The Role of Demographics in Cross-Cohort Lifetime Income Differences

Fergal Hanks & Laura Murphy

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1 Introduction

Guvenen et al. (2021) have found that American men's lifetime median incomes have followed a hump-shaped pattern: rising with each cohort entering the labour market from the late 1950s until the 1970s, and subsequently falling. As shown in Figure 1, population demographics have traced a similar pattern, lagging the path of median lifetime wages by $10 - 15$ years. We will explore how firm organization and promotion structures, in tandem with declining cohort size, help explain the fall in median male lifetime incomes.

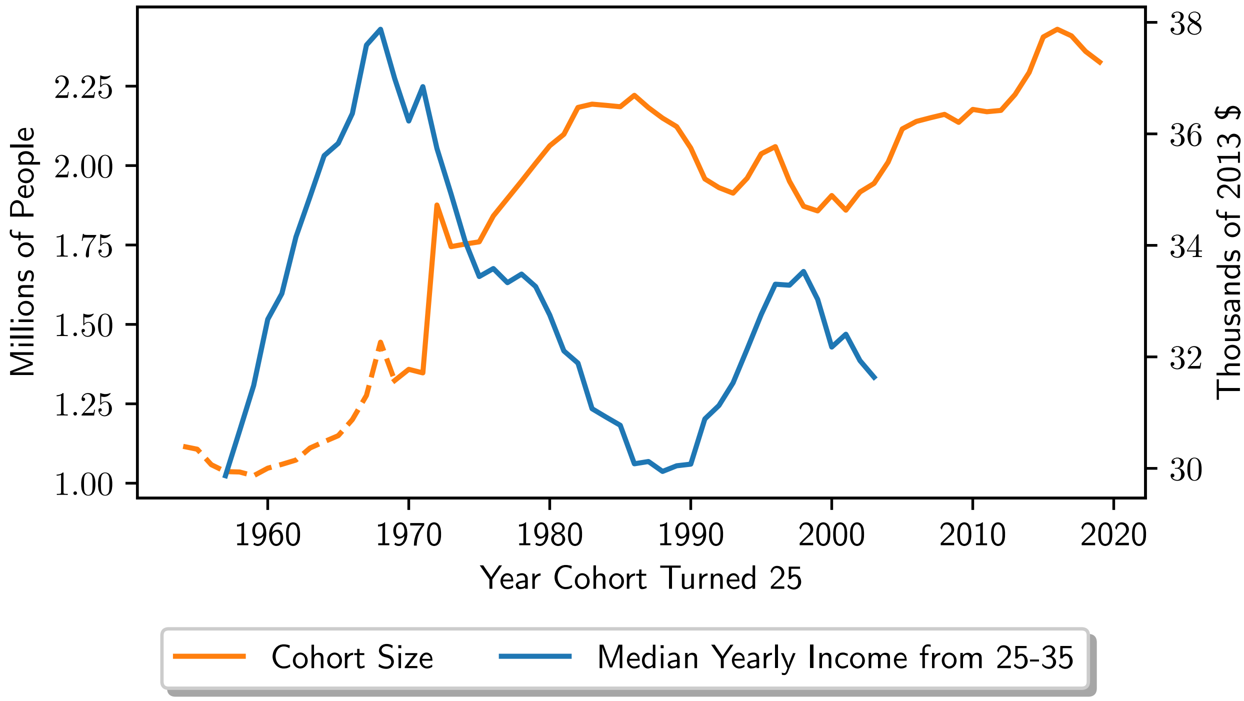

Figure 1: Earnings and cohort dynamics

The path of an individual's wage over their lifetime is heavily linked to career progression **To Cite**. Career progression is often synonymous with taking on more management or supervisory responsibility.As a worker climbs the career ladder, their time shifts away from tasks involved directly with production towards production-enhancing tasks, such as management or training. The availability of these tasks, and the potential for higher wages associated with them, will increase with the number of people to manage. Thus, there is a link between population and wage dynamics across cohorts. For those born prior to large generations, supervisory roles may be easier to obtain, as there are both more young people to manage and relatively few same-age competitors for these roles. In contrast, those born within or after large generations, may face reduced promotion opportunities and thus lower lifetime income. Being born within a large cohort may reduce management opportunities as more individuals are seeking the same, limited, supervisory roles. Being born after a large cohort may further reduce promotion opportunities as prior generations may 'congest' limited management roles.

An empirical literature documents the existence of career spillovers between workers in nearby rungs of firm hierarchies. Bianchi et al. (2021) uses the 2011 Italian pension reform, which delayed the retirement age by three years, to explore the effects of delayed retirement on the promotion opportunities and wage growth of younger workers. They find that both wage growth and promotion probabilities are harmed by retirement delays. These effects are concentrated amongst older workers, i.e. those whose seniority is just below the lingering retirement-aged workers. Symmetrically, Jäger & Heining (2019) show that following unexpected worker deaths in Germany on average remaining workers' wages and retention probabilities increase. This effect is more pronounced for workers in the same occupation, or those who are most likely to replace the deceased.

For promotion opportunities to impact lifetime earnings requires that compensation is at least partially driven by luck rather than productivity alone. Another strand of empirical literature documents how lifetime earnings of workers are, at least partially, determined by luck. For instance, Oreopoulos et al. (2012) shows that workers who begin their careers in recessions are less likely to do well in the long run. Other papers showing the relationship between career outcomes and luck include Kahn (2010), von Wachter & Bender (2006), and Lazear et al. (2016). Suandi (2021) provides empirical evidence supporting our mechanism directly, that luck in promotions is a key determinant to lifetime earnings. Using U.S. Navy personnel data from WWII, Suandi (2021) shows how luck in early promotions shapes future career outcomes. He exploits exogenous promotions associated with successful submarines to look at long-term welfare of sailors who survived the war, finding that promoted sailors tend to live longer and die in wealthier ZIP codes.

The mechanism we outline above links promotion opportunities to population dynamics. While Guvenen et al. (2021) do not explore mechanisms for the pattern of falling lifetime median wages, they do note that a state-level panel regression indicates that median lifetime income seems closely linked to entry cohort size, which supports our hypothesis. They suggest this is consistent with models of imperfect substitutability of labour across age groups, such as Card & Lemieux (2001), or more recently Jeong et al. (2015). This argument for large cohorts driving down wages relies on the decreasing marginal productivity of a specific age-type of worker; thus when a firm must hire a big cohort, wages fall for markets to clear. This mechanism implies that as cohort sizes fall, newer generations should have relatively higher wages. This is inconsistent with the lag shown in our Figure 1, where earnings and cohort sizes were rising in tandem during the late 1950s through to the early 1970s. Thus, we need a different framework to understand why wages continue to fall despite falling cohort sizes. Our proposed mechanism looks at how adjacent cohort sizes affect the promotion path opportunities, and thus lifetime wages, for a worker of a given generation.

The rest of the paper is structured as follows. Section 2 presents a simple model highlighting how limited promotion prospects coupled with hump-shaped population cohorts can help explain falling lifetime incomes. Section **??** extends this simple model by allowing for firms' to endogenously change their organizational structure in response to demographic changes.

2 Exogenous hierarchy

To begin, we set up a simple model with two key features. The first is a representative firm with an organizational structure which consists of multiple promotion rungs, where higher rungs correspond to higher wage jobs. The second is overlapping generations of workers with potentially different sizes. With this framework we can test how changing the distribution of the population across the generations (i.e. more young vs. more old) changes the patterns of lifetime wages for those in each cohort.

2.1 Model Set-Up

To simplify the model we assume a firm with fixed shares of production along each rung of the promotion ladder. In other words, there is a Leontif production function across promotion rungs. This assumption will be relaxed in Section ??. There are I rungs, each indexed by $i \in \{1, \cdots, I\}$. The production share of each rung is denoted γ_i, with $\boldsymbol{\gamma}$ denoting the vector of the shares of all rungs.

There are G generations, indexed by $g \in \{1, \cdots, G\}$. Each period the eldest generation of the previous period dies, and a new generation is born. Both births and deaths can create promotion opportunities. Births do so by increasing the total size of the population, and thus the number of available positions at each rung to keep rung proportions the same. Deaths leave vacancies at all rungs. The firm will fill open job opportunities in a given rung by hiring at random from the rung below. All members of the newly born generation are hired into the bottom rung of firm's promotion ladder, and are unable to be promoted in the period they are born. For simplicity, in this section we will assume that the number of generations and promotion steps are equal (i.e. $I = G$).

We are interested in how the organizational structure of the firm interacts with population demographics to change lifetime income. To determine lifetime income of a worker in this model we need to solve for the promotion probabilities of each rung. With these promotion probabilities, average lifetime income for a worker in a given cohort can be calculated as their wage at a given rung times their average time at that rung over their lifetime. Section 2.2 calculates these promotion probabilities in steady-state, i.e. for a flat age demographics where all cohorts are equal in size. Section 2.3 will calculate these promotion probabilities for non-equal generation sizes, and show how lifetime income differs when cohorts follow a hump-shaped population pattern like the one described in the introduction.

2.2 Steady-State

As mentioned above, the key object of interest is the promotion probabilities, which determine the lifetime wages of each cohort. In this section we will solve for these probabilities in a steady-state environment. Here steady-state is where all the generations are of equal size. ω_g denotes the mass in generation g, where $\boldsymbol{\omega}$ is the $G \times 1$ vector of these shares. We normalise the total population in a steady-state with equal cohort sizes to 1, thus in steady-state $\omega_g = \frac{1}{G} \, \forall g$.

p_i represents the probability of promotion from rung i to $i+1$. This can be otherwise stated as the share of those in rung i with $g < G$ who are in rung $i+1$ in the next period. The promotion probability is random within a rung, and thus is the same across cohorts at the same promotion level. We denote $\boldsymbol{p}$ the $I-1 \times 1$ vector of promotion probabilities. It is also useful for us to represent these promotion probabilities as a transition matrix, $\boldsymbol{P}$, faced by agents in the economy:

$$\boldsymbol{P} = \begin{bmatrix} 1-p_1 & 0 & \cdots & 0 & 0 \\ p_1 & 1-p_2 & \cdots & 0 & 0 \\ 0 & p_2 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1-p_{I-1} & 0 \\ 0 & 0 & \cdots & p_{I-1} & 1 \end{bmatrix}$$

This transition matrix is $I \times I$, where columns denote the rung you are coming from, and rows the you are going to. By assumption, workers can move up by at most one rung per period and can't be demoted.

In solving this model we take $I, G, \boldsymbol{\gamma}$ as given and are interested in solving for the endogenous promotion probabilities, $\boldsymbol{p}$. To do so we can write $\boldsymbol{\gamma}$ in terms of the promotion probabilities. Promotions determine the share of each generation in a given rung, which must add up to the share of the overall workforce in that rung. This is summarized in equation 1.

$$\begin{aligned} \boldsymbol{\gamma} &= \frac{1}{G}\left(I + \boldsymbol{P} + \boldsymbol{P}^2 + \cdots + \boldsymbol{P}^{G-1}\right)\mathbb{1}_1 \\ &= \frac{1}{G}(I-\boldsymbol{P})^{-1}(I-P^G)\mathbb{1}_1 \end{aligned} \tag{1}$$

Where $\mathbb{1}_j$, is defined to be a $I \times 1$ vector with a 1 in the j_{th} position and zeros elsewhere, where the first position is indexed by a 1. Here we will have a system of I equations and $I-1$ unknowns, where one of our equations becomes redundant.

With these promotion probabilities in hand, we can also calculate the distribution of cohorts over rungs. Let $\alpha_{i,g}$ denote the share of generation g employed in rung i, where $\boldsymbol{\alpha}$ is the $I \times G$ matrix of all these shares. Note that $\boldsymbol{\alpha}$ will be upper-triangular, since no cohort can be in a rung greater than their age. Equation 2 summarizes how to calculate each column of $\boldsymbol{\alpha}$ using $\boldsymbol{P}$.

$$\boldsymbol{\alpha}\mathbb{1}_g = \boldsymbol{P}^{G-1}\mathbb{1}_1 \tag{2}$$

2.3 Dynamics

This steady-state solution abstracts from population dynamics of the kind highlighted in the introduction. This section outlines how to solve the model out of steady state. In particular where $\omega_g \neq \frac{1}{G}\forall g$. All model objects will now also be denoted with a subscript t for time.

Let $c_{i,t}$ be the population mass at the beginning of period t at rung i before the new generation arrives/promotions occur, but after the old generation has retired at the end of $t-1$. We define $m_{i,t}$ as the moves into rung i, in terms of mass, where by definition $m_{1,t}$ is the mass of the newly born generation. We further define the total mass of the population at time t as M_t. Then, for each rung $i \in \{1, \cdots, I\}$ the following is true:

$$M_t\gamma_i = c_{i,t} - m_{i+1,t} + m_{i,t}$$

where $m_{I+1,t} = 0$ by construction.

In vector for this becomes:

$$M_t \boldsymbol{\gamma} = \boldsymbol{C}_t - \boldsymbol{S}_1 \boldsymbol{m}_t + \boldsymbol{I}_I \boldsymbol{m}_t \tag{3}$$

Where $\boldsymbol{S}_1$ with ones on its first super-diagonal and zeros elsewhere, and $\boldsymbol{I}_I$ is the $I \times I$ identity matrix. Then, the probability of moving from rung i to $i+1$ in period t will be $p_{i,t} = \frac{m_{i+1,t}}{c_i,t}$. The definitions of $\boldsymbol{p}_t$ and $\boldsymbol{P}_t$ are analogous to those in Section 2.2, but with the addition of time subscripts on each interior term. As in equation 1 in Section 2.2, the above equation 3 can be used to solve for the promotion probabilities.

With the promotion probabilities, we can calculate the distribution of generations across rungs, $\boldsymbol{\alpha}_t$. A given column of $\boldsymbol{\alpha}_t$ can be written as:

$$\boldsymbol{\alpha} \mathbb{1}_g = \prod_{j=0}^{g-1} \boldsymbol{P}_{t-g} \mathbb{1}_1 \tag{4}$$

This construction is intuitive in that the a given column of $\boldsymbol{\alpha}_t$ is the distribution of that generation across the rungs. Thus it depends on the history of promotion probabilities that generation has faced, represented by history of promotion matrices it has faced.

Average rung ($\bar{r}$) of a generation that retires at the end of period T can be written as follows:

$$\begin{aligned}
\bar{r} =& \frac{1}{G} \sum_{g=1}^{G} \mathbf{1}^\intercal \boldsymbol{\alpha}_{T-G+g} \mathbf{1}_g \\
=& \frac{1}{G} \left(\mathbf{1}^\intercal \mathbf{1}_1 \mathbf{1}_1^\intercal \mathbf{1}_1 + \mathbf{1}^\intercal \boldsymbol{P}_{T-G+1} \mathbf{1}_1 \mathbf{1}_2^\intercal \mathbf{1}_1 + \mathbf{1}^\intercal \boldsymbol{P}_{T-G+2} \boldsymbol{P}_{T-G+1} \mathbf{1}_1 \mathbf{1}_3^\intercal \mathbf{1}_2 + \dots \right) \\
=& \frac{1}{G} \left(\mathbf{1}^\intercal \mathbf{1}_1 + \mathbf{1}^\intercal \boldsymbol{P}_{T-G+1} \mathbf{1}_1 + \mathbf{1}^\intercal \boldsymbol{P}_{T-G+2} \boldsymbol{P}_{T-G+1} \mathbf{1}_1 + \dots \right) \\
=& \frac{1}{G} \mathbf{1}^\intercal \left(1 + \boldsymbol{P}_{T-G+1} + \boldsymbol{P}_{T-G+2} \boldsymbol{P}_{T-G+1} + \dots \right) \mathbf{1}_1
\end{aligned} \tag{5}$$

2.4 Three-rung example

To highlight how population dynamics impact lifetime wages in our model we construct a three-rung version. We then shock one generation to be above steady-state level, and see how earnings differ for this and the surrounding cohorts. Parameter values for this experiment can be found in Table 1. In particular, the parameters we need to set are the number of rungs/generations, the shape of the promotions structure ($\boldsymbol{\gamma}$), and the size of the shock (ϵ).

After calculating the promotion probabilities at each time period, we translate this into average lifetime rung, capturing how long on average a given cohort spends at each rung. Higher average lifetime rungs correspond to higher average lifetime wages. The results can be seen in Figure 2. Those born before the shocked generation experience higher wages relative to steady-state, as they are promoted more quickly than they would be in steady-state. This is because the new large generation must work in the bottom rung in their youth, and so the elder generations are promoted quickly to ensure the correct share of workers in management. These promotion probabilities are shown in Figure 3.

The shocked generation receives lifetime earnings substantially below steady-state levels. This is explained by depressed promotion probabilities that occur for the entirety of their middle and old ages.

Parameter	Description	Model	Value
$G = I$	Number of rungs/generations	Promotions	3
γ_1	Rung 1 share labour	Promotions	$\frac{1}{2}$
γ_2	Rung 2 share labour	Promotions	$\frac{1}{3}$
γ_3	Rung 3 share labour	Promotions	$\frac{1}{6}$
σ	Elasticity of Substitution between Young and Old	CES	0.7
ρ	Elasticity of Substitution between Middle Aged and Young or Old	CES	0.5
α_x	CES coefficient on Middle aged	CES	$\frac{1}{3}$
α_m	CES coefficient on Young in lower nest	CES	$\frac{1}{2}$
ϵ	Cohort size shock	CES and Promotions	0.01

Table 1: Calibration

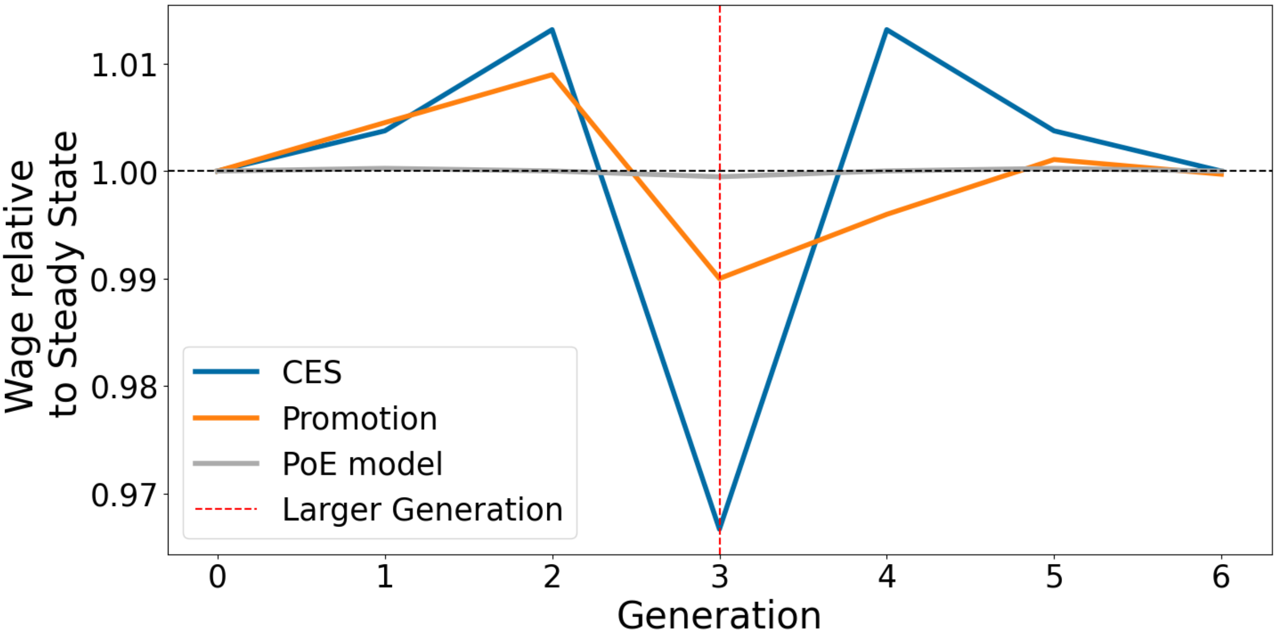

Figure 2: Shocking a generation size: three-rung example

The generations born after the large cohort are also affected though less significantly than the larger generation As the large cohort ages, promotion probabilities go down, since the top rungs are quickly filled with this larger cohort. After they die, these promotion probabilities rise once again due to the large number of vacancies in higher rungs created. For the generation 4 the first effect dominates while for the 5th generation the second effect dominates and they receive lifetime incomes higher than in steady state The quick recovery in lifetime incomes is partially a function of the short-livedness of the large generation, due to the three-period assumption. If we were to extend this to a 30-rung model, while keeping the generation shock the same, more cohorts directly following the large generation would also see depressed lifetime earnings. This is because they will not live long during the probability probability 'rebound' associated with the retirement of the large generation. This is highlighted in Figure 4.[1]

[1] Details of the construction of Figure 4 can be found in Appendix ??.

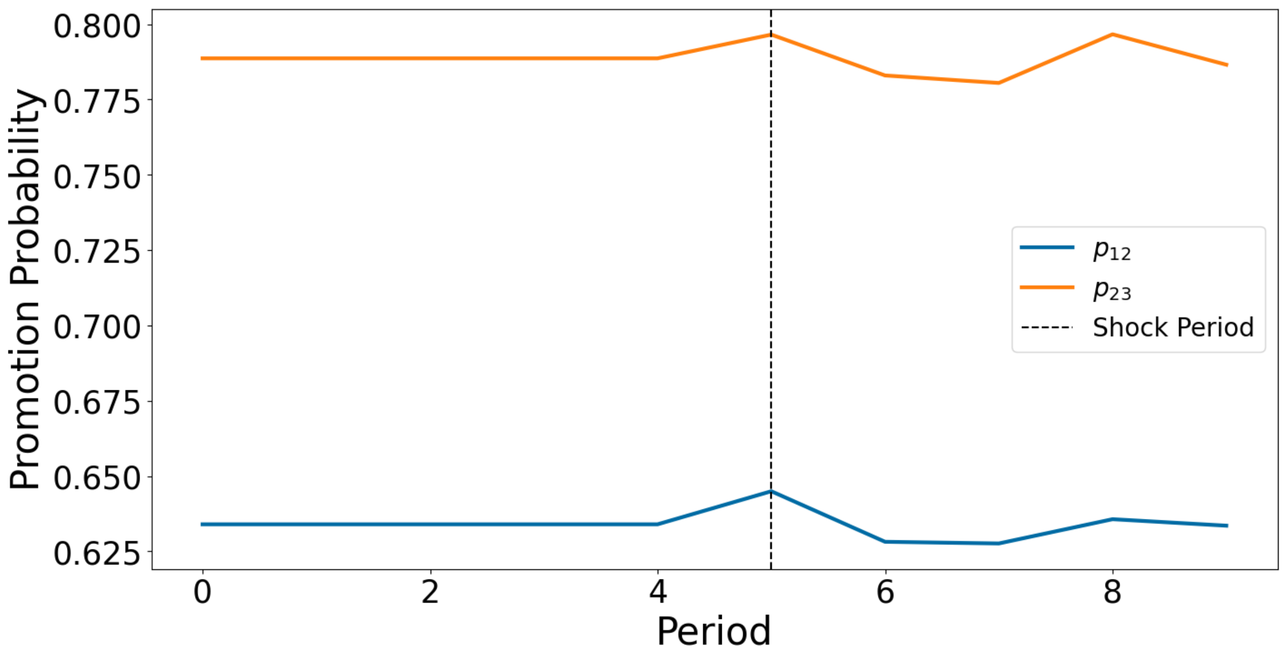

Figure 3: Promotion Probabilities

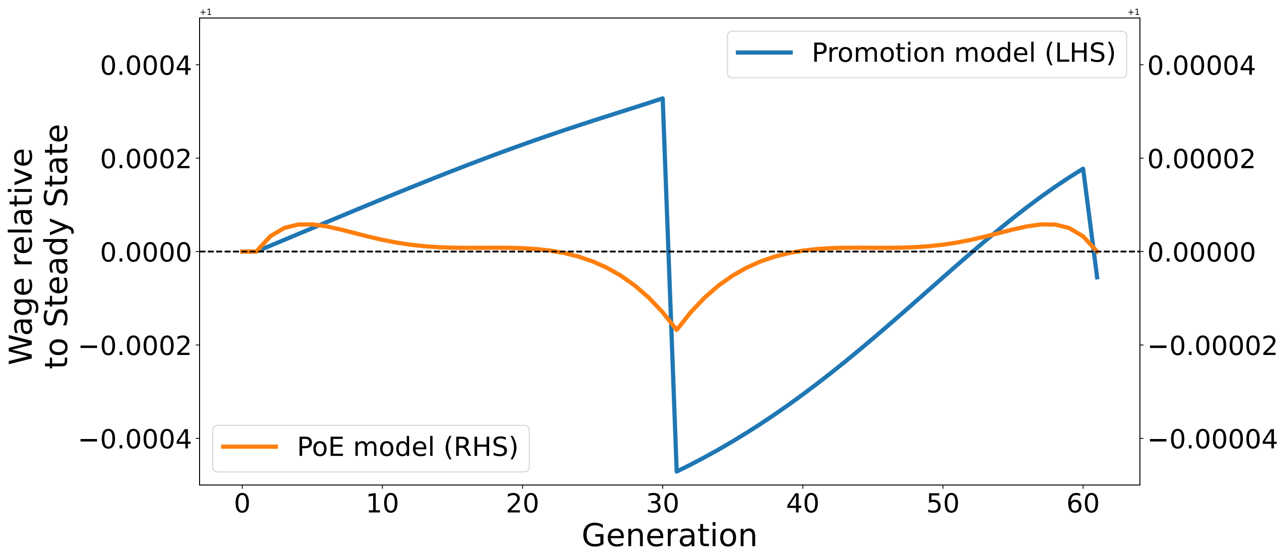

Figure 4: Shocking a generation size: thirty-rung example

Our simple model is able to capture the intuition and external empirical evidence discussed in the introduction: that cohorts born before large generations will receive higher wages via promotion, and those born in or after will be harmed by congestion effects. The authors who established our motivating fact suggested lifetime earnings patterns may be explained by a CES with imperfect substitution of labour across generations. In the following two subsections, sections 2.4.1 and 2.4.2, we will compare the performance of our promotions model with two versions of a CES model.

2.4.1 Comparison with Simple CES

As discussed in Section 1, the literature has argued that the negative relationship between wages and cohort size is consistent with a model where cohorts act as imperfect substitutes to production (for instance, via a CES production function). Before showing the results of an experiment in our promotions model, we will also set up a simple CES framework to show how the two behave differently.

Consider a CES model that combines the labour of three generations of workers, M,X,B from young to old.

$$Y = \left(\alpha_x^{\frac{1}{\rho}} X^{\frac{\rho-1}{\rho}} + (1-\alpha_x)^{\frac{1}{\rho}} \left(\alpha_m^{\frac{1}{\sigma}} M^{\frac{\sigma-1}{\sigma}} + (1-\alpha_m)^{\frac{1}{\sigma}} B^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\rho-1}{\rho}\frac{\sigma}{\sigma-1}} \right)^{\frac{\rho}{\rho-1}} \tag{6}$$

This nested structure allows for a different degree of substitutability between young and old than between young and middle aged or old and middle aged. However, the substitutability of the middle aged for old is constraint to be the same as for young.

In perfectly competitive markets, wages for each cohort will be equal to their marginal product:

$$W_X = \left(\alpha_x^{\frac{1}{\rho}} + (1-\alpha_x)^{\frac{1}{\rho}} \left(\alpha_m^{\frac{1}{\sigma}} \left(\frac{M}{X}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_m)^{\frac{1}{\sigma}} \left(\frac{B}{X}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\rho-1}{\rho}\frac{\sigma}{\sigma-1}} \right)^{\frac{1}{\rho-1}} \alpha_x^{\frac{1}{\rho}}$$

$$W_M = \left(\alpha_x^{\frac{1}{\rho}} \left(\frac{X}{\left(\alpha_m^{\frac{1}{\sigma}} M^{\frac{\sigma-1}{\sigma}} + (1-\alpha_m)^{\frac{1}{\sigma}} B^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}} \right)^{\frac{\rho-1}{\rho}} + (1-\alpha_x)^{\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \cdots$$

$$\cdots \times \left(\alpha_m^{\frac{1}{\sigma}} + (1-\alpha_m)^{\frac{1}{\sigma}} \left(\frac{B}{M}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha_m^{\frac{1}{\sigma}} (1-\alpha_x)^{\frac{1}{\rho}}$$

$$W_B = \left(\alpha_x^{\frac{1}{\rho}} \left(\frac{X}{\left(\alpha_m^{\frac{1}{\sigma}} M^{\frac{\sigma-1}{\sigma}} + (1-\alpha_m)^{\frac{1}{\sigma}} B^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}} \right)^{\frac{\rho-1}{\rho}} + (1-\alpha_x)^{\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \cdots$$

$$\cdots \times \left(\alpha_m^{\frac{1}{\sigma}} \left(\frac{M}{B}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_m)^{\frac{1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1-\alpha_m)^{\frac{1}{\sigma}} (1-\alpha_x)^{\frac{1}{\rho}}$$

As can be seen in the wage equations above, a given cohort's wages are decreasing in its size, but increasing in the size of the other cohorts. The size of this effect is depends on the degree of substitutability between the cohorts. The more substitutable, the lower the impact will be on other cohort's wages.

As with our simple promotions model, we shock one generation to be above steady-state level, and see how lifetime earnings differ for this and the surrounding cohorts as a result. Parameter values for this experiment can be found in Table 1.

The results of this experiment can be seen in Figure 2. While CES is able to generate a positive effect on lifetime earnings for those born before the large generation, it is unable to account for normal or worse wages in the cohorts born after. As highlighted in the wage equations above, this is because one large generation benefits all other generations coexisting with them by driving up their marginal product. This increase in the marginal productivity occurs regardless of being born before or after the large generation. Conversely, our promotions model is able to generate asymmetry.

2.4.2 Comparison with Price of Experience Model

Another prominent paper using CES is Jeong et al. (2015). This model uses two inputs to production: labour (L_t) and experience (E_t). Workers of different ages, education, gender and years of work are able to supply these inputs differentially.

The aggregate production function is:

$$Y_t = (L_t^{\mu} + \delta E_t^{\mu})^{\frac{1}{\mu}}$$

Where L_t and E_t are aggregate labour and experience, respectively, summing over the supplies of labour and experience of all generations. In particular, older workers, who also have more labour market experience, supply relative more 'experience' units whereas younger workers who have no experience supply mainly labour. The supplies of labour and experience as a function of age in our comparison are summarized in Figure 5.[2]

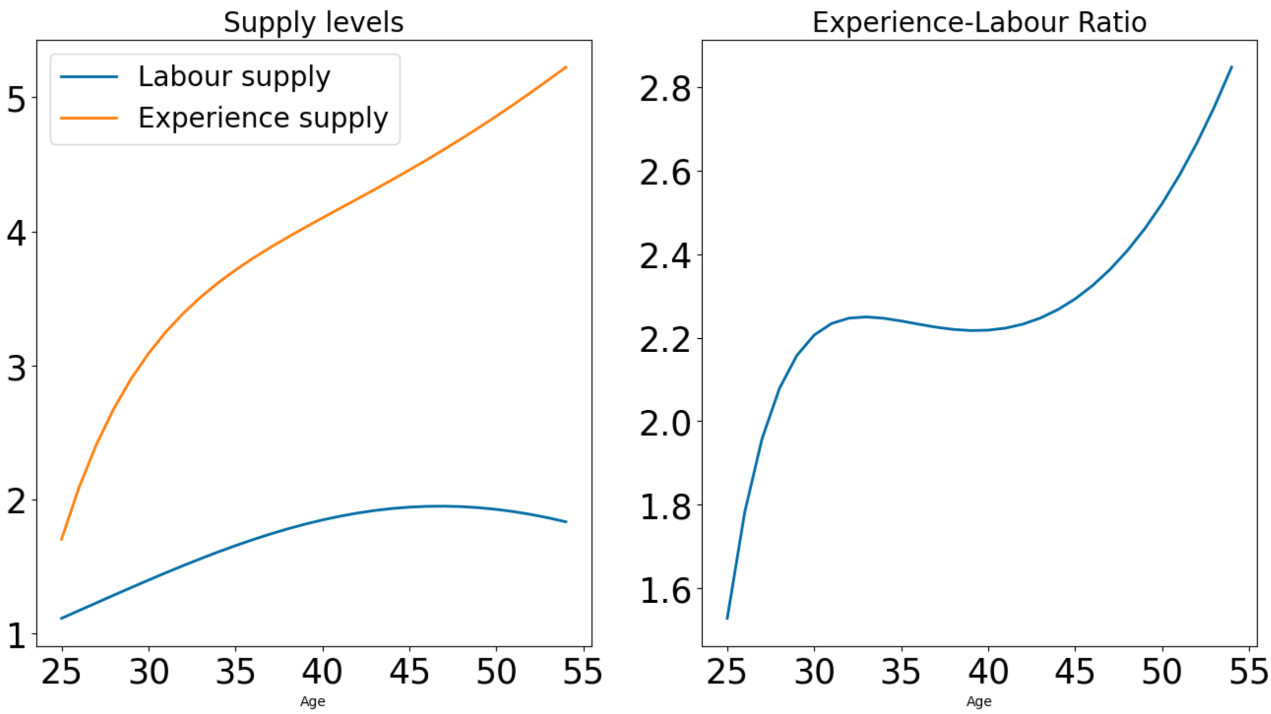

Figure 5: Supply Schedules of Labour and Experience

[2] Jeong et al. (2015) estimate the labour and experience supply schedules as a function of age and time worked. We assume our population to be made up of college-educated men of various ages, and calculate the labour and experience supply schedules using the parameters estimated for this demographic group. The paper assumes that labour supplied is determined directly by age, whereas total experience supplied is comprised of a level of experience as a function of total years worked and a return to experience determined by age. Given an individual's age, j_{it}, and years of experience, e_{it}, Jeong et al. (2015) estimate the following supply schedule parameters:

$$\begin{aligned}
\hat{\ell}_{it} &= \exp\left(\lambda_{0,\ell} + \lambda_{1,\ell} j_{it}^2 + \lambda_{2,\ell} j_{it}^2\right) \\
\hat{e}_{it} &= \underbrace{\exp\left(\lambda_{0,e} + \lambda_{1,e} j_{it}^2 + \lambda_{2,e} j_{it}^2\right)}_{\text{Return to Experience}} \underbrace{\left(e_{it} + \theta_1 e_{it}^2 + e_{it}^3 + e_{it}^4\right)}_{\text{Level of Experience}} \\
&= \hat{\ell}_{it} \exp\left(\lambda_{0,\frac{e}{\ell}} + \lambda_{1,\frac{e}{\ell}} j_{it}^2 + \lambda_{2,\frac{e}{\ell}} j_{it}^2\right) \left(e_{it} + \theta_1 e_{it}^2 + e_{it}^3 + e_{it}^4\right)
\end{aligned}$$

The parameters for the experience supply schedule can be normalized by the labour supply schedule parameters. It is these re-normalized parameters we report in Table **to add**. We assume all workers work continuously beginning at the age 22, thus $e_{it} = a_{it} - 22$.

These age-supply schedules allow for more complex patterns of substitution between age groups than the simple CES model. As noted in Appendix A9 of Jeong et al. (2015), the effect of an increase in the size of one cohort on the wage of another is governed by the relative supply of labour and experience of both groups in aggregate. This is because the returns to labour and experience depend directly on the aggregate labour–experience ratio:

$$R_{L,t} = \left(1 + \delta \left(\frac{E_t}{L_t}\right)^{\mu}\right)^{\frac{1-\mu}{\mu}} \qquad\qquad R_{E,t} = \delta \left(\left(\frac{E_t}{L_t}\right)^{-\mu} + \delta\right)^{\frac{1-\mu}{\mu}}$$

If both cohorts supply relatively more of the same input relative to the aggregate ratio, then they are substitutable. In this case an increase in size of one age group will lower the wage of the other. On the other hand if a cohort's labour to experience ratio is above the aggregate ratio, and the other's is below, then the two age groups are complements. Here, an increase in size of one age group will raise the wage of the other. Due to this dependence on the aggregate labour–experience ratio, the size of a third cohort will affect the wage derivative of the first cohort with respect to the size of the second.Even the sign of this derivative is dependent of the distribution of age group sizes.

To compare with our promotions model, suppose there are three generations alive simultaneously, indexed by $g \in \{1, 2, 3\}$. We take the ages of the three generations to be $25, 40, 55$ to best cover the prime age period. We also simplify the model for this comparison by assuming all workers are college educated men who work uninterruptedly from the age of 22 onwards. This implies the years of work of each generation are $3, 18, 28$.

The results of this experiment can also be seen in Figure 2. The three-cohort price of experience experiment shares some qualitative similarities with the simple CES model results. First, the response of the generations before and after the shocked cohort are close to symmetric. This is due to the symmetry of signs of the cross derivate of wages and cohort size. So, a larger aged 25 group will have the same signed effect on the wage of the aged 40 group as a large aged 40 group on aged 25 wages. Second, the lifetime wages of all non-shocked generations are higher than steady state. However, for the price of experience model this is a result of our choice of cohort ages. In particular, the large age gaps between the three cohorts give them very different labour–experience ratios, making them complements in production. If we were to increase the number of age groups between 25 and 55, the groups close to the shocked cohort will behave as complements, and their lifetime wages will go down. This can be seen in the 30-generation example in Figure 4. So both the generation above and the one below a larger generation will experience lower lifetime wages. However, these experiments show that these CES-based models are unable to generate the asymmetry in the effect of a large cohort, which our promotions model succeeds in doing.

3 Promotions model & Lifetime Income Dynamics

In this section we run an experiment using the actual population path to see whether our model can account for the lifetime earnings dynamics documented by Guvenen et al. (2021) in Figure 1. To do so we will need to choose a richer calibration than what we used above to highlight the model mechanisms. For the population data we use age by sex data from the National Cancer Institute's Surveillance Epidemiology and End Results Program (SEER) to calculate the size of annual male cohorts. Note that in this section we denote all cohorts

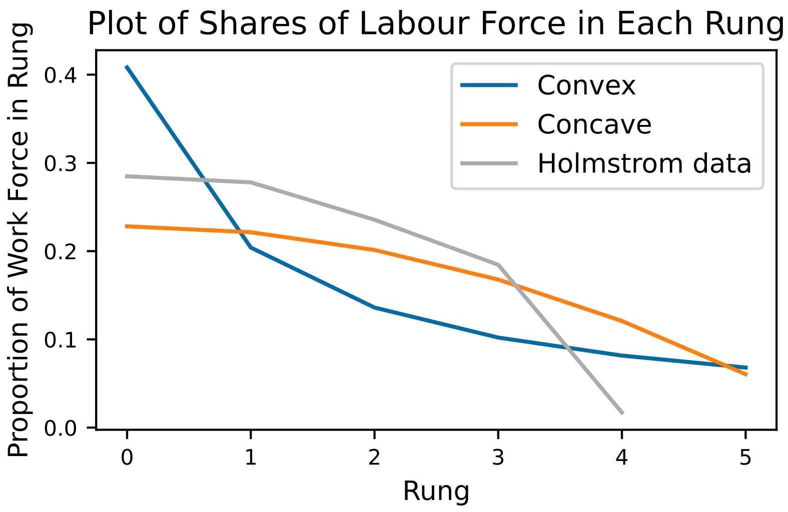

Figure 6: γ distribution from Baker et al. (1994)

by their year of entry into the labour market (i.e. the year they turned 25), rather than their birth year. with the number of rungs chosen to match Baker et al. (1994).

The SEER dataset starts in 1969, so we interpolate back from cohorts over the age of 25 in 1969 to predict the missing cohort sizes. To do this we adjust each cohort's 1969 population by the average relative difference between cohort size at 25 and 50 for all cohorts who turn 25 in 1969 or after. To estimate the model, we begin the economy in steady-state where all preceding cohorts are equal in size to the 1969 cohort. We then feed the labour-force entry size of cohorts as a time series. This time series of cohort sizes can be seen in Figure 1.

In addition to the population path, we need to calibrate the number of rungs as well as their sizes. To do so, we make use of the work of Baker et al. (1994), who use decades of a firm's personnel data to identify promotion rungs within the firm. In addition to identifying these rungs, they also estimate their sizes sizes and average wages at each rung. Based on their work, we set the number of rungs to five ($I = 5$)), and set the distribution across rungs as pictured in Figure 6. We also use their work to translate rung sizes to wages, calculating average lifetime wages as the time spent in each rung times the average wage associated witht hat rung from Baker et al. (1994). Lastly, we assume that generations work for 30 years, from ages 25–55 ($G = 30$).

The results for average lifetime wages across cohorts can be seen in Figure 7. The average wage features a hump shape, with those who turned 25 in the early 1970s achieving the highest lifetime earnings. These results match the empirical evidence from Guvenen et al. (2021) well, with the decline after falling much faster and below the level of the earliest cohorts.

These results are also fairly robust to the parameterization of γ. In Figure 8 we compare the results using the same population path but with different alternative shapes of the rung distribution. The alternative shapes of the rung distribution we try can be viewed in Figure 6. The overall results still hold.

However, this exercise assumes that the structure of the firm's hierarchy is invariant to the population distribution, which may be unreasonable. If workers improve their management skills with experience as the workforce ages there will be a greater supply of management skill. Firms may react by increasing the size of the highest rungs which could drive down their value, and by extension the wages paid to them. This endogenous reaction would then dampen the mechanical force on earnings highlighted in this section. In our next section,

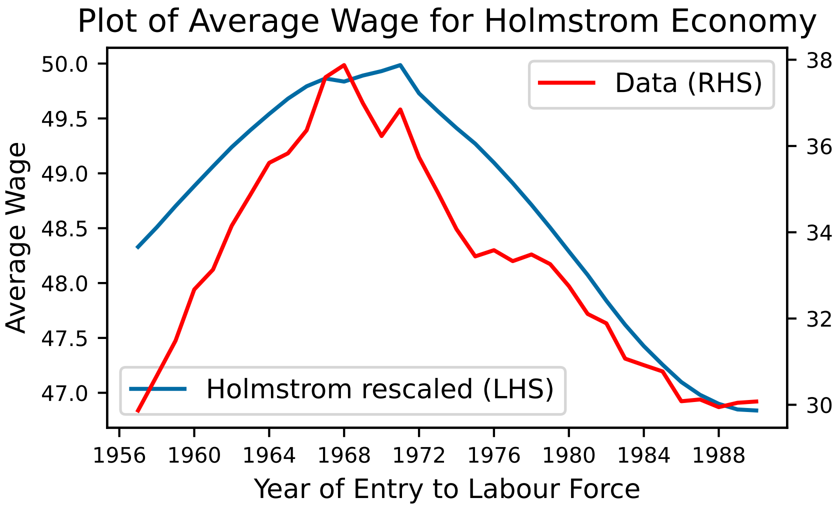

Figure 7: Promotions Model Lifetime Incomes with True Population Path

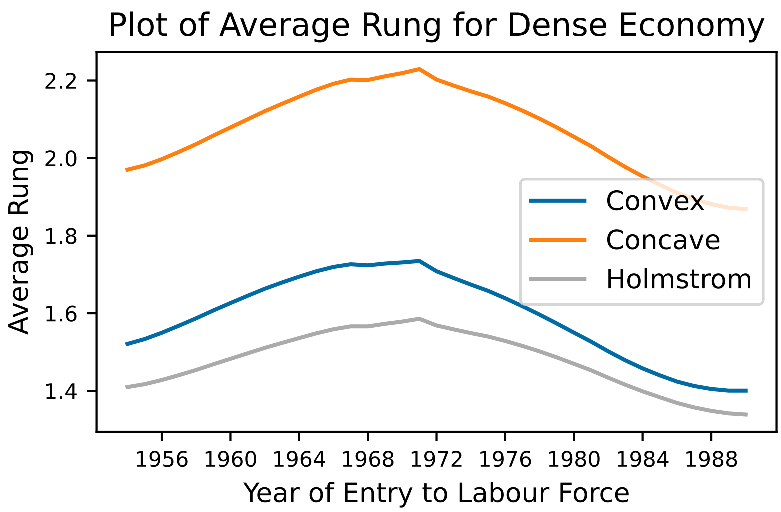

Figure 8: Promotions Model Lifetime Incomes: Alternative Rung Structures

we endogenize the distribution of workers across rungs, $\boldsymbol{\gamma}$, to address this issue.

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