1a) $v = v_f + x v_{fg}$

$= 0.0008421 + 0.8(0.026622 - 0.0008421)$

$= 0.02146602 \, m^3$

$m = \dfrac{0.01}{0.02146602}$

$= 0.4658525428 \, kg$

$m_g = 0.4658525428 \times 0.8$

$= 0.3726820342$

$\approx 0.373 \, kg$

$m_f = 0.4658525428 \times 0.8$

$= 0.09317050855$

$\approx 0.0932 \, kg$

1b) $\Delta P = 1400 - 1000$

$= 400 \, kPa$

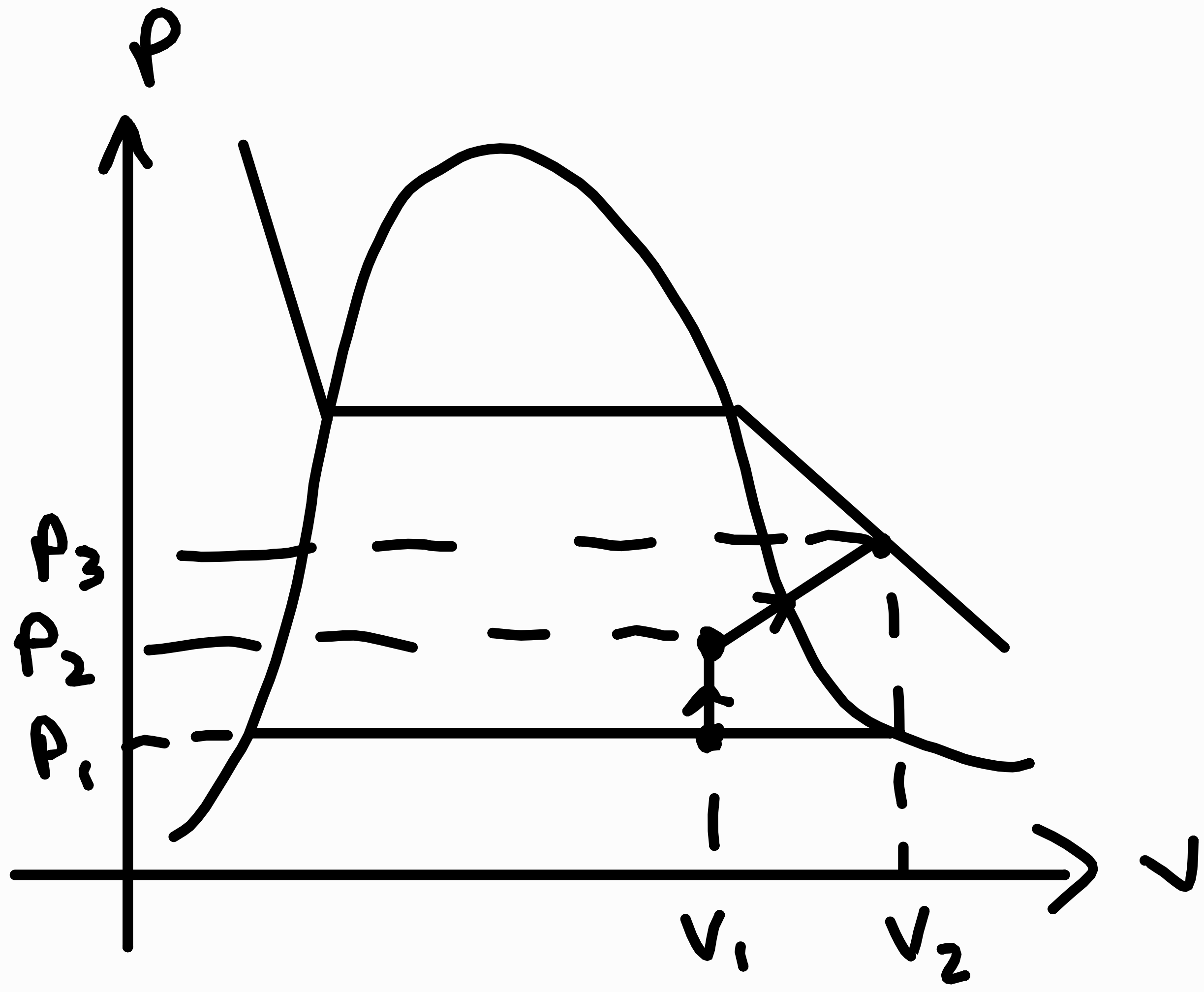
$F = kx$

$400 \times 10^3 \times 0.03 = 600 \times 10^3 x$

$x = 0.02 \, m$

1c) $WD = \frac{1}{2}(1000+1400)(0.02 \times 0.03)$

$= 0.72\ kJ$



(d) For subsystem A,

$$V_{final} = 0.01 + 0.02 \times 0.03$$
$$= 0.0106\,m^3$$
$$v_{final} = \frac{0.0106}{0.4658525428}$$
$$= 0.0227539812\,m^3/kg$$

$$\text{Volume ratio} = \frac{0.023355 - 0.0227539812}{0.023355 - 0.022636}$$
$$= 0.8359093185$$

$$\frac{363.51 - u_{final}}{363.51 - 353.37} = 0.8359093185$$

$$u_{final} = 355.0338795$$

$$Q_A - W = m(u_{final} - u_{initial})$$
$$Q_A = 0.4658525428(355.0338795 - (92.93 + 0.8(153.22))) + 0.72$$
$$= 65.71941746$$
$$\approx 65.72\,kJ$$

1d) For subsystem B,

$$v_{initial} = v_f + x v_{fg}$$
$$= 0.001004 + 0.0119(32.879 - 0.001004)$$
$$= 0.3922521524 \, m^3$$

$$u_{initial} = u_f + x u_{fg}$$
$$= 125.73 + 0.0119(2290.2)$$
$$= 152.98338 \, kJ$$

Since $v_{initial} = v_{final}$,

$$\text{Volume ratio} = \frac{0.34648 - 0.3922521524}{0.34648 - 0.39248}$$

$$= 0.9950467913$$

$$\frac{2563.5 - u_{final}}{2563.5 - 2559.1} = 0.9950467913$$

$$u_{final} = 2559.121794 \, kJ$$

$$Q_B - W = m(u_{final} - u_{initial})$$
$$Q_B = 0.01(2559.121794 - 152.98338)$$

$$= 24.0613841 4 \, kJ$$

(d) $Q = Q_A + Q_B$

$= 65.71941746 + 24.06138414$

$= 89.7808016 \text{ kJ}$

$\approx 89.78 \text{ kJ}$

(e) For subsystem B,

$$\frac{155 - T_{final}}{155 - 150} = 0.9950467913$$

$$T_{final} = 150.024766 °C$$

$$\approx 150 °C$$

For subsystem A,

$$\frac{160 - T_{final}}{160 - 150} = 0.8359093185$$

$$T_{final} = 151.6409068$$

$$\approx 151 °C$$

2a) $V_{initial} = \dfrac{0.5 \times 0.287 \times 10^3 \times 303}{100 \times 10^3}$

$= 0.434805 m^3$

$\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}$

Since $V_1 = V_2$

$\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$

$\dfrac{100}{303} = \dfrac{500}{T_2}$

$T_2 = 1515K$

$Q = mc_v \Delta T$

$= 0.5 \times 0.721 \times (1515 - 303)$

$= 436.926 kJ$

$Q_{total} = 436.926 + 500 \times 5 \times 60 \times 10^{-3}$

$= 586.926 kJ$

$Q = mc\Delta T$

$586.926 = 0.1 \times 5 \times 60 \times 4.18 \Delta T$

$\Delta T = 4.680430622°C$

2a) Final $T_w = 100 - 4.680430622$

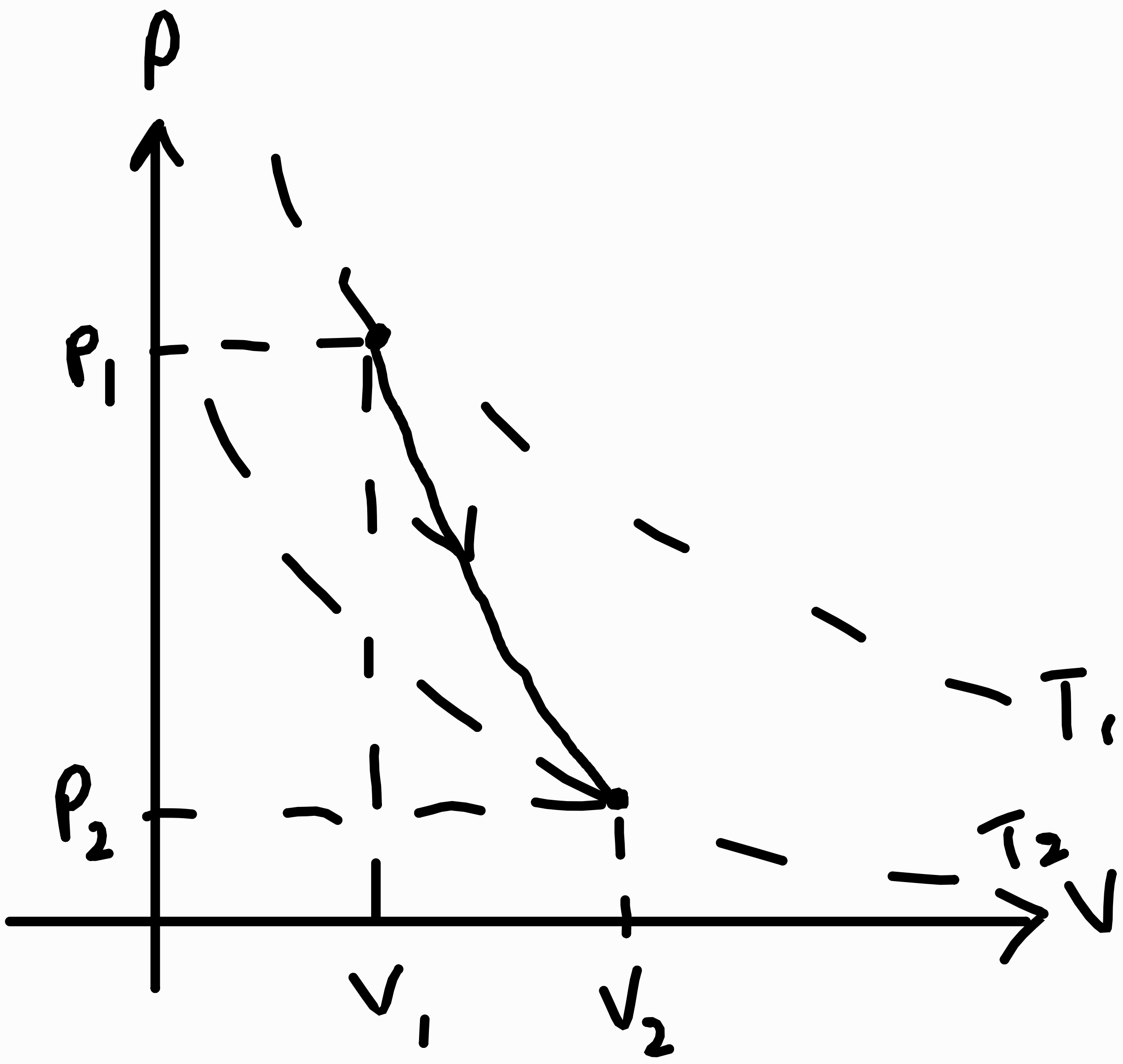
$= 95.31956938°C$

$\approx 95.32°C$

2b) $500 \times 10^3 \times 0.1 = 100 \times 10^3 \times 0.1 + 2 \times 9.81 + F_L$

$F_L = 39980.38 N$

2c) The air expands in an adiabatic process until the pressure reaches 100kPa. This is due to the process being instantaneous, which means no heat is transferred to the surroundings. Hence the process is adiabatic.



3ai) $F_H = \rho g h_c$ $\qquad z_H = \frac{2}{3}H$ from the fluid surface

$= \rho g \frac{H}{2}$

$= \frac{\rho g H}{2}$

3aii) $F_V = \rho g V$ $\qquad r_v = \frac{L}{2}$

$= \rho g H L W$

bi) $\frac{\partial p}{\partial r} = \rho r w^2$ $\qquad \frac{\partial p}{\partial z} = -\rho(g + a_z)$

$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} dz$

$dp = \rho r w^2 dr - \rho(g + a_z) dz$

At B, $r = 0$, $a_z = 0$

$dp = -\rho g dz$

$\therefore P = \rho g H$

3bi) At D, $a_z = 0$,

$$dp = \rho r w^2 dr - \rho(g + a_z)dz$$

$$dp = \rho r w^2 dr - \rho g dz$$

$$P = \int_0^L \rho r w^2 dr - \int_0^0 \rho g dz$$

$$= \frac{\rho L^2 w^2}{2} - 0$$

$$= \frac{\rho L^2 w^2}{2}$$

3bii) Let x be a point on the surface BC

$$P = \int_0^x \rho r w^2 dr - \int_H^0 \rho g dz$$

$$= \frac{\rho x^2 w^2}{2} + \rho g H$$

3biii) $F = \int P \, dA$

$$= \int_0^L \frac{\rho x^2 \omega^2}{2} W \, dx + \int_0^{LW} \rho g H \, dA$$

$$= \left[\frac{\rho \omega^2 x^3}{6} W \right]_0^L + \rho g H L W$$

$$= \frac{W \rho \omega^2 L^3}{6} + \rho g H L W$$

3biv) $r_v = \dfrac{\sum A \bar{y}}{\sum A}$

$$= \frac{H\cancel{L}\cancel{W}\cancel{\rho}g \times \frac{L}{2} + \frac{\cancel{W}\cancel{\rho}\omega^2 L^3}{6} \times \frac{3\cancel{L}}{4}}{H\cancel{L}\cancel{W}\cancel{\rho}g + \frac{\cancel{W}\cancel{\rho}\omega^2 L^{\cancel{3}2}}{6}}$$

$$= \frac{\frac{1}{2} H L g + \frac{1}{8} \omega^2 L^3}{H g + \frac{\omega^2 L^2}{6}}$$

3c i) $$\frac{\rho_b}{\rho_w} = \frac{\text{Volume of water displaced}}{\text{volume of block}}$$

$$= \frac{Ah}{AH}$$

$$\rho_b = \frac{h}{H}\rho_w$$

(ii) For stable configuration

$$\overline{GM} = \frac{I_o}{V} - \overline{CG}, \quad \text{for } \overline{GM} > 0$$

$$I_o = \frac{\pi}{4}ab^3$$

$$V = \pi abh$$

$$\overline{CG} = \frac{H}{2} - \frac{h}{2}$$

$$= \frac{H-h}{2}$$

$$\overline{GM} = \frac{\frac{\cancel{\pi}}{4}\cancel{a}b^{\cancel{3}2}}{\cancel{\pi}\cancel{a}\cancel{b}h} - \frac{H-h}{2}$$

$$= \frac{b^2}{4h} + \frac{h-H}{2}$$

$$= \frac{b^2 + 2h(h-H)}{4h} = \frac{b^2+2h^2-2hH}{4h}$$

3cii) To be always stable,

$$b^2 + 2h^2 - 2hH > 0,$$

$$\therefore B^2 - 4AC < 0$$

$$(-2H)^2 - 4(2)(b^2) < 0$$

$$2H^2 - 8b^2 < 0$$

$$2H^2 < 8b^2$$

$$H^2 < 4b^2$$

$$H < 2b$$

$\therefore$ the configuration is stable as long as $H < 2b$.

4 a i) $V_1 = \frac{Q}{A_1}$

$= \frac{Q}{\frac{\pi D_1^2}{4}}$

$= \frac{4Q}{\pi D_1^2}$

$V_3 = \frac{Q}{A_3}$

$= \frac{Q}{\frac{\pi D_3^2}{4}}$

$= \frac{4Q}{\pi D_3^2}$

ii) Bernoulli from 1B to 5B

$$\frac{P_{1B}}{\rho g} + \frac{V_{1B}^2}{2g} \cancel{+ z_1}^{0} = \frac{P_{5B}}{\rho g} + \frac{V_{5B}^2}{2g} + z_{5B}$$

$$\frac{P_{5B}}{\rho g} = -\frac{V_{5B}^2}{2g} - \left(D_1 + H + \frac{D_3}{2} - \frac{D_5}{2}\right) + \frac{V_{1B}^2}{2g} + \frac{P_{1B}}{\rho g}$$

$$\frac{P_{5B}}{\rho g} = -\frac{V_{1B}^2}{2g} \times \left(\frac{D_1}{D_5}\right)^4 + \frac{V_{1B}^2}{2g} + \frac{P_{1B}}{\rho g} - \left(D_1 + H + \frac{D_3}{2} - \frac{D_5}{2}\right)$$

$$\frac{P_{5B}}{\rho g} = \frac{V_{1B}^2}{2g}\left(1 - \frac{D_1^4}{D_5^4}\right) + \frac{P_{1B}}{\rho g} - \left(D_1 + H + \frac{D_3}{2} - \frac{D_5}{2}\right)$$

$$P_{5B} = \frac{\rho V_{1B}^2}{2}\left(1 - \frac{D_1^4}{D_5^4}\right) + P_{1B} - \rho g\left(D_1 + H + \frac{D_3}{2} - \frac{D_5}{2}\right)$$

$$P_{5B} = P_{1B} + \frac{8\rho Q^2}{\pi^2 D_1^4}\left[1 - \frac{D_1^4}{D_5^4}\right] - \rho g\left(D_1 + H + \frac{D_3}{2} - \frac{D_5}{2}\right)$$

4aiii) When flow rate is small, pressure due to hydrostatic pressure is significant.

Since 3T is the highest point, P_{3T} is the lowest pressure.

Bernoulli from 1T to 3T

$$\frac{P_{1T}}{\rho g} + \frac{V_{1T}^2}{2g} + z_{1T} = \frac{P_{3T}}{\rho g} + \frac{V_{3T}^2}{2g} + z_{3T}$$

$$\frac{P_{3T}}{\rho g} = \frac{P_{1T}}{\rho g} + \frac{V_{1T}^2}{2g} + \cancel{D_1} - \frac{V_{3T}^2}{2g} - (\cancel{D_1} + H + D_3)$$

$$\frac{P_{3T}}{\rho g} = \frac{P_{1T}}{\rho g} + \frac{V_{1T}^2}{2g} - \frac{V_{1T}^2}{2g}\left(\frac{D_1}{D_3}\right)^4 - H - D_3$$

$$\frac{P_{3T}}{\rho g} = \frac{P_{1T}}{\rho g} + \frac{V_{1T}^2}{2g}\left(1 - \frac{D_1^4}{D_3^4}\right) - H - D_3$$

$$P_{3T} = P_{1T} + \frac{8\rho Q^2}{\pi D_1^4}\left(1 - \frac{D_1^4}{D_3^4}\right) - H - D_3$$

$$P_{3T} = P_{1B} + \frac{8\rho Q^2}{\pi D_1^4}\left(1 - \frac{D_1^4}{D_3^4}\right) - D_1 - H - D_3$$

4aiii) For very large flow rate, pressure due to velocity is significant.

Highest velocity = lowest pressure

$\therefore P_{1T}$ is the lowest

Bernoulli from 1B to 1T

$$\frac{P_{1B}}{\rho g} + \cancel{\frac{V_{1B}^2}{2g}} + \cancel{z_{1B}}^{0} = \frac{P_{1T}}{\rho g} + \cancel{\frac{V_{1T}^2}{2g}} + z_{1T}$$

$$P_{1T} = P_{1B} - \rho g D_1$$

4bi) Bernoulli from water surface to small hole:

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2}^{0}$$

$$\frac{V_2^2}{2g} = h$$

$$V_2 = V_{out} = \sqrt{2gh}$$

4bii) Flow rate $= Q - \pi r_0^2 \sqrt{2gh}$

$$\frac{dh}{dt} = \frac{Q - \pi r_0^2 \sqrt{2gh}}{\pi R_0^2}$$

$$\int_h^{h_0} \frac{\pi R_0^2}{Q - \pi r_0^2 \sqrt{2gh}} dh = \int 1 dt$$

$$t = \int_h^{h_0} \frac{\pi R_0^2}{Q - \pi r_0^2 \sqrt{2gh}} dh$$

$$t = \frac{\pi R_0^2}{Q - \pi r_0^2 \sqrt{2gh}} (h_0 - h)$$

4biii) $\frac{dV}{dt} = 0$

$$Q - \pi r_0^2 \sqrt{h_s - h_0} \sqrt{2gh_s} = 0$$

$$Q = \pi r_0^2 \sqrt{h_s - h_0} \sqrt{2gh_s}$$

$$\frac{Q}{\pi r_0^2} = \sqrt{h_s - h_0} \sqrt{2gh_s}$$

4b iii) $$\frac{Q^2}{\pi^2 r_0^4} = (h_s - h_0)(2gh_s)$$

$$\frac{Q^2}{2g\pi^2 r_0^4} = h_s^2 - h_0 h_s$$

$$h_s^2 - h_0 h_s - \frac{Q^2}{2g\pi^2 r_0^4} = 0$$

$$h_s = \frac{h_0 \pm \sqrt{h_0^2 - \overset{2}{\cancel{4}}(1)\left(\frac{Q^2}{2g\pi^2 r_0^4}\right)}}{2(1)}$$

$$= \frac{h_0 \pm \sqrt{h_0^2 + \frac{2Q^2}{g\pi^2 r_0^4}}}{2}$$

$$= \frac{h_0 + \sqrt{h_0^2 + \frac{2Q^2}{g\pi^2 r_0^4}}}{2} \quad \because h_s > 0$$