Stat 134: Section 22

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Problem 1

Let X have uniform distribution on $\{-1, 0, 1\}$ and let $Y = X^2$. Are X and Y uncorrelated? Are X and Y independent? Explain carefully.

Ex 6.4.5 in Pitman's Probability

Problem 2

Let T_1 and T_3 be the times of the first and third arrivals in a Poisson process with rate λ.

a. Do you expect $Corr(T_1, T_3)$ to be positive or negative?

b. Find $Corr(T_1, T_3)$.

c. Generalize this result to find $Corr(T_m, T_n)$, where $m, n \in \{1, 2, \ldots\}$, $m < n$.

Ex 6.4.11 in Pitman's Probability

Problem 3

Let A and B be two possible results of a trial, not necessarily mutually exclusive. Let N_A and N_B be the number of times A and B respectively occur in the same n trials. True or false, and explain: if N_A and N_B are uncorrelated, then they are independent.
Ex 6.4.13 in Pitman's Probability

Problem 4

I toss a coin which lands heads with probability p. Let W_H be the number of tosses till I get a head, W_{HH} the number of tosses till I get two heads in a row, and W_{HHH} until three heads in a row. Find:

a. $\mathbb{E}(W_H)$;

b. $\mathbb{E}(W_{HH})$;

c. $\mathbb{E}(W_{HHH})$.

Ex 6.rev.21 in Pitman's Probability

Hint: condition on whether the first toss was heads or tails.

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