1) $s = ut + \frac{1}{2}at^2$

$15 = 5t + \frac{1}{2}(9.81)t^2$

$4.905t^2 + 5t - 15 = 0$

$t = 1.311821357s$

$\approx 1.31s$

$v = u + at$

$= 5 + 9.81(1.311821357)$

$= 17.86896751$

$\approx 17.9ms^{-1}$

2) $x = t^4 - 12t^2 - 40$

$v = \frac{dx}{dt} = 4t^3 - 24t$

$a = \frac{dv}{dt} = 12t^2 - 24$

when $t = 2$,

$x = (2)^4 - 12(2)^2 - 40$

$= -72\,m$

$v = 4(2)^3 - 24(2)$

$= -16\,ms^{-1}$

$a = 12(2)^2 - 24$

$= 24\,ms^{-2}$

$\therefore \vec{r}(2) = -72\underset{\sim}{i}$

$\vec{v}(2) = -16\underset{\sim}{i}$

$\vec{a}(2) = 24\underset{\sim}{i}$

3) $a = 25 - 3x^2$, $v_0 = 0$, $x_0 = 0$ at $t = 0$

$$a = v\frac{dv}{dx}$$

$$25 - 3x^2 = v\frac{dv}{dx}$$

$$\int 25 - 3x^2 \, dx = \int v \, dv$$

$$25x - x^3 = \frac{v^2}{2} + c$$

when $x_0 = 0$, $v_0 = 0$,

$$0 = 0 + c$$

$$\therefore c = 0$$

$$\therefore v^2 = 50x - 2x^3$$

$$v = \sqrt{50x - 2x^3}$$

a) When $x = 2\,mm$,

$$v = \sqrt{50(2) - 2(2)^3}$$

$$= 9.1651513\,9ms^{-1}$$

$$\approx 9.17ms^{-1}$$

3b) When $v = 0$,

$$0 = \sqrt{50x - 2x^3}$$

$$25 - x^2 = 0$$

$$x = \sqrt{25}$$

$$= 5$$

$$\therefore x = 5\,mm$$

c) To maximise v, maximise v^2 as $v^2 \geq 0$

$$v^2 = 50x - 2x^3$$

$$\frac{dv^2}{dx} = 50 - 6x^2$$

when $\frac{dv^2}{dx} = 0$,

$$0 = 50 - 6x^2$$

$$6x^2 = 50$$

$$x = \sqrt{\frac{25}{3}}$$

x	$\sqrt{\frac{25}{3}}^-$	$\sqrt{\frac{25}{3}}$	$\sqrt{\frac{25}{3}}^+$
$\frac{dv^2}{dx}$	0^+	0	0^-
slope			

$$\therefore x = 2.886751346\,mm$$

$$\approx 2.89\,mm$$

4) $$a = \frac{g(v_f^2 - v^2)}{v_f^2}$$

$$\frac{dv}{dt} = \frac{g(v_f^2 - v^2)}{v_f^2}$$

$$\frac{v_f^2}{g(v_f^2 - v^2)} \frac{dv}{dt} = 1$$

$$\frac{v_f^2}{g} \int \frac{1}{v_f^2 - v^2} dv = \int 1 dt$$

$$\frac{v_f^{\cancel{2}}}{g} \left(\frac{1}{2\cancel{v_f}} \ln \left| \frac{v_f + v}{v_f - v} \right| \right) = t + c$$

$$\frac{v_f}{2g} \ln \left| \frac{v_f + v}{v_f - v} \right| = t + c$$

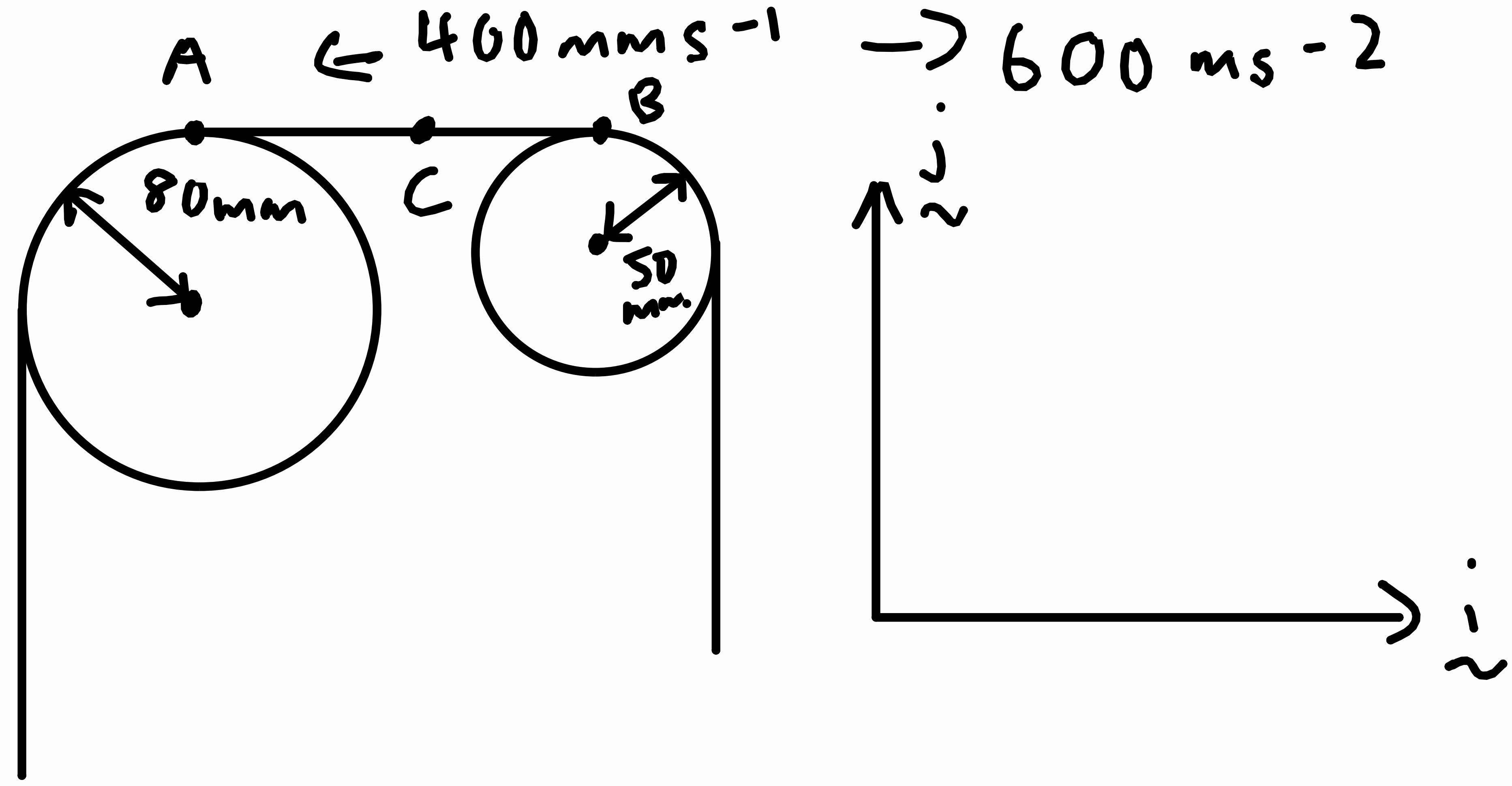
when $v = 0$, $t = 0$,

$$\frac{v_f}{2g} \ln \left| \frac{v_f + 0}{v_f - 0} \right| = 0 + c$$

$$c = 0$$

$$\therefore \; t = \frac{v_f}{2g} \ln \left| \frac{v_f + v}{v_f - v} \right|$$

5)



$\vec{v}_C = -400\underset{\sim}{i}$, $\vec{a}_C = 600\underset{\sim}{i}$ $\vec{r}_A = 80\underset{\sim}{j}$ $\vec{r}_B = 50\underset{\sim}{j}$

$\vec{v}_A = \vec{v}_B = \vec{v}_C = -400\underset{\sim}{i}$

$\vec{a}^{\,t}_A = \vec{a}^{\,t}_B = \vec{a}_C = 600\underset{\sim}{i}$

a) Assuming $\vec{w}_1 = w_1\underset{\sim}{k}$, $\vec{w}_2 = w_2\underset{\sim}{k}$, $\vec{\alpha}_1 = \alpha_1\underset{\sim}{k}$, $\vec{\alpha}_2 = \alpha_2\underset{\sim}{k}$

$\vec{v}_A = \vec{w}_1 \times \vec{r}_A$

$-400\underset{\sim}{i} = \vec{w}_1 \times 80\underset{\sim}{j}$

$-400\underset{\sim}{i} = w_1\underset{\sim}{k} \times 80\underset{\sim}{j}$

$-400\underset{\sim}{i} = -80w_1\underset{\sim}{i}$

$w_1 = 5\ rads^{-1}$

$\vec{w}_1 = 5\underset{\sim}{k}\ rads^{-1}$

Tangential acceleration at A is $\vec{a}^{\,t}_A$

$\vec{a}^{\,t}_A = \vec{\alpha}_1 \times \vec{r}_B$

$600\underset{\sim}{i} = \alpha_1\underset{\sim}{k} \times 80\underset{\sim}{j}$

$600\underset{\sim}{i} = -80\alpha_1\underset{\sim}{i}$

$\alpha_1 = -7.5\ rads^{-2}$

$\vec{\alpha}_1 = -7.5\underset{\sim}{k}\ rads^{-2}$

5a) $\vec{v}_B = \vec{w}_2 \times \vec{r}_B$ Tangential acceleration at B is $\vec{a}_B^t$

$-400\underset{\sim}{i} = w_2\underset{\sim}{k} \times 50\underset{\sim}{j}$

$-400 = -50w_2\underset{\sim}{i}$

$w_2 = 8 \text{ rads}^{-1}$

$\vec{w}_2 = 8\underset{\sim}{k} \text{ rads}^{-1}$

$\vec{a}_B^t = \vec{\alpha}_2 \times \vec{r}_B$

$600\underset{\sim}{i} = \alpha_2\underset{\sim}{k} \times 50\underset{\sim}{j}$

$600\underset{\sim}{i} = -50\alpha_2\underset{\sim}{i}$

$\alpha_2 = -50 \text{ rads}^{-1}$

$\vec{\alpha}_2 = -50\underset{\sim}{k} \text{ rads}^{-2}$

b) The total acceleration at A is the sum of the tangential and centripetal acceleration $\vec{a}_A^c$

$$\vec{a}_A = \vec{a}_A^t + \vec{a}_A^c$$

$$= 600\underset{\sim}{i} + (-w_1^2\vec{r}_A)$$

$$= 600\underset{\sim}{i} + \left[-(5^2)(80\underset{\sim}{j})\right]$$

$$= 600\underset{\sim}{i} - 2000\underset{\sim}{j}$$

$$= 2088.061302 \angle -73.30075577^\circ$$

$$\underset{\sim}{\approx} 2088 \angle -73.3^\circ$$

5b) The total acceleration at B is the sum of the tangential and centripetal acceleration $\vec{a}_B^c$

$$\vec{a}_B = \vec{a}_B^{\,t} + \vec{a}_B^{\,c}$$

$$= 600\underset{\sim}{i} + \left[-\omega_2^2 \vec{r}_B\right]$$

$$= 600\underset{\sim}{i} + \left[-(8^2)(50\underset{\sim}{j})\right]$$

$$= 600\underset{\sim}{i} - 3200\underset{\sim}{j}$$

$$= 3255.764119 \angle -79.38034472^\circ$$

$$\approx 3256 \angle -79.3^\circ$$

6) Let $\vec{x} = \underset{\sim}{i}$, $\vec{y} = \underset{\sim}{j}$, $\vec{z} = \underset{\sim}{k}$

$$\vec{\alpha} = 5\underset{\sim}{k}$$

$$\vec{w} = w_0 + 5(t)\underset{\sim}{k}$$

Since $w_0 = 0$,

$$\vec{w} = 5t\underset{\sim}{k}$$

Let $\vec{a}_T$ be the tangential acceleration,

$\vec{a}_c$ be the centripetal acceleration

$$\vec{a} = \vec{a}_T + \vec{a}_c$$

$$= r\alpha\hat{e}_T + w^2 r\hat{e}_c$$

$$= 250\times10^{-3}(5)\hat{e}_T + (5t)^2(250\times10^{-3})\hat{e}_c$$

$$= 1.25\hat{e}_T + 6.25t^2\hat{e}_c$$

$$|\vec{a}| = \sqrt{1.25^2 + (6.25t^2)^2}$$

$$4^2 = \frac{25}{16} + \frac{625}{16}t^4$$

$$t^4 = \frac{231}{625}$$

$$t = 0.7797097961$$

$$\approx 0.7797\text{s}$$

6) $\vec{w} = 5t\underset{\sim}{k}$

when $t = 0.7797$,

$$\vec{w} = 5(0.7797)\underset{\sim}{k}$$

$$= 3.898548981\underset{\sim}{k}$$

$$\approx 3.90\underset{\sim}{k} \text{ rad}s^{-1}$$