1)

$f(t)$	$g(t)$	$-f(t)$	$f(t)+g(t)$	$f(t)-g(t)$	$f(t)\times g(t)$	$\frac{f(t)}{g(t)}$
even	even	even	even	even	even	even
odd	odd	odd	odd	odd	even	even
even	odd	even	?	?	odd	odd
odd	even	odd	?	?	odd	odd

2a) $f(t)$ is a periodic function with period T.

When $t=0, f(t)=0$

$t=\frac{T}{4}, f(t)=1$

$f(t)=mt+c$

$0=m(0)+c$

$\therefore c=0$

$f(t)=mt$

$1=m\left(\frac{T}{4}\right)$

$m=\frac{4}{T}$

$\therefore f(t)=\frac{4}{T}t$ for $0\le t\le\frac{T}{4}$

When $t=\frac{T}{4}, f(t)=1$

$t=\frac{T}{2}, f(t)=0$

$t=\frac{3T}{4}, f(t)=-1$

$f(t)=mt+c$

$0=m\left(\frac{T}{2}\right)+c$

$c=-m\frac{T}{2}$ $-(1)$

$f(t)=mt-m\frac{T}{2}$

$f(t)=m\left(t-\frac{T}{2}\right)$

$1=m\left(\frac{T}{4}-\frac{T}{2}\right)$

$1=m\left(-\frac{1}{4}T\right)$

$m=-\frac{4}{T}$

$\therefore f(t)=-\frac{4}{T}\left(t-\frac{T}{2}\right)$

$=\frac{4}{T}\left(\frac{T}{2}-t\right)$ for $\frac{T}{4}\le t\le\frac{3T}{4}$

2a) When $t = \frac{3T}{4}$, $f(t) = -1$

$t = T$, $f(t) = 0$

$$f(t) = mt + c \qquad f(t) = mt - mT$$

$$0 = mT + c \qquad f(t) = m(t - T)$$

$$c = -mT \qquad -1 = m\left(\frac{3T}{4} - T\right)$$

$$-1 = m\left(-\frac{1}{4}T\right)$$

$$m = \frac{4}{T}$$

$$\therefore f(t) = \frac{4}{T}(t - T) \text{ for } \frac{3T}{4} \leq t \leq T$$

$$f(t) = \begin{cases} \frac{4}{T}t & \text{for } 0 \leq t \leq \frac{T}{4} \\ \frac{4}{T}\left(\frac{T}{2} - t\right) & \text{for } \frac{T}{4} \leq t \leq \frac{3T}{4} \\ \frac{4}{T}(t - T) & \text{for } \frac{3T}{4} \leq t \leq T \end{cases}$$

b) The function is an odd periodic function, as $f(-t) = f(t)$. Hence, the function is symmetric.

2c) Yes, the function can be represented by a Fourier Series.

$C_0 = 0$, $A_n = 0$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

Since $f(t)$ and $\sin\left(\frac{2\pi nt}{T}\right)$ are both odd functions, $f(t)\sin\left(\frac{2\pi nt}{T}\right)$ will be an even function.

$$\therefore B_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$= \frac{4}{T}\int_0^{\frac{T}{4}} \frac{4}{T}t\sin\left(\frac{2\pi nt}{T}\right)dt + \frac{4}{T}\int_{\frac{T}{4}}^{\frac{T}{2}} \frac{4}{T}\left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$= \frac{16}{T^2}\left[\int_0^{\frac{T}{4}} t\sin\left(\frac{2\pi nt}{T}\right)dt + \int_{\frac{T}{4}}^{\frac{T}{2}}\left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi nt}{T}\right)dt\right]$$

Let $X_n = \frac{16}{T^2}\int_0^{\frac{T}{4}} t\sin\left(\frac{2\pi nt}{T}\right)dt$,

$Y_n = \frac{16}{T^2}\int_{\frac{T}{4}}^{\frac{T}{2}}\left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi nt}{T}\right)dt$

2c) $X_n = \frac{16}{T^2}\int_0^{\frac{T}{4}} t\sin\left(\frac{2\pi n t}{T}\right)dt$

$$= \frac{\overset{8}{\cancel{16}}}{T\cancel{2}}\left[-\left[\frac{\cancel{T}t}{\cancel{2}\pi n}\cos\left(\frac{2\pi n t}{T}\right)\right]_0^{\frac{T}{4}} + \frac{\cancel{T}}{\cancel{2}\pi n}\int_0^{\frac{T}{4}}\cos\left(\frac{2\pi n t}{T}\right)dt\right]$$

$$= \frac{8}{\pi n T}\left[\left[-t\cos\left(\frac{2\pi n t}{T}\right)\right]_0^{\frac{T}{4}} + \int_0^{\frac{T}{4}}\cos\left(\frac{2\pi n t}{T}\right)dt\right]$$

$$= \frac{8}{\pi n \cancel{T}}\left[\left[-\frac{\cancel{T}}{4}\cos\left(\frac{\cancel{2}\pi n\left(\frac{\cancel{T}}{\cancel{4}}\right)}{\cancel{T}}\right)+0\right] + \frac{\cancel{T}}{2\pi n}\left[\sin\left(\frac{2\pi n t}{T}\right)\right]_0^{\frac{T}{4}}\right]$$

$$= \frac{8}{\pi n}\left(-\frac{1}{4}\cos\left(\frac{\pi n}{2}\right) + \frac{1}{2\pi n}\left[\sin\left(\frac{\cancel{2}\pi n\left(\frac{\cancel{T}}{\cancel{4}_2}\right)}{\cancel{T}}\right) - \cancel{\sin 0}\right]\right)$$

$$= \frac{8}{\pi n}\left(-\frac{1}{4}\cos\left(\frac{\pi n}{2}\right) + \frac{1}{2\pi n}\sin\left(\frac{\pi n}{2}\right)\right)$$

$$= \frac{4}{(\pi n)^2}\sin\left(\frac{\pi n}{2}\right) - \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right)$$

2c) $Y_n = \frac{16}{T^2}\int_{\frac{T}{4}}^{\frac{T}{2}} \left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi n t}{T}\right)dt$

$$= \frac{\cancel{16}^{\,8}}{T^{\cancel{2}}}\left[\left[-\frac{\cancel{T}}{\cancel{2}\pi n}\cos\left(\frac{2\pi n t}{T}\right)\left(\frac{T}{2}-t\right)\right]_{\frac{T}{4}}^{\frac{T}{2}} + \frac{\cancel{T}}{2\pi n}\int_{\frac{T}{4}}^{\frac{T}{2}} -\cos\left(\frac{2\pi n t}{T}\right)dt\right]$$

$$= \frac{8}{\pi n T}\left[\left[\left(t-\frac{T}{2}\right)\cos\left(\frac{2\pi n t}{T}\right)\right]_{\frac{T}{4}}^{\frac{T}{2}} - \int_{\frac{T}{4}}^{\frac{T}{2}} \cos\left(\frac{2\pi n t}{T}\right)dt\right]$$

$$= \frac{8}{\pi n T}\left[\cancel{\left(\frac{T}{2}-\frac{T}{2}\right)\cos\left(\frac{2\pi n\left(\frac{T}{2}\right)}{T}\right)} - \left(\frac{T}{4}-\frac{T}{2}\right)\cos\left(\frac{2\pi n\left(\frac{\cancel{T}}{\cancel{2}4}\right)}{\cancel{T}}\right)\right]$$

$$- \frac{T}{2\pi n}\left[\sin\left(\frac{2\pi n t}{T}\right)\right]_{\frac{T}{4}}^{\frac{T}{2}}$$

$$= \frac{8}{\pi n \cancel{T}}\left[\frac{\cancel{T}}{4}\cos\left(\frac{\pi n}{2}\right) - \frac{\cancel{T}}{2\pi n}\left[\cancel{\sin\left(\frac{2\pi n\left(\frac{T}{2}\right)}{T}\right)}^{\,0} - \sin\left(\frac{2\pi n\left(\frac{T}{4}\right)}{\cancel{T}}\right)\right]\right]$$

$$= \frac{8}{\pi n}\left[\frac{1}{4}\cos\left(\frac{\pi n}{2}\right) + \frac{1}{2\pi n}\sin\left(\frac{\pi n}{2}\right)\right]$$

$$= \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2}\sin\left(\frac{\pi n}{2}\right)$$

$\therefore B_n = X_n + Y_n$

$$= \frac{4}{(\pi n)^2}\sin\left(\frac{\pi n}{2}\right) \cancel{- \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right)} + \frac{4}{(\pi n)^2}\sin\left(\frac{\pi n}{2}\right)$$

$$= \frac{8}{(\pi n)^2}\sin\left(\frac{\pi n}{2}\right)$$

$$\sin\left(\frac{\pi n}{2}\right) = \begin{cases} 0, & n = 2k \\ 1, & n = 4k+1 \\ -1, & n = 4k-1 \end{cases}$$

$\therefore$ The Fourier Series is :

$$F(t) = \frac{8}{\pi^2}\left[\sin\left(\frac{2\pi}{T}t\right) - \frac{1}{3^2}\sin\left(\frac{6\pi}{T}t\right) + \frac{1}{5^2}\sin\left(\frac{10\pi}{T}t\right) + \dots\right]$$

2d) The peak amplitude is 1.
The peak-to-peak amplitude is 2.

e) When the peak amplitude is changed to A, the Fourier Series will be multiplied by A, as the original function had an amplitude 1.

$$\therefore f(t) = \begin{cases} \frac{4A}{T}t, & 0 \le t \le \frac{T}{4} \\ \frac{4A}{T}\left(\frac{T}{2}-t\right), & \frac{T}{4} \le t \le \frac{3T}{4} \\ \frac{4A}{T}(t-T), & \frac{3T}{4} \le t \le T \end{cases}$$

$$B_n = A\left(\frac{8}{(\pi n)^2}\sin\left(\frac{\pi n}{2}\right)\right)$$

$$= \frac{8A}{(\pi n)^2}\sin\left(\frac{\pi n}{2}\right)$$

$$F(t) = \frac{8A}{\pi^2}\left[\sin\left(\frac{2\pi}{T}t\right) - \frac{1}{3^2}\sin\left(\frac{6\pi}{T}t\right) + \frac{1}{5^2}\sin\left(\frac{10\pi}{T}t\right) + \dots\right]$$

3a) $\frac{1}{2}T = 0.01$

$T = 0.02$

$$w = \frac{2\pi}{T}$$

$$= \frac{2\pi}{0.02}$$

$$= 100\pi$$

$\therefore f(t) = 10\sin(100\pi t), \quad 0 \le t \le 0.02$

b) $F(t) = 10\sin(100\pi t), \quad 0 \le t \le 0.02$

C_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
0	10	0	0	0	0	0	0	0	0	0

c) $g(t) = A\sin(100\pi t)$

$G(t) = A\sin(100\pi t)$

4a) $f(t) = 10|\sin(100\pi t)|$

b) $f(t) = \begin{cases} 10\sin(100\pi t), & 0 \leq t \leq \frac{T}{2} \\ -10\sin(100\pi t), & \frac{T}{2} \leq t \leq T \end{cases}$

Since the function is even,

$B_n = 0$

$C_0 = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\,dt$

$A_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(100n\pi t)\,dt$

$C_0 = \frac{1}{T}\int_0^{T} 10\sin(100\pi t)\,dt$

$= \frac{1}{T}\left[\int_0^{\frac{T}{2}} 10\sin(100\pi t)\,dt + \int_{\frac{T}{2}}^{T} -10\sin(100\pi t)\,dt\right]$

$= \frac{2}{T}\int_0^{\frac{T}{2}} 10\sin(100\pi t)\,dt$

$= \frac{20}{100\pi T}\left[-\cos\left(100\pi\left(\frac{T}{2}\right)\right) - (-\cos 0)\right]$

$= \frac{20}{100\pi T}\left[1 - \cos(50\pi T)\right]$

$= \frac{\cancel{2}0}{100\pi(0.0\cancel{2})}\left[1 - \cos(50\pi(0.02))\right]$

$= \frac{10}{\pi}\left[1 - (-1)\right]$

$= \frac{20}{\pi}$

4b) $A_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(100n\pi t)\,dt$

$$= \frac{4}{T}\int_0^{\frac{T}{2}} 10\sin(100\pi t)\cos(100n\pi t)\,dt$$

$$= \frac{20}{T}\int_0^{\frac{T}{2}} \sin(100\pi t(1+n)) + \sin(100\pi t(1-n))\,dt$$

$$= -\frac{20}{T}\left[\frac{\cos(100\pi t(1+n))}{100\pi(1+n)} + \frac{\cos(100\pi t(1-n))}{100\pi(1-n)}\right]_0^{\frac{T}{2}}$$

$$= -\frac{20}{T}\left[\frac{\cos(100\pi t(n+1))}{100\pi(n+1)} - \frac{\cos(100\pi t(1-n))}{100\pi(n-1)}\right]_0^{\frac{T}{2}}$$

$$= \frac{-20}{T}\left[\frac{\cos(50\pi T(n+1))}{100\pi(n+1)} - \frac{\cos(50\pi T(1-n))}{100\pi(n-1)} - \left(\frac{\cos 0}{100\pi(n+1)} - \frac{\cos 0}{100\pi(n-1)}\right)\right]$$

$$= -\frac{20}{0.02}\left[\frac{\cos(50\pi(0.02)(n+1))}{100\pi(n+1)} - \frac{\cos(50\pi(0.02)(1-n))}{100\pi(n-1)} - \frac{1}{100\pi(n+1)} + \frac{1}{100\pi(n-1)}\right]$$

$$= -\frac{10}{(n+1)\pi}\left[\cos((n+1)\pi) - 1\right] + \frac{10}{(n-1)\pi}\left[\cos((1-n)\pi) - 1\right]$$

When n is even, $\cos((n+1)\pi) = -1$, $\cos((1-n)\pi) = -1$

When n is odd, $\cos((n+1)\pi) = 1$, $\cos((1-n)\pi) = 1$

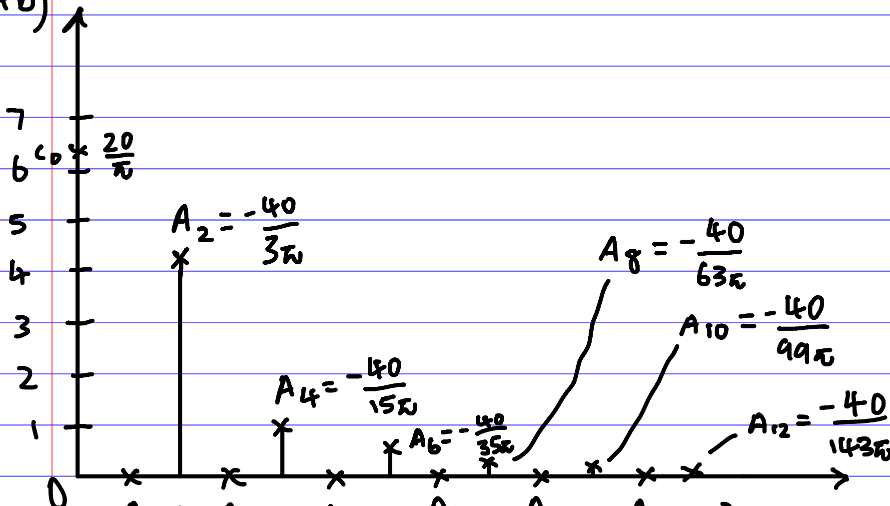
Since the function is even,

$$A_n = -\frac{10}{(n+1)\pi}[-1-1] + \frac{10}{(n-1)\pi}[-1-1]$$

$$= \frac{20}{(n+1)\pi} - \frac{20}{(n-1)\pi}$$

$$\therefore F(t) = \frac{20}{\pi} + \sum_{n=2,4,\dots}^{\infty} \frac{20}{(n+1)\pi} - \frac{20}{(n-1)\pi}$$

4b)



$A_1 = A_3 = A_5 = A_7 = A_9 = A_{11} = 0$

$$A_n = \frac{20}{(n+1)\pi} - \frac{20}{(n-1)\pi}$$

$$= \frac{20(n-1) - 20(n+1)}{(n+1)(n-1)\pi}$$

$$= \frac{-40}{\pi(n^2-1)}$$

$$A_{2k} = \frac{-40}{\pi(4k^2-1)}$$

4c) $g(t) = A|\sin(50\pi t)|$ $\begin{cases} A\sin(50\pi t), & 0 \le t \le \frac{T}{2} \\ -A\sin(50\pi t), & \frac{T}{2} \le t \le T \end{cases}$

$C_0 = 0, \; A_n = 0$

$$B_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(50\pi nt)\,dt$$

$$= \frac{4}{T}\int_0^{\frac{T}{2}} \sin(50\pi t)\sin(50\pi nt)\,dt$$

$$= \frac{2}{T}\int_0^{\frac{T}{2}} \cos(50\pi t - 50\pi nt) - \cos(50\pi t + 50\pi nt)\,dt$$

$$= \frac{2}{T}\int_0^{\frac{T}{2}} \cos(50\pi t(1-n)) - \cos(50\pi t(1+n))\,dt$$

$$= \frac{2}{50\pi T}\left[\frac{-\sin(50\pi(\frac{T}{2})(1-n))}{(1-n)} - \frac{-\sin(50\pi(\frac{T}{2})(1+n))}{(1+n)} - \cancel{\left(\frac{-\sin 0}{(1-n)} - \frac{-\sin 0}{(1+n)}\right)}\right]$$

$$= \frac{2}{50\pi T}\left[\frac{\sin(25\pi T(1+n))}{1+n} - \frac{\sin(25\pi T(1-n))}{1-n}\right]$$

$$= \frac{\sin((1+n)\pi)}{\pi(1+n)} - \frac{\sin((1-n)\pi)}{\pi(1-n)}$$

$$= 0$$