10^{-6})$$
$$= 70\text{ MPa}$$



$$\sigma_B = E\varepsilon_B$$
$$= 200\times10^{9}(-70\times10^{-6})$$
$$= -14\text{ MPa}$$

$$I = \frac{1}{12}bh^3$$
$$= \frac{1}{12}(25\times10^{-3})(90\times10^{-3})^3$$
$$= 1.51875\times10^{-6}\ m^3$$

$$\sigma_A = \frac{F_p}{A} + \frac{My_A}{I}$$

$$\sigma_A = \frac{F_p}{A} + \frac{My_A}{I}$$

$$\sigma_A = \frac{F_p}{A} + \frac{F_p d y_A}{I}$$

$$F_p = \frac{\sigma_A}{\frac{1}{A} + \frac{dy_A}{I}} \quad -(1)$$

4.122) $\sigma_B = \dfrac{F_P}{A} - \dfrac{M y_B}{I}$

$$\sigma_B = F_P\left(\frac{1}{A} - \frac{d y_B}{I}\right)$$

$$F_P = \frac{\sigma_B}{\frac{1}{A} - \frac{d y_B}{I}} \qquad - (2)$$

Equating (1) and (2):

$$\sigma_A\left(\frac{1}{A} - \frac{d y_B}{I}\right) = \sigma_B\left(\frac{1}{A} + \frac{d y_A}{I}\right)$$

$$\frac{1}{A}(\sigma_A - \sigma_B) = \frac{\sigma_B d y_A}{I} + \frac{\sigma_A d y_B}{I}$$

$$\sigma_A - \sigma_B = \frac{dA}{I}(\sigma_B y_A + \sigma_A y_B)$$

$$(70-(-14)) \times 10^{\cancel{6}3} = \frac{d(90\times25)\cancel{\times10^{-6}}}{1.51875\cancel{\times10^{-6}}}\left(-14\times15+70\times30\right)\cancel{\times10^{6}}^{3}$$

$$d = 0.03\,m$$
$$= 30\,mm$$

$$F_P = \frac{-14\times10^6}{\frac{10^6}{90\times25} - \frac{0.03\times0.03}{1.51875\times10^{-6}}}$$

$$= 94.5\,kN$$

4.144) For W150×24 :

$$A = 3060\,mm^2 = 3060\times10^{-6}\,m^3$$

$$I_z = 13.4\times10^{6}\,mm^4 = 13.4\times10^{-6}\,m^4$$

$$I_y = 1.83\times10^{6}\,mm^4 = 1.83\times10^{-6}\,m^4$$

$$\sigma = -\frac{50\times10^3}{3060\times10^{-6}} = -\frac{2500}{153}\times10^{6}\,Pa$$

$$\sigma_{bz} = -\frac{M_z y}{I_z} = \frac{-50\times75\times80\times10^{-3}}{13.4\times10^{-6}} = -\frac{1500}{67}\times10^{6}\,Pa$$

$$-90\times10^{6} = -\frac{M_y y}{I_y} - \frac{2500}{153}\times10^{6} - \frac{1500}{67}\times10^{6}$$

$$\left(-90 + \frac{2500}{153} + \frac{1500}{67}\right)\cancel{\times10^{6}} = -\frac{50a\left(\frac{102}{2}\times10^{-3}\right)}{1.83\cancel{\times10^{-6}}}$$

$$a = 36.79525097$$

$$\approx 36.8\,mm$$