1a) $m_s v_s + m_c v_c = m_s v_f + m_c v_f$

$12(2.5) + 30(0) = 12 v_f + 30 v_f$

$30 = 42 v_f$

$v_f = \frac{5}{7}$

$\approx 0.714285714 3 \, ms^{-1}$

b) $E_i = \frac{1}{2} m_s v_s^2$

$= \frac{1}{2}(12)(2.5)^2$

$= 37.5 \, J$

$E_f = \frac{1}{2} m_s v_f^2 + \frac{1}{2} m_c v_f^2$

$= \frac{1}{2}(12)\left(\frac{5}{7}\right)^2 + \frac{1}{2}(30)\left(\frac{5}{7}\right)^2$

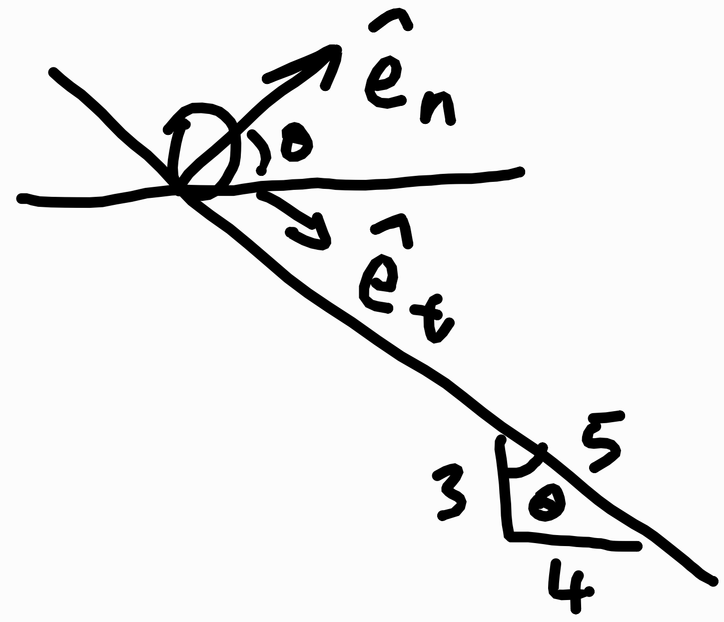
$= \frac{75}{7} \, J$

$\frac{E_f}{E_i} = \frac{\frac{75}{7}}{37.5}$

$= \frac{2}{7}$

≈ 0.286

2) $\hat{e}_n = \frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}$



$\cos\theta = \frac{3}{5}$ $\sin\theta = \frac{4}{5}$

$\hat{e}_t = \angle(\theta - 90°)$

$= \frac{4}{5}\underset{\sim}{i} - \frac{3}{5}\underset{\sim}{j}$

$\underset{\sim}{i} \cdot \hat{e}_n = (1,0) \cdot \left(\frac{3}{5}, \frac{4}{5}\right)$

$= \frac{3}{5}$

$\underset{\sim}{i} \cdot \hat{e}_t = (1,0) \cdot \left(\frac{4}{5}, -\frac{3}{5}\right)$

$= \frac{4}{5}$

$\underset{\sim}{i} = \frac{3}{5}\hat{e}_n + \frac{4}{5}\hat{e}_t$

$\underset{\sim}{j} = \frac{4}{5}\hat{e}_n - \frac{3}{5}\hat{e}_t$

$\vec{v}_0 = v_0(-\underset{\sim}{j})$

$= -0.8v_0\hat{e}_n + 0.6v_0\hat{e}_t$

$\vec{v}_1 = v_1^n\hat{e}_n + v_1^t\hat{e}_t$

$\vec{v}_1^{\,t} = \vec{v}_0^{\,t} = 0.6v_0\hat{e}_t$

2) $\vec{v}_1^{\,n} - 0 = -e(\vec{v}_0^{\,n} - 0)$

$$\vec{v}_1^{\,n} = -0.8(-0.8 v_0 \hat{e}_n$$

$$= 0.64 v_0 \hat{e}_n$$

$$\vec{v}_1 = 0.64 v_0 \hat{e}_n + 0.6 v_0 \hat{e}_t$$

$$\vec{v}_1 = 0.864 v_0 \underset{\sim}{i} + 0.152 v_0 \underset{\sim}{j}$$

$$x = 0.864 v_0 t$$

$$y = (0.152 v_0) t - \frac{1}{2} g t^2$$

$$\frac{x}{y} = \frac{0.864 v_0 t}{0.152 v_0 t - \frac{1}{2} g t^2}$$

$$4\left(0.152 v_0 t - \frac{1}{2} g t^2\right) = -3(0.864 v_0 t)$$

$$0.608 v_0 \cancel{t} - 2 g t^{\cancel{2}} = -2.592 v_0 \cancel{t}$$

$$0.608 v_0 - 2gt = -2.592 v_0$$

$$3.2 v_0 = 2gt$$

$$t = \frac{3.2 v_0}{2g}$$

2) $\frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gh$

$$v_0^2 = 2g(1.2)$$

$$v_0 = \sqrt{2.4g}$$

$$\therefore t = \frac{3.2\sqrt{2.4g}}{2g}$$

$$= \frac{3.2\sqrt{2.4 \times 9.81}}{2 \times 9.81}$$

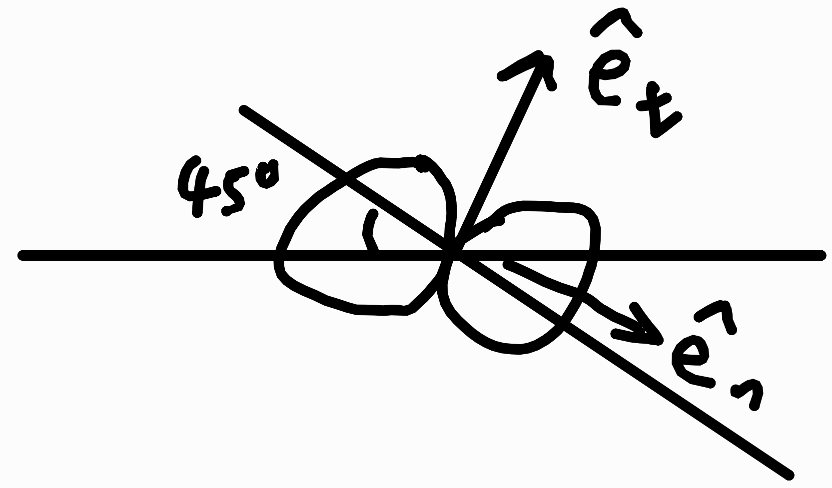
$$= 0.7913909869\,s$$

$$d = \frac{5}{4}x = \frac{5}{4}\left(0.864\sqrt{2.4 \times 9.81}\right)(0.79139)$$

$$= 4.1472\,m$$

3) $\hat{e}_n = \angle(-45°)$

$= \frac{1}{\sqrt{2}}\underset{\sim}{i} - \frac{1}{\sqrt{2}}\underset{\sim}{j}$



$\hat{e}_t = \angle 45°$

$= \frac{1}{\sqrt{2}}\underset{\sim}{i} + \frac{1}{\sqrt{2}}\underset{\sim}{j}$

$\underset{\sim}{i} = \frac{1}{\sqrt{2}}\hat{e}_n + \frac{1}{\sqrt{2}}\hat{e}_t$

$\underset{\sim}{j} = -\frac{1}{\sqrt{2}}\underset{\sim}{j} + \frac{1}{\sqrt{2}}\hat{e}_t$

$\vec{v}_{A0} = v_0\underset{\sim}{i} = \frac{1}{\sqrt{2}}v_0\hat{e}_n + \frac{1}{\sqrt{2}}v_0\hat{e}_t$

$v_{A1}^t = v_{A0}^t = \frac{v_0}{\sqrt{2}}$

$v_{B1}^t = v_{B0}^t = 0$

$m v_{A1}^n + m v_{B1}^n = m v_{A0}^n + \cancel{m v_{B0}^n}$

$v_{A1}^n + v_{B1}^n = \frac{v_0}{\sqrt{2}}$

$v_{A1}^n - v_{B1}^n = -e(v_{A0}^n - \cancel{v_{B0}^n})$

$v_{A1}^n - v_{B1}^n = -0.8\frac{v_0}{\sqrt{2}}$

3) $v_{A1}^{n} = 0.0707106781 2 v_0$

$\approx 0.07 v_0$

$v_{B1}^{n} = 0.6363961031 v_0$

$\approx 0.6394 v_0$

$\vec{v}_{A1} = 0.07 v_0 \hat{e}_n + \frac{1}{\sqrt{2}} v_0 \hat{e}_t$

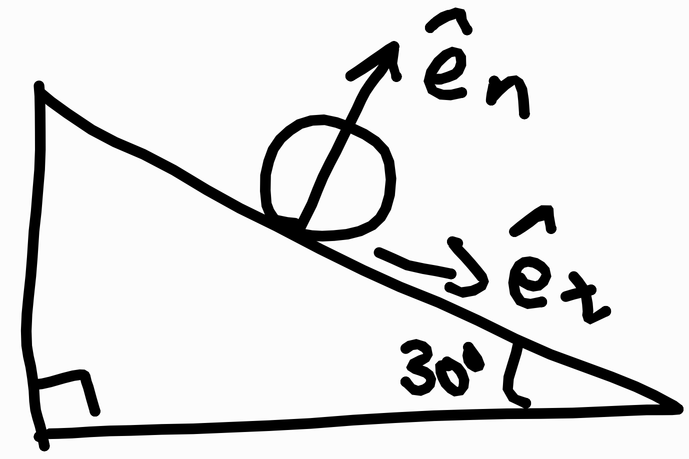
$= 0.55 v_0 \underset{\sim}{i} + 0.45 v_0 \underset{\sim}{j}$

$\vec{v}_{B1} = 0.6394 v_0 \hat{e}_n$

$= 0.45 \underset{\sim}{i} - 0.45 \underset{\sim}{j}$

4) $\hat{e}_n = \angle 60^\circ$

$= \frac{1}{2}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\underset{\sim}{j}$



$\hat{e}_t = \angle -30^\circ$

$= \frac{\sqrt{3}}{2}\underset{\sim}{i} - \frac{1}{2}\underset{\sim}{j}$

$\vec{v}_{A1} = v_{A1}^n \hat{e}_n + v_{A1}^t \hat{e}_t$

$\underset{\sim}{i} = \frac{1}{2}\hat{e}_n + \frac{\sqrt{3}}{2}\hat{e}_t$

$\vec{v}_{B1} = -v_B \underset{\sim}{i}$

$= -\frac{1}{2} v_{B1}\hat{e}_n - \frac{\sqrt{3}}{2} v_{B1}\hat{e}_t$

$\underset{\sim}{j} = \frac{\sqrt{3}}{2}\hat{e}_n - \frac{1}{2}\hat{e}_t$

$\vec{v}_0 = -3\underset{\sim}{j}$

$\vec{v}_0 = -\frac{3\sqrt{3}}{2}\hat{e}_n + \frac{3}{2}\hat{e}_t$

$v_{A1}^t = v_{A0}^t = \frac{3}{2}\hat{e}_t$

$v_{B1}^t = v_{B0}^t = 0$

$m_A v_{A0_x} + m_B v_{B0_x} = m_A v_{A1_x} + m_B v_{B1_x}$

$m_A\left(\frac{1}{2}v_{A1}^n + \frac{\sqrt{3}}{2}v_{A1}^t\right) + m_B(-v_{B1}) = 0$

$v_{A1}^n + \frac{3\sqrt{3}}{2} - 6v_{B1} = 0$

$v_{A1}^n - 6v_{B1} = -\frac{3\sqrt{3}}{2} \quad -(1)$

4) $(v_{A1}^n - v_{B1}^n) = -e(v_{A0}^n - v_{B0}^n)$

$$v_{A1}^n - v_{B1}^n = -0.8\left(-\frac{3\sqrt{3}}{2}\right)$$

$$v_{A1}^n - v_{B1}^n = \frac{6\sqrt{3}}{5}$$

$$v_{A1}^n + \frac{1}{2}v_{B1} = \frac{6\sqrt{3}}{5} \quad - (2)$$

Solving (1), (2)

$$v_{A1}^n = 1.71872734\,ms^{-1}$$

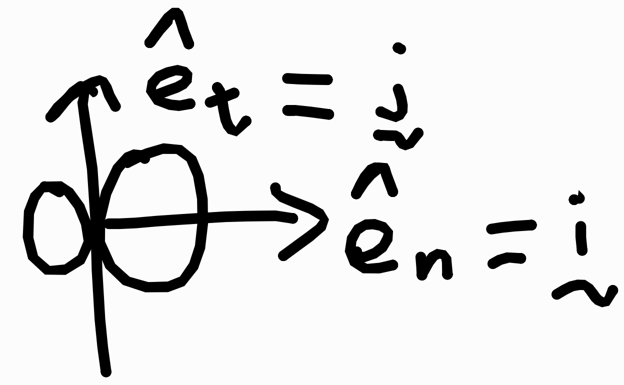
$$v_{B1} = 0.7194672585\,ms^{-1}$$

$$\vec{v}_{A1} = 1.5\hat{e}_t + 1.719\hat{e}_n$$

$$= 2.158401776\underset{\sim}{i} + 0.738465586\underset{\sim}{j}$$

$$\vec{v}_{B1} = -0.7194672585\underset{\sim}{i}$$

5) $m_A v_{A0} + \cancel{m_B v_{B0}} = m_A(-v_{A1}) + m_B v_{B1}$



$$1.5 v_{A0} = 3 v_{B1} - 1.5 v_{A1}$$

$$-1.5 v_{A1} + 3 v_{B1} = 1.5 v_{A0} \quad - (1)$$

$$(-v_{A1} - v_{B1}) = -e(v_{A0} - \cancel{v_{B0}})$$

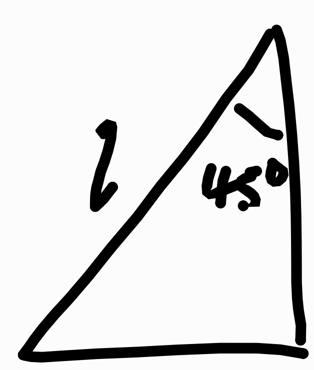
$$-v_{A1} - v_{B1} = -0.75 v_{A0}$$

$$v_{A1} + v_{B1} = 0.75 v_{A0} \quad - (2)$$

Solving (1), (2)

$$v_{A1} = \frac{1}{6} v_{A0}$$

$$v_{B1} = \frac{7}{12} v_{A0}$$



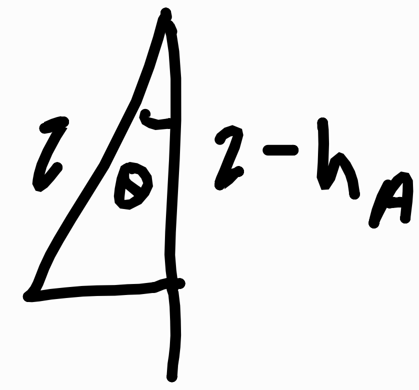
$$\frac{1}{2} \cancel{m} v_{A0}^2 = \cancel{m} g(l - l\cos 45°)$$

$$v_{A0}^2 = 2gl\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

$$v_{A0}^2 = (2-\sqrt{2})gl$$

$$v_{A0} = \sqrt{(2-\sqrt{2})gl}$$

5) $\frac{1}{2} \cancel{m_A} v_{A1}^2 = \cancel{m_A} g h_A$



$$h_A = \frac{1}{2g} v_{A1}^2$$

$$h_A = \frac{1}{72\cancel{g}} (2-\sqrt{2}) \cancel{g} l$$

$$= \frac{2-\sqrt{2}}{72} l$$

$$\theta_A = \cos^{-1}\left(\frac{\cancel{l}^{\,1} - \frac{2-\sqrt{2}}{72}\cancel{l}}{\cancel{l}}\right)$$

$$= 7.313679543^\circ$$
$$\approx 7.3^\circ$$

$\frac{1}{2} \cancel{m_B} v_B^2 = \cancel{m_B} g h_B$

$$h_B = \frac{1}{2g} v_B^2$$

$$h_B = \frac{49}{288\cancel{g}} (2-\sqrt{2}) \cancel{g} l$$

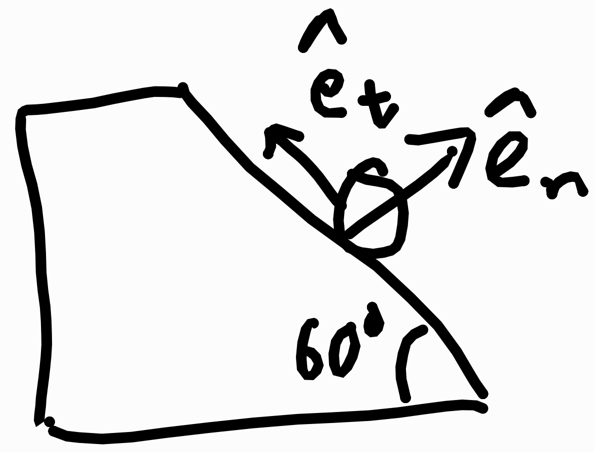
$$= \frac{49}{288} (2-\sqrt{2}) l$$

$$\theta_B = \cos^{-1}\left(\frac{\cancel{l}^{\,1} - \frac{49}{288}(2-\sqrt{2})\cancel{l}}{\cancel{l}\ 1}\right)$$

$$= 25.79787057^\circ \approx 25.8^\circ$$

6) $\hat{e}_n = \angle 30^\circ$

$$= \frac{\sqrt{3}}{2}\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$$



$\hat{e}_t = \angle 120^\circ$

$$= -\frac{1}{2}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\underset{\sim}{j}$$

$$\underset{\sim}{i} = \frac{\sqrt{3}}{2}\hat{e}_n + \frac{1}{2}\hat{e}_t$$

$$\underset{\sim}{j} = \frac{1}{2}\hat{e}_n - \frac{\sqrt{3}}{2}\hat{e}_t$$

$$\vec{v}_{A0} = -15\underset{\sim}{i}$$

$$= -\frac{15\sqrt{3}}{2}\hat{e}_n + 7.5\hat{e}_t$$

$$\vec{v}_{A1} = v_{A1}^{n}\hat{e}_n + v_{A1}^{t}\hat{e}_t$$

$$\vec{v}_{B1} = -v_B\underset{\sim}{i}$$

$$= -\frac{\sqrt{3}}{2}v_B\hat{e}_n - \frac{1}{2}v_B\hat{e}_t$$

$$v_{A1}^{t} = v_{A0}^{t} = 7.5$$

$$m_A v_{A0x} + \cancel{m_B v_{B0x}} = m_A v_{A1x} + m_B v_{B1}$$

$$1.5(-15) = 1.5\left(\frac{\sqrt{3}}{2}v_{A1}^{n} + \frac{1}{2}v_{A1}^{t}\right) + 4(-v_B)$$

$$\frac{3\sqrt{3}}{4}v_{A1}^{n} - \frac{3}{4}v_{A1}^{t} - 4v_B = -22.5$$

$$\frac{3\sqrt{3}}{4}v_{A1}^{n} - 4v_B = -\frac{135}{8} \quad -(1)$$

6) $(v_{A1}^{n} - v_{B1}^{n}) = -e(v_{A0}^{n} - \cancel{v_{B0}^{n}})$

$$v_{A1}^{n} + \frac{\sqrt{3}}{2} v_B = -0.75\left(-\frac{15\sqrt{3}}{2}\right)$$

$$v_{A1}^{n} + \frac{\sqrt{3}}{2} v_B = \frac{45}{8}\sqrt{3} \quad -(2)$$

Solving (1), (2),

$$v_B = \frac{945}{164}$$

$$\frac{1}{2} k x^2 = \frac{1}{2} m_B v_B^2$$

$$x^2 = \frac{m_B v_B^2}{k}$$

$$x = \sqrt{\frac{4\left(\frac{945}{164}\right)^2}{5\times10^3}}$$

$$x = 0.1629794898$$

$$\approx 0.163\,m$$