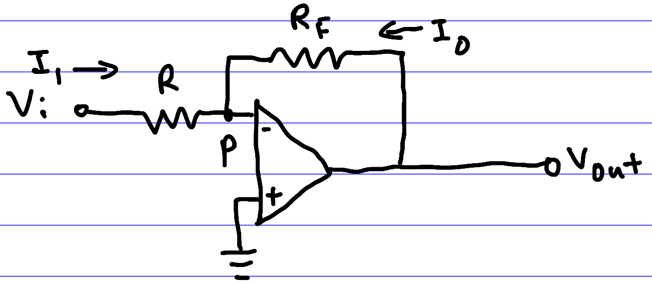
1) Op-amp with infinite input resistance and zero output resistance: $I_0 = -I_1$



So,

$$\frac{V_p - V_o}{R_F} = -\frac{V_p - V_i}{R}$$

$$R(V_p - V_o) = -R_F(V_p - V_i)$$

$$V_p(R_F + R) = RV_o - V_iR_F.$$

Finite loop gain, $A = -\frac{V_o}{V_p}$,

$$V_p = -\frac{V_o}{A}$$

$$\therefore -\frac{V_o}{A}(R_F + R) = RV_o - V_iR_F$$

$$-V_o(R_F + R + AR) = V_iAR_F$$

By definition, op-amp gain, $G = \frac{V_o}{V_i}$

$$-V_o(R_F + R + AR) = V_iAR_F$$

$$\frac{V_o}{V_i} = \frac{-AR_F}{R_F + R + AR}$$

$$G = \frac{-AR_F}{R_F + R + AR} \text{ (shown)}$$

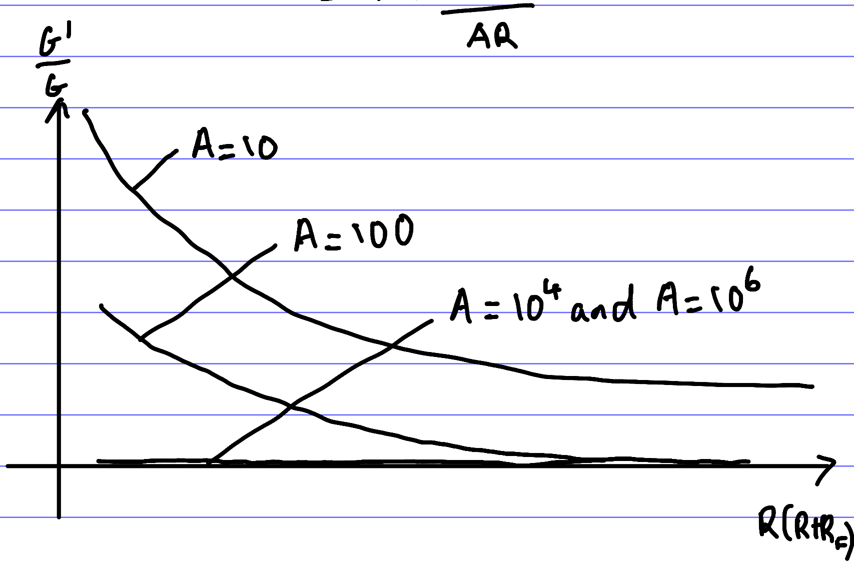
1) For large open loop gain A, $AR >>> (R+R_F)$,

$$G' = -\frac{R_F}{R}$$

$$\frac{G'}{G} = \frac{R\cancel{_F}}{R} \times \frac{R_F + R + AR}{AR\cancel{_F}}$$

$$= \frac{R_F + R + AR}{AR}$$

$$= 1 + \frac{R + R_F}{AR}$$



A practical op-amp has very high input impedance ($Z_i > 10^7\ \Omega$), low output impedance ($Z_o < 100\ \Omega$), and high open-loop gain (A about $10^5 - 10^6$), and the values of R and R_F are typically in the kΩ to the MΩ range, so it is valid to use the second equation.

2a) $G = 1 + \frac{R_F}{R}$

$V_p = \left(1 + \frac{R_2}{R_1}\right) V_i$

$= \left(1 + \frac{6}{1}\right)(1)$

$= 7V$

$V_o = \left(1 + \frac{R_4}{R_3}\right) V_p$

$= \left(1 + \frac{1}{1}\right)(7)$

$= 14V$

b) $V_p = -\left(\frac{R_3}{R_1} V_1 + \frac{R_3}{R_2} V_2\right)$

$= -\left(\frac{10}{1}(0.1) + \frac{10}{10}(1)\right)$

$= -2V$

$V_o = -\left(\frac{R_6}{R_4} V_p + \frac{R_6}{R_5} V_3\right)$

$= -\left(\frac{10}{1}(-2) + \frac{10}{1}(1)\right)$

$= 10V$

3) Since signals above 2.5kHz are not wanted, a low pass filter is needed. The cut-off frequency must be at least 2.5kHz and less than about 3kHz where the accelerometer response becomes more than 1.

$$f_c = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_c}$$

$$RC = \frac{1}{2\pi(2500)}$$

$$= 6.366197724 \times 10^{-5}$$

$$\approx 6.37 \times 10^{-5}$$

$\therefore$ Let $R = 63.7\,k\Omega$, $C = 1\,nF$

$\frac{1}{f_c}$ is a good approximation of the time constant.

$$\therefore T \approx \frac{1}{f_c}$$

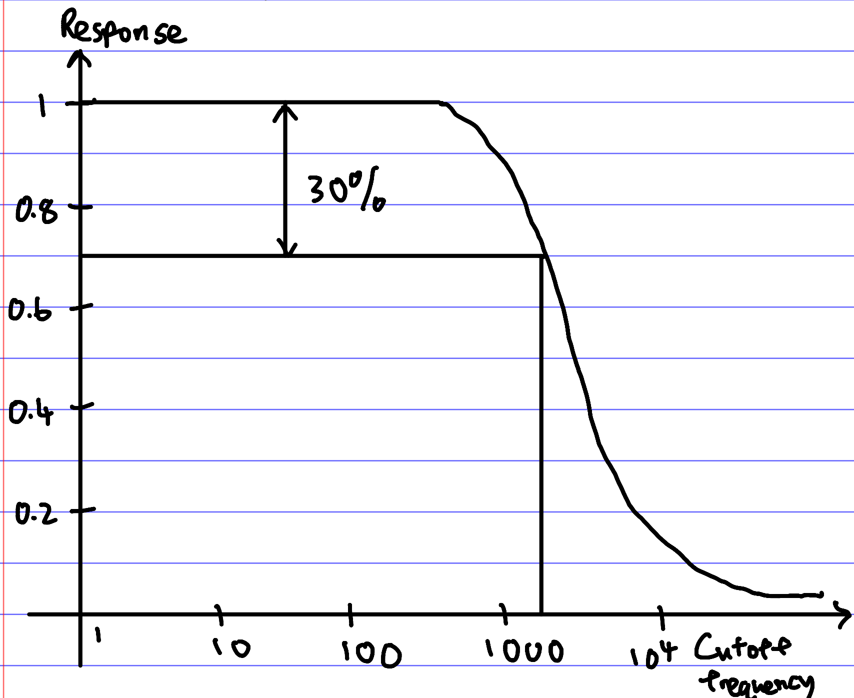
3) The frequency response of a filter is:

$$M(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{2500}{2500}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\approx 0.707$$



3) Frequency Response Curve