Stat 134: Section 19

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Problem 1

Suppose $X \sim \text{Unif}\,(-1,1)$, and $Y \sim \text{Unif}\,(0,1)$. Find the density of $Z = X + Y$, using:

a. $P(Z \in dz)$, and drawing the relevant region;

b. the C.D.F. of Z.

Problem 2

A system consists of two components. Each component is subject to failure at a constant rate λ, independently of the other, up to when the first component fails. After that, the remaining component is subject to additional load and to failure at constant rate 2λ.

a. Find the distribution of the time until both components have failed.

b. What are the mean and variance of this distribution?

Ex 5.4.4 in Pitman's Probability

Problem 3

Let X and Y be independent exponential variables with rates α and β, respectively. Find the C.D.F. of X/Y.
Ex 5.4.8 in Pitman's Probability

Problem 4

Suppose $U_1, U_2 \overset{\text{i.i.d.}}{\sim}$ Unif $(0, 1)$. Let $Z = Y - X$, where $X = U_{(1)}$, $Y = U_{(2)}$. Note that Z represents the range of our random variables.

a. Find the joint density $f(x, y)$ of X, Y.

b. Find the C.D.F. of Z, $F_Z(z)$.

Hint: Draw the region of interest. It may be easier to work with $P(Z \geq z)$.

c. Use part (b) to find the density of Z.

d. It can be shown that for the range $Z_n = U_{(n)} - U_{(1)}$ of n i.i.d. Unif $(0, 1)$ random variables, the CDF of Z_n is given by $F_{Z_n}(z) = z^n + nz^{n-1}(1 - z)$. Using what we know about order statistics, explain why this is the case.