

# HW 8 元柏漸數值方法

$x$	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3
$y$	102.6	113.2	130.1	142.1	169.5	195.1	224.9	256.8

$$\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & \sum x_i^0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum x_i^2 y \\ \sum x_i y \\ \sum y \end{bmatrix}$$

$$\begin{bmatrix} 5843.815 & 1087.488 & 206.74 \\ 1087.488 & 206.74 & 40.2 \\ 206.74 & 40.2 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 317691.573 \\ 17011.21 \\ 1332.3 \end{bmatrix}$$

↘  
-1

$$a = 6.691184$$

$$b = -1.883746$$

$$c = 3.086393$$

$$6.691184x^2 - 1.883746x + 3.086393$$

$$\text{總平方誤差} = 0.00525$$

↘  
C++  
算

$$y = be^{ax}$$

$$\ln\left(\frac{y}{b}\right) = ax$$

$$\underbrace{\ln y}_{\downarrow Y} = \underbrace{ax}_{\downarrow Ax} + \underbrace{\ln b}_{\downarrow B}$$

$$\begin{bmatrix} \sum x^2 & \sum x \\ \sum x & \sum x^0 \end{bmatrix} \begin{bmatrix} a \\ \ln b \end{bmatrix} = \begin{bmatrix} \sum x \ln y \\ \sum \ln y \end{bmatrix}$$

$$\begin{bmatrix} 206.74 & 40.2 \\ 40.2 & 8 \end{bmatrix} \begin{bmatrix} a \\ \ln b \end{bmatrix} = \begin{bmatrix} 205.6168 \\ 40.5433 \end{bmatrix}$$

$$a = 0.398462$$

$$\ln b = 3.065641$$

$$b = 21.4482$$

$$y = 21.4482 e^{0.398462 x}$$

C-1+  $\frac{1}{n}$

$$a = 0.398445$$

$$b = 21.444544$$

$$y = 21.444544 e^{0.398445 x}$$

$$\text{總平方誤差} = 94.98$$

$$y = b x^n$$

$$\ln \frac{y}{b} = n \ln x$$

$$\ln y = \underbrace{n \ln x} + \underbrace{\ln b}$$

$$Y = A X + B$$

$$\begin{bmatrix} \sum (\ln x)^2 & \sum (\ln x) \\ \sum (\ln x) & \sum (1) \end{bmatrix} \begin{bmatrix} n \\ \ln b \end{bmatrix} = \begin{bmatrix} \sum \ln x \ln y \\ \sum \ln y \end{bmatrix}$$

$$\begin{bmatrix} 20.9371 & \\ 12.8225 & 8 \end{bmatrix} \begin{bmatrix} n \\ \ln b \end{bmatrix} = \begin{bmatrix} 65.3571 \\ 40.5433 \end{bmatrix}$$

$$n = 2.0201$$

$$\ln b = 1.830099$$

$$b = 6.234366$$

C++ 算

$$n = 2.019634$$

$$b = 6.238952$$

$$\underline{y = 6.238952 \cdot x^{2.019634} \quad *}$$

C++ 算

$$\text{总平方误差} = 0.011721$$

#

2.  $(-1, 1)$   $f(x) = \frac{1}{2} \cos x + \frac{1}{4} \sin 2x$

$$\begin{bmatrix} \int_{-1}^1 x^4 & \int_{-1}^1 x^3 & \int_{-1}^1 x^2 \\ \int_{-1}^1 x^3 & \int_{-1}^1 x^2 & \int_{-1}^1 x \\ \int_{-1}^1 x^2 & \int_{-1}^1 x & \int_{-1}^1 x^0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \int_{-1}^1 x^2 f(x) dx \\ \int_{-1}^1 x f(x) dx \\ \int_{-1}^1 f(x) dx \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{5} & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.23913 \\ 0.21770 \\ 0.84147 \end{bmatrix}$$

$$a = -0.23265$$

$$b = 0.32655$$

$$c = 0.498285$$

$$-0.23265 x^2 + 0.32655 x + 0.498285 \quad \#$$

$$\begin{aligned} & \left. \frac{1}{2} \sin x + -\frac{1}{8} \cos 2x \right|_{-1}^1 \\ &= \frac{1}{2} \sin 1 - \sin 1 - 0 \\ &= \sin 1 \\ &= 0.84147 \end{aligned}$$

3.

 $m=16$ 

$$f(x) = x^2 \sin x$$

$$[0, 1] \Rightarrow x = \frac{j}{16}$$

$$S_4 = a_0 + \sum_{k=1}^4 a_k \cos(2\pi kx) + b_k \sin(2\pi kx)$$

$$a_k = \frac{2}{m} \sum_{j=0}^{m-1} f(x_j) \cos(2\pi kx_j) \Rightarrow \begin{array}{l} \text{計算機} \\ k=1 \quad a_1 = 0.072827 \\ k=2 \quad a_2 = -0.022262 \\ k=3 \quad a_3 = -0.038390 \\ k=4 \quad a_4 = -0.043865 \end{array}$$

$$b_k = \frac{2}{m} \sum_{j=0}^{m-1} f(x_j) \sin(2\pi kx_j) \Rightarrow \begin{array}{l} k=1 \quad b_1 = -0.237249 \\ k=2 \quad b_2 = -0.123859 \\ k=3 \quad b_3 = -0.0177809 \\ k=4 \quad b_4 = -0.052223 \end{array}$$

$$a_0 = \frac{1}{m} \sum_{j=0}^{m-1} f(x_j) \Rightarrow 0.197672$$

$$S_4 = 0.197672 + 0.072827 \cos(2\pi x) + -0.237249 \sin(2\pi x)$$

$$-0.022262 \cos(4\pi x) + -0.123859 \sin(4\pi x)$$

$$-0.038390 \cos(6\pi x) + -0.043865 \sin(6\pi x)$$

$$-0.043865 \cos(8\pi x) + -0.052223 \sin(8\pi x)$$

$$(a) \int_0^1 S_4(x) dx = 0.197672 + \begin{array}{ccc} 0 & + & 0 \\ 0 & + & 0 \\ 0 & + & 0 \\ 0 & + & 0 \end{array} = \underline{0.197672} \quad \#$$

$$(b) \int_0^1 x^2 \sin x dx = 0.223244 \quad \text{計算機} \quad \text{error} = |0.223244 - 0.197672| = 0.025572$$

$$(c) E(S_4) = \sum_{j=0}^{15} [f(x_j) - S_4(x_j)]^2 \stackrel{\text{計算機}}{=} \underline{0.0174016} \quad \#$$