Stat 134: Section 16

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Problem 1

Let X be an arbitrary random variable with a strictly increasing cdf F_X. Show that the random variable given by the transformation $F_X(X)$ is uniform on $(0,1)$.

Hint: Look at the cdf of $F_X(X)$.

Problem 2

Let $U_{(1)}, \ldots, U_{(n)}$ be the values of n i.i.d. Uniform (o,1) variables arranged in increasing order. For $0 < x < y < 1$, find simple formulae for:

a. $P(U_{(1)} > x, U_{(n)} < y)$

b. $P(U_{(1)} > x, U_{(n)} > y)$

c. $P(U_{(1)} < x, U_{(n)} < y)$

d. $P(U_{(1)} < x, U_{(n)} > y)$

Ex 4.6.3 in Pitman's Probability

Problem 3

Four people agree to meet at a cafe at noon. Suppose each person arrives at a time chosen uniformly at random between 11:45 am and 12:15 pm, independently of the others.

a. What is the chance that the first person to arrive at the cafe gets there before 11:50?

b. What is the chance that some of the four have not arrived by 12:10?

c. Suppose that if all 4 people have shown up before 12:10, the waiter takes their orders as soon as the fourth person arrives. Otherwise, the waiter takes the orders of whoever is there at 12:10 pm. Let T represent the time at which the waiter takes their order. Find and sketch the cdf of T.

From Ex 4.6.1 in Pitman's Probability

Problem 4: Sample Quiz Question

For any given hour, a train station is either busy or uncrowded, (each equally likely). If the train station is busy, then trains arrive at an average rate of twenty per hour, according to a Poisson arrival process. If the train station is uncrowded, then trains arrive at an average rate of ten per hour, according to a Poisson arrival process. Suppose no trains show up in the first 10 minutes of the hour. What is the probability that this hour is busy?