

# Skill-Biased Reallocation\*

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## **Abstract**

Workers displaced by the reallocation of labour demand across industries suffer persistent earnings losses, in a large part due to higher unemployment risk. This paper quantifies the aggregate unemployment implications of a reallocation of labour demand. I develop a search and matching model with multiple industries and industry specific skill that is calibrated to the US economy. In the model a reallocation shock leads to up to a 0.8 percentage points rise in unemployment. The combination of industry specific skill and the substitutability between workers of different skill levels are key to this result. A policy of subsidies for vacancy posting can reduce this rise but only when targeted by skill level.

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# 1 Introduction

Many economic shocks, such as automation and trade, cause the reallocation of labour demand across industries. In response, displaced workers move to industries with increasing labour demand in search of higher wages and better employment prospects. Recent papers in macroeconomics [Huckfeldt \(2022\)](#) and trade [Traiberman \(2019\)](#) have emphasised the importance of industry-specific human capital in explaining the income losses of displaced workers. However, it has not been so prominent in the case of aggregate unemployment effects. Reallocation across industries with a high degree of industry-specific skills implies greater destruction of human capital than reallocation across industries with less specific skills. So the impact of reallocation may vary with this specificity.

In this paper, I show that skills indeed matter, but a second essential factor is the degree of substitutability between workers with different levels of industry-specific skills. If the inputs of skilled workers are complementary to the inputs of unskilled workers, then reallocation will increase unemployment. This is because the entry of workers into the industry increases the supply of unskilled workers. Then the marginal product of the entering workers declines due to their complementarity with skilled workers and slows the rate at which these workers are hired. In contrast, if skilled workers and unskilled workers are close substitutes the industry can absorb new workers quickly and the effect of reallocation on unemployment will be small.

To assess the quantitative importance of this mechanism I build a quantitative search and matching model with multiple industries. Workers accumulate industry-specific skill while employed in a stochastic manner. If a worker switches industries, they lose their accumulated skill. This acts as a mobility friction as workers who have accumulated skill are less likely to move as they would lose the wage premium associated with their accumulated skill. Then, instead of assuming perfect substitutability, I assume the industry-level production function that has constant elasticity of substitution over workers of different skill levels. A low elasticity of substitution corresponds with the case where skilled workers are doing different and complementary work to that of unskilled workers.

In the model, I allow for four industries, two of which have highly specific skills and two of which have less specific skills. To calibrate the model, I use heterogeneity in the observed returns to tenure and transition probabilities across industries. I target the industries I label high skill specificity to have higher returns to tenure and lower transition probabilities. The calibration results in the high-skill specificity industries having a higher wage differential between skilled and unskilled workers as well as a slower rate of skill accumulation. Thus skill is more desirable but harder to obtain in these industries.

I use the quantitative model to assess the impact of a reallocation shock. I formally model this as a shock that raises productivity in some industries and lowers it in others but doesn't affect aggregate employment in the long

run. The magnitude of the productivity shocks is set to match the average decadal dispersion in industry share growth rates. I find the shock leads to a rise in unemployment of up to 0.8 percentage points. Unemployment primarily occurs for skilled workers in the shrinking industry and unskilled workers in the growing industries. Skilled workers in the declining sector experience reduced marginal product and thus unemployment due to the productivity shock and the outflows of unskilled workers. They stay, however, as switching to the growing industry would mean a loss of skill and the associated wage premium. Unskilled workers in the growing industry face declining marginal product due to an influx of workers from other industries. Despite the possibility of unemployment, workers are attracted by the potential for high wages if they can acquire skill.

The elasticity of substitution between workers of different skill levels in production is the key parameter in the model governing the magnitude of the effect of reallocation on unemployment. Taking this elasticity to infinity which is the case of perfect substitutability, the effect of reallocation on unemployment becomes negligible. As the elasticity of substitution increases, the marginal product of workers of different skill levels is less dependent on the relative employment of workers of different skill levels. Thus when unskilled workers move to the growing industry the firm is willing to hire more of them as their marginal product declines only a little. Taking the elasticity of substitution from 2 to 0.5 leads to over a 50% reduction in the level of transitory unemployment induced by the shock.

[Şahin et al. \(2014-11-01\)](#) propose a measure of the degree of unemployment that occurs due to mismatch between unemployed workers and vacancies. Replicating their measure in the model, it attributes only 1% of the peak of unemployment to mismatch. This is because unemployed workers are not mismatched in the sense that they are not searching for jobs in the wrong industry. Instead, they don't have the skills that firms are posting vacancies for. Extending the measure of mismatch to include skill increases the share of unemployment due to mismatch to 17.5% at the peak. Here mismatch is occurring within industry rather than across industries.

I then analyse the extent to which policy can alleviate unemployment caused by reallocation. There is an externality separate from the usual search externalities, as firms fail to fully internalise the social benefit of hiring unskilled workers. If a match breaks up after a worker has accumulated skill they retain their skill and so take it into their next match. This skill is valuable to the new firm but this future benefit is not internalised by the old firm. So in the decentralised steady state firms post inefficiently few vacancies for unskilled workers leading to inefficiently high unemployment. A planner with the ability to subsidise vacancy posting funded by lump-sum taxes can achieve higher welfare than the decentralised equilibrium. It does this by incentivising firms to post more vacancies for unskilled workers and taxing vacancies for skilled workers. The planner's steady state has higher employment and more skilled workers than the decentralised equilibrium. This also affects the response to a reallocation shock, through the increased

posting of vacancies for unskilled workers. This speeds up the speed of aggregate skill accumulation as there are more unskilled workers employed who have the potential to gain skill. This both speeds up the recovery but also reduces the peak level of unemployment in transition.

**Literature Review** There is a large literature studying the effect of sectoral reallocation on unemployment. Starting with [Lilien \(1982\)](#) and [Rogerson \(1987\)](#) but also including [Dvorkin \(2014\)](#), [Pilossoph \(2012\)](#), [Chodorow-Reich and Wieland \(2020\)](#) and [Carrillo-Tudela and Visschers \(2023\)](#). These papers all use search and matching models with multiple sectors to study the impact of reallocation on unemployment. I contribute to this literature in two ways, the first of which is I allow for heterogeneity in the degree of skill specificity across sectors. Both [Carrillo-Tudela and Visschers \(2023\)](#) and [Kambourov \(2009\)](#) allow for occupational-specific skills but don't allow the accumulation process to differ across sectors. [Wiczer \(2015\)](#) allows the skill level that workers who have just entered an occupation have relative to higher tenure workers to differ across occupations. The speed of accumulation is fixed, however, at one model period. This addition reveals substantial heterogeneity reallocations of the same magnitude can have depending on the industries affected.

The second main contribution concerns the substitutability between workers of different skill levels. While the previous literature has assumed perfect substitutability, I allow for imperfect substitutability. [Kambourov and](#)

[Manovskii \(2009\)](#) contains a model with occupational-specific skill and a CES sectoral production function but does not consider the impact of reallocation on unemployment. Instead, it focuses on the link between occupational mobility and wage inequality. Assuming workers are perfect substitutes aids the solving of the model as the equilibrium will be block recursive as in [Menzio and Shi \(2010\)](#). I show that this assumption is impactful on the quantitative outcomes of the model. For estimates of the substitutability in the range of the empirical literature, the effect changes greatly. This observation has implications beyond this specific application to any model of labour search with heterogeneous workers. If the heterogeneous types are substitutes in production then the implications of the model may be very different.

This paper is also related to the literature on mismatch unemployment. [Shimer \(2007\)](#), [Şahin et al. \(2014-11-01\)](#) study how mismatch between workers and the industry they are searching in can lead to unemployment. This paper highlights how there can be mismatch on skills demanded and supplied within an industry.

This paper also relates to the literature on the impact of trade shocks when there are costs to switching sectors starting with [Artuç, Chaudhuri and McLaren \(2010\)](#). [Traiberman \(2019\)](#) highlights the importance of specific skills for explaining the distribution of income responses to a trade shock, however, they don't consider the impact on unemployment. [Dix-Carneiro et al. \(2023\)](#) considers the response to a trade shock in a model with costs of switching industries and search frictions within industries leading to involun-

tary unemployment. What they label as unemployment due to reallocation is due to the reallocation of labour demand from industries with lower frictional unemployment to industries with higher frictional unemployment. Thus they find eliminating the costs of switching increases the unemployment response to a trade shock in the US. While they do highlight the importance of the heterogeneity of industries, the mechanism in this paper generates transitory unemployment due to the reallocation of labour demand across industries which does not occur in their model.

This paper also relates to the literature on Diamond, Mortensen and Pissarides style models with imperfect substitutability between workers. [Mercecan, Schoefer and Sedláček \(forthcoming\)](#) assume that newly hired workers are imperfectly substitutable with incumbent workers in the initial period they are hired. There are two major differences. First, workers take longer to become skilled than in their model in which it takes a quarter. Secondly, in this paper workers retain their skill if they remain within the industry. Thus a separation shock would not have a large effect as the skilled workers would be quickly rehired. However, if the separations shocks they identify are driven by shocks that also cause reallocation then there would be a slow recovery of employment in the model from this paper.



## 2 Quantitative Model

I build a search and matching model in which workers can switch industries if separated. Following [Artuç, Chaudhuri and McLaren \(2010\)](#) I model this switching decision as a discrete choice subject to taste shocks. The main contrast of this model from the literature is allowing for the marginal product of a worker to depend on the distribution of skill within the industry.

### 2.1 Labour Market

There is a separate labour market for each skill level - industry pair. Firms can post vacancies  $v(k, s)$  in the market of their choosing. There is also a pool of unemployed workers  $u(k, s)$  for each worker skill - industry pair. The market tightness for a given labour market is defined as usual as vacancies divided by unemployed workers  $\theta(k, g) = \frac{v(k, g)}{u(k, g)}$ . The labour markets have a matching friction in the form of the standard cobb-douglas matching function

$$m(u, v) = \mu u^\eta v^{1-\eta}$$

Where  $\eta$  is the elasticity of matching and  $\mu$  is a matching efficiency parameter. The cobb-douglas matching function can produce more matches than either the number of unemployed workers or vacancies. In these cases, I truncate the number of matches to the minimum of the number of unem-

employed workers or vacancies. Given this matching function and the definition of labour market tightness, the job finding rate can be written  $f(\theta(k, g)) = \frac{m(u(k, g), v(k, g))}{u(k, g)} = \theta(k, g)^{1-\eta}$ . The vacancy fill rate can similarly be written as  $q(\theta(k, g)) = \theta^{-\eta}$

## 2.2 Workers

There is a unit mass of workers, who are risk neutral and discount at rate  $\beta$ . They can either be employed or unemployed and are at all times attached to an industry. Workers employed in an industry at the start of a period keep their job with a fixed probability  $(1 - \delta)$  and lose it with probability  $\delta$ . This timing is based on [Christiano, Eichenbaum and Trabandt \(2016\)](#). It allows for the possibility of workers switching jobs without a period of unemployment consistent with the large numbers of job-to-job transitions observed in the data. Those that lose their job at the beginning of the period or who were unemployed at the start of the period face a choice over whether to change industries. I model this as a discrete choice where workers choose the sector  $k'$  that maximises their utility

$$S_t(k, s, \zeta) = \max\{U_t(k, s) + \zeta_{i,0}, \max_{k' \neq k} U_t(k', 0) - \alpha_{k,k'} + \zeta_{i,k'}\}$$

Where  $U(k', 0)$  is the expected utility from being in sector  $k'$  with no industry-specific skill,  $\alpha_{k,k'}$  is a utility cost of switching from sector  $k, k'$  and

$\zeta_{i,k'}$  is the type 1 extreme value taste shock for sector  $k'$  which is iid across sectors and time and has variance  $\sigma_\zeta$ . The type 1 extreme value taste shocks generate a motive for gross moves. Some workers in industry  $k$  will draw a high taste shock for industry  $k'$  and so will want to switch to that industry and vice versa. Additionally the shocks and mean that the probability of switching from  $k$  to  $k'$  can be written tractably as

$$P(k \rightarrow k'|s) = \frac{e^{(U_t(k',0) - \alpha_{k,k'})/\sigma_\zeta}}{e^{U_t(k,s)/\sigma_\zeta} \sum_{\hat{k}=k} e^{(U_t(\hat{k},0) - \alpha_{k,\hat{k}})/\sigma_\zeta}}$$

The expected value function when making the choice has the following form

$$S_t(k, s) = E_\zeta[S_t(k, s, \zeta)] = \sigma_\zeta(\gamma + \log(e^{U_t(k,s)/\sigma_\zeta} \sum_{\hat{k}=k} e^{(U_t(\hat{k},0) - \alpha_{k,\hat{k}})/\sigma_\zeta}))$$

I add a search cost of  $\sigma_\zeta\gamma + \sigma_\zeta \log(n_k)$  to eliminate most of the gains in utility from search due to the type 1 extreme value shocks. The first part  $\sigma_\zeta\gamma$  reflects the mean type I extreme value while  $\sigma_\zeta \log(n_k)$  eliminates the gains due to more alternatives which increase the expected value of the maximum shock. This ensures workers don't prefer to be unemployed in order to be exposed to the taste shocks.

Once industry switching decisions are made, unemployed workers search

for a job in the job market associated with their current industry and level of skill human capital. They thus find a job with probability  $f(\theta(k, s))$  and remain unemployed with probability  $1 - f(\theta(k, s))$ . After this production occurs, the employed receive a wage  $w(k, s)$  and the unemployed receive unemployment benefits  $b$ . Finally, at the end of the period, two events can occur. First employed workers potentially gain human capital in their current industry with probability  $\psi_k$ . On the other hand, unemployed workers lose their industry-specific human capital with probability  $\rho$ . The second event is that a proportion  $d$  of workers die and are replaced by unemployed workers in the same industry with no industry-specific skill. I add death to the model as I will target wage growth in calibrating the human capital parameters. As workers experience wage growth over the lifecycle, not adding death will lead to too many workers with human capital in the steady state distribution. Given this the values of employment  $V$  and unemployment  $U$  are

$$\begin{aligned}
V_t(k, s) &= \delta S_t(k) + (1 - \delta)(w_t(k) \\
&\quad + m_t(1 - d)((1 - \psi(k))V_{t+1}(k, s) + \psi(k)V_{t+1}(k, s + 1))) \\
U_t(k, s) &= f(\theta_t(k))(w_t(k) + m_t(1 - d)((1 - \psi(k))V_{t+1}(k, s) + \psi(k)V_{t+1}(k, s + 1))) \\
&\quad + (1 - f(\theta_t(k)))(b + m_t(1 - d)((1 - \rho(k))S_{t+1}(k, s) + \rho(k)S_{t+1}(k, s - 1)))
\end{aligned}$$

## 2.3 Firms

There is a continuum of firms in each industry which each employs one worker. A firm must post a vacancy in order to hire a worker. The cost of posting a vacancy for a worker of skill  $s$  is denoted  $\kappa(k, s)$  and there is free entry into the market for vacancies. This implies the free entry condition for firms

$$\kappa = q(\theta)E[J_t(k, s)]$$

Where  $J_t(k, s)$  is the value of a filled vacancy, which solves the following Bellman equation.

$$J_t(k, s) = (y(k, s) - w(k, s)) + \beta(1 - d)(1 - \delta)[(1 - \psi(k))J_{t+1}(k, s) + \psi(k)J_{t+1}(k, s + 1)]$$

Where  $y(k, s)$  is the revenue generated by a match of worker with skill  $s$  in industry  $k$ . The interpretation of  $\kappa$  is of effective vacancy posting cost as the productivity of the matching function will not be separately pinned down in the calibration.

I assume wages are set by Nash bargaining between the firm and worker with equal bargaining weights. So in steady-state the wage can be calculated using the equation

$$\begin{aligned}
J(k, s) &= w(k, s) - b + \beta * (1 - d) ([ (1 - \psi(k)) V(k, s) + \psi(k) V(k, s + 1) ] \\
&\quad - [(1 - \rho) S(k, s) + \rho S(k, s - 1)]) \\
w(k, s) &= J(k, s) + b - \beta * (1 - d) ([ (1 - \psi(k)) V(k, s) + \psi(k) V(k, s + 1) ] \\
&\quad - [(1 - \rho) S(k, s) + \rho S(k, s - 1)])
\end{aligned}$$

I assume for each industry there is a Constant Elasticity of Substitution (CES) aggregator of the output of different skill types with each worker employed in an industry producing one unit of industry-skill-specific output.

$$Y_k = A_k \left( \sum_s \tau_{k,s} e[k, s]^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

I assume the production function is constant returns to scale implying  $\sum_s \tau_{k,s} = 1$ . The industry-skill CES parameters  $\tau_{k,s}$  determine the relative marginal product of different skill levels in each industry and thus influence the relative wages. This combined with the probability of gaining skill  $\phi(k)$  determines the expected returns to staying in a given industry for a long time. The industrial productivity  $A_k$  affects the relative wage across industries and therefore the relative size of different industries. Later in the quantitative exercise I will shock these productivities to induce reallocation of workers across industries. The elasticity of substitution across skills  $\eta$  is an

important parameter governing the response of unemployment in the model to reallocation as controls how the relative marginal products of workers of different skill levels respond to changes in the relative supply of workers of different skill levels within an industry. So if workers move into an industry this will increase the relative supply of unskilled workers and thus decrease the marginal product of unskilled workers. If  $\eta$  is large the change in marginal product will be small but if  $\eta$  is close to 0 then the change in marginal product will be large and thus the value to firms of posting vacancies for these workers will fall greatly.

## 2.4 Household

All workers are members of the representative household. The household's preferences over the output of each industry are given by a constant elasticity of substitution (CES) aggregator over industry output.

$$U(\{c_k\}_{k \in \{1, \dots, n_k\}}) = \left( \sum_k \omega_k^{\frac{1}{\sigma}} c_k^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Where the  $\omega_k$  are the CES weights and  $\sigma$  is the elasticity of substitution over industry output. The profits of the firms are paid out as dividends to the household as well as the wages of the workers.

### 3 Calibration

I take the model period to be a month. This allows for a reasonable frequency of churn across jobs, skill levels, employment and industries while not being so divorced from some of the data which is only available at the annual level. I set the number of industries to 4 and the number of skill levels to 2 which I label skilled and unskilled. Of the four industries, I label two high skill specificity and two low skill specificity, which will differ in their productivity  $A_k$ , CES production weights  $\tau_{k,s}$  and skilling rate  $\psi(k)$ . As I will discuss later in the calibration I will use heterogeneity in the returns to industry tenure and industry mobility to differentiate between the two types of industry. High-specificity industries will feature higher returns to tenure and lower mobility than low-specificity industries. In essence, I will use industry returns as a proxy for the unobserved skill specificity of the industry. I set the number of skill levels to 2 as for each skill level I need a moment of returns to industry tenure and longer periods of tenure are more noisy due to the smaller sample size of people with long tenure. Despite allowing for only two types of industries, I still require more than two industries, as the steady state in the two-industry economy implies absolute flows from and to each industry must be equal. This would imply that the heterogeneity in flows across industries would determine the relative size of the industries. Thus in order to be able to compare the impact of a shock to same-sized industries with differential mobility I allow for two industries for each type. Thus one



type of industry can be observed to have higher mobility in steady state than the other because more workers flow between the two industries of that type than between the two industries of the other type.

I first start by fixing some parameters to values that are standard in the literature. I set the discount factor  $\beta$  to 0.996 which implies an annual discount rate of approximately 5%. The parameter for the probability of losing skill while unemployed  $\rho$  I take from [Carrillo-Tudela and Visschers \(2023\)](#) as 0.02. The model in this paper doesn't have the idiosyncratic heterogeneity in productivity which enables [Carrillo-Tudela and Visschers \(2023\)](#) to explain duration dependence of unemployment which they use to calibrate this parameter. However, for the aggregates of interest in this paper, the results will not be sensitive to reasonable choices of this parameter. This is because only a small percentage of workers are unemployed each period and reasonable estimates of  $\rho$  are of a similar magnitude so the changes in skill driven by skill loss while unemployed are small compared to other sources of skill change. I set the probability of death  $d$  to  $\frac{1}{480}$  which implies an average working life of 40 years. I set the elasticity of substitution across industry output in the household's utility function to 4 which is within the range estimated by [Broda and Weinstein \(2006\)](#). I set the productivity of the matching function  $\mu$  to be 0.1. As I will calibrate the vacancy posting cost  $\kappa$  to match the unemployment rate, this is essentially a normalisation. If I increase  $\mu$  then the calibration will increase  $\kappa$  such that  $\frac{\kappa}{\mu} = q(\theta_{k,s})J(k, s)$  is unchanged. This is only not the case if either  $q(\theta)$  or  $f(\theta)$  are truncated which a low value of  $\mu$

helps avoid.

I then calibrate the rest of the parameters. For the cost of vacancy posting  $\kappa$  I assume it to be constant across industries and skill levels. I then calibrate it to match an aggregate unemployment rate of 4%. Then for the flow benefit of unemployment  $b$  I calibrate it to match the estimates from [Chodorow-Reich and Karabarbounis \(2016\)](#) that the flow benefits of unemployment are 55% of wages. The rest of the parameters fall into one of two categories. First are the parameters relating to skill and production  $A_k$ ,  $\tau_{k,s}$  and  $\psi(k)$ . Second are the parameters relating to industry choice  $\sigma_\zeta$  and  $\alpha$ . To calibrate them I will use moments of returns to career tenure, differential mobility across industries, a normalisation of average wages to 2 and an assumption that in the initial steady state, all industries have the same number of workers attached. I estimate the returns to career tenure using the NLSY79 data following [Pavan \(2011\)](#). I make two major changes to the specification, first I use OLS estimates rather than IV. This is because the selection effect that biases OLS occurs in the model and is informative about industry choice parameters. Secondly, I allow the returns to vary depending on 1 digit industry and I take the 25th and 75th percentiles of the estimated returns as my targets I take the estimates of industry mobility from [Dvorkin \(2021\)](#). I again take the 25th and 7th percentiles of the industry transition probabilities as my targets. To do this I consider the data for each industry from each period for which [Dvorkin \(2021\)](#) estimates a transition probability as an independent data point. The normalisation of the average wage of the

employed to 2 rather than 1 is done for numerical reasons to avoid wages going negative for some guesses causing discontinuities in the returns to career tenure moments. Finally, I assume that in the initial steady state all industries have the same number of workers attached so that in the quantitative exercise I can compare how the impact of the shock varies depending on the type of industry hit.

While intuitively one might think that the returns to career tenure will primarily inform the skill parameters and the industry mobility data will primarily inform the industry choice parameters, these parameters and moments are heavily interrelated. In the case of industry returns to tenure, the OLS estimates in the data are contaminated by selection bias as workers who experience lower returns may be more likely to leave the industry. The model also has this selection bias as workers who have accumulated skill in an industry are less likely to leave. The degree to which mobility is selective is partially determined by  $\sigma_\zeta$  the variance of the taste shocks. If  $\sigma_\zeta$  is low, the staying probability will be more sensitive to the value of staying. Thus skilled workers will be much less likely to move than unskilled workers, so selection bias will be high. Similarly, fixing mobility parameters, the skill and production parameters will affect the degree of mobility. The CES production weights  $\tau_{k,s}$  determine the wage premium to skill and therefore the returns to staying in an industry relative to moving. The skilling rate  $\psi(k)$  will play two roles, first it changes the proportion of workers who are skilled for a given mobility rate and skilled workers will move less. Secondly, it lowers the cost of moving

to a new industry as workers will accumulate skill faster and so the earnings loss from moving is lower.

Another subtlety of the identification of the parameters is in the relative magnitude of  $\psi(k)$ , the probability of gaining skill, between the high and low skill specificity industries. A higher  $\psi(k)$ , all else held equal, will lead to higher returns to tenure as workers will gain skill faster and therefore it might be expected that the high-skill specific industry will have a higher  $\psi$ . This need not be the case, however, as in order the calibration must also match the lower mobility in the high-specificity industry. In steady state net flows must be zero and thus in-migration must be lower. A low initial wage plus slow skill accumulation would make the industry unattractive to workers who would enter as unskilled. Also too high a  $\psi(k)$  would lead to many workers being skilled in the high specificity industry and given the high returns these workers would be unlikely to leave leading to excessively low outmigration.

### 3.1 Calibrated Parameters

<i>Calibrated parameters</i>			
	Parameter	Value	
Sectoral Productivity	$A_k$	[13.2, 13.5]	
CES Production Weights	$\tau_{k,0}$	[0.13,0.15]	
Skilling Rate	$\psi(k)$	[0.027, 0.042]	
Utility Cost of Switching	$\alpha$	5.13	
Variance of Taste Shocks	$\sigma_\zeta$	5.13	
Vacancy Posting Cost	$\kappa$	0.0026	

<i>Model and Data Moments</i>			
Moment		Model	Data
2 year returns to industry tenure high specificity		8.14%	7.5 %
2 year returns to industry tenure low specificity		6.9%	6.3 %
5 year returns to industry tenure high specificity		13.4%	16.2 %
5 year returns to industry tenure low specificity		9.7%	12.1 %
Average wage		2.15	2
Transition probability away high specificity		6.7%	7.0%
Transition probability away low specificity		9.0%	10.6%
Unemployment rate		4.1%	4%

The model in general does a good job matching the moments of the data. The tension that stops the model from completely matching the moments is that for the returns to tenure to be high the wage premium must be high.

However, this reduces the transition probabilities of workers due to the high opportunity cost of losing skill. Raising  $\sigma_{\eta}$  the variance of the type 1 extreme value shocks is limited by the fact this reduces the gains from being employed. Increasing sectoral productivity would lead to an increase in the wage which is attempting to be normalised.

The high specificity industry actually has a lower rate of skill accumulation compared to the low specificity industry. This makes the specific skill in the high specificity industry harder to acquire but more valuable. This occurred because the mobility effects of the slower skill accumulation help the calibration match the data more than the increased tenure returns from faster skill accumulation

The low value of the  $\kappa$ , the vacancy posting cost can only be understood when taking into account the productivity of the matching function which I take to be 0.1. Given the steady state vacancy fill rates cost per match ranges between 0.11 and 0.73. Compared with a marginal product of a match ranging between 1.9 and 2.26.

<i>Model and Data Moments</i>		
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Transition probability away low specificity	9.0%	10.6%
Unemployment rate	4.1%	4%

## 4 Quantitative Experiment

In order to understand the effect of reallocation in the model I study the response to a shock to the productivity of two industries in the economy. One industry receives a positive productivity shock and the other a negative productivity shock. The shock takes the form of an unanticipated MIT shock which takes effect in a linear manner over a decade. I determine the shock size by finding the negative shock to a high-skill specific industry and positive shock to the other high-skill specific industry that leads to the same steady state employment as the initial steady state and a change in industry shares in line with decadal changes in industry shares. I then take the same sized negative shock and solve for the positive shock that leads to the same steady state employment for all other combinations of industry types getting shocks.

I plot the results for unemployment in [Figure 1](#). There are two main takeaways from this figure. First, is that the reallocation shock can lead to a substantial increase in unemployment in this model. The lowest trough in unemployment is 0.8 percentage points below steady state, which is a 17% increase from steady state unemployment. Additionally, the unemployment response is highly persistent, with the recovery taking a decade to complete. It is important to note that unlike [Chodorow-Reich and Wieland \(2020\)](#) this effect does not require a coinciding negative aggregate demand shock nor downward nominal wage rigidity.

Secondly, the impact of the reallocation shock is heterogeneous in both magnitude as well as dynamics with reallocation involving high skill specificity industries having larger effects. In particular if the growing industry is high specificity this leads to an persistent rise in unemployment. This is because skill accumulation is slower in the high specificity industry and so it takes a long time to reach the new steady state level of skilled workers in that industry. The size of the shock in the very short run is larger if the shrinking industry is high skill specificity. This is due to the skilled workers in the high specificity industry being less willing to move and therefore exposed to the negative shock to their industry.

These effects are driven by the dynamics of marginal productivity of different workers. To show this I plot in [Figure 2](#) the dynamics of workers' marginal product to the shock that reallocates between the high skill specificity industries. The marginal products of the skilled workers in the



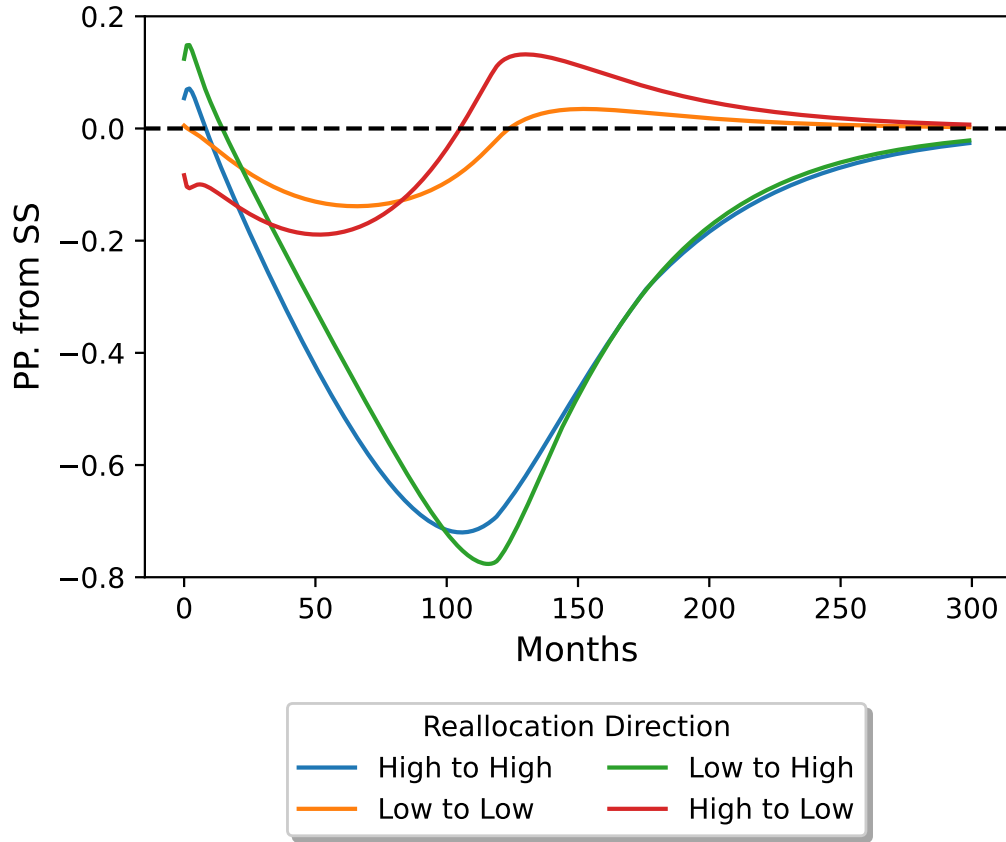
shrinking industry and the unskilled workers in the growing industry both fall in response to the shock before slowly recovering. This happens to the skilled workers in the shrinking industry due to both the decline in productivity but also the exit of unskilled workers from the industry. This drives down the marginal product of skilled workers as the relative supply of skilled workers increases. While for the unskilled workers in the growing industry, the productivity shock is positive for their marginal product however this is dominated by the negative effect entry of workers into the industry. For unskilled workers in the shrinking industry and skilled in the growing their marginal product increases driven primarily by their relative share of industry employment falling.

#### 4.1 The Role of Substitutability Between Skills

The importance of changing relative supplies of different skills to the effects points to  $\eta$  the elasticity of substitution between workers of different skill levels as a key parameter in the model to generate unemployment in response to reallocation. In this subsection, I illustrate the impact of this parameter on the results of the model as well as the mechanism through which it operates. I rerun the counterfactual experiment with different values of  $\eta$  and plot the results in [Figure 3](#).

As can be seen in the figure, the impact of the demand shock on employment is decreasing in  $\eta$  and the effect is substantial. Going from an  $\eta$  of 0.5 to an  $\eta$  of 10 reduces the size of the unemployment response by over 50% The

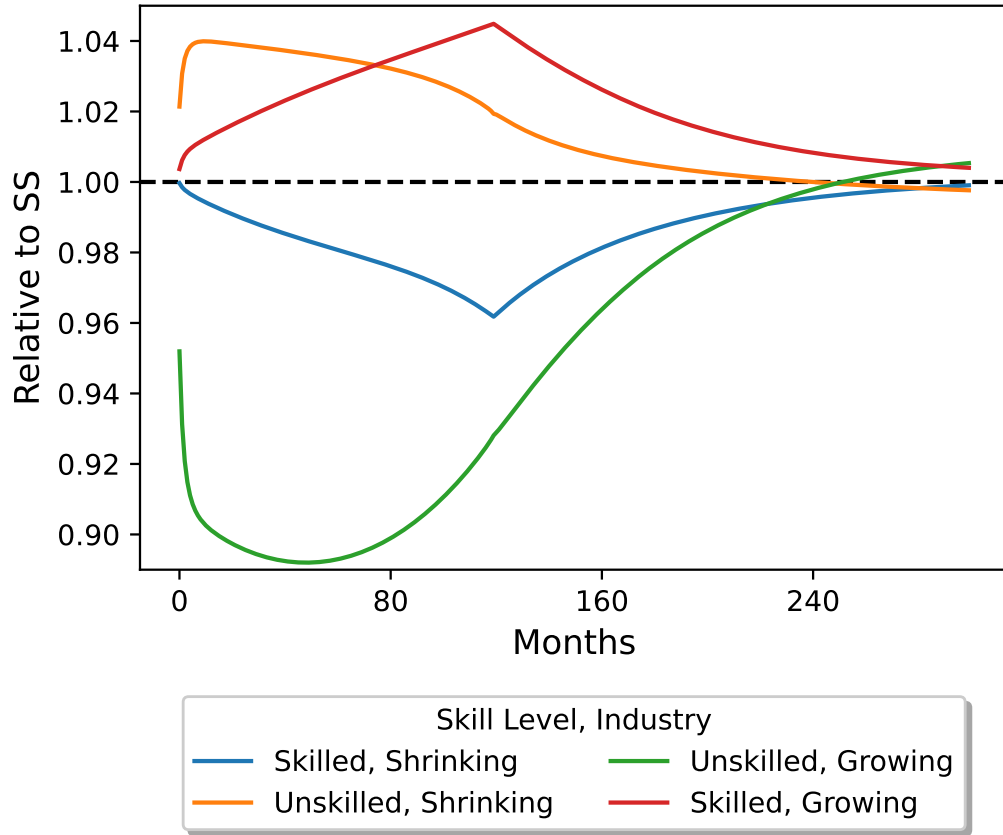
Figure 1: Comparison of Employment Depending Industries Shocked



All lines are in response to productivity shock to the two industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

higher the elasticity of substitution the less responsive the relative marginal products of different skill levels are to changes in the ratio of workers of different skill levels. So when unskilled workers leave the industry with the negative productivity shock, the relative marginal product of skilled workers falls by less the higher  $\eta$ . Similarly the relative marginal product of unskilled

Figure 2: Dynamics of Workers' Marginal Products

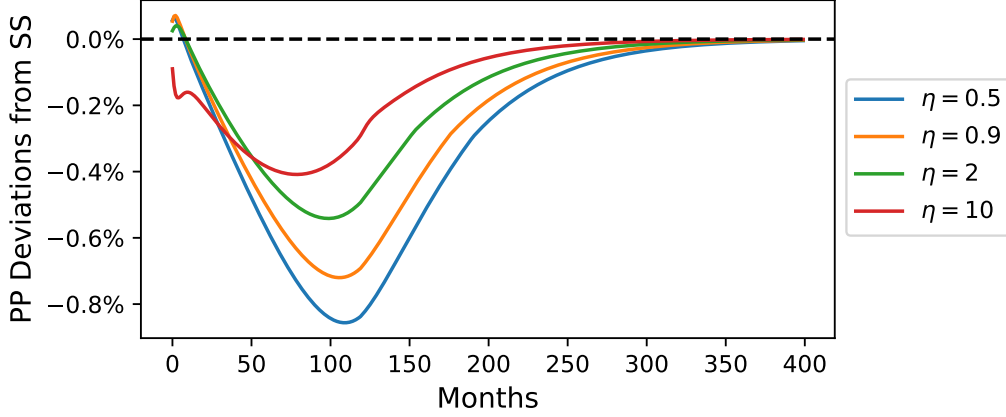


The shock is to the two high-specificity industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

workers in the industry with the positive productivity shock fall by less the higher  $\eta$  as workers enter the industry. As these are the locations where most of the unemployment occurs, the more their marginal products fall the more unemployment there is.

This is the important difference from [Carrillo-Tudela and Visschers \(2023\)](#). In their model workers have idiosyncratic productivity for the sector they are

Figure 3: Comparison of Employment depending of  $\eta$



All lines are in response to productivity shock to the two high-specificity industries. One hit positively and one negatively. I resolve the steady state for each  $\eta$ . The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

in which accumulates as well as having stochastic variation. However, in their model workers with different productivities are perfect substitutes so when there is a demand shock to a sector the marginal product of workers of all skills will rise no matter the skill distribution. Thus the value to a firm of the low productivity workers still increases when workers without industry specific skill enter. So firms post enough vacancies to absorb these incoming workers. So despite their model having specific skill, reallocation does not increase unemployment.

## 4.2 Mismatch Unemployment

[Şahin et al. \(2014-11-01\)](#) proposed an index of mismatch  $\mathcal{M}_t$  defined as below.

$$\mathcal{M}_t = 1 - \sum_k \left( \frac{\phi_{k,t}}{\bar{\phi}_t} \right) \left( \frac{v_{k,t}}{v_t} \right)^\eta \left( \frac{u_{k,t}}{u_t} \right)^{1-\eta} \quad (1)$$

Where  $\phi_{k,t}$  is the industry-level matching efficiency, which in this paper is constant  $\mu$ .  $\bar{\phi}_t$  is the economy-wide matching efficiency, which is also constant at  $\mu$ .  $v_t$  and  $u_t$  are the aggregate number of vacancies and unemployed workers.

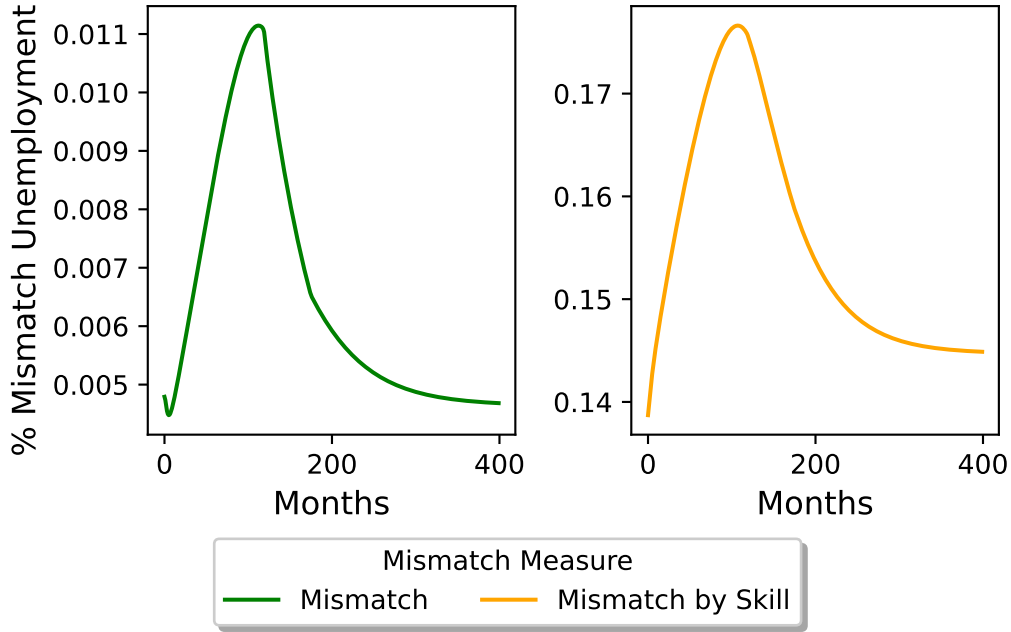
[Şahin et al. \(2014-11-01\)](#) found that this index could explain only one-third of the rise in unemployment during the great recession. I calculate this index for the model economy in response to reallocation between the two high-skill specificity industries and plot it in [Figure 4](#). In the left hand side figure is the dynamics of the [Şahin et al. \(2014-11-01\)](#) index. It does rise in response to the shock however it remains small peaking at just over 1% of total unemployment being ascribed to mismatch. As the shock generates additional unemployment of around 17% the index correctly shows that the unemployment generated is not due to mismatch across industries.

The mismatch in this model is between workers of different skill levels within an industry. Thus the correct index to use is one that accounts for mismatch across skills.

$$\mathcal{M}_t = 1 - \sum_k \sum_s \left( \frac{\phi_{k,s,t}}{\bar{\phi}_t} \right) \left( \frac{v_{k,s,t}}{v_t} \right)^\eta \left( \frac{u_{k,s,t}}{u_t} \right)^{1-\eta} \quad (2)$$

I plot the dynamics of this index in response to the shock in the right hand side of Figure 4. Here the measure increases by much more than in the previous case, peaking at almost 18% of total unemployment. However given it starts from a much higher level of 14% this rise still doesn't fully explain the rise in unemployment.

Figure 4: Comparison of Employment depending of  $\eta$



All lines are in response to productivity shock to the two high-specificity industries. One hit positively and one negatively. The mismatch indexes are as described in the text

## 5 Inefficiency and Optimal Policy

The model features an externality different from the usual search externalities of thick markets and congestion. Firms fail to fully internalise the value of

the human capital that a worker will accumulate during a match when they post a vacancy for a unskilled worker. This is because if the worker gains skill while matched to a firm and then the match breaks up, the firm will not gain the additional surplus that the firm that next matches with that worker will receive due to the higher skill of the worker. This can't be contracted on as the next firm that hires a worker is not a party to the negotiations between the original firm and the worker. Thus they cannot be bound to compensate the original firm for the value of the skill.

I restrict the planner to only be able to subsidise vacancy posting funded by lump sum taxes on the household. I impose this restriction as the fully unrestricted planner who sets both vacancies and wages can set wages infinitely high or low. This is because the profits or losses of the firm from those wages will cancel out in the budget constraint on the household. As the planner raises wages this raises the income of the representative household but the increase in wages is funded by lump sum taxation which reduces the available income of the household. I assume the subsidy is paid to the firm upon a match but not conditional on bargaining being successful as this avoids the complication of a wage premium in the first period of a match as in [Mortensen and Pissarides \(2003\)](#). This changes the free entry condition to

$$\kappa_{k,s} = q(\theta_{k,s})(J(k,s) + \lambda(k,s))$$

Where  $\lambda(k, s)$  is the subsidy for a vacancy in industry  $k$  for a worker of skill  $s$ . This allows the planner to pick the  $\theta_{k,s}$  but maintains Nash bargaining as the wage-setting mechanism.

Given these restrictions, the planner's problem is to choose subsidy levels  $\lambda(k, s)$  to maximise the utility of the economy.

$$\max_{\{\lambda(k,s)\}} \text{Consumption} + \text{Flow Utility from Unemployment} + \text{Utility from Discrete Choice}$$

Where consumption is total output minus the costs of vacancy posting. The total flow utility of unemployment is  $b$  the amount of utility each unemployed worker gets, times the number of unemployed workers. The utility from discrete choice comes from the taste shocks plus the utility costs of switching industries incurred.

I then consider four forms of subsidies. A subsidy that varies across both industries and skill levels, a subsidy that varies only across skills, a subsidy that varies only across industries and a subsidy that is constant across industries and skill levels. The motivation for considering the more restricted subsidies is that they may be more feasible to implement in practise. For instance, the skill level of a worker is not easily observable and firms and workers could potentially collude in reporting the wrong skill level to obtain a higher subsidy. On the other hand, firms often don't belong to a single clear industry and so may manipulate their assigned industry label in order



to qualify for subsidies.

In section 5 I list the optimal subsidies and the steady state unemployment rate for each subsidy. I list the unemployment rate rather than welfare as this is more representative of the size of the gains. In the model going from no unemployment to the decentralised steady state leads to a welfare gain of less than 100% which is implausibly low. As such utility gains from different subsidy regimes are small compared to the unemployment changes. Unsurprisingly the most flexible subsidy has the lowest unemployment rate which at 3.34% is 15% lower than without a subsidy. Allowing the subsidy to vary only across skill groups achieves a very similar level of unemployment, though the difference is more noticeable in utility terms. On the other hand, allowing the subsidy to vary across industries does not lead to lower unemployment compared to a completely flat subsidy. The reason can be seen by looking at the subsidy levels of the fully flexible subsidy. The subsidy is positive for the unskilled vacancies and negative for the skilled vacancies. Due to the externality, the planner wants to subsidise the vacancies for unskilled workers to increase the amount of skill being acquired in the economy. The vacancies for skilled workers are taxed as the externality is much smaller for these vacancies. Posting additional vacancies for skilled workers reduces the proportion of skilled workers who lose their skills while unemployed. However, as the chance of skill loss in unemployment is low the planner only wants to subsidise the vacancies for unskilled workers by a small amount. But subsidising the vacancies for unskilled workers already acts as a sub-

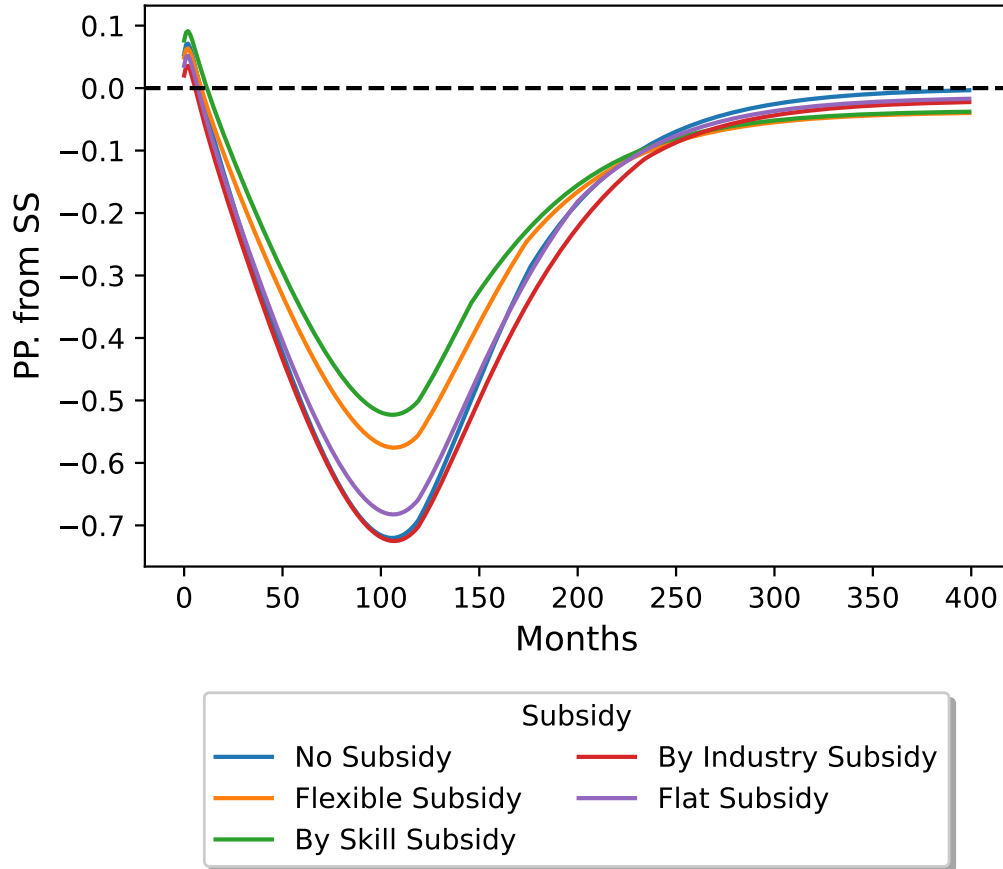
sidy due to the complementary in production. Therefore the planner finds it optimal to tax the vacancies for skilled workers to offset this implicit subsidy.

<i>Optimal Subsidies and Unemployment Rates</i>		
Subsidy	$\lambda(k, s)$	Unemployment Rate
Fully Flexible	$[\lambda_{h,0}, \lambda_{h,1}, \lambda_{l,0}, \lambda_{l,1}] = [0.085, 0.013, 0.72, -0.087]$	3.49%
Skill Varying	$[\lambda_0, \lambda_1] = [0.078, -0.040]$	3.49%
Industry Varying	$[\lambda_h, \lambda_l] = [0.049, 0.0037]$	3.78%
Flat	$\lambda = 0.024$	3.78%
No Subsidy	N/A	4.12%

I then consider the impact of the subsidy on the response of unemployment to the reallocation shock. I compare the response to the same shock as in the previous section but starting from the steady state with the respective subsidy. Despite it being the same shock, the long-run employment impact is not necessarily 0 as the subsidy will change both the initial and long run steady state.

What can be seen from the Figure 5 is that the fully flexible subsidy leads to a smaller transitory rise in unemployment, This is despite the shock leading to a long run rise in unemployment when fully flexible subsidies are in place. The reason for this is that the flexible subsidy leads to greater posting of vacancies for unskilled workers. Since a large proportion of the transitory unemployment is unskilled workers, this mitigates the rise. There

Figure 5: Dynamics Unemployment With Different Subsidies



All lines are in response to productivity shock to the two high-specificity industries. One hit positively and one negatively. I resolve the steady state for each subsidy. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state for the no subsidy case. For comparison I use the same shock in each case.

is also an indirect effect from as greater hiring of unskilled workers causes the absolute number of workers gaining skill in the growing industry to be higher. So the transition is quicker so the decline in the marginal product of unskilled workers smaller and recovers earlier further promoting vacancy

posting in this submarket.

For the same reasons, the subsidy that varies by skill achieves a sizable reduction in transitory unemployment. It does slightly better in unemployment terms than the flexible subsidy due to the subsidy on unskilled vacancies in the low specificity industries being higher so they absorb more of the workers leaving the shrinking industry along the transition. A fully flexible subsidy that responds to the shock would likely be able to further reduce the transitory unemployment. On the other hand, the flat and industry varying subsidies lead to only a small reduction in the transitory rise in unemployment. This is because of the small size of the subsidy which means it incentivises little additional vacancy posting for unskilled workers. So the path of marginal products of these workers is similar to case without a subsidy leading to similar unemployment dynamics.

This paper argues that the reallocation of labour demand can have consequences for aggregate unemployment. This result comes from allowing for a realistic structure of substitutability between workers of different skill levels. When different skill levels are not perfect substitutes the demand for unskilled workers will be lower in the transition than in steady state. As this is also where there is a greater supply of workers during the transition this can lead to transitory unemployment.

That substitutability between workers is important for the response to shocks may also apply to other cases. Many modern macro models assume the marginal product of a match is independent of the distribution of matches

in the economy for tractability. So there is a need to better understand when this powerful assumption is a good approximation to the real world.

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