12.3 Natural Frequency and Resonance

Theoretically, waves can extend forever. Realistically, they are limited by the size of the system. Boundaries create conditions that favor special frequencies or wavelengths. Just as the length of the string set the period of the pendulum, the boundaries and properties of the system make certain waves much more powerful than others. The concepts of *resonance* and *natural frequency* apply to a huge range of natural and human-made systems. These two powerful ideas are the key to understanding the tides of the ocean, the way our ears separate sound, and even how a microwave oven works.

Natural frequency

The natural frequency is the frequency at which a system oscillates when it is disturbed.

Natural frequency

What is natural frequency?

If you pluck a guitar string in the middle it vibrates back and forth. If you pluck the same string 10 times in a row and measure the frequency of vibration you find that it is always the same. When plucked, the string vibrates at its **natural frequency**. The pendulum also had a natural frequency.

Why natural frequency is important

The natural frequency is important for many reasons:

1 All things in the universe have a natural frequency, and many things have more than one.

2 If you know an object's natural frequency, you know how it will vibrate.

3 If you know how an object vibrates, you know what kinds of waves it will create.

4 If you want to make specific kinds of waves, you need to create objects with natural frequencies that match the waves you want.

Microwave ovens, musical instruments, and cell phones all use the natural frequency of an oscillator to create and control waves. Musical instruments work by adjusting the natural frequency of vibrating strings or air to match musical notes. The A string on a guitar has a natural frequency of 440 hertz.

Changing the natural frequency

The natural frequency depends on many factors, such as the tightness, length, or weight of a string. We can change the natural frequency of a system by changing any of the factors that affect the size, inertia, or forces in the system. For example, tuning a guitar changes the natural frequency of a string by changing its tension.

Figure 12.12: *A guitar uses the natural frequency of the strings to make the correct notes. Once it is tuned, the A string, when plucked, will always vibrate at 440 hertz.*

Resonance

The response of an oscillator

To keep a system oscillating, we apply an oscillating force. For example, if you want to get a jump rope going, you shake the end up and down. What you are really doing is applying an oscillating force to the rope. The response of the rope is to oscillate up and down with the same frequency of your applied force.

If you try this, you notice that at certain frequencies your force is noticeably more effective at making the rope oscillate. For example, shaking the end up and down twice per second (1.6 Hz) results in an amplitude of a few centimeters. Slowing down to once per second (1 Hz) makes an amplitude of more than a meter! Slowing even more, to once every two seconds (0.5 Hz), causes the amplitude to drop back down again. Your experiment shows that the frequency of 1 hertz is *many times* more effective than any other frequency.

Resonance

The extra-strong response at 1 hertz is an example of **resonance**. You can think of resonance as having the natural frequency of the system exactly in tune with your force. Each cycle of your force exactly matches each cycle of the system. As a result, each push adds to the next one and the amplitude of the oscillation grows (figure 12.13). Resonance happens when something is vibrated at its natural frequency (or a multiple of the natural frequency). Resonance is an important idea because it is used to transfer power into all kinds of waves from lasers to microwave ovens to musical instruments.

A swing is a good example of resonance

The example of a swing (that you might sit on at the park) is one of the best ways to describe resonance. With a swing, small pushes applied over time build up a large amplitude of motion. This happens because each push is synchronized to the natural motion of the swing. A forward push is always given when the swing is as far back as it can go. The swing is like a pendulum, which has a natural frequency. By applying small pushes at a frequency matched to the natural frequency, we are able to create a large motion. The interaction of the repeating pushes and the natural motion of the swing is what creates resonance. The effect of the resonance is that the swing's motion gets large even though the pushes are small. Resonance is not a single thing. Resonance is an interaction between a wave, a driving force, and the boundaries of the system.

An example of resonance

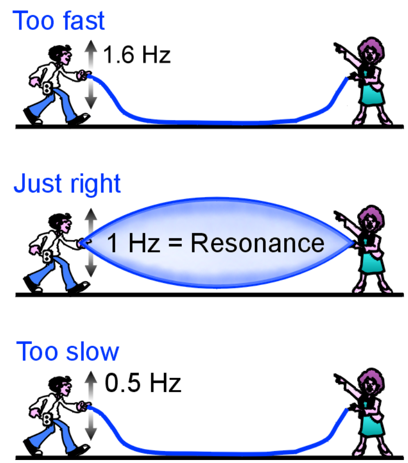


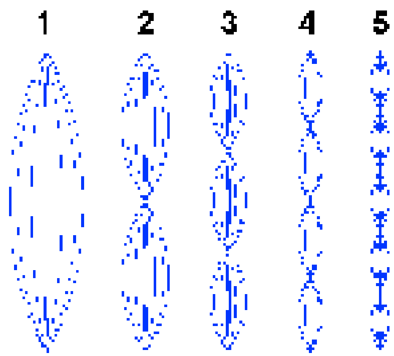
Figure 12.13: *A jump rope is a good experiment for resonance. If you shake it at the right frequency, it makes a big wave motion. If your frequency is not just right, the rope will not make the wave pattern at all.*

Standing waves on a string

What is a standing wave?

Although waves usually travel, it is possible to make a wave stay in one place. A wave that is trapped in one spot is called a **standing wave**. It is possible to make standing waves of almost any kind, including sound, water, and even light. A vibrating string is a great example for doing experiments with standing waves. Vibrating strings are what make music on a guitar or piano.

Harmonics are multiples of the natural frequency of a standing wave



The first five harmonics of the vibrating string

Standing waves occur at frequencies that are multiples of the **fundamental**, which is the natural frequency of the string. The fundamental and multiples of its frequency are called **harmonics**. The diagram to the left shows the first five harmonics. You can tell the harmonic number by counting the number of "bumps" on the wave. The first harmonic has one bump, the second has two bumps, the third has three, and so on. If the frequency of the first harmonic is 10 hertz, then the second will be at a frequency of 20 hertz, the third will be at 30 hertz, and so on.

Wavelength

A vibrating string moves so fast that your eye averages out the image and you see a wave-shaped blur (figure 12.14). At any one moment the string is really in only one place within the blur. The wavelength is the length of one complete "S" shape on the string. Higher frequency waves have shorter wavelengths.

Why are standing waves useful?

Standing waves are useful because we can control their frequency and wavelength. Because the wave is trapped, it is easy to put power into it and make large amplitudes. In your microwave oven, there is a device called a magnetron. Inside the magnetron is a standing wave driven by electricity. A small hole in the boundary lets some of the wave's energy out to cook food. The shape of the magnetron forces the standing wave to oscillate at exactly 2.4 billion cycles per second (2.4 gigahertz). Energy that leaks out at the same frequency is perfectly matched to heat water molecules in food.

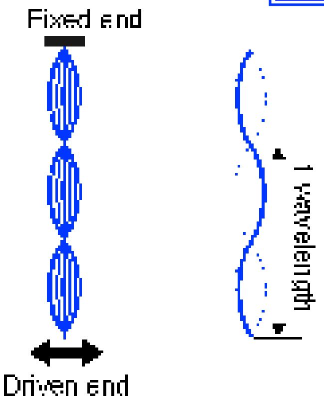


Figure 12.14: *A standing wave on a vibrating string. The wavelength is the length of one complete **S** shape of the wave.*

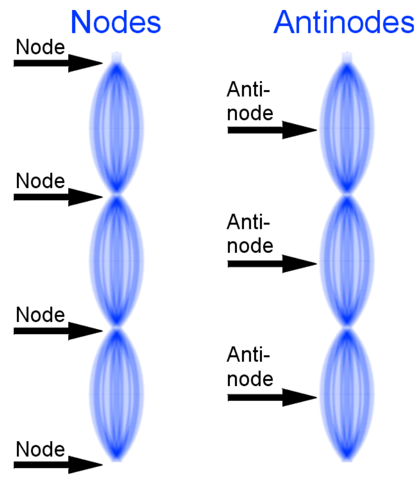


Figure 12.15: *Nodes and antinodes for the third harmonic of the vibrating string. Nodes are points where the string does not move. Antinodes are points of the greatest amplitude.*

Interference

What is interference? Interference happens when two or more waves come together. Because there are so many waves around us, they often interfere with each other. In fact, radio and television use the interference of two waves to carry music and video. The resonance of a vibrating string can be understood using the interference of waves. Sometimes on the ocean, two big waves add up to make a gigantic wave that may only last a few moments but is taller than ships, and can have a terrible impact.

Constructive interference Suppose you make two wave pulses on the stretched spring. One comes from the left and the other comes from the right. When they meet in the middle, they combine to make a single large pulse. This is called **constructive interference**. Constructive interference occurs when waves add up to make a larger amplitude (figure 12.16).

Destructive interference There is another way to launch the two pulses. If we make pulses on opposite sides of the cord, something different happens. When the pulses meet in the middle they cancel each other out! One wants to pull the string up and the other wants to pull it down. The result is that the string is flat and both pulses vanish for a moment. This is called **destructive interference**. In destructive interference waves add up to make a smaller amplitude (figure 12.17).

After they interfere, both wave pulses separate again and travel on their own. This is surprising if you think about it. For a moment, the middle of the cord is flat in the example of destructive interference. A moment later, two wave pulses come out of the flat part and race away from each other. Waves still store energy, even when they interfere.

Waves at the atomic level Down at the scale of atoms, there are many extremely strong waves. Because there are so many and they are tiny and random, they interfere destructively on average. We don't see the wavelike nature of atoms because of large-scale destructive interference. In special cases, like with a magnetic resonance imaging (or MRI) machine, or a laser, we create constructive interference of atomic waves. The result is very powerful and useful technology.

Constructive interference

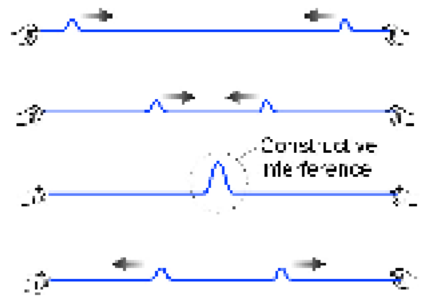


Figure 12.16: *Two wave pulses on the same side add up to make a single, bigger pulse when they meet. This is an example of constructive interference.*

Destructive interference

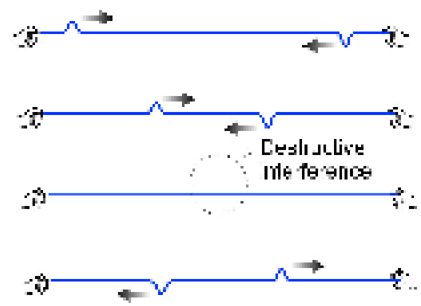


Figure 12.17: *Two equal wave pulses on opposite sides subtract when they meet. The upward movement of one pulse exactly cancels with the downward movement of the other. For a moment there is no pulse at all. This is an example of destructive interference.*