1) Film temperature, $T_f = \frac{1}{2}(T_s + T_\infty)$

$$= \frac{1}{2}(65 + 15)$$

$$= 40°C$$

Air properties @ 40°C:

$k = 0.02662 \ Wm^{-1}K^{-1}$

$\rho = 1.127 \ kg \ m^{-3}$

$\mu = 1.918 \times 10^{-5} \ kg \ m s^{-1}$

$v = 1.702 \times 10^{-5} \ m^2 s^{-1}$

$Pr = 0.7255$

$$Re_x = \frac{\rho U_\infty x}{\mu}$$

$$= \frac{1.127(3)(0.3)}{1.918 \times 10^{-5}}$$

$$= 5.288 \times 10^4 < 5 \times 10^5$$

$\therefore$ The flow is laminar up to the point.

a) Hydrodynamic boundary layer thickness, m:

$$\delta_x = \frac{4.91x}{\sqrt{Re_x}} = \frac{4.91(0.3)}{\sqrt{5.288 \times 10^4}}$$

$$= 0.0064053643$$

$$\approx 0.0064 \ m$$

1b) Local friction coefficient:

$$C_{f,x} = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{\sqrt{5.288\times10^4}}$$

$$= 0.0028874147 3$$

$$\approx 0.00289$$

c) Average friction coefficient: $\bar{C}_f = \dfrac{1.328}{\sqrt{Re_L}}$

$$= 2C_{f,x}$$

$$= 0.0057748294 6$$

$$\approx 0.00577$$

d) $F_f = \bar{C}_f A_s \dfrac{\rho V^2}{2}$

$$= 0.00577(0.3)^2\left(\frac{1.127(3)^2}{2}\right)$$

$$= 0.0026358 3428\,N$$

$$\approx 0.00264\,N$$

e) Local convection heat transfer coefficient:

$$Nu_x = \frac{h_x x}{k}$$

$$h_x = \frac{Nu_x k}{x}$$

$$Nu_x = 0.332 Re^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$= 0.332\left(5.288\times10^4\right)^{\frac{1}{2}}(0.7255)^{\frac{1}{3}}$$

$$= 68.60295148 \approx 68.6$$

1e) $\therefore h_x = \dfrac{68.6(0.02662)}{0.3}$

$= 6.087368562 \, Wm^{-2}K^{-1}$

$\approx 6.09 \, Wm^{-2}K^{-1}$

f) $\overline{Nu}_L = 0.664 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}} = 2Nu_{x=0.3}$

$\bar{h}_L = \dfrac{\overline{Nu}_L k}{L}$

$= 2(h_x)$

$= 12.17473712$

$\approx 12.2 \, Wm^{-2}K^{-1}$

g) Rate of convective heat transfer:

$\dot{Q} = \bar{h}_L A (T_s - T_\infty)$

$= 12.2(0.3)^2(65-15)$

$= 54.78631705$

$\approx 54.8 \, W$

2) $$\bar{h}_L = \frac{1}{L}\int_0^L h_x \, dx$$

$$= \frac{1}{L}\int_0^L \frac{Nu_x k}{L} \, dx$$

$$= \frac{1}{L}\int_0^L C Re_x^{\,m} Pr^n \left(\frac{k}{x}\right) dx \quad \text{where } C, m, n \in \mathbb{R}$$

$$= \frac{1}{L}\int_0^L C\left(\frac{U_\infty x}{\nu}\right)^m Pr^n \left(\frac{k}{x}\right) dx$$

$$= \frac{kCU_\infty^{\,m}}{L\nu^m} Pr^n \int_0^L x^{m-1} \, dx$$

$$= \frac{kCU_\infty^{\,m}}{L\nu^m} Pr^n \left[\frac{x^m}{m}\right]_0^L$$

$$= \frac{kC}{L}\left(\frac{U_\infty}{\nu}\right)^m Pr^n \left(\frac{L^m}{m} - 0\right)$$

$$= \frac{kCL^m}{Lm}\left(\frac{U_\infty}{\nu}\right)^m Pr^n$$

$$Nu_x = \frac{h_x x}{k}$$

$$h_x = \frac{Nu_x k}{x}$$

At $x = L$

$$h_L = \frac{Nu_L k}{L}$$

$$= \frac{0.035 Re_L^{\,0.8} Pr^{\frac{1}{3}}}{L}$$

2) $$\bar{h}_L = \frac{kCL^m}{Lm}\left(\frac{u_\infty}{\nu}\right)^m Pr^n$$

$$= \frac{kC}{Lm}\left(\frac{u_\infty L}{\nu}\right)^m Pr^n$$

$$= \frac{kC}{Lm} Re_L^m Pr^n$$

$$= \frac{kNu_L}{Lm}$$

$$\frac{\bar{h}_L}{h_L} = \frac{\cancel{k}\cancel{Nu_L}}{\cancel{L}m} \div \frac{\cancel{Nu_L}k}{\cancel{L}}$$

$$= \frac{1}{m}$$

when $m = 0.8$,

$$\frac{\bar{h}_L}{h_L} = \frac{1}{0.8}$$

$$= 1.25$$

3) $Re_x = \frac{\rho U_\infty x}{\mu}$

$T_f = \frac{1}{2}(10+15)$

$= 12.5°C$

Properties of air @ 12.5°C, 100kPa:

$k = 0.02458 \, W m^{-1} K^{-1}$

$\nu = 1.448 \times 10^{-5} \, m^2 s^{-1}$

$Pr = 0.7330$

$Re_{cr} = \frac{V x_{cr}}{\nu} = 5 \times 10^5$

$\frac{(2) x_{cr}}{1.448 \times 10^{-5}} = 5 \times 10^5$

$x_{cr} = 3.62 m$

$\bar{h}_L = \frac{\overline{Nu}_L k}{x}$

$= \frac{0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} k}{x}$

3a) $Re_1 = \frac{Vx}{\nu} = \frac{2(1)}{1.448 \times 10^{-5}}$

$= 138121.547$

$$\bar{h}_1 = \frac{0.664(138121.547)^{\frac{1}{2}}(0.733)^{\frac{1}{3}}(0.02458)}{1}$$

$= 5.469093309 \, W m^{-2} K^{-1}$

$\approx 5.47 \, W m^{-2} K^{-1}$

$\dot{Q}_1 = \bar{h}_1 A (T_s - T_\infty)$

$= 5.47(1 \times 4)(15 - 10)$

$= 109.3818662 \, W$

b) $Re_3 = \frac{2(3)}{1.448 \times 10^{-5}} = 414364.6409$

$$\bar{h}_3 = \frac{0.664(414364.6409)^{\frac{1}{2}}(0.733)^{\frac{1}{3}}(0.02458)}{3}$$

$= 3.15758294 \, W m^{-2} K^{-1}$

$\dot{Q}_{1 \to 3} = 3.158(3 \times 4)(15 - 10)$

$= 189.4549497 \, W$

$Re_2 = \frac{2(2)}{1.448 \times 10^{-5}} = 276243.0939$

$$\bar{h}_2 = \frac{0.664\sqrt{276243.0939}(0.733)^{\frac{1}{3}}(0.02458)}{2}$$

$= 3.867232966 \, W m^{-2} K^{-1}$

3b) $\dot{Q}_{1\to 2} = 3.867(2\times 4)(15-10)$
$= 154.6893186\,W$

$\dot{Q}_3 = 189.4549497 - 154.6893186$
$= 34.76563106\,W$
$\approx 34.8\,W$

4) Properties of air at $25°C$, $100kPa$:

$k = 0.02551\,W\,m^{-1}K^{-1}$
$\nu = 1.562\times 10^{-5}\,m^2 s^{-1}$
$Pr = 0.7296$

Air speed: $$U_\infty = \frac{110\times 10^3}{60\times 60} = \frac{275}{9}\,ms^{-1}$$

$$Re_L = \frac{U_\infty L}{\nu} = \frac{\frac{275}{9}(6)}{1.562\times 10^{-5}} = 11737089.2 > 5\times 10^5$$

$\therefore$ the flow is turbulent.

$$\overline{Nu_L} = 0.037 Re_L^{\frac{4}{5}} Pr^{\frac{1}{3}}$$
$$= 0.037(11737089.2)^{\frac{4}{5}}(0.7296)^{\frac{1}{3}}$$
$$= 15073.41501$$

$$\overline{h_L} = \frac{\overline{Nu_L}k}{L} = \frac{15073.41501(0.02551)}{6}$$
$$= 64.08713617\,W m^{-2}K^{-1}$$

4) $\dot{Q}_R = \bar{h}_L A(T_s - T_\infty)$

$$\frac{\dot{Q}_R}{\bar{h}_L A} = T_s - T_\infty$$

$$T_s = T_\infty + \frac{\dot{Q}_R}{\bar{h}_L A}$$

$$= 25 + \frac{\frac{1}{2} \times \frac{-633}{60} \times 10^3}{64.09(2)(6 \times 2.8 + 6 \times 2.1 + 2.8 \times 2.1)}$$

$$= 23.83347775$$

$$\approx 23.8°C$$

$$T_f = \frac{1}{2}(25 + 23.8)$$

$$= 24.41673887$$

$$\approx 24.4°C$$

$\therefore$ the assumption is good, as the actual film temperature of 24.4°c is close to 25°c.

5) Assumptions: Steady-state
Constant properties at film temperature
Pipe surface is smooth

$$T_f = \frac{1}{2}(75+5)$$
$$= 40°C$$

$$u_\infty = \frac{10\times10^3}{60^2}$$
$$= \frac{25}{9}\ ms^{-1}$$

Properties of air @40°C, 100kPa:

$k = 0.02662\ Wm^{-1}K^{-1}$

$\nu = 1.702\times10^{-5}\ m^2s^{-1}$

$Pr = 0.7255$

$$Re_D = \frac{u_\infty D}{\nu} = \frac{\frac{25}{9}(10\times10^{-2})}{1.702\times10^{-5}}$$
$$= 16320.66849 < 2\times10^5$$

$$Nu_{cyl} = 0.3 + \frac{0.62Re_D^{\frac{1}{2}}Pr^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\left[1+\left(\frac{Re_D}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$
$$= 0.3 + 62.58535199(1.13265554)$$
$$= 71.18764565$$

$$\bar{h} = \frac{\overline{Nu}_{cyl}k}{D}$$
$$= \frac{71.19(0.02662)}{10\times10^{-2}}$$
$$= 18.95015127\ Wm^{-2}K^{-1}$$

5) $\dot{Q}_{rad} = \sigma \varepsilon A(T_s^4 - T_{sur}^4)$

$= (5.67\times10^{-8})(0.8)(\pi(10\times10^{-2})(12))$
$((75+273)^4 - 273^4)$

$= 1558.113689\,W$

$\dot{Q}_{conv} = \bar{h}A(T_s - T_\infty)$

$= 18.95(\pi(10\times10^{-2})(12))(75-5)$

$= 5000.827105\,W$

$\dot{Q}_{total} = 5000.827105 + 1558.113689$
$= 6558.940794\,W$
$\approx 6.56\,kW$

Heat loss (1 day) $= 6.56\times10\times60^2$
$= 236.1218686\,MJ$

Heat loss (1 year) $= 236.1218686\times365$
$= 86.18448204\,GJ$

For 80% efficiency,

Total $Q_{loss} = \dfrac{86.18}{0.8}$

$= 107.7306025\,GJ$

Heat loss saved with insulation $= 0.9(107.7306025)$
$= 96.95754229\,GJ$

Money saved $= \dfrac{96.95754229\times10^6}{105500}\times1.05$

$= \$964.98$