HW 8 阮相瀚數值方法

x	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3
y	102.6	113.2	130.1	142.1	167.5	195.1	224.9	256.8

$$\begin{bmatrix} \Sigma x_i^4 & \Sigma x_i^3 & \Sigma x_i^2 \\ \Sigma x_i^3 & \Sigma x_i^2 & \Sigma x_i \\ \Sigma x_i^2 & \Sigma x_i & \Sigma x_i^0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \Sigma x^2 y \\ \Sigma x y \\ \Sigma y \end{bmatrix}$$

$$\begin{bmatrix} 5843.815 & 1089.488 & 206.74 \\ 1087.488 & 206.74 & 40.2 \\ 206.74 & 40.2 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 37691.573 \\ 7011.21 \\ 1332.3 \end{bmatrix}$$

$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ 的 -1

$a = 6.691164$

$b = -1.483746$

$c = 3.086393$

$6.691184x^2 - 1.483746x + 3.086393$ ※

殘差平方誤差 $= 0.00525$ ※

↳ C++ 算

$y = be^{ax}$

$\ln\left(\frac{y}{b}\right) = ax$

$\ln y = ax + \ln b$

$Y = AX + B$

$$\begin{bmatrix} \Sigma x^2 & \Sigma x \\ \Sigma x & \Sigma x^0 \end{bmatrix} \begin{bmatrix} a \\ \ln b \end{bmatrix} = \begin{bmatrix} \Sigma x \ln y \\ \Sigma \ln y \end{bmatrix}$$

$$\begin{bmatrix} 206.74 & 40.2 \\ 40.2 & 8 \end{bmatrix} \begin{bmatrix} a \\ \ln b \end{bmatrix} = \begin{bmatrix} 205.6168 \\ 40.5433 \end{bmatrix}$$

$a = 0.398462$

$\ln b = 3.065641$

$b = 21.4482$

~~$y = 21.4482 e^{0.398462x}$~~ ※

C++ 算

$a = 0.398495$

$b = 21.444544$

$y = 21.444544 \times e^{0.398495x}$ ※

総合平方誤差 = 94.98 ※

$$y = bx^n$$

$$\ln \frac{y}{b} = n \ln x$$

$$\ln y = n \ln x + \ln b$$

$$Y = AX + B$$

$$\begin{bmatrix} \sum (\ln x)^2 & \sum (\ln x) \\ \sum (\ln x) & \sum (8) \end{bmatrix} \begin{bmatrix} n \\ \ln b \end{bmatrix} = \begin{bmatrix} \sum \ln x \ln y \\ \sum \ln y \end{bmatrix}$$

$$\begin{bmatrix} 20.7371 & \\ 12.8225 & 8 \end{bmatrix} \begin{bmatrix} n \\ \ln b \end{bmatrix} = \begin{bmatrix} 65.3571 \\ 40.5433 \end{bmatrix}$$

$n = 2.0201$

$\ln b = 1.830077$

$b = 6.234366$

$y = 6.238952 \times x^{2.019634}$ *

C++算

$n = 2.019634$

$b = 6.238952$

C++算

絕對平方誤差 = 0.011721 #

2. $(-1, 1)$ $\quad f(x) = \frac{1}{2}\cos x + \frac{1}{4}\sin 2x$

$$\begin{bmatrix} \int_{-1}^{1} x^4 & \int_{-1}^{1} x^3 & \int_{-1}^{1} x^2 \\ \int_{-1}^{1} x^3 & \int_{-1}^{1} x^2 & \int_{-1}^{1} x \\ \int_{-1}^{1} x^2 & \int_{-1}^{1} x & \int_{-1}^{1} x^0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \int_{-1}^{1} x^2 f(x)\,dx \\ \int_{-1}^{1} x\, f(x)\,dx \\ \int_{-1}^{1} f(x)\,dx \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{5} & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.23913 \\ 0.21770 \\ 0.84147 \end{bmatrix}$$

$a = -0.23265$

$b = 0.32655$

$c = 0.498285$

$-0.23265x^2 + 0.32655x + 0.498285$ ※

$\frac{1}{2}\sin x + -\frac{1}{8}\cos 2x \Big|_{-1}^{1}$

$= \frac{1}{2}\sin 1 - \sin -1 \quad - \quad 0$

$= \sin 1$

$= 0.84147$

3.

$m=16 \qquad f(x)=x^2\sin x \qquad [0,1] \Rightarrow x=\frac{j}{16}$

$$S_4 = a_0 + \sum_{K=1}^{4} a_K \cos(2\pi K x) + b_K \sin(2\pi K x)$$

$$a_K = \frac{2}{m}\sum_{j=0}^{m-1} f(x_j)\cos(2\pi K x_j) \Rightarrow$$ 計算機

$K=1 \quad a_1 = 0.072827$

$K=2 \quad a_2 = -0.022262$

$K=3 \quad a_3 = -0.038390$

$K=4 \quad a_4 = -0.043865$

$$b_K = \frac{2}{m}\sum_{j=0}^{m-1} f(x_j)\sin(2\pi K x_j) \Rightarrow$$

$K=1 \quad b_1 = -0.237249$

$K=2 \quad b_2 = -0.123859$

$K=3 \quad b_3 = -0.017709$

$K=4 \quad b_4 = -0.052223$

$$a_0 = \frac{1}{m}\sum_{j=0}^{m-1} f(x_j) \Rightarrow 0.197672$$

$$S_4 = 0.197672 + 0.072827\cos(2\pi x) + -0.237249\sin(2\pi x)$$
$$-0.022262\cos(4\pi x) + -0.123859\sin(4\pi x)$$
$$-0.038390\cos(6\pi x) + -0.043865\sin(6\pi x)$$
$$-0.043865\cos(8\pi x) + -0.052223\sin(8\pi x)$$

(a) $\int_0^1 S_4(x)dx = 0.197672 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = \underline{0.197672}$ ※

(b) $\int_0^1 x^2\sin x\,dx = 0.223244$ 計算機

$error = |0.223244 - 0.197672| = 0.025572$

(c) $E(S_4) = \sum_{j=0}^{15}\left[f(x_j) - S_4(x_j)\right]^2 = 0.017401$ C++算 ※

3.

$m=16 \quad f(x)=x^2\sin x \quad [0,1] \quad 2m-1=31 \quad x_i = 0+\frac{i}{31}$

$$S_4 = \frac{1}{2}a_0 + a_n\cos nx + \sum_{k=1}^{3} a_k\cos kx + b_k\sin kx$$

$$S_4 = \frac{1}{2}a_0 + a_4\cos 4z + \sum_{k=1}^{3} a_k\cos kz + b_k\sin kz \qquad x=\frac{1}{2}\left(\frac{z}{\pi}+1\right)$$

$$a_0 = \frac{1}{m}\sum_{i=0}^{2m-1} f(z_i) \qquad z \Rightarrow [-\pi, \pi]$$

$$a_n = \frac{1}{m}\sum_{i=0}^{2m-1} f(z_i)\cos nz_i \qquad z = \pi\left(2\frac{x-0}{1-0}-1\right)$$

$$b_n = \frac{1}{m}\sum_{i=0}^{2m-1} f(z_i)\sin nz_i \qquad = \pi\left(2\frac{i}{31}-1\right)$$

$$a_0 = \frac{1}{16}\sum_{i=0}^{31}\left(\frac{i}{31}\right)^2\sin\left(\frac{i}{31}\right) = 0.459205$$

$$a_1 = \frac{1}{16}\sum_{i=0}^{31}\left(\frac{i}{31}\right)^2\sin\left(\frac{i}{31}\right)\cos\left(1\,\pi\left(\frac{2i}{31}-1\right)\right) = -0.146756 \qquad b_1 = 0.232287$$

$a_2 = 0.054608 \qquad b_2 = -0.124941$

$a_3 = -0.038929 \qquad b_3 = 0.082932$

$a_4 = 0.033542$

$$S_4 = \frac{0.459205}{2} - 0.146756\cos z + 0.232287\sin z + 0.054608\cos 2z - 0.124941\sin 2z - 0.038929\cos 3z + 0.082932\sin 3z + 0.033542\cos 4z$$

1)

$$\int_0^1 S_4(x)\,dx \qquad dz = 2\pi\,dx \qquad z=\pi(2x-1)$$

(a) $\int_{-\pi}^{\pi} S_4(z)\frac{1}{2\pi}\,dz = 0.229603$

(b) $\int_0^1 x^2\sin x\,dx = 0.223244$ $\quad$ error $= \left|0.229603-0.223244\right| = 6.359\times10^{-3}$

(SSE)

(c) $E(S_4) = \sum_{j=0}^{31}\left[f(x_j)-S_4(x_j)\right]^2 = 2.43421$