Stat 134: Section 8

Adam Lucas

February 12th, 2018

Problem 1

Suppose the Stat department teaches 15 classes a semester: 2 have 60 students, 1 has 300 students, and 12 have 20 students. Each course is taught by a different professor.

a. For a randomly selected professor, what is the expected size of the class they teach?

b. For a randomly selected student, what is the expected size of the class they are in? How does this compare to part (a)?

Problem 2

Let A and B be independent events, with indicator random variables I_A and I_B.

a. Describe the distribution of $(I_A + I_B)^2$ in terms of $P(A)$ and $P(B)$.

b. What is $\mathbb{E}\left[(I_A + I_B)^2\right]$?

c. Suppose we now have a set of identical but not necessarily independent indicators $I_1, I_2, \ldots, I_n$. Derive a useful formula for $\mathbb{E}\left[(I_1 + I_2 + \ldots + I_n)^2\right]$.

Hint: Expand the polynomial, then use linearity of expectations.

Ex 3.2.10 in Pitman's Probability

Problem 3

In a well-shuffled standard deck of cards, we are interested in the number of adjacent pairs; i.e., cards which are the same as the card before or after them in the deck. Calculate the expected number of adjacent pairs.

Hint: consider the probability that a card is the same as the card before it.

Problem 4: Sample Quiz Question

Suppose we have a collection of $2n + 1$ characters, of which there are n unique pairs and 1 unique character. (E.g., the collection aabbccd). If the characters are shuffled randomly, find the probability that the resulting sequence is a palindrome. (A palindrome is a sequence of characters which reads the same forwards and backwards.)

Prepared by Brian Thorsen