1) $T_{avg} = \frac{1}{2}(37+25)$

$= 31°C$

$\rho = 996\,kg\,m^{-3}$, $k = 0.617\,W\,m^{-1}K^{-1}$

$c = 4.178\,kJ\,kg^{-1}$

$$L_c = \frac{V}{A_s}$$

$$= \frac{\pi(15\times10^{-2})^2 \times 1.7}{2\pi(15\times10^{-2})^2 + 30\pi\times10^{-2}\times1.7}$$

$$= \frac{51}{740}$$

$$Bi = \frac{hL_c}{k}$$

$$= \frac{8\left(\frac{51}{740}\right)}{0.617}$$

$= 0.8936002453 > 0.1$

1) $\tau = \dfrac{\rho V c}{h A}$

$= \dfrac{\rho c L_c}{h}$

$= \dfrac{996 \times 4.178 \times 10^3 \times \frac{51}{740}}{8}$

$= 35848.93378 s^{-1}$

$t = -\tau \ln\left[\dfrac{T - T_\infty}{T_i - T_\infty}\right]$

$= -35849 \ln\left[\dfrac{25-20}{37-20}\right]$

$= 43870.4441 s$

$= 12.18640123 h$

The person died at roughly 5am.

2) $L_c = \dfrac{V}{A_s}$

$$= \dfrac{\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}}}{\cancel{4}\cancel{\pi}\cancel{r^2}}$$

$$= \frac{1}{3} r$$

$$0.99(T_\infty - T_i) = T(5) - T_i$$

$$0.99T_\infty - 0.99T_i = T(5) - T_i$$

$$0.99T_\infty - T_\infty - 0.99T_i + T_i = T(5) - T_\infty$$

$$T(5) - T_\infty = 0.01T_i - 0.01T_\infty$$

$$T(5) - T_\infty = 0.01(T_i - T_\infty)$$

$$\frac{T(5) - T_\infty}{T_i - T_\infty} = 0.01$$

$$t = -\tau \ln\left[\frac{T(t) - T_\infty}{T_i - T_\infty}\right]$$

2) when $t=5$,

$$5=-\tau\ln(0.01)$$

$$\tau=1.085736205$$

$$1.085736=\frac{\rho c L_c}{h}$$

$$1.085736=\frac{\rho c \frac{1}{3}r}{h}$$

$$1.085736=\frac{8500\times320\times0.25\times10^{-3}}{3h}$$

$$h=208.7677151\,Wm^{-2}$$

$$h=2.2\sqrt{\frac{V}{D}}$$

$$208.7677151=2.2\sqrt{\frac{V}{0.5\times10^{-3}}}$$

$$2.121903648=\sqrt{V}$$

$$V=4.50247509\,ms^{-1}$$

$$\approx4.5\,ms^{-1}$$

3) $L_{c_{sphere}} = \dfrac{\cancel{4}\over{3}\cancel{\pi} r^{\cancel{3}}}{\cancel{4}\cancel{\pi r^2}}$

$$= \frac{r}{3}$$

$$= \frac{1}{120} \text{ m}$$

$$L_{c_{cube}} = \frac{V}{A_s}$$

$$= \frac{(5\times10^{-2})^3}{6\times(5\times10^{-2})^2}$$

$$= \frac{1}{120} \text{ m}$$

$$L_{c_{prism}} = \frac{V}{A_s}$$

$$= \frac{4\times5\times6\times10^{-2\times3}}{[2(4\times5)+2(5\times6)+2(4\times6)]\times10^{-4}}$$

$$= \frac{3}{370} \text{ m}$$

$$Bi_{sphere,cube} = \frac{hL_c}{K}$$

$$= \frac{12\left(\frac{1}{120}\right)}{429}$$

$$= \frac{1}{4290} < 0.1$$

3) $$B_{i_{prism}} = \frac{hL_c}{k}$$

$$= \frac{12\left(\frac{3}{370}\right)}{429}$$

$$= \frac{6}{26455} < 0.1$$

$$\tau_{sphere,\, cube} = \frac{\rho c L_c}{h}$$

$$= \frac{10{,}500 \times 0.235 \left(\frac{1}{120}\right) \times 10^3}{12}$$

$$= \frac{41125}{24} s^{-1}$$

$$\tau_{prism} = \frac{\rho c L_c}{h}$$

$$= \frac{10{,}500 \times 0.235 \left(\frac{3}{370}\right) \times 10^3}{12}$$

$$= \frac{123375}{74} s^{-1}$$

3) $t_{sphere,cube} = -\tau \ln\left[\frac{T - T_\infty}{T_i - T_\infty}\right]$

$= -\frac{41125}{24} \ln\left[\frac{25-33}{0-33}\right]$

$= 2428.201669\,s$

$= 40.47002782\,min$

$\approx 40.5\,min$

$t_{prism} = -\tau \ln\left[\frac{T - T_\infty}{T_i - T_\infty}\right]$

$= -\frac{123375}{74} \ln\left[\frac{25-33}{0-33}\right]$

$= 2362.574597\,s$

$= 39.37624329\,min$

$\approx 39.4\,min$

4) $$\frac{dE}{dt} = \cancel{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_{gen}$$

$$\frac{d}{dt}(mcT) = \dot{E}_{gen} - \dot{Q}_{loss,conv}$$

$$\frac{d}{dt}(mcT) = \dot{E}_{gen} - hA(T - T_\infty)$$

$$\frac{dT}{dt} = \frac{\dot{E}_{gen}}{mc} - \frac{hA}{mc}(T - T_\infty)$$

$$= \frac{\dot{E}_{gen}}{\rho V c} - \frac{hA}{\rho V c}(T - T_\infty)$$

Let $A = \dfrac{\dot{E}_{gen}}{\rho V c}$, $B = \dfrac{hA}{\rho V c}$

$$\frac{d(T - T_\infty)}{dt} = A - B(T - T_\infty)$$

$$\frac{1}{A - B(T - T_\infty)} \frac{d(T - T_\infty)}{dt} = 1$$

$$\int \frac{1}{A - B(T - T_\infty)} \frac{d(T - T_\infty)}{dt} dt = \int 1 dt$$

$$-\frac{\ln|A - B(T - T_\infty)|}{B} + C = t$$

4) $B(C-t) = \ln|A-B(T-T_\infty)|$

$$D - Bt = \ln|A-B(T-T_\infty)|$$

when $t=0$, $T=T_\infty$

$$D = \ln|A|$$

$$\ln|A| - Bt = \ln|A-B(T-T_\infty)|$$

$$-Bt = \ln|A-B(T-T_\infty)| - \ln A$$

$$-Bt = \ln\left|1-\frac{B}{A}(T-T_\infty)\right|$$

$$e^{-Bt} = 1-\frac{B}{A}(T-T_\infty)$$

$$\frac{B}{A}(T-T_\infty) = 1-e^{-Bt}$$

$$T-T_\infty = \frac{A}{B}\left(1-e^{-Bt}\right)$$

$$T = \frac{A}{B}\left(1-e^{-Bt}\right)+T_\infty$$

$$= \frac{\dot{E}_{gen}}{\cancel{\rho V c}} \times \frac{\cancel{\rho V c}}{hA}\left(1-e^{-\frac{hA}{\rho V c}t}\right)+T_\infty$$

$$= \frac{\dot{E}_{gen}}{hA}\left(1-e^{-\frac{t}{\tau}}\right)+T_\infty$$

4) $\tau = \frac{\rho V c}{hA}$

$= \frac{mc}{hA}$

$= \frac{20 \times 10^{-3} \times 850}{12 \times 4 \times 10^{-4}}$

$= \frac{10625}{3} s^{-1}$

When $t = 5 \times 60$,

$T = \frac{20}{12 \times 4 \times 10^{-4}} \left(1 - e^{-\frac{300}{\left(\frac{10625}{3}\right)}}\right) + 25$

$= 363.406355 °C$

$\simeq 363.4 °C$

4) With heat sink

$$\tau = \frac{(\rho V c)_T}{h A_T}$$

$$= \frac{mc + m_{hs} c_{hs}}{h(A + A_{hs})}$$

$$= \frac{20\times10^{-3} \times 850 + 200\times10^{-3} \times 875}{12(4+80)\times10^{-4}}$$

$$= \frac{40,000}{21} s^{-1}$$

Final Temperature:

$$T = \frac{20}{12(4+80)\times10^{-4}}\left(1 - e^{-\frac{300\times21}{40,000}}\right) + 25$$

$$= 53.91333063$$

$$\approx 53.9°C$$