Stat 134: Section 15

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March 14, 2018

HAPPY PI(E) DAY!

Problem 1

Suppose U has uniform $(0,1)$ distribution. Let $W = -\log U$. Find the density of W.

Do you recognize the distribution of W?

Problem 2

Suppose X has uniform $(-1,2)$ distribution. Find the density of X^2.

Ex 4.4.5 in Pitman's Probability

Is this a one-to-one transformation?

Problem 3

Let Z be a standard normal random variable. Find formulae for the densities of each of the following random variables:

a. $|Z|$;

b. Z^2;

c. $1/Z^2$.

Ex 4.4.10 in Pitman's Probability

Problem 4

Geometric from Exponential

a. Show that if T has exponential distribution with rate λ, then $int(T)$, the greatest integer less than or equal to T, has a geometric(p) distribution on $\{0, 1, 2, ...\}$, and find p in terms of λ.

b. Let $T_m = int(mT)/m$, the greatest multiple of $1/m$ less than or equal to T. Show that T has exponential distribution on $(0, \infty)$ for some λ, if and only if for every m there is some p_m such that mT_m has geometric(p_m) distribution on $\{0, 1, 2, ...\}$. Find p_m in terms of λ.

Ex 4.2.10 in Pitman's Probability

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