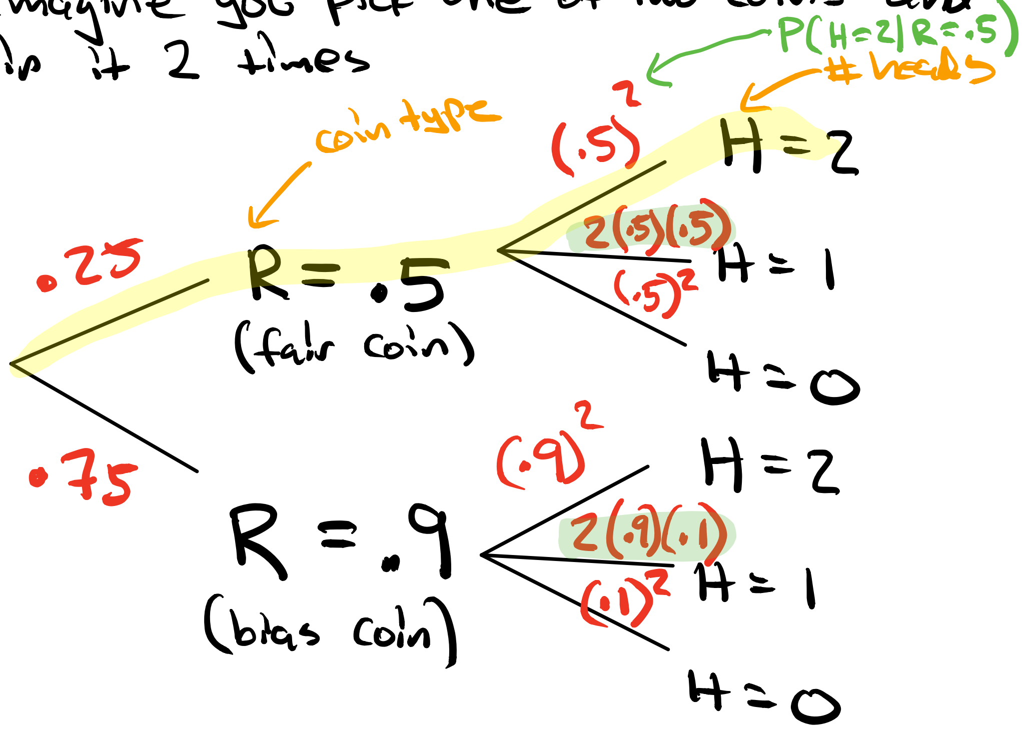
Stat 134 lec 33

Sec 6.1 Continuous Distributions

Discrete case:

Imagine you pick one of two coins and flip it 2 times



Joint distribution:

	R=.5	R=.9	Sum (marginal H)
H=2	$(.25)(.5)^2=0.0625$	0.6075	0.67
H=1	0.125	0.1350	0.26
H=0	0.0625	0.0075	0.07
Sum (marginal R)	0.25	0.75	1

Bayes rule (backwards conditioning)

Fix x

Conditional dist of Y given x

$$\boxed{P(Y=y \mid X=x) = \frac{P(X=x, Y=y)}{P(X=x)}}$$ Bayes' rule

$$P(R=.5 \mid H=0) = \frac{P(R=.5, H=0)}{P(H=0)} = \frac{.0675}{.07} = .89$$

$$P(R=.9 \mid H=0) = \frac{P(R=.9, H=0)}{P(H=0)}$$

$$= \frac{.0075}{.07} = .11$$

What is the probability get $H=1$? We condition on R.

$$P(Y=y) = \sum_{\text{all } x} P(Y=y \mid X=x) \cdot P(X=x)$$

$$P(H=1) = P(H=1 \mid R=.5) P(R=.5) + P(H=1 \mid R=.9) P(R=.9)$$

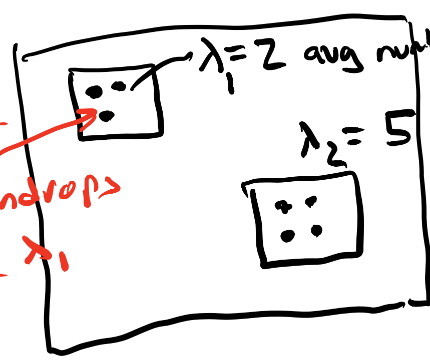
$$= 2(.5)(.5)(.25) + 2(.9)(.1)(.75)$$

ex $\underline{6.1.6}$ Conditioning independent Poisson variables on their sum.

$N_1 \sim \text{Pois}(\lambda_1)$
$N_2 \sim \text{Pois}(\lambda_2)$ } indep

recall λ is avg intensity rate

(i.e. expected # points / unit)



Think of λ_1, λ_2 as mean # arrivals in 1 sq inch in diff parts of the plane

What is the conditional joint distribution of $(N_1, N_2) \mid N_1 + N_2 = n$?

$$P((N_1 = k_1, N_2 = k_2 \mid N_1 + N_2 = n)$$

Only nonzero if $k_1 + k_2 = n$.

$, N_2 = n - k_1$

$$= \frac{P(N_1 = k_1, N_2 = k_2, N_1 + N_2 = n)}{P(N_1 + N_2 = n)}$$

recall, $P(N_1 + N_2 = n) = \dfrac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}{n!}$

$$= \frac{P(N_1 = k_1)P(N_2 = k_2)}{P(N_1+N_2 = n)} = \frac{\dfrac{e^{-\lambda_1}\lambda^{k_1}}{k_1!} \cdot \dfrac{e^{-\lambda_2}\lambda_2^{n-k_1}}{(n-k_1)!}}{\dfrac{e^{-(\lambda+\lambda_2)}(\lambda_1+\lambda_2)^n}{n!}}$$

$(\lambda_1+\lambda_2)^{k_1}(\lambda_1+\lambda_2)^{n-k_1}$

$$= \binom{n}{k_1}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{k_1}\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k_1}$$

$$\Rightarrow (N_1, N_2) \mid N_1 + N_2 = n \sim \text{Bin}\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$$

Sec 6.2

Stat 134

Wednesday April 11 2018

1. Below is a joint distribution of S and T. What is the expectation of T given S=7?

	T=3	T=4	Sum
S=7	0.3	0.1	0.4
S=6	0.2	0.2	0.4
S=5	0.1	0.1	0.2
Sum	0.6	0.4	1.0

Sum ← marginal of S

marginal of T

$$E(T) = \sum_{t \in T} t \cdot P(T=t)$$

$$= 3(.6) + 4(.4)$$

$$= 3.4$$

a 1.3

b 3.25

c 3.5

d none of the above

$$E(T|S=7) = \sum_{t \in T} t \cdot P(T=t|S=7)$$

$$P(T=t|S=7) = \frac{P(T=t, S=7)}{P(S=7)}$$

$$= 3 \cdot \frac{(.3)}{(.4)} + 4 \frac{(.1)}{.4} = 3.25$$

The value of T is still 3 and 4 just the probability of these value changes given that S=7.

$$\left.\begin{aligned} E(T \mid S=7) &= 3.25 \\ E(T \mid S=6) &= 3.5 \\ E(T \mid S=5) &= 3.5 \end{aligned}\right\} \text{function of } S$$

Two main points:

① $E(T|S)$ is a function of S

② $E(T|S)$ is a RV so it has an expectation.

Next time will explore the expectation of $E(T|S)$.