1) $y''(t) - 4y'(t) + 3y(t) = 6t - 8$

$$\mathcal{L}\{y''(t) - 4y'(t) + 3y(t)\} = \mathcal{L}\{6t - 8\}$$

$$s^2\mathcal{L}\{y(t)\} - \cancel{sy(0)} - \cancel{y'(0)} - 4s\mathcal{L}\{y(t)\} - \cancel{4y(0)} +$$

$$3\mathcal{L}\{y(t)\} = \frac{6}{s^2} - \frac{8}{s}$$

$$x = \mathcal{L}\{y(t)\}$$

$$s^2x - 4sx + 3x = \frac{6-8s}{s^2}$$

$$x(s^2 - 4s + 3) = \frac{6-8s}{s^2}$$

$$x = \frac{6-8s}{s^2(s^2-4s+3)}$$

$$x = \frac{6-8s}{s^2(s-3)(s-1)}$$

1) $$\frac{6-8s}{s^2(s-3)(s-1)} = \frac{As+B}{s^2} + \frac{C}{s-3} + \frac{D}{s-1}$$

$$6-8s = (As+B)(s-3)(s-1) + Cs^2(s-1) + Ds^2(s-3)$$

when $s=1$,

$$6-8 = -2D$$

$$D = 1$$

when $s=3$,

$$6-24 = 18C$$

$$C = -1$$

when $s=0$,

$$6 = 3B$$

$$B = 2$$

when $s=2$,

$$6-16 = (2A+2)(-1)(1) + 4(-1)(1) + 4(1)(-1)$$

$$-10 = -2A-2-4+4$$

$$A = 0$$

1) $\therefore \dfrac{6-8s}{s^2(s-3)(s-1)} = \dfrac{2}{s^2} - \dfrac{1}{s-3} + \dfrac{1}{s-1}$

$$\mathcal{L}\{y(t)\} = \frac{2}{s^2} - \frac{1}{s-3} + \frac{1}{s-1}$$

$$y(t) = 2t - e^{3t} + e^{t}$$

2a) $\mathcal{L}\{g(t)\} = G(s) = \frac{1}{s^2+4s}$

$$\frac{1}{s}F(s) = \frac{1}{s}\left(\frac{1}{s+4}\right)$$

$$F(s) = \frac{1}{s+4}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s+4}$$

$$f(t) = e^{-4t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\frac{1}{s+4}\right\} = \int_0^t f(\tau)\,d\tau$$

$$= \int_0^t e^{-4\tau}\,d\tau$$

$$= \frac{e^{-4t}-1}{-4}$$

$$= \frac{1}{4}\left(1-e^{-4t}\right)$$

2b) $G(s) = \frac{1}{s(s^2+w^2)}$

$$F(s) = \frac{1}{s^2+w^2}$$

$$f(t) = \frac{1}{w}\sin(wt)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+w^2)}\right\} = \frac{1}{w}\int_0^t \sin(w\tau)\,d\tau$$

$$= -\frac{1}{w}\left[\frac{\cos(w\tau)}{w}\right]_0^t$$

$$= -\frac{1}{w^2}\left(\cos(wt) - 1\right)$$

$$= \frac{1-\cos(wt)}{w^2}$$

3a) $g(t) = t^2 u(t-1)$

$f(t-1) = t^2$

$f(t) = (t+1)^2$

$F(s) = \mathcal{L}\{(t+1)^2\}$

$= \mathcal{L}\{t^2+2t+1\}$

$= \frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}$

$\mathcal{L}\{t^2 u(t-1)\} = e^{-s}\left(\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}\right)$

3b) $g(t) = \sin(wt)\left[u(t) - u\left(t-\frac{\pi}{w}\right)\right]$

$$= u(t)\sin(wt) - u\left(t-\frac{\pi}{w}\right)\sin(wt)$$

$$= u(t)\sin(wt) - u\left(t-\frac{\pi}{w}\right)\sin\left(w\left(t-\frac{\pi}{w}+\frac{\pi}{w}\right)\right)$$

$$= u(t)\sin(wt) - u\left(t-\frac{\pi}{w}\right)\sin\left(w\left(t-\frac{\pi}{w}\right)+\pi\right)$$

$$= u(t)\sin(wt) + u\left(t-\frac{\pi}{w}\right)\sin\left(w\left(t-\frac{\pi}{w}\right)\right)$$

$$F(s) = \mathcal{L}\{\sin(wt)\}$$

$$= \frac{w}{s^2+w^2}$$

$$\mathcal{L}\{g(t)\} = \mathcal{L}\left\{u(t)\sin(wt) + u\left(t-\frac{\pi}{w}\right)\sin\left(w\left(t-\frac{\pi}{w}\right)\right)\right\}$$

$$= \frac{w}{s^2+w^2} + \frac{we^{-\frac{\pi s}{w}}}{s^2+w^2}$$

4) $$\frac{3(1-e^{-\pi s})}{s^2+9} = \frac{3(1-e^{-\pi s})}{s^2+3^2}$$

$$= \frac{3}{s^2+3^2}(1-e^{-\pi s})$$

$$= \frac{3}{s^2+3^2} - \frac{3e^{-\pi s}}{s^2+3^2}$$

$$g(t) = \mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2} - \frac{3e^{-\pi s}}{s^2+3^2}\right\}$$

$$= \sin(3t) - u(t-\pi)\sin(3t)$$

5) Let $f(t) = \cosh(\pi t)$

$$F(s) = \mathcal{L}\{f(t)\}$$

$$= \mathcal{L}\{\cosh(\pi t)\}$$

$$= \frac{s}{s^2 - \pi^2}$$

$$\frac{dF}{ds} = \frac{(s^2-\pi^2) - s(2s)}{(s^2-\pi^2)^2}$$

$$= -\frac{s^2+\pi^2}{(s^2-\pi^2)^2}$$

$$\frac{d^2F}{ds^2} = -\frac{2s(s^2-\pi^2)^2 - 2(2s)(s^2-\pi^2)(s^2+\pi^2)}{(s^2-\pi^2)^4}$$

$$= -\frac{2s(s^2-\pi^2)^{\cancel{2}} - 4s\cancel{(s^2-\pi^2)}(s^2+\pi^2)}{(s^2-\pi^2)^{\cancel{4}3}}$$

$$= \frac{4s(s^2+\pi^2) - 2s(s^2-\pi^2)}{(s^2-\pi^2)^3}$$

$$= \frac{4s^3 + 4s\pi^2 - 2s^3 + 2s\pi^2}{(s^2-\pi^2)^3}$$

$$= \frac{2s^3 + 6s\pi^2}{(s^2-\pi^2)^3}$$

$$= \frac{2(s^3+3s\pi^2)}{(s^2-\pi^2)^3}$$

5) $\therefore \mathcal{L}\{t^2\cosh(\pi t)\} = (-1)^2\left[\dfrac{2(s^3+3s\pi^2)}{(s^2-\pi^2)^3}\right]$

$$= \frac{2(s^3+3s\pi^2)}{(s^2-\pi^2)^3}$$

6) $\mathcal{L}\{f(t)\} = p(s) = \ln\left[\dfrac{s^2+1}{(s-1)^2}\right]$

$$= \ln(s^2+1) - 2\ln(s-1)$$

$$F(s) = \ln(s^2+1) - 2\ln(s-1)$$

$$\frac{dF}{ds} = \frac{2s}{s^2+1} - \frac{2}{s-1}$$

$$\mathcal{L}\{t f(t)\} = \frac{2}{s-1} - \frac{2s}{s^2+1}$$

$$t f(t) = \mathcal{L}^{-1}\left\{\frac{2}{s-1} - \frac{2s}{s^2+1}\right\}$$

$$t f(t) = 2(e^t - \cos t)$$

$$f(t) = \frac{2}{t}(e^t - \cos t)$$