

Task 3

True ①

$$-2x^2 + 3 = a(x^2 + 3x) + b(2x^2 + 4x - 1)$$

$$(-2, 0, 3) = a(1, 3, 0) + b(2, 4, -1)$$

$$(-2, 0, 3) = (a + 2b, 3a + 4b, -b)$$

$$\begin{cases} a + 2b = -2 \\ 3a + 4b = 0 \\ -b = 3 \end{cases}$$

$$\begin{cases} a + 2b = -2 \\ -2b = 6 \\ -b = 3 \end{cases}$$

$$\begin{cases} a = -4 - 8 \\ b = 3 \end{cases}$$

$$\text{III} \quad b = 3, a = 4$$

$$-2x^2 + 3 = 4(x^2 + 3x) + 3(2x^2 + 4x - 1)$$

$$-2x^2 + 3 = 4(x^2 + 3x) + (-3)(2x^2 + 4x - 1)$$

True ②

$$x^2 + 2x - 3 = a(-3x^2 + 2x + 1) + b(2x^2 - x - 1)$$

$$(1, 2, -3) = a(-3, 2, 1) + b(2, -1, -1)$$

$$= (-3a + 2b, 2a - b, a - b)$$

$$\begin{cases} -3a + 2b = 1 \\ 2a - b = 2 \\ a - b = -3 \end{cases}$$

$$\begin{cases} a - b = -3 \\ 2a - b = 2 \\ -b = -8 \end{cases}$$

$$\begin{cases} a - b = -3 \\ b = 8 \end{cases}$$

$$a = 5, b = 8$$

$$x^2 + 2x - 3 = 5(-3x^2 + 2x + 1) + 8(2x^2 - x - 1)$$

False ③

$$3x^2 + 4x + 1 = a(x^2 - 2x + 1) + b(-2x^2 - x + 1)$$

$$(3, 4, 1) = a(1, -2, 1) + b(-2, -1, 1)$$

$$= (a - 2b, -2a - b, a + b)$$

$$\begin{cases} a - 2b = 3 \\ -2a - b = 4 \\ a + b = 1 \end{cases}$$

$$\begin{cases} a + b = 1 \\ b = 6 \\ -3b = 2 \end{cases}$$

$$\begin{cases} a + b = 1 \\ b = 6 \end{cases}$$

Can't be expressed as a linear combination of these polynomials