Stat 134 lec 23

Note 2 prob densities f, g represent the same prob distribution if they differ by only a countable number of points.

ex $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{else} \end{cases}$, $g(x) = \begin{cases} 2x & 0 \le x < 1 \\ 0 & \text{else} \end{cases}$

are equivalent densities

Hence I may be sloppy about end pts when defining densities.

sec 4.4 Change of variable for densities.

linear change of variable.

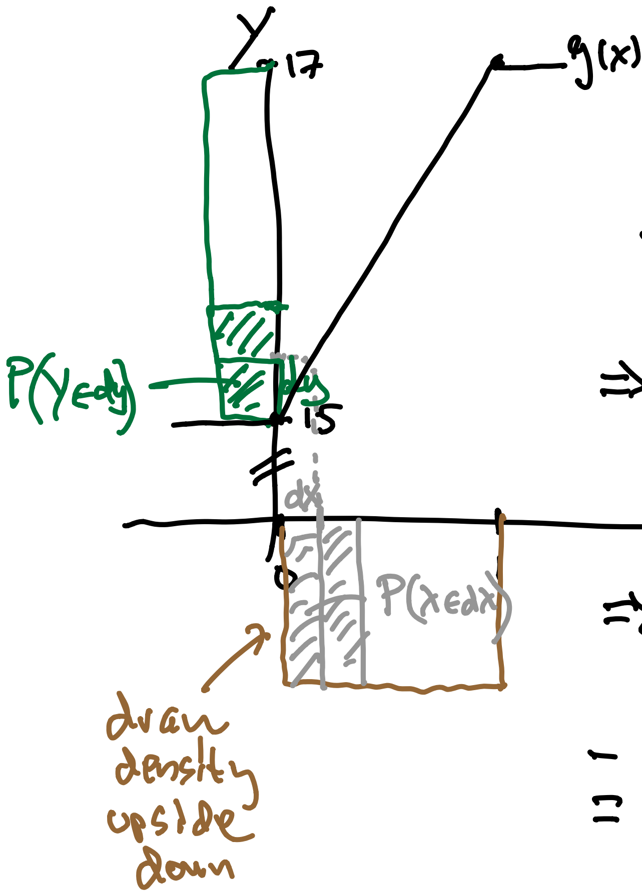
X cont. RV w/ known density f_X

$Y = g(X) = aX + b$ a, b constants, $a > 0$

Find density of Y. — $\boxed{f_Y(y) = \frac{1}{a} f_X\left(\frac{y-b}{a}\right)}$

ex $X = \text{Unif}(0,1)$

$Y = 2X + 15$ — 15 - 17 nonzero



$$\frac{dy}{dx} = \frac{d(2x+15)}{dx} = 2$$

$\Rightarrow dy = 2dx$

$X \in dx \Leftrightarrow Y \in dy$

since g increasing

$\Rightarrow P(X \in dx) = P(Y \in dy)$

$$\Rightarrow f_Y(y)\,dy = f_X(x)\,dx$$

$$= f_Y(y)\frac{dy}{dx} = f_X(x)$$

$$= f_Y(y) \times \frac{1}{\underbrace{\frac{dy}{dx}}_{2}} \underbrace{f_X(x)}_{=1}$$

$x = \frac{y-15}{2}$

$$\Rightarrow \boxed{f_Y(y) = \begin{cases} \frac{1}{2} & \text{for } 15 < y < 17 \\ 0 & \text{else} \end{cases}}$$

$Y \sim \text{unif}(15, 17)$

$\underline{\text{ex}}$ $Z \sim N(0,1)$, $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

$X = \sigma Z + \mu \quad (\sigma > 0)$

Find density of X.

$$f_X(x) = \frac{1}{\sigma} f_Z\left(\frac{x-\mu}{\sigma}\right)$$

$$= \frac{1}{\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

This generalizes to non linear g !

Rigorous arguement:

X : density f_X, g smooth, increasing

$Y = g(X)$. Find density of Y.

cdf $F_Y(y) = P(Y \le y) = P(g(X) \le y)$

$$= P(X \le g^{-1}(y))$$

g increasing so can find inverse.

$$= F_X(g^{-1}(y))$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X\left(g^{-1}(y)\right)$$

$$= f_X\left(g^{-1}(y)\right) \cdot \frac{d}{dy}\left(\underbrace{g^{-1}(y)}_{=x}\right)$$

$$\Rightarrow f_Y(y) = f_X(x) \frac{dx}{dy}$$

ex Find the density of the area of a disc whose radius is uniform on $(0,2)$

let X = area of disc. — $X = \pi R^2$
R = radius of disc.

Steps

1. Poss values: $X \in (0, 4\pi)$
2. $x = \pi r^2$ $\quad \frac{dx}{dr} = 2\pi r$
3. $r = \sqrt{\frac{x}{\pi}}$ $(R > 0)$
4. find $f_X(x) = \frac{1}{2\pi r} \cdot f_R(r)$

$$= \begin{cases} \frac{1}{2\pi\sqrt{\frac{x}{\pi}}} \cdot \frac{1}{2} & \text{for } 0 < x < 4\pi \\ 0 & \text{else} \end{cases}$$

Stat 134

Monday March 12 2018

1. Let T be the time in minutes it takes a customer service rep to respond to 10 telephone inquiries. $T \sim Unif(8, 12)$. Let R denote the average rate, in customers per minute, at which the representative responds to inquiries.

   Which of the following is the density function of the random variable R on the interval $(\frac{10}{12} \leq r \leq \frac{10}{8})$?

   a $\frac{12}{5}$

   b $\frac{10}{r^2}$

   c $\frac{5}{2r^2}$

$T \sim Unif(8,12)$

$R = 10/T \quad R' = -\frac{10}{T^2}$

$\frac{10}{12} < r < \frac{10}{8}$

$f_R(r) = \frac{1}{\frac{10}{t^2}} \cdot f_T(t) \quad t = \frac{10}{r}$

make positive if negative, so density is positive.

$= \frac{\left(\frac{10}{r}\right)^2}{10} \cdot \frac{1}{4} = \frac{10}{r^2} \cdot \frac{1}{4}$

$= \frac{5}{2r^2}$

$f_T(t) = \frac{1}{4}$ for $8 < t < 12$