

Stat 134: Conditional Probabilities, Distributions, & Expectations Review

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Problem 1

Let $X_1 \sim \text{Geom}(p_1)$, $X_2 \sim \text{Geom}(p_2)$, $X_1 \perp X_2$, both on $\{1, 2, \dots\}$.

Find:

- $P(X_1 \leq X_2)$
- $P(X_1 = x \mid X_1 \leq X_2)$. Recognize $X_1 \mid X_1 \leq X_2$ as a named distribution, and state the parameter(s).
- $P(X_1 = k \mid X_1 + X_2 = n)$ in the case $p_1 = p_2$. Recognize this as a named distribution and state the parameter(s).

Problem 2

Let $Y \sim \text{Beta}(r, s)$. Conditioned on $Y = y$, let $X \sim \text{Geometric}(y)$ on $\{0, 1, 2, \dots\}$. For simplicity, assume $r, s > 1$.

- What is $\mathbb{E}(X \mid Y = y)$?
- Find $\mathbb{E}(X)$.
- Find $P(X = x)$, for $x \in \{0, 1, 2, \dots\}$.

Problem 3

Suppose a proportion p of a population has a gene m that makes them predisposed to migraines. Of these people, the number of migraines they experience in a year follows a Poisson process with rate λ_m per year, whereas the rest of the population experiences migraines according to a Poisson process with rate λ_x .

- What is the probability that a randomly selected individual experiences no migraines in a given year?
- Let N_t denote the number of migraines a randomly selected individual experiences in t years. Find $\mathbb{E}(N_t)$.
- Find $\text{Var}(N_t)$.
- Given that a person experienced k migraines in a year, find the expected number of migraines they will have next year.
- Challenge: Find $\text{Corr}(N_{[0,1]}, N_{[1,2]})$, i.e. the correlation between number of migraines in consecutive years.

Hint: Condition on whether the individual has gene m .

Hint: Update the likelihood they have gene m using Bayes' Rule.