Stat 134: Section 20

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Problem 1

Suppose I toss three coins. Some of them land heads and some land tails. Those that land tails I toss again. Let X be the number of heads showing after the first tossing, Y the total number showing after the second tossing, including the X heads appearing on the first tossing. So X and Y are random variables such that $0 \leq X \leq Y \leq 3$ no matter how the coins land. Write out distribution tables for each of the following distributions:

a. the distribution of X;

b. the conditional distribution of Y given $X = x$ for $x = 0, 1, 2, 3$;

c. the joint distribution of X and Y;

d. the distribution of Y;

e. the conditional distribution of X given $Y = y$ for $y = 0, 1, 2, 3$;

f. What is the best guess of the value of X given $Y = y$ for $y = 0, 1, 2, 3$. That is, for each y, choose x depending on y to maximize $P(X = x | Y = y)$.

Ex 6.1.1 in Pitman's Probability

Problem 2

Let X_1 and X_2 be independent Poisson random variables with parameters λ_1 and λ_2. Show that for every $n \geq 1$, the conditional distribution of X_1, given $X_1 + X_2 = n$, is binomial, and find the parameters of this binomial distribution.
Ex 6.1.5 in Pitman's Probability

Problem 3

Poissonization of the binomial distribution. Let N have Poisson (λ) distribution. Let X be a random variable with the following property: for every n, the conditional distribution of X given $(N = n)$ is binomial (n, p). Show that the unconditional distribution of X is Poisson, and find its parameter.
Ex 6.1.7 in Pitman's Probability