HW 7 數值方法 E84116049

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 1 & -1 \\ -1 & 0 & 0 & 4 & -1 & -1 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 9 \\ 4 \\ 8 \\ 6 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix} \qquad D^{-1} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \qquad L+U = \begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{4} \begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

Jacobi Iteration

$$\vec{x} = -D^{-1}(L+U)\vec{x} + D^{-1}\vec{b}$$
$$= D^{-1}(\vec{b} - (L+U)\vec{x})$$

new old

$$x_1^{(n+1)} = \frac{1}{4}(0 + x_2 + x_4)$$
$$x_2^{(n+1)} = \frac{1}{4}(-1 + x_1 + x_3 + x_5)$$
$$x_3^{(n+1)} = \frac{1}{4}(9 + x_2 - x_5 + x_6)$$
$$x_4^{(n+1)} = \frac{1}{4}(4 + x_1 + x_5 + x_6)$$
$$x_5^{(n+1)} = \frac{1}{4}(8 + x_2 + x_4 + x_6)$$
$$x_6^{(n+1)} = \frac{1}{4}(6 + x_3 + x_5)$$

$x_1 = 1.17479$ $\quad x_4 = 3.05598$

$x_2 = 1.64317$ $\quad x_5 = 3.94966$

$x_3 = 2.44825$ $\quad x_6 = 3.09948$

C++ 算

Gauss-seidel method

$$\vec{x} = -(D+L)^{-1} U \vec{x} + (D+L)^{-1} b$$

$$(D+L)\vec{x} = -U\vec{x} + \vec{b}$$

$$(D+L) = \begin{bmatrix} 4 \\ -1 & 4 \\ 0 & -1 & 4 \\ -1 & 0 & 0 & 4 \\ 0 & -1 & 0 & -1 & 4 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix}$$

$$4x_1^{(n+1)} = x_2 + x_4 + 0$$

$$4x_2^{(n+1)} = x_1^{(n+1)} + x_3 + x_5 + -1$$

$$4x_3^{(n+1)} = x_2^{(n+1)} - x_5 + x_6 + 9$$

$$4x_4^{(n+1)} = x_1^{(n+1)} + x_5 + x_6 + 4$$

$$4x_5^{(n+1)} = x_2^{(n+1)} + x_4^{(n+1)} + x_6 + 8$$

$$4x_6 = x_3^{(n+1)} + x_5^{(n+1)} + 6$$

C++算 $\Rightarrow$

$x_1 = 1.17479$

$x_2 = 1.64317$

$x_3 = 2.44825$

$x_4 = 3.05598$

$x_5 = 3.94966$

$x_6 = 3.09948$

SOR method $w = 1.25$

$$D\vec{x} + w(A)\vec{x} = D\vec{x} + w\vec{b}$$

$$(D+wL)\vec{x} = [(1-w)D - wU]\vec{x} + w\vec{b}$$

$$4x_1^{(n+1)} = wx_2 + wx_4 + w0 + 4(1-w)\vec{x}_1$$

$$4x_2^{(n+1)} = w(x_1^{(n+1)} + x_3 + x_5 - 1) + 4(1-w)\vec{x}_2$$

$$4x_3^{(n+1)} = w(x_2^{(n+1)} - x_5 + x_6 + 9) + 4(1-w)\vec{x}_3$$

$$4x_4^{(n+1)} = w(x_1^{(n+1)} + x_5 + x_6 + 4) + 4(1-w)\vec{x}_4$$

$$4x_5^{(n+1)} = w(x_2^{(n+1)} + x_4^{(n+1)} + x_6 + 8) + 4(1-w)\vec{x}_5$$

$$4x_6^{(n+1)} = w(x_3^{(n+1)} + x_5^{(n+1)} + 6) + 4(1-w)\vec{x}_6$$

C++算 $\Rightarrow$

$x_1 = 1.17479$

$x_2 = 1.64317$

$x_3 = 2.44825$

$x_4 = 3.05598$

$x_5 = 3.94966$

$x_6 = 3.09948$

$$A\vec{x} = \vec{b}$$

$$t = \frac{\vec{v}^{T}\vec{v}}{\vec{v}^{T}A\vec{v}}$$

$$A = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 1 & -1 \\ -1 & 0 & 0 & 4 & -1 & -1 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \neq A^{T}$$ (可能有問題)

set $\vec{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad b = \begin{bmatrix} 0 \\ -1 \\ 9 \\ 4 \\ 8 \\ 6 \end{bmatrix}$ $\quad v = b - Ax$

$$\vec{v}^{(1)} = \vec{b} - A\vec{x}^{(1)}$$

$$\vec{v}^{(1)} = \begin{bmatrix} 0 \\ -1 \\ 9 \\ 4 \\ 8 \\ 6 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 6 \\ 3 \\ 7 \\ 4 \end{bmatrix}$$

$$\underbrace{\vec{v}^{(1)T}\vec{v}^{(1)}}_{\vec{v}\cdot\vec{v}} = 118 \qquad \underbrace{\vec{v}^{(1)T}A\vec{v}^{(1)}}_{\vec{v}\cdot A\vec{v}^{(1)}} =$$

$$t = \frac{\vec{v}^{T}\vec{v}}{\vec{v}^{T}A\vec{v}} =$$

$$\vec{x}^{(n+1)} = \vec{x}^{(n)} + t_n v^{(n)}$$

① $\vec{A}\vec{x}$

② $\vec{v} = \vec{b} - A\vec{x}$

③ $\vec{A}\vec{v}$

④ $\vec{v}^{T}\vec{v}$

⑤ $\vec{v}^{T}A\vec{v}$

⑥ $t = \frac{\vec{v}^{T}\vec{v}}{\vec{v}^{T}A\vec{v}}$

⑦ $x^{(n+1)} = \vec{x}^{(n)} + t_n\vec{v}^{(n)}$

⑧ $\text{error} = \sqrt{(\text{new}\,\vec{x} - \vec{x})^2}$

$\Downarrow$ C++ 算

$x_1 = 1.117479$

$x_2 = 1.64317$

$x_3 = 2.44825$

$x_4 = 3.05598$

$x_5 = 3.94966$

$x_6 = 3.09946$