$\underset{\sim}{F}$ is conservative means $\underset{\sim}{F} = \text{grad } f$

Line Integrals

Arc length:

$$\int_C ds = \int_a^b \|\underset{\sim}{r}'(t)\| dt$$

Line integral of real valued function f:

$$\int_C f(x)\, ds = \int_a^b f(\underset{\sim}{r}(t)) \|\underset{\sim}{r}'(t)\| dt$$

Line integral (work) of vector field $\underset{\sim}{F}$:

$$\int_C \underset{\sim}{F} \cdot \overbrace{\underset{\sim}{U}}^{\text{unit tangent to the vector field}} ds = \int_a^b \underset{\sim}{F}(\underset{\sim}{r}(t)) \cdot \underset{\sim}{r}'(t)\, dt$$

$$\text{curl } \underset{\sim}{F} = \nabla \times \underset{\sim}{F}$$

$$\text{div } \underset{\sim}{F} = \nabla \cdot \underset{\sim}{F}$$

Surface integrals

Surface area:

$$\iint_S dS = \iint_R \|\underset{\sim}{r}_u \times \underset{\sim}{r}_v\| \, du\,dv$$

Surface integral of real valued function f:

$$\iint_S f(x)\,dS = \iint_R f(\underset{\sim}{r}(t))\,\|\underset{\sim}{r}_u \times \underset{\sim}{r}_v\| \, du\,dv$$

Surface integral (flux) of vector field $\underset{\sim}{F}$:

unit normal vector to the surface

$$\iint_S \underset{\sim}{F} \cdot \underset{\sim}{u}\,dS = \iint_R \underset{\sim}{F}(\underset{\sim}{r}(u,v)) \cdot \underset{\sim}{r}_u \times \underset{\sim}{r}_v \, du\,dv$$

Common unit normal vectors

Cylinder: $(x, y, 0)$

Sphere: (x, y, z)

Green's Theorem

Positively oriented boundary ∂D

$\underset{\sim}{F}$ has continuous partial derivatives on both D and ∂D, then

$$\oint_{\partial D} P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dx\,dy$$

Stoke's Theorem

Positively oriented boundary ∂S

$\underset{\sim}{F}$ has continuous partial derivatives on both S and ∂S, then

$$\oint_{\partial S} P\,dx + Q\,dy + R\,dz = \iint \operatorname{curl} \underset{\sim}{F} \cdot d\underset{\sim}{S}$$

Gauss' Divergence Theorem

Surface oriented with normal vector pointing away from Q.

$\underset{\sim}{F}$ has continuous partial derivatives on both Q and ∂Q, then:

$$\iint_{\partial Q} \underset{\sim}{F} \cdot d\underset{\sim}{S} = \iiint_Q \operatorname{div} \underset{\sim}{F}\,dx\,dy\,dz$$

Triangular and diagonal matrices

Upper triangular matrix:

$$A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 3 & -2 & 9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Lower triangular matrix:

$$B = \begin{bmatrix} 53 & 0 & 0 & 0 & 0 \\ -4 & 3 & 0 & 0 & 0 \\ 1 & 0 & \pi & 0 & 0 \\ 0 & -1 & 2 & \sqrt{2} & 0 \\ 1 & 3 & 5 & 7 & 9 \end{bmatrix}$$

Diagonal matrix:

$$C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 29 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Matrix transpose

$$(A^T)_{ij} = (A)_{ji}$$

Symmetric matrices

$$A^T = A$$

Matrix arithmetic involving transposes

1. $(A^T)^T = A$
2. $(A \pm B)^T = A^T \pm B^T$
3. $(\alpha A^T) = \alpha (A^T)$
4. $\boxed{(AB)^T = B^T A^T}$

Invertible matrices

A matrix A being invertible means there exists a matrix B such that:

$$AB = BA = I$$

A being invertible also means

1. $A\underset{\sim}{x} = \underset{\sim}{0}$ has only the trivial solution (the solution $\underset{\sim}{x} = \underset{\sim}{0}$)
2. For any $\underset{\sim}{b} \in \mathbb{R}^n$, $A\underset{\sim}{x} = \underset{\sim}{b}$ has exactly one solution
3. $\det A \neq 0$

Subspace check

W is a subspace if

1. $W \neq \emptyset$
2. W is closed under addition, i.e. $\underset{\sim}{u} + \underset{\sim}{v} \in W$, $\underset{\sim}{u}, \underset{\sim}{v} \in W$
3. W is closed under multiplication, i.e. $k\underset{\sim}{u} \in W$, $\underset{\sim}{u} \in W$

Span

Let $S = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots \underset{\sim}{x}_n\}$

$$\text{Span } S = \{k_1\underset{\sim}{x}_1 + k_2\underset{\sim}{x}_2 + \dots + k_n\underset{\sim}{x}_n, k_1, \dots k_n \in \mathbb{R}\}$$

$$\text{Span } \emptyset = \{\underset{\sim}{0}\}$$

Linear independence

$$k_1\underset{\sim}{x}_1 + k_2\underset{\sim}{x}_2 + \dots + k_n\underset{\sim}{x}_n = 0$$

$$k_1 = k_2 = \dots = k_n = 0$$