A study of Recursive Least Squares (RLS) adaptive filter algorithm in noise removal from ECG signals

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***Abstract*—Electrocardiogram (ECG) is a diagnostic procedure that measures and records the electrical activity of heart in detail. By reviewing an ECG report, one's condition of heart can be evaluated. But ECG signals are often affected and altered by the presence of various noises that degrade the accuracy of an ECG signal and thus misrepresents the recorded data. To filter out these noises conventional digital filters have been used for decades. Yet noise cancellation with finite and determined coefficients has often been unsuccessful due to the non-stationary nature of ECG signal. Adaptive filters adapt their filter coefficients with the continuous change of signal using adaptive algorithms, providing the optimum noise removal features for non-stationary signals like ECG. In this study, the adaptive filter algorithm, RLS has been used in cancellation of various noises in ECG signals. We have also performed noise removal using LMS adaptive filter algorithm to compare the performance of RLS algorithm. We have used MATLAB® to simulate different noise signals and process the noises. The ECG signals used here have been taken from the PhysioNet ECG-ID database. The simulation results depict that RLS algorithm renders a much better performance in removing noises from the ECG signals than LMS algorithm.**

Keywords- ECG signal; Noise removal; Adaptive filter; RLS; SNR; MSE; PRD; PSD; Spectrogram.

I. INTRODUCTION

ECG is generated by the electrical activity of the heart and can be measured by connecting electrodes on the skin surface of specific parts of the body. As the ECG is a recording of the electrical activities of the heart, it can help one get a comprehensive idea of a person's heart activities and can also help in detecting aberrations in heart activity such as cardiac infractions or unequal beat intervals [1]. Hence the ECG signals are required to be very accurate. Even slight distortions in the ECG waveforms can impair the understanding of the patient's heart conditions. But due to some inherent measurement and instrumentation faults, some noises get induced to the ECG signals, thus distorting the information carried by the signals. These noises typically are: white noise, various harmonics of power line interference, baseline wander noises, electrode movement noise and muscle artifacts. To filter out these noises, conventional digital filters like Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) have been employed [2]-[3]. But as these filters have fixed number of filter coefficients, they have been of little avail in removing the noises successfully. To achieve better noise removal from non-stationary signals like ECG, various adaptive filter algorithms have been employed [4]-[10]. As adaptive filters do not have fixed filter coefficients, these filters can change their filter coefficients to reduce the noise present in the signal through adaptation. Even the most basic adaptive filter algorithms, like the least mean squares (LMS), have shown promising performance in noise removal from non-stationary signals. However, in this study we have performed noise removal with the help of recursive least squares (RLS) adaptive filter algorithm. It is a more complex algorithm than the LMS. To understand how well the RLS algorithm performs in removing various noises from ECG signals, we have corrupted ECG signals with specific noises and removed those noises with both LMS and RLS algorithm based adaptive filters. Afterwards, we have compared the filtered signals in terms of Signal to Noise Ratio (SNR) Improvement, Mean Square Error (MSE), Percentage Root-Mean-Square Difference (PRD), plots of convergence, plots of Power Spectral Density (PSD) and Spectrograms [11]-[12]. Depending on these performance criteria it has been observed that the RLS algorithm has removed all types of noises more effectively than the LMS algorithm.

II. THE ADAPTIVE NOISE CANCELLER

The adaptive noise canceller is the circuit configuration that has been used in this experiment to remove the various noises from ECG and improve the signal quality. This act of noise cancelling depends upon subtraction of noise from a received signal which is controlled in an adaptive manner. The simplified block diagram of an adaptive filter used in the adaptive noise canceller setting is given in Fig. 1.

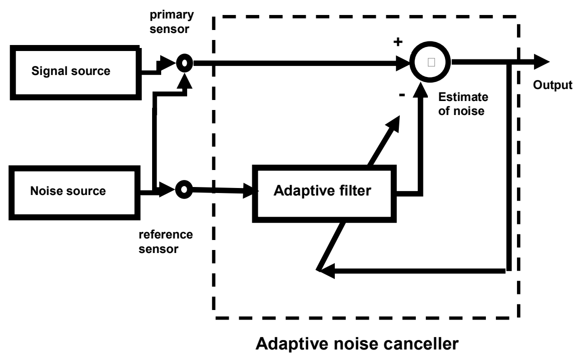


Figure 1. An adaptive noise canceller system setting.

The adaptive noise canceller depicted here is basically a dual input closed loop feedback system. The primary sensor receives the corrupted ECG signal *d(n),* which contains the clean ECG signal *s(n)* and additive noise $v_1(n)$. The signals *s(n)* and $v_1(n)$ are uncorrelated. We write *d(n)* as,

$$d(n) = v_1(n) + s(n) \qquad (1)$$

The reference sensor receives a noise $v_2(n)$ which is uncorrelated with *s(n)* but is correlated with $v_1(n)$. The reference signal is processed by the adaptive filter to produce an output *y(n)*, which is the estimate of noise. This *y(n)* is given as,

$$y(n) = \sum_{k=0}^{M-1} w_k v_2(n-k) \qquad (2)$$

Here, w_k are the adjustable tap weights or filter coefficients. The filter output *y(n)* gets subtracted from the primary signal *d(n)* and produces the error signal *e(n)*. The error signal *e(n)* is defined as,

$$e(n) = d(n) - y(n) \qquad (3)$$

By substituting Eq. (1) into Eq. (3) we get,

$$e(n) = s(n) + v_1(n) - y(n) \qquad (4)$$

This error signal is used to update the filter coefficients of the adaptive filter. Also this error signal comprises the overall system output. From Eq. (4) we see that the noise component present in the system output is $v_1(n) - y(n)$. The adaptive filter attempts to minimize the average power of the error signal *e(n)* which leaves the clean ECG signal largely unaffected. As the clean ECG signal remains unaffected, minimizing the average power of the output error signal *e(n)* is equivalent to minimizing the average power of the noise $v_1(n) - y(n)$ that was present on the signal. In this way the noises present in the ECG signal gets removed [13]-[15].

In the experimental setting of Fig. 1, two adaptive filters based on LMS and RLS algorithms [13]-[15] have been used. The setting has been used to remove five types of noises from ECG signal. After noise removal, the output signals of the two filters have been compared in terms of SNR Improvement, MSE, PRD, Convergence Plots, PSD Plots and Spectrogram Plots.

III. EXPERIMENTAL FINDINGS AND DISCUSSION

The ECG signals that have been used in this experiment have been collected from the ECG-ID database [16] of PhysioNet [17]. The duration of each signal is 10 seconds. In the digitized form the signal has been represented by 5000 samples. The amplitude of the signals are in millivolts (mV) range. The plot of one of the ECG signals that have been used is given in Fig. 2.

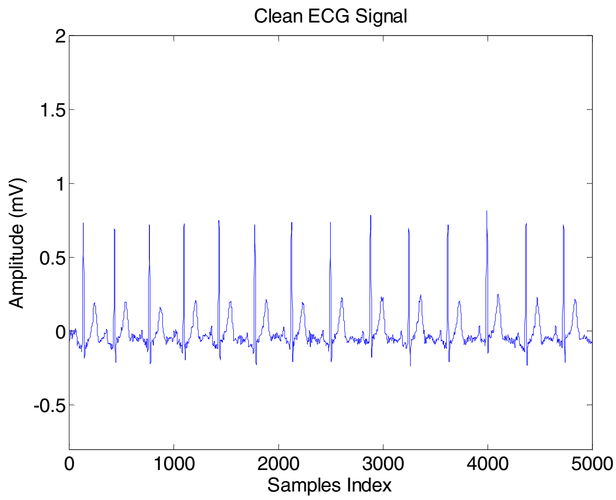
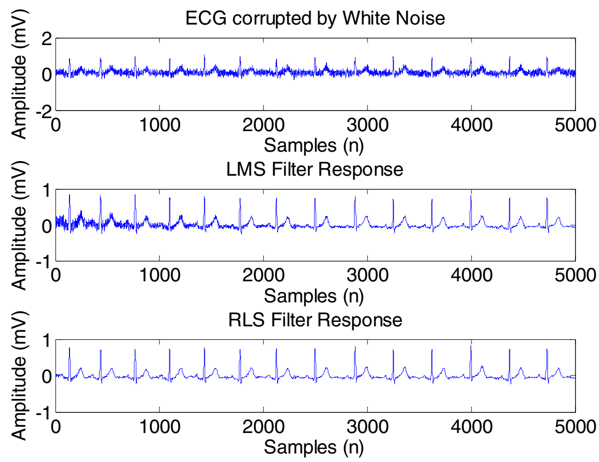
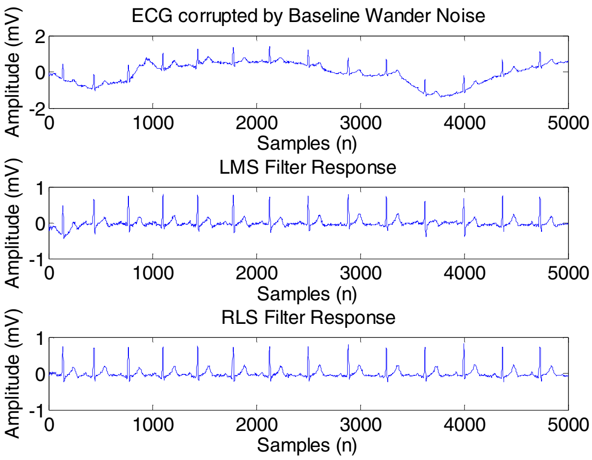


Figure 2. The plot of a clean ECG signal.

The noises that have been used to corrupt the ECG signal are: white noise, baseline wander noise, power line interference (PLI), electrode movement noise and muscle artifact. Out of these five noises, white noise and PLI have been simulated with the help of MATLAB and the other three noises have been collected from MIT-BIH Noise Stress Test Database [18]. The noise corrupted ECG signal is applied through the primary sensor of the adaptive noise canceller. An estimate of the respective noises is then applied to the reference sensor. The output of the adaptive noise canceller is the filtered response. These responses for the RLS and LMS algorithms are graphically represented in Fig. 3.



(a)



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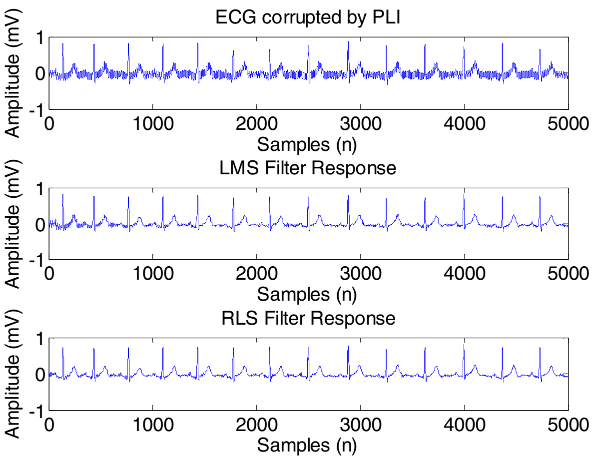

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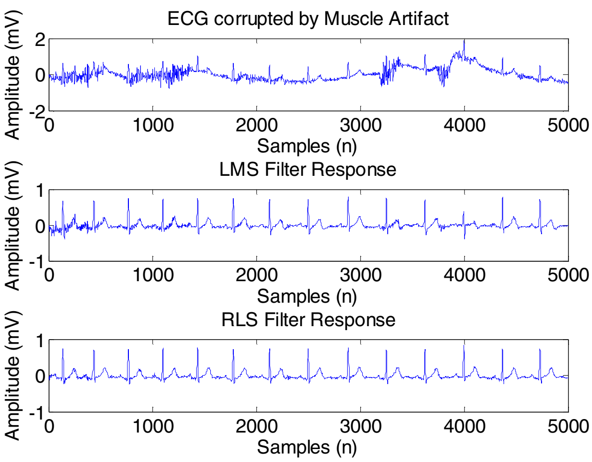

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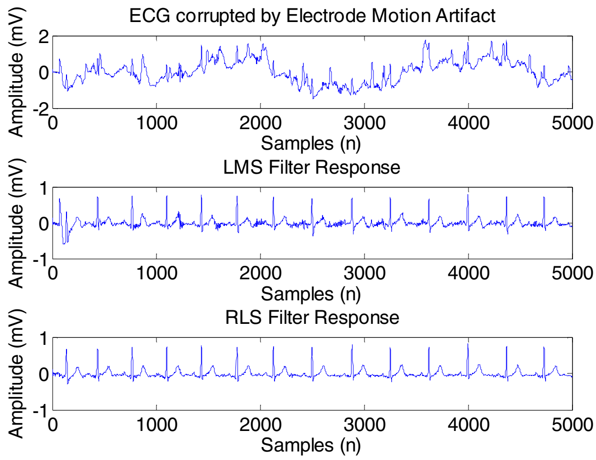


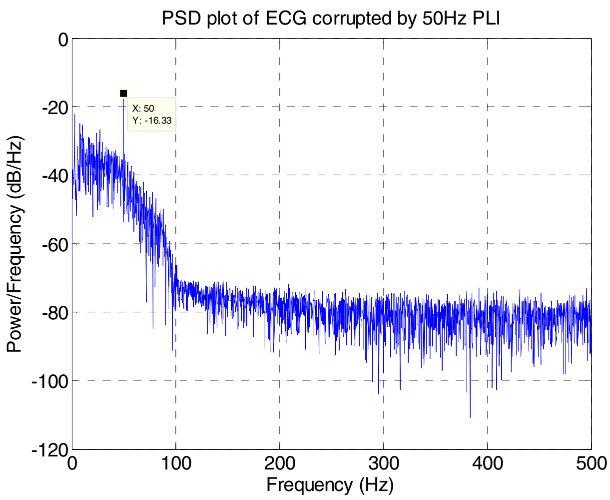
(e)

Figure 3. Graphical representation of ECG signals filtered by LMS and RLS algorithm based adaptive filters in removing (a) White Noise, (b) Baseline Wander Noise, (c) PLI, (d) Muscle Artifact and (e) Electrode Motion Artifact.

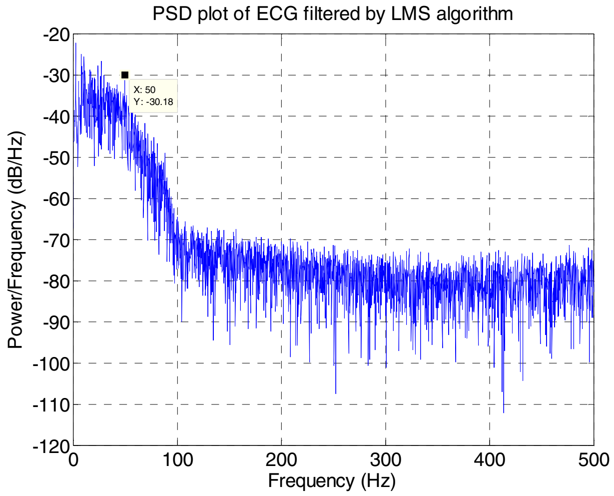
In plotting the waveforms in the Fig. 3, optimum values for filter step size has been used for the LMS algorithm. For RLS algorithm, a forgetting factor of 1 has been used. If this forgetting factor of RLS algorithm is decreased by even a little amount, the output signal quality deteriorates very quickly.

From the plots of Fig. 3 it is clearly evident that the RLS algorithm has removed all types of noises much more efficiently than the LMS algorithm.

To visually comprehend the performance measures of the two algorithms in removing noises, we have used PSD plots, Spectrogram plots and convergence plots of the noisy and clean signals. They are given in the subsequent figures. Fig. 4 gives the comparison of two algorithms in terms of PSD plots.



(a)



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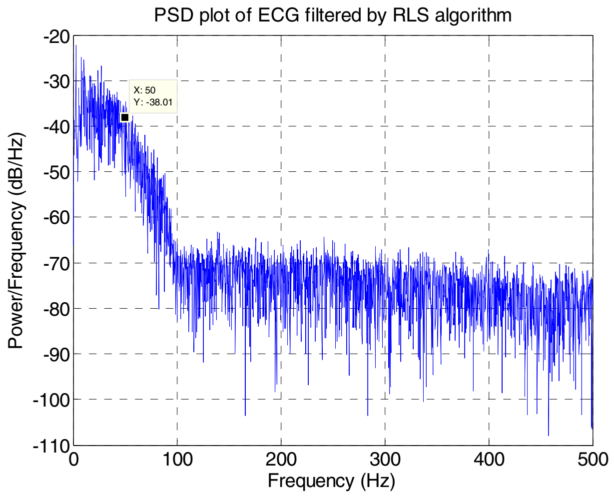
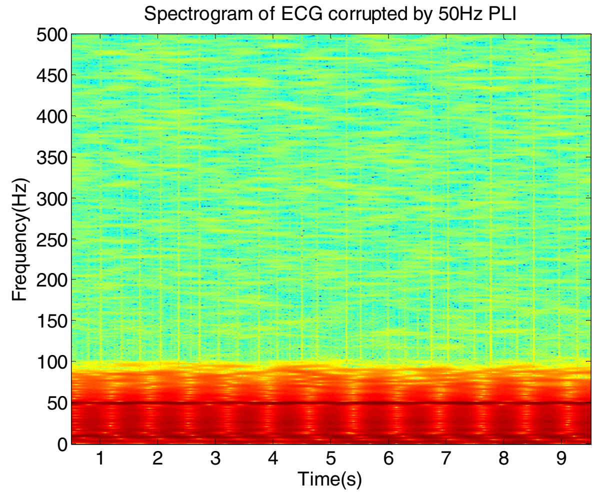


(c)

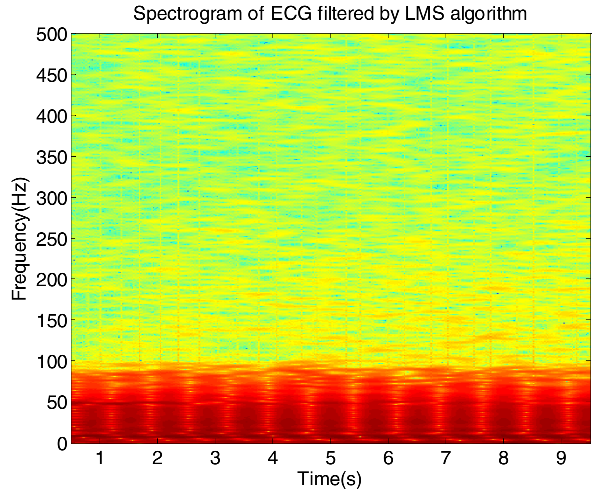
Figure 4. PSD plots of (a) ECG corrupted by 50Hz PLI, (b) ECG filtered by LMS algorithm and (c) ECG filtered by RLS algorithm.

Fig. 4 depicts the performance analysis of the two algorithms in terms of PSD plots. In Fig. 4(a), the PSD plot of ECG corrupted by 50Hz PLI is shown. Inspecting this figure, a sharp spike at 50Hz spectral component is observed which represents the induced PLI. From Fig. 4(b) and Fig. 4(c) we see that both the LMS and RLS algorithms have reduced this spike. But the RLS algorithm has reduced the specific spectral component to such an extent that the noise component has been rendered imperceptible.

The spectrogram plots of noisy and filtered ECG signals are given in Fig. 5. The 50Hz PLI noise is indicated in the Fig. 5(a) by the dark red band at the 50Hz. Fig. 5(b) and Fig. 5(c) represent the LMS and RLS filtered ECGs respectively. The figures denote that both the LMS and RLS algorithms have removed the PLI noise successfully but the RLS algorithm has removed the noise comparatively better than the LMS algorithm.



(a)



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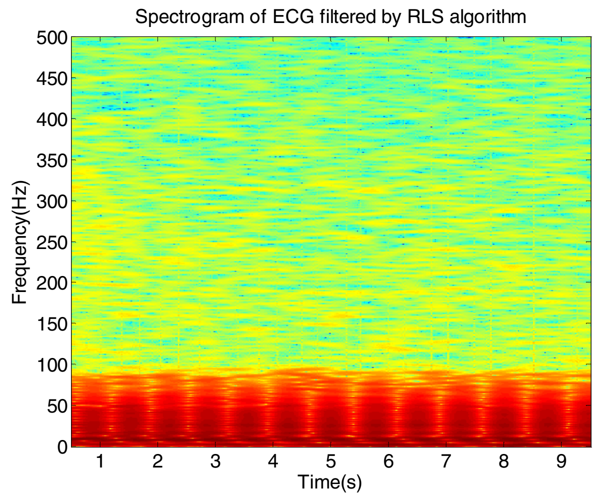

(c)

Figure 5. Plots of spectrogram of (a) ECG corrupted by 50Hz PLI, (b) ECG filtered by LMS algorithm and (c) ECG filtered by RLS algorithm.

Table I, Table II and Table III show the numerical analysis of LMS and RLS algorithms in terms of SNR Improvement, MSE and PRD respectively. From these tables it can be inferred that the RLS algorithm removes all noise better than LMS algorithm.

TABLE I. ANALYSIS IN TERMS OF SNR IMPROVEMENT

Noise Type	**Algorithm**	**SNR Improvement of ECG**			
		ECG 1	*ECG 2*	*ECG 3*	*Average*
White	LMS	11.1057	11.3282	11.6098	11.3479
	RLS	24.2556	22.8172	16.7664	22.2797
Baseline Wander	LMS	19.6563	20.2374	18.8886	19.5941
	RLS	34.2318	34.2752	30.6881	33.0651
PLI	LMS	11.2977	10.0901	11.0896	10.8258
	RLS	17.8375	20.9356	13.5546	17.4426
Muscle Artifact	LMS	15.5753	16.8904	14.7878	15.7512
	RLS	30.8448	31.5943	27.7240	30.0544
Electrode Motion	LMS	19.2091	20.1806	18.5415	19.3104
	RLS	33.5082	37.3850	26.1218	32.3383

TABLE II. ANALYSIS IN TERMS OF MSE

Noise Type	**Algorithm**	**MSE of reconstructed ECG**			
		ECG 1	*ECG 2*	*ECG 3*	*Average*
White	LMS	0.0015	0.0015	0.0014	0.0015
	RLS	0.0001	0.0001	0.0004	0.0002
Baseline Wander	LMS	0.0037	0.0033	0.0043	0.0038
	RLS	0.0001	0.0001	0.0003	0.0002
PLI	LMS	0.0004	0.0005	0.0004	0.0004
	RLS	0.0001	0.0001	0.0002	0.0001
Muscle Artifact	LMS	0.0029	0.0022	0.0034	0.0028
	RLS	0.0001	0.0001	0.0002	0.0001
Electrode Motion	LMS	0.0054	0.0046	0.0059	0.0053
	RLS	0.0002	0.0001	0.0009	0.0004

TABLE III. ANALYSIS IN TERMS OF PRD

Noise Type	**Algorithm**	**PRD of reconstructed ECG**			
		ECG 1	*ECG 2*	*ECG 3*	*Average*
White	LMS	7.4707	8.5124	6.7356	7.5729
	RLS	1.5662	1.5727	2.5016	1.8802
Baseline Wander	LMS	17.0819	18.5932	13.8885	16.5212
	RLS	2.3443	2.3968	2.7159	2.4857
PLI	LMS	4.0746	4.7770	3.2038	4.0185
	RLS	1.5648	1.2147	2.2049	1.6615
Muscle Artifact	LMS	25.1796	19.6581	23.1673	22.6683
	RLS	1.9928	1.7766	2.0509	1.9401
Electrode Motion	LMS	21.7234	11.6901	20.1434	17.8523
	RLS	3.6517	1.4767	8.2388	4.4557

The convergence speed of an algorithm determines how fast the noise present in the signal gets removed with increasing sample numbers. Fig. 6 presents the plot of error magnitude vs. sample index of the two algorithms. From the plots of Fig. 6, the convergence speed of the two algorithms can be compared.

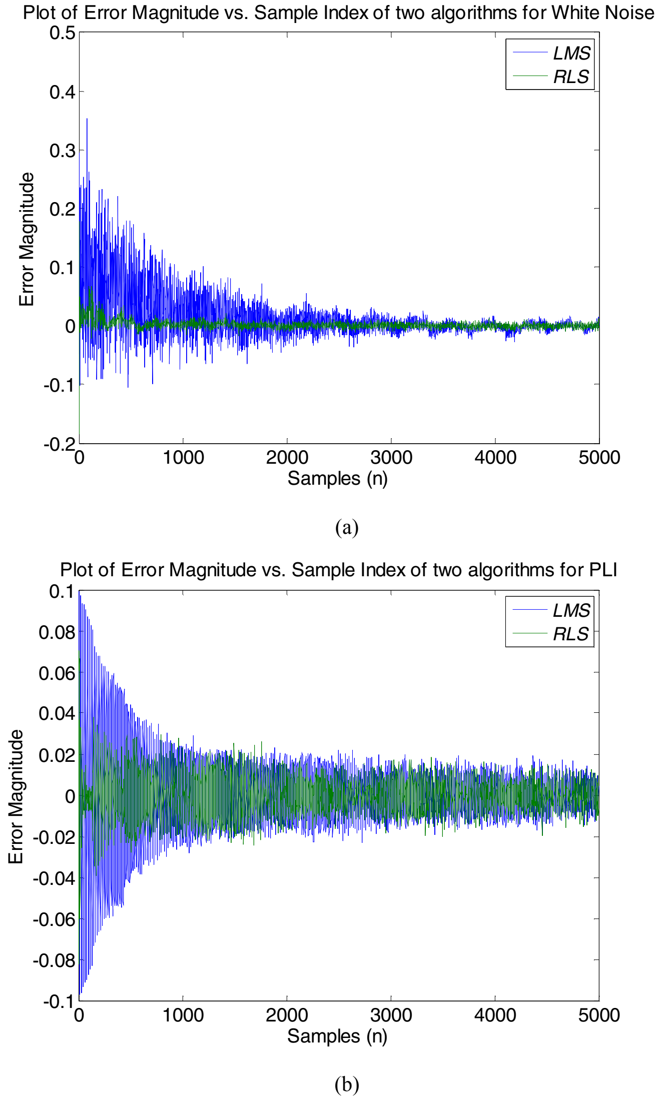

Figure 6. Plots of error magnitude vs. samples for (a) White Noise and (b) PLI.

From Fig. 6(a) and Fig. 6(b) it can be observed that the RLS algorithm reduces the error magnitude, which reflects the noise present in the signal, more quickly than the LMS algorithm. For both cases, the step size giving the optimum value for the LMS algorithm has been selected. For RLS algorithm, the forgetting factor is again set to 1.

IV. CONCLUSION

From the above analyses we can conclude that both the LMS and RLS algorithms remove various noises from ECG signals successfully. The LMS algorithm is a rudimentary adaptive filter algorithm whereas, RLS is comparatively much more complex [13]-[15]. From the results of graphical representations, SNR Improvement, MSE, PRD, PSD plots, Spectrograms and convergence plots we can conclude that this complex nature of RLS algorithm has caused it to remove noises from ECG signals with more efficacy than the LMS algorithm.

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