Stat 134: Section 18

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Problem 1: C.D.F. of the beta distribution for integer parameters

a. Let $X_1, X_2, X_3, \ldots, X_n$ be independent Uniform (0,1) random variables, and let $X_{(k)}$ be the kth order statistic of the X's. Find the c.d.f. of $X_{(k)}$ by expressing the event $X_{(k)} \leq x$ in terms of the number of X_i that are $\leq x$.

b. Use a. to show that for positive integers r and s, the c.d.f. of the beta(r, s) distribution is given by

$$\sum_{i=r}^{r+s-1} \binom{r+s-1}{i} x^i (1-x)^{r+s-1-i} (0 \leq x \leq 1)$$

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Ex 4.6.5 in Pitman's Probability

Problem 2

Let X and Y be independent standard normal random variables. Let $M = min(X, Y)$. Find:

a. $P(3X + 2Y > 5)$;

b. $P(M < 1)$;

c. $P(|M| < 1)$;

d. $P(M > max(X, Y) - 1)$.

Ex 5.3.6 in Pitman's Probability

Problem 3: Einstein's model for Brownian motion

Suppose that the X coordinate of a particle performing Brownian motion has normal distribution with mean 0 and variance σ^2 at time 1. Let X_t be the X displacement after time t. Assume the displacement over any time interval has a normal distribution with parameters depending only on the length of the interval, and that displacements over disjoint time intervals are independent.

a. Find the distribution of X_t;

b. Let (X_t, Y_t) represent the position at time t of a particle moving in two dimensions. Assume that X_t and Y_t are independent Brownian motions start at $t = 0$. Find the distribution of $R_t = \sqrt{X_t^2 + Y_t^2}$, and give the mean and standard deviation in terms of σ and t;

c. Suppose a particle performing Brownian motion (X_t, Y_t) as in part b. has an X coordinate after one second which has mean 0 and standard deviation one millimeter (mm). Calculate the probability that the particle is more than 2 mm from the point $(0, 0)$ after one second.

Ex 5.3.11 in Pitman's Probability