Amplitude linearity:

$$V_{out}(t) - V_{out}(0) = \alpha\left[V_{in}(t) - V_{in}(0)\right]$$

where α is a constant

1a) $V_{out}(t) = 5V_{in}(t)$

$\alpha = 5$

$\therefore V_{out}(t) = 5V_{in}(t)$ is linear

b) $\dfrac{V_{out}(t)}{V_{in}(t)} = 5t$

$V_{out}(t) = 5tV_{in}(t)$

$\alpha = 5t$

$\therefore \dfrac{V_{out}(t)}{V_{in}(t)} = 5t$ is not linear

c) $V_{out}(t) = V_{in}(t) + 5$

$\alpha = 1$

$\therefore V_{out}(t) = V_{in}(t) + 5$ is linear

d) $V_{out}(t) = V_{in}(t) + V_{in}(t)$

$V_{out}(t) = 2V_{in}(t)$

$\alpha = 2$

$\therefore V_{out}(t) = V_{in}(t) + V_{in}(t)$ is linear

1e) $V_{out}(t) = V_{in}(t) \times V_{in}(t)$

$V_{out}(t) = (V_{in}(t))^2$

$V_{out}(t) = (V_{in}(t))^2$ does not follow the equation for amplitude linearity, so $V_{out}(t) = V_{in}(t) \times V_{in}(t)$ is not linear.

f) $V_{out}(t) = V_{in}(t) + 10t$ does not follow the equation for amplitude linearity, so $V_{out}(t) = V_{in}(t) + 10t$ is not linear.

g) $V_{out}(t) = V_{in}(t) + \sin(5)$

$\alpha = 1$

$\therefore V_{out}(t) = V_{in}(t) + \sin(5)$ is linear.

Fourier Series:

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n w_0 t) + \sum_{n=1}^{\infty} B_n \sin(n w_0 t)$$

$$w_0 = \frac{2\pi}{T} = 2\pi f_0$$

2a) $f(t) = 5\sin(2\pi t)$

$n = 1$

$w_0 = 2\pi = 2\pi f_0$

$\therefore f_0 = 1$

$F(t) = 5\sin(2\pi t)$

b) $f(t) = 5\cos(2\pi t)$

$n = 1$

$w_0 = 2\pi = 2\pi f_0$

$\therefore f_0 = 1$

$F(t) = 5\cos(2\pi t)$

c) $f(t) = -5\sin(2\pi t)$

$n = 1$

$w_0 = 2\pi = 2\pi f_0$

$\therefore f_0 = 1$

$F(t) = -5\sin(2\pi t)$

3) $f = \frac{1}{T} = 60$

$60T = 1$

$T = \frac{1}{60}$

$= 0.01\dot{6}$

$\approx 0.0167s$

Fourier Series:

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n w_0 t) + \sum_{n=1} B_n \sin(n w_0 t)$$

4) $$y(t) = 4 + \sum_{n=1}^{\infty} \frac{2n\pi}{10} \cos \frac{n\pi}{4} t + \frac{120n\pi}{30} \sin \frac{n\pi}{4} t$$

a) $w_0 = 2\pi f_0 = \frac{\pi}{4} \text{ rads}^{-1}$

$f_0 = \frac{1}{8} Hz$

b) $f = \frac{1}{T}$

$\frac{1}{8} = \frac{1}{T}$

$T = 8s$

4c) let $C_n = \sqrt{A_n^2 + B_n^2}$

$\phi_n = \arctan\left(\frac{A_n}{B_n}\right)$

$$F(t) = C_0 + \sum_{n=1}^{\infty} C_n \sin(n w_0 t + \phi_n)$$

$$C_n = \sqrt{\left(\frac{2n\pi}{10}\right)^2 + \left(\frac{120n\pi}{30}\right)^2}$$

$$= \sqrt{\frac{4n^2\pi^2}{100} + 16n^2\pi^2}$$

$$= \sqrt{16.04}\, n\pi$$

$$\phi_n = \arctan\left(\frac{\frac{2\cancel{n\pi}}{\cancel{10}}}{\frac{120\cancel{n\pi}}{3\cancel{30}}}\right)$$

$$= \arctan\left(\frac{1}{20}\right)$$

$$\therefore y(t) = 4 + \sum_{n=1}^{\infty} \sqrt{16.04}\, n\pi \sin\left[\frac{n\pi}{4}t + \arctan\left(\frac{1}{20}\right)\right]$$

$$\approx 4 + 4\pi \sum_{n=1}^{\infty} n \sin\left(\frac{n\pi}{4}t + 0.05\right)$$