1) $\vec{v}_B = \vec{v}_{B|f} + \vec{v}_{B'}$

$v_B \angle -\beta = 0 + \vec{v}_{B'}$

$v_B\left(\cos(-\beta)\underset{\sim}{i} + \sin(-\beta)\underset{\sim}{j}\right) = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B'A}$ $\quad$ i j k i j

$v_B\left(\cos(-\beta)\underset{\sim}{i} + \sin(-\beta)\underset{\sim}{j}\right) = v_A\underset{\sim}{i} + \omega_{AB}\underset{\sim}{k} \times L\left(\cos(-\theta)\underset{\sim}{i} + \sin(-\theta)\underset{\sim}{j}\right)$

$v_B\cos(-\beta)\underset{\sim}{i} + v_B\sin(-\beta)\underset{\sim}{j} = v_A\underset{\sim}{i} + \omega_{AB}L\cos(-\theta)\underset{\sim}{j} - \omega_{AB}L\sin(-\theta)\underset{\sim}{i}$

$v_B\cos\beta\underset{\sim}{i} - v_B\sin\beta\underset{\sim}{j} = v_A\underset{\sim}{i} + \omega_{AB}L\cos\theta\underset{\sim}{j} + \omega_{AB}L\sin\theta\underset{\sim}{i}$

$v_B\cos\beta\underset{\sim}{i} - \omega_{AB}L\sin\theta\underset{\sim}{i} - v_B\sin\beta\underset{\sim}{j} - \omega_{AB}L\cos\theta\underset{\sim}{j} = v_A\underset{\sim}{i}$

$v_B\cos\beta - \omega_{AB}L\sin\theta = v_A$ $\qquad$ $-v_B\sin\beta - \omega_{AB}L\cos\theta = 0$

$v_B\sin\beta + \omega_{AB}L\cos\theta = 0$

$$\boxed{\begin{aligned} v_B\cos\beta - \omega_{AB}L\sin\theta &= v_A \\ v_B\sin\beta + \omega_{AB}L\cos\theta &= 0 \end{aligned}}$$

Solving $L = 450$, $\theta = 30$, $\beta = 80$, $v_A = 300$

$v_B = 404.1889066$

$\approx 404.19\ mms^{-1}$

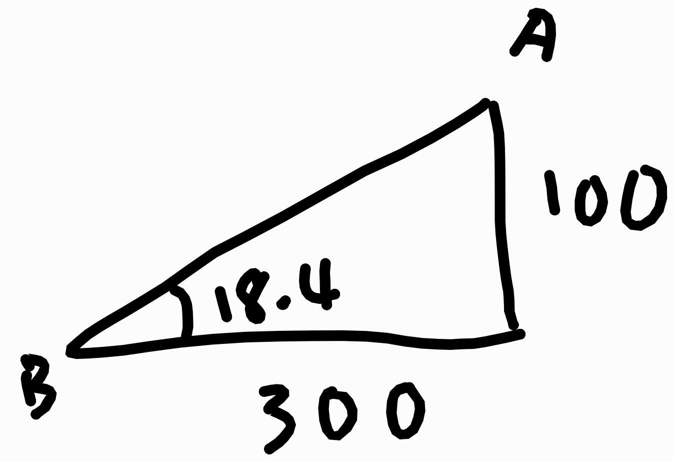
$\omega_{AB} = -1.02139259l$

$\approx -1.02\ rads^{-1}$

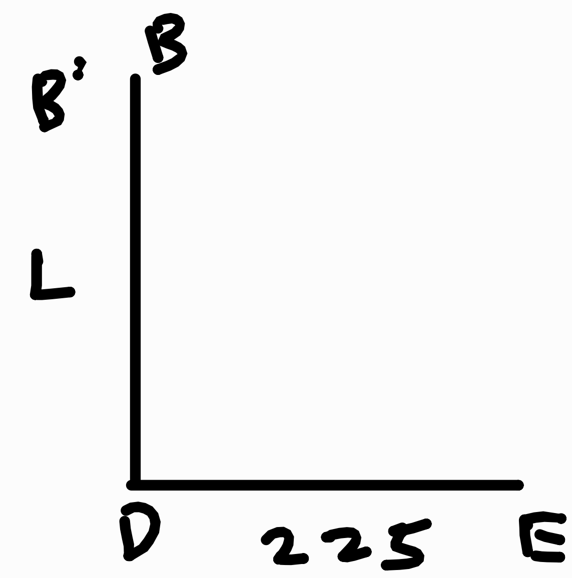
2) $\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{BA}$

$= \omega_{AB} \underset{\sim}{k} \times (-300\underset{\sim}{i} - 100\underset{\sim}{j})$

$= 100\omega_{AB}\underset{\sim}{i} - 300\omega_{AB}\underset{\sim}{j}$



i j k → i j



$\vec{v}_B = \vec{v}_{B/f} + \vec{v}_{B'}$

$= 0 + \vec{v}_D + \vec{\omega}_{BD} \times \vec{r}_{B'D}$

$= \vec{v}_D + \omega_{BD}\underset{\sim}{k} \times (L\underset{\sim}{j})$

$= \vec{\omega}_{DE} \times \vec{r}_{DE} - \omega_{BD} L \underset{\sim}{i}$

$= \omega_{DE}\underset{\sim}{k} \times (-225\underset{\sim}{i}) - \omega_{BD} L \underset{\sim}{i}$

$= -\omega_{BD} L \underset{\sim}{i} - 225\omega_{DE}\underset{\sim}{j}$

$100\omega_{AB}\underset{\sim}{i} - 300\omega_{AB}\underset{\sim}{j} = -\omega_{BD} L\underset{\sim}{i} - 225\omega_{DE}\underset{\sim}{j}$

$100\omega_{AB} = -\omega_{BD} L$

$\omega_{BD} = \dfrac{-100\omega_{AB}}{L}$

$-300\omega_{AB} = -225\omega_{DE}$

$\omega_{DE} = \dfrac{300}{225}\omega_{AB}$

$= \dfrac{4}{3}\omega_{AB}$

$$\boxed{\begin{aligned} \omega_{BD} &= \frac{-100\omega_{AB}}{L} \\ \omega_{DE} &= \frac{4}{3}\omega_{AB} \end{aligned}}$$

Solving $L = 400$, $\omega_{AB} = 4$,

$\omega_{BD} = -1 \text{ rad s}^{-1}$

$\omega_{DE} = \frac{16}{3} \text{ rad s}^{-1}$

3) $\vec{v}_A = \vec{w}_{AO} \times \vec{r}_{AO}$ $\quad$ i j k i j

$= w_{AO}\underset{\sim}{k} \times (L\underset{\sim}{i} + 3\underset{\sim}{j})$

$= -3w_{AO}\underset{\sim}{i} + w_{AO}L\underset{\sim}{j}$

$\vec{v}_A = \vec{v}_{A/f} + \vec{v}_{A'}$

$= 0 + \vec{v}_E + \vec{w}_{AE} \times \vec{r}_{AE}$

$= v_E\underset{\sim}{i} + w_{AE}\underset{\sim}{k} \times (-8\underset{\sim}{i} + 8\underset{\sim}{j})$

$= v_E\underset{\sim}{i} - 8w_{AE}\underset{\sim}{i} - 8w_{AE}\underset{\sim}{j}$

$-3w_{AO}\underset{\sim}{i} + w_{AO}L\underset{\sim}{j} = v_E\underset{\sim}{i} - 8w_{AE}\underset{\sim}{i} - 8w_{AE}\underset{\sim}{j}$

$8w_{AE}\underset{\sim}{i} - 3w_{AO}\underset{\sim}{i} + 8w_{AE}\underset{\sim}{j} + w_{AO}L\underset{\sim}{j} = v_E\underset{\sim}{i}$

$$\boxed{\begin{array}{ll} 8w_{AE} - 3w_{AO} = v_E & \vec{v}_A = v_E\underset{\sim}{i} - 8w_{AE}\underset{\sim}{i} - 8w_{AE}\underset{\sim}{j} \\ 8w_{AE} + w_{AO}L = 0 & \end{array}}$$

Solving with $L = 4$, $v_E = 1.2$

$w_{AE} = w_{ABCD} = \frac{3}{35}\ \text{rads}^{-1}$

$w_{AO} = -\frac{6}{35}$

$\vec{v}_A = 1.2\underset{\sim}{i} - 8\left(\frac{3}{35}\right)\underset{\sim}{i} - 8\left(\frac{3}{35}\right)\underset{\sim}{j}$

$= \frac{18}{35}\underset{\sim}{i} - \frac{24}{35}\underset{\sim}{j}$

4) $\vec{v}_B = \vec{v}_{B/f} + \vec{v}_{B'}$

$i \; j \; k \; i \; j$

$= v\underset{\sim}{i} + \vec{w} \times \vec{r}_{B'A}$

$= v\underset{\sim}{i} + w(-\underset{\sim}{k}) \times (-250\underset{\sim}{i} - 200\underset{\sim}{j})$

$= v\underset{\sim}{i} - 200w\underset{\sim}{i} + 250w\underset{\sim}{j}$

$$\boxed{v_{Bx} = v - 200w \qquad v_{By} = 250w}$$

Solving with $v = 600$, $w = 2$,

$v_{Bx} = 200\,mms^{-1}$ $\quad v_{By} = 500\,mms^{-1}$

5) Cable length

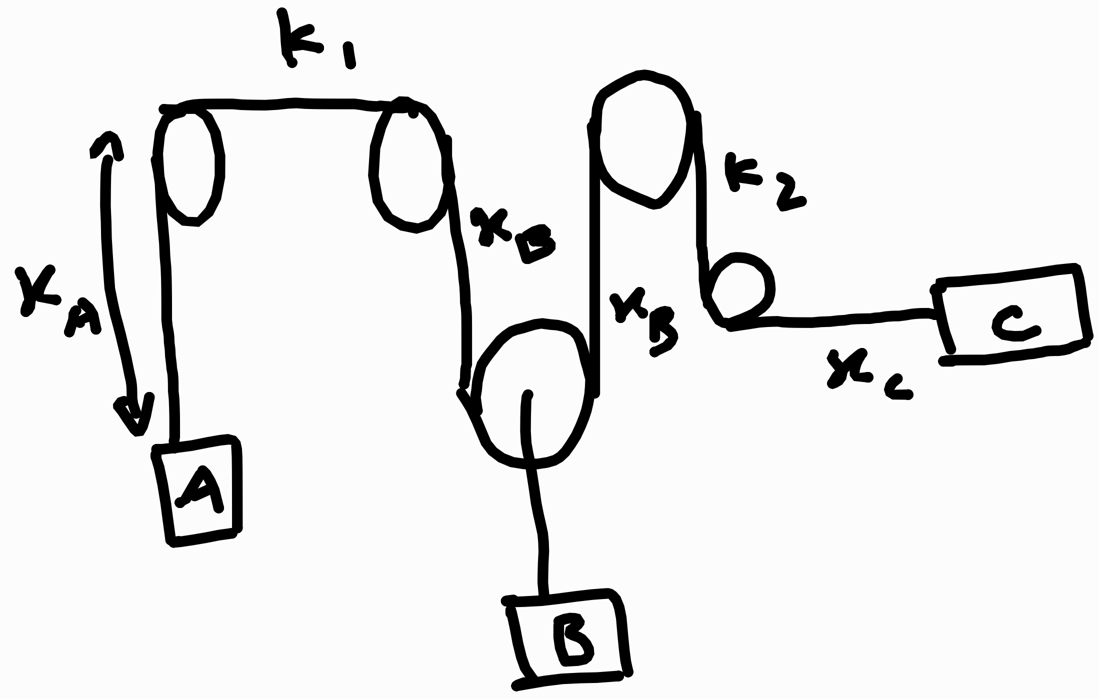
$= x_A + k_1 + 2x_B + k_2 + x_c$

Differentiate w.r.t t,

$0 = \dot{x}_A + 2\dot{x}_B + \dot{x}_c$

Differentiate w.r.t t,

$\ddot{x}_A + 2\ddot{x}_B + \ddot{x}_c = 0$ - (1)



$$\ddot{x}_A = \frac{m_A g - T}{m_A}$$

$$\ddot{x}_A = g - \frac{T}{m_A} \quad - (2)$$

$$m_B \ddot{x}_B = m_B g - 2T$$

$$\ddot{x}_B = \frac{m_B g - 2T}{m_B}$$

$$\ddot{x}_B = g - \frac{2T}{m_B} \quad - (3)$$

$$N = m_c g$$

$$f = \mu_k N = \mu_k m_c g$$

$$\ddot{x}_c = \frac{\mu_k m_c g - T}{m_c}$$

$$\ddot{x}_c = \mu_k g - \frac{T}{m_c}$$

Sub (2), (3), (4) into (1),

$$g - \frac{T}{m_A} + 2g - \frac{4T}{m_B} + \mu_k g - \frac{T}{m_c} = 0$$

$$\frac{T}{m_A} + \frac{4T}{m_B} + \frac{T}{m_c} = (\mu_k + 3)g$$

$$\boxed{T = \frac{(\mu_k + 3)g}{\left(\frac{1}{m_A} + \frac{4}{m_B} + \frac{1}{m_c}\right)}}$$

5) Solving when $m_A = 200$, $m_B = 300$, $m_C = 50$, $\mu_k = 0.2$,

$$T = 818.9217391\,\text{N}$$

$$\ddot{x}_A = 5.715391304\,\text{ms}^{-2}$$

$$\ddot{x}_B = 4.350521739\,\text{ms}^{-2}$$

$$\ddot{x}_C = -14.41643478\,\text{ms}^{-2}$$

6) Conservation of total energy:

$$KE + E_{GPE} + E_{EPE}$$

$$= \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

$$E_A = mg2R + \frac{1}{2}k(\sqrt{2}R - R)^2$$

$$E_B = \frac{1}{2}mv_B^2 + mgR$$

$$E_C = \frac{1}{2}mv_C^2$$

$$\frac{1}{2}\cancel{m}v_C^2 = \cancel{m}g2R + \frac{1}{2}k(\sqrt{2}R - R)^2$$

$$v_C^2 = 4Rg + \frac{k}{m}(\sqrt{2}R - R)^2$$

$$\boxed{v_C = \sqrt{4Rg + \frac{k}{m}(\sqrt{2}R - R)^2}}$$

$$\frac{1}{2}\cancel{m}v_B^2 + \cancel{mgR} = \cancel{m}g\cancel{2}R + \frac{1}{2}k(\sqrt{2}R - R)^2$$

$$\frac{1}{2}v_B^2 = 2Rg + \frac{k}{2m}(\sqrt{2}R - R)^2$$

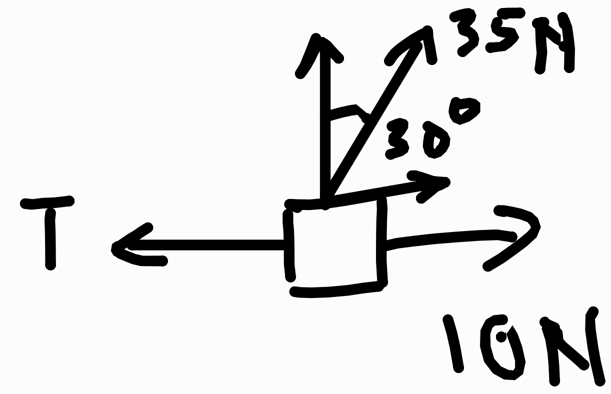
$$v_B^2 = 2Rg + \frac{k}{m}(\sqrt{2}R - R)^2$$

$$\boxed{v_B = \sqrt{2Rg + \frac{k}{m}(\sqrt{2}R - R)^2}}$$

Solving for $R = 3$, $k = 400$, $m = 10$,

$v_C = 13.3972473\ ms^{-1}$ $\quad$ $v_B = 10.98299755\ ms^{-1}$

7) In the $\underset{\sim}{i}$ direction:



$$-mr\omega^2 = -T + 35\sin 30° + 10$$

$$T = 27.5 + mr\omega^2$$

$$T = 27.5 + \frac{50}{9.81}(1.2)\omega^2$$

$$T = 27.5 + \frac{2000}{327}\omega^2$$

$$T_{max} = 27.5 + \frac{2000}{327}\omega_{max}^2$$

$$\omega_{max}^2 = \frac{327(T_{max} - 27.5)}{2000}$$

$$\omega_{max} = \sqrt{\frac{327(T_{max} - 27.5)}{2000}}$$

$$\boxed{v_{max} = r\omega_{max} = 1.2\sqrt{\frac{327(T_{max} - 27.5)}{2000}}}$$

Solving with $T_{max} = 260$,

$$v_{max} = 7.398635009 \, ms^{-1}$$

In the $\underset{\sim}{j}$ direction:

$$mr\alpha = 35\cos 30°$$

$$\alpha = \frac{35\cos 30°}{\frac{50}{9.81}(1.2)}$$

$$= \frac{2289}{400}\cos 30°$$

7) $\omega_{max} = \omega_i + \alpha t$

$$\omega_{max} = 0 + \alpha t$$

$$\omega_{max} = \alpha t$$

$$t = \frac{\omega_{max}}{\alpha}$$

$$\boxed{t = \sqrt{\frac{327(T_{max} - 27.5)}{2000}} \div \left(\frac{2289}{400} \cos 30^\circ\right)}$$

Solving when $T_{max} = 260$,

$$t = 1.244096087\text{s}$$