Stat 134: Change of Variables and Operations - Review

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Conceptual Review

a. Let X be a discrete random variable and set $Y = g(X)$, what is a formula for $\mathbb{P}(Y = y)$?

b. Let now X be a continuous random variable with density f_X and set again $Y = g(X)$. What is a formula for the density f_Y of Y?

c. Which steps do we need to follow when applying this formula?

d. What is the density of a sum of two continuous random variables $X + Y$?

e. If X and Y are discrete, how can we find an expression for $\mathbb{P}(X + Y = z)$?

f. What is the density of the ratio of two positive continuous random variables $\frac{X}{Y}$?

Problem 1

Let X and Y be exponentially distributed with parameters λ, resp. μ. Find the density of $R = \frac{X}{Y}$.

1. Solve this question using the formula for densities of ratios.

2. Try to relate the problem to competing exponentials.

Problem 2

Let X and Y be i.i.d. uniform on $(0, e^{-1})$. Determine the distribution of $\log(XY)$.

Step 1 This is not an operation of two random variables we immediately know how to deal with. Try to get it into a different form.

Step 2 Find the density of $V = \log(X)$.

Step 3 Can you recognize the distribution of V? If yes, use this to determine the distribution of $Z = V + W$. If not, skip to the next step.

Step 4 Use the formula for densities of sums of random variables to find the density of $Z = V + W$.

Problem 3

Assume that we first flip a coin until we get heads, where the probability of getting head at a toss is p. Let T be the number of tosses we need. Given $T = t$, we toss a coin with success probability $\frac{1}{t}$ until we get heads for the first time. Let S denote the number of tosses we need this time. What is the distribution of $Z = T + S$?

Step 1: What is the range of Z?

Step 2: For z in the range of Z, find an expression for $\mathbb{P}(Z = z)$.