1) All radiation from surface 1 will be captured by surface 3

Reciprocity rule: $A_3 F_{31} = A_1 F_{13}$

$$F_{31} = \frac{A_1}{A_3} F_{13}$$

$$= \frac{A_1}{A_3}$$

$$= \frac{\frac{\cancel{\pi} d^2}{\cancel{4}\,2}}{\frac{\cancel{\pi} D^2}{\cancel{2}}}$$

$$= \frac{d^2}{2D^2}$$

2) Reciprocity rule: $A_3 F_{31} = A_1 F_{13}$

$$F_{13} = \frac{A_3}{A_1} F_{31}$$

Summation rule: $F_{11} + F_{13} = 1$

$$F_{11} = 1 - F_{13}$$

$$F_{11} = 1 - \frac{A_3}{A_1} F_{31}$$

$$= 1 - \frac{\frac{\cancel{\pi} D^{\cancel{2}}}{\cancel{4}2}}{\frac{\cancel{\pi}\cancel{D}l}{\cancel{\pi}}}$$

$$= 1 - \frac{D}{2l}$$

$$= 1 - \frac{100}{2\sqrt{100^2 + 50^2}}$$

$$= 1 - \frac{\cancel{100}}{\cancel{2(50)}\sqrt{5}}$$

$$= 1 - \frac{1}{\sqrt{5}}$$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{A_2}{A_1} F_{21}$$

$$= \frac{A_3}{A_1} F_{21}$$

$$= \frac{1}{\sqrt{5}} F_{21}$$

2) $F_{21} = F_{23}$

Compute graph parameters:

$$\frac{L}{r_1} = \frac{L}{\frac{1}{2}D_2} = 2$$

$$\frac{r_2}{L} = \frac{\frac{1}{2}D_3}{L} = 0.5$$

From the graph,

$$F_{23} = 0.17$$

$$\therefore F_{12} = \frac{0.17}{\sqrt{5}}$$

3) $\dot{Q} = hA(T_s - T_{surr}) + \sigma\varepsilon A(T_s^4 - T_{surr}^4)$

$\dot{Q}_{old} = \dot{Q}_{new}$

$h\cancel{A}(\cancel{T_s} - T_{\infty_1}) + \sigma\varepsilon\cancel{A}(\cancel{T_s^4} - T_{surr_1}^4) = h\cancel{A}(\cancel{T_s} - T_{\infty_2}) + \sigma\varepsilon\cancel{A}(\cancel{T_s^4} - T_{surr_2}^4)$

$-hT_{\infty_1} - \sigma\varepsilon T_{surr_1}^4 = -hT_{\infty_2} - \sigma\varepsilon T_{surr_2}^4$

$-3.1(22) - 5.67\times10^{-8}(0.95)(22+273)^4 = -3.1T_{\infty_2} - 5.67\times10^{-8}(0.95)(18+273)^4$

$T_{\infty_2} = 28.99339484°C$
$\approx 29.0°C$

4) Summation rule: $F_{22} + F_{21} = 1$

$F_{22} = 1 - F_{21}$

Reciprocity rule: $A_1F_{12} = A_2F_{21}$

$F_{21} = \frac{A_1}{A_2}F_{12}$

$= \frac{A_1}{A_2}$

$= 0.5$

$\therefore F_{22} = 0.5$

Energy balance for surface 1: $\dot{Q} - \cancel{W} = \cancel{\frac{dU}{dt}}$

4) Steady state: $\dot{Q}_{in} - \dot{Q}_{loss} = 0$

$$\dot{Q}_{in} - Q_{rad,1,2} = 0$$

$$\dot{Q}_{in} - A_1 \underset{\to 1}{F_{12}} \sigma (T_1^4 - T_2^4) = 0$$

Heat flux: $\dot{q}_{in} - \sigma(T_1^4 - T_2^4) = 0$

$$\dot{q}_{in} = \sigma(T_1^4 - T_2^4)$$

$$T_1^4 = \frac{\dot{q}_{in}}{\sigma} + T_2^4$$

$$T_1 = \left(\frac{\dot{q}_{in}}{\sigma} + T_2^4\right)^{0.25}$$

$$= \left(\frac{1600}{5.67\times10^{-8}} + 500^4\right)^{0.25}$$

$$= 548.8127577K$$

$$\approx 548.8K$$