1a) $\sqrt{x+yz}$

$x = t^2$ $\quad$ $\frac{dx}{dt} = 2t$ $\quad$ $z = t$ $\quad$ $\frac{dz}{dt} = 1$

$y = 3t$ $\quad$ $\frac{dy}{dt} = 3$

$= \sqrt{t^2 + 3t(t)}$

$= \sqrt{t^2 + 3t^2}$

$= \sqrt{4t^2}$

$= 2t$

$$\int_C \sqrt{x+ydz}\, ds$$

$$= \int_1^2 2t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$= \int_1^2 2t \sqrt{4t^2 + 9 + 1}\, dt$$

$$= \int_1^2 2t \sqrt{4t^2 + 10}\, dt$$

$$= \left[\frac{(4t^2+10)^{\frac{3}{2}}}{6}\right]_1^2$$

$$= \frac{1}{6}\left(26\sqrt{26} - 14\sqrt{14}\right) = \frac{1}{3}\left(13\sqrt{26} - 7\sqrt{14}\right)$$

1b) $\int_C xy\, ds$

$$= \int_0^{\frac{\pi}{2}} \cos t \sin t \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2t\, dt$$

$$= \frac{1}{2}\left[-\frac{\cos 2t}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[-(-1) - [-(-1)]\right]$$

$$= \frac{1}{2}$$

1c) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 5-1 \\ 0-2 \\ 2-3 \end{bmatrix}$

$\therefore x = 1+4t$ $\qquad \frac{dx}{dt} = 4$

$y = 2-2t$ $\qquad \frac{dy}{dt} = -2$

$z = 3-t$ $\qquad \frac{dz}{dt} = -1$

$$\int_C x^2 + yz \, ds$$

$$= \int_0^1 \left[(1+4t)^2 + (2-2t)(3-t)\right] \sqrt{4^2 + (-2)^2 + (-1)^2} \, dt$$

$$= \sqrt{21} \int_0^1 1 + 8t + 16t^2 + 6 - 2t - 6t + 2t^2 \, dt$$

$$= \sqrt{21} \int_0^1 18t^2 + 7 \, dt$$

$$= \sqrt{21} \left[6t^3 + 7t\right]_0^1$$

$$= \sqrt{21}(13)$$

$$= 13\sqrt{21}$$

2) $y = x^{\frac{3}{2}}$, $0 \le x \le 1$, $z = 0$

$x = t$, $\frac{dx}{dt} = 1$

$y = t^{\frac{3}{2}}$, $\frac{dy}{dt} = \frac{3}{2}\sqrt{t}$

$$\int_C ds = \int_0^1 \sqrt{1^2 + \left(\frac{3}{2}\sqrt{t}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{1 + \frac{9}{4}t}\, dt$$

$$= \frac{4}{9}\int_0^1 \frac{9}{4}\sqrt{1 + \frac{9}{4}t}\, dt$$

$$= \frac{4}{9}\left[\frac{\left(1 + \frac{9}{4}t\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \frac{8}{27}\left[\frac{13}{4}\sqrt{\frac{13}{4}} - 1\right]$$

$$= \frac{13}{27}\sqrt{13} - \frac{8}{27}$$

3a) $\underset{\sim}{F} = (2y, -z, x)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} -1-0 \\ 0-1 \\ 5-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$$

$\therefore x = -t \qquad \frac{dx}{dt} = -1$

$y = 1 - t \qquad \frac{dy}{dt} = -1$

$z = 2 + 3t$

$\frac{dz}{dt} = 3$

$$\int_c \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_0^1 (2-2t, -2-3t, -t) \cdot (-1, -1, 3)\, dt$$

$$= \int_0^1 2t \cancel{-2+2} + \cancel{3t - 3t}\, dt$$

$$= \int_0^1 2t\, dt$$

$$= \left[t^2 \right]_0^1$$

$$= 1$$

3b) $\underset{\sim}{F} = (x, -2y, z)$, $(x, y, z) = (\sin t, 2\cos t, t)$

$$\left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = (\cos t, -2\sin t, 1)$$

$$\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_0^{\frac{\pi}{4}} (\sin t, -4\cos t, t) \cdot (\cos t, -2\sin t, 1)\, dt$$

$$= \int_0^{\frac{\pi}{4}} \sin t\cos t + 8\cos t\sin t + t\, dt$$

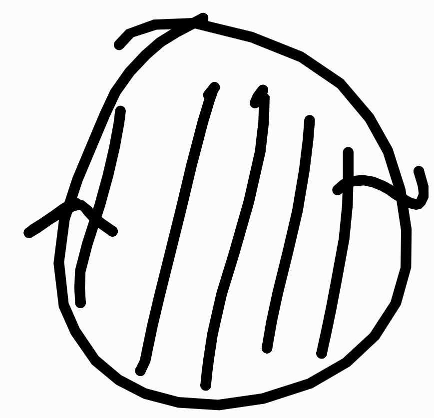
$$= \int_0^{\frac{\pi}{4}} \frac{1}{2}\sin 2t + 4\sin 2t + t\, dt$$

$$= \left[-\frac{9}{4}\cos 2t + \frac{t^2}{2}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi^2}{32} + \frac{9}{4}$$

$$= \frac{72 + \pi^2}{32}$$

4) Let $\underset{\sim}{F} = (f(x,y), g(x,y))$
$= ((3x^2+2)y, x^3-x)$



$$\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_C (3x^2+2)y\,dx + (x^3-x)\,dy$$

$$= -\iint_R \cancel{3x^2} - 1 - \cancel{3x^2} - 2 \; dx\,dy$$

$$= -\iint_R -3\,dx\,dy$$

$$= -\int_{r=0}^{r=2} \int_{\theta=0}^{\theta=2\pi} -3r\,dr\,d\theta$$

$$= -\int_0^{2\pi} \left[-\frac{3r^2}{2}\right]_0^2 d\theta$$

$$= \int_0^{2\pi} 6\,d\theta$$

$$= [6\theta]_0^{2\pi}$$

$$= 12\pi$$

5) $\underset{\sim}{F} = (f(x,y), g(x,y)) = (xy, xy)$

$$\text{Region} = \cancel{\int_{(0,0)-(2,0)} x\overset{\overset{0}{\uparrow}}{y}\,dx + xy\,\underset{\underset{0}{\downarrow}}{dy}} + \int_{(2,0)-(1,1)} xy\,dx + xy\,dy + \int_{(1,1)-(0,0)} xy\,dx + xy\,dy$$

$(2,0) + t(1,-1)$ $\quad$ $(1,1) + t(1,1)$

$$= \cancel{\int_0^1 (2+t)(-t)(1) + (2+t)(-t)(-1)\,dt}$$

$$+ \int_0^1 (1+t)^2 + (1+t)^2\,dt$$

$$= \int_0^{-1} 2(1+t)^2\,dt$$

$$= \left[\frac{2(1+t)^3}{3}\right]_0^{-1}$$

$$= -\frac{2}{3}$$

5) Using Green's Theorem:

$$\int_C xy\,dx + xy\,dy = \iint_R (y-x)\,dx\,dy$$

$$= \int_{x=0}^{x=2} \int_{y=0}^{y=x} (y-x)\,dx\,dy$$

$$= \int_0^2 \left[\frac{y^2}{2} - xy \right]_0^x dx$$

$$= \int_0^2 \frac{x^2}{2} - x^2\,dx$$

$$= \int_0^2 -\frac{1}{2}x^2\,dx$$

$$= \left[-\frac{1}{6}x^3 \right]_0^2$$

$$= -\frac{2}{3}$$