MATH 104 - WEEKLY ASSIGNMENT 4

DUE 26 SEPTEMBER 2016, BY 16:00

(1) Find a sequence $(a_n)_{n\in\mathbb{N}}$ in $\mathbb{R}$, with $a_{n+1} - a_n \to 0$, but $(a_n)_{n\in\mathbb{N}}$ not Cauchy.

(2) Let $(a_n)_{n\in\mathbb{N}}$ be a sequence in $\mathbb{R}$. Show that $(a_n)_{n\in\mathbb{N}}$ converges if and only if: $(a_{2n})_{n\in\mathbb{N}}$ and $(a_{2n-1})_{n\in\mathbb{N}}$ converge and have the same limit.

(3) (i) Show that $\sum_{k=1}^{+\infty} \frac{1}{k}$ diverges using the Cauchy condensation test.

(ii) Show that, if $a_k \geq 0$ for all $k \in \mathbb{N}$, then the series $\sum_{k=1}^{+\infty} \frac{a_k}{1+k^2 a_k}$ converges.

(iii) Test for convergence the series $\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots$.

(4) Test for convergence the following series:

(i) $\sum_{k=1}^{+\infty} \frac{k^2-k}{k^2}$

(ii) $\sum_{k=1}^{+\infty} \frac{k!}{k^k}$

(iii) $\sum_{k=1}^{+\infty} \frac{(-1)^k \sin(k^{100})}{k^{\frac{3}{2}}}$

(iv) $\sum_{k=5}^{+\infty} \frac{k+1}{k^4-5k^3+8}$

(v) $\sum_{k=1}^{+\infty} \frac{3^{k+1}+k}{4^k-k}$

(vi) $\sum_{k=1}^{+\infty} \frac{k+\sqrt{k}}{2k^3-1}$

(vii) $\sum_{k=1}^{+\infty} \frac{\cos^k k}{k^{\frac{3}{2}}}$

(viii) $\sum_{k=1}^{+\infty} \frac{1}{k^p-k^q}$, $0 < q < p$

(ix) $\sum_{k=1}^{+\infty} k(1+k^2)^p$, $p \in \mathbb{R}$

(x) $\sum_{k=1}^{+\infty} \frac{k^2 x^k}{2^k}$, $x \in \mathbb{R}$

(5) (i) Show that $\sum_{k=1}^{+\infty}(\sqrt[k]{k} - 1)$ diverges. (Maybe you can take some inspiration from the proof of $\sqrt[k]{k} \to 1$).

(ii) Show that $\sum_{k=1}^{+\infty} \frac{1}{k(k+1)(k+2)}$ converges. Find its limit.