Skill-Biased Reallocation

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Abstract

Workers displaced by the reallocation of labor demand across industries suffer persistent earnings losses, in large part due to higher unemployment risk. This paper quantifies the aggregate unemployment implications of a reallocation of labor demand. I develop a search and matching model with multiple industries and industry specific skill that is calibrated to the US economy. In the model a reallocation shock leads to up to a 0.5 percentage points rise in unemployment. The combination of industry specific skill and imperfect substitutability between workers of different skill levels are key to this result.

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1 Introduction

Many economic forces, such as automation and trade, cause reallocation of labor demand across industries. I define a reallocation of labor demand as a change to the industry composition of employment that does not change the long run level of aggregate unemployment. This displaces workers from shrinking industries to other industries. A large literature[1] has documented large worker level costs from reallocation in the form of earnings losses in part from higher unemployment risk. Recent papers on these earnings losses such as Huckfeldt (2022) and Traiberman (2019) have emphasized the importance of skill in explaining the losses. Previous work, however, on the impact of reallocation on aggregate unemployment has found no aggregate effects even when considering skill. This is despite the movement of workers across industries leading to skill destruction when the skills of the workers are industry specific.

In this paper, I study how aggregate unemployment evolves along the transition in response to a reallocation of labor demand when skill is industry specific. I find that skills indeed matter, but a second essential factor is the degree of substitutability between workers with different levels of industry-specific skills. In a model with both of these features calibrated to the US economy, I find the typical magnitude of changes in industry employment shares over a decade can raise the unemployment rate by up to 0.5 percentage points.

The substitutability between workers with different levels of industry-specific skills is key as it determines how willing an industry is to hire incoming workers. When a reallocation of labor demand occurs between two industries there is a net movement of workers to the growing industry. As the entering workers cannot transfer their industry specific skills they enter as unskilled. Thus the supply of unskilled workers in the growing industry increases but in the short run the supply of skilled workers is inelastic. Due to imperfect substitutability in production, this increase in relative supply causes the marginal product of the entering

[1] See Davis and Von Wachter (2011), Autor, Dorn and Hanson (2016),Neal (1995), and Walker (2013) for some examples across different topics

workers to decline. Thus firms in the industry are not willing to hire all the incoming workers, leading to unemployment. In the long run as the workers who moved develop industry specific skill, unemployment returns to its steady state level.

To assess the quantitative importance of this mechanism I build a quantitative search and matching model with multiple industries. Workers accumulate industry-specific skill while employed in a stochastic manner. If a worker switches industries, they lose their accumulated skill. This acts as a mobility friction, as workers who have acquired industry-specific skill are less likely to move as they would lose the wage premium. Then, instead of assuming perfect substitutability, I assume the industry-level production function has constant elasticity of substitution over workers of different skill levels. A low elasticity of substitution corresponds with the case where skilled workers are doing different and complementary work to that of unskilled workers. Additionally, I allow firms to direct their vacancies by the skill level of the workers.

To calibrate the model, I use heterogeneity in the observed returns to industry tenure and transition probabilities across industries. These moments discipline the skill accumulation and its returns in the model, as well as the ease of switching industries. I then validate this calibration by comparing earnings losses of displaced workers to those estimated by Huckfeldt (2022). The model estimates match the on impact decline in earnings as well as the dynamics for displaced workers re-employed in the same industry and those not.

Then, I use the quantitative model to assess the impact of reallocation of labor demand on aggregate unemployment. I formally model the reallocation as being caused by a shock that raises productivity in one industry and lowers it in another. The magnitude of the productivity shocks is set to match the average decadal dispersion in industry employment share growth rates as well as to keep steady state unemployment constant. I find the shock leads to a rise in unemployment of up to 0.5 percentage points. Additionally, there is a large amount of heterogeneity in the impact of reallocation. When the growing industry is one in which specific skill is less important, the magnitude of the rise in unemployment is less than

0.1 percentage points.

The elasticity of substitution between workers of different skill levels in production is a key determinant of the magnitude of the rise in unemployment. The estimated unemployment response declines by just under half when the elasticity of substitution is increased from 0.5 to 10. The is because as the elasticity of substitution increases, the marginal product of workers of different skill levels becomes less dependent on the relative employment of workers of different skill levels. Thus when unskilled workers move to the growing industry the firm is willing to hire more of them as their marginal product declines only a little.

I then consider how much of the unemployment effect is due to mismatch between workers and vacancies or an aggregate decline in vacancies. Şahin et al. (2014) propose a measure of the degree of unemployment that occurs due to a mismatch between unemployed workers and vacancies. Replicating their measure in the model, mismatch explains very little of unemployment both in steady state and the changes due to reallocation. This is because unemployed workers are not mismatched in terms of the industry they are searching in. Instead, they don't have the specific skills that firms are posting vacancies for. Extending the measure of mismatch to include mismatches skill increases the share of the total unemployment response explained by mismatch to $\frac{2}{5}$. Mismatch across skills rather than across industries thus plays an important role but is not the main channel.

Instead the majority of the rise in unemployment is due to an aggregate decline in vacancies. This occurs as the match surplus of unskilled workers is smaller. So when marginal products for those workers decline due to imperfect substitutability this causes a larger proportional decline in their match surplus than the corresponding rise for skilled workers. This asymmetric effect leads to an overall decline in vacancy posting and therefore the increase in unemployment seen in the model.

Finally I consider how the results change if the assumption of directed search over skill is replaced with random search. Calibrating the random search model to the same moments as the directed search model, I find no large rise in unemployment in response to the same shock.

This is because the effect of the marginal products changing cancel out in the vacancy posting decision. As the value of unskilled workers declines, the value of skilled workers increases thus leaving the value of a lottery between them unchanged. Additionally, the distribution of skill among the unemployed converges quickly to the overall distribution in the industry as there is no selection in which workers are hired or separated.

Literature Review The study of the aggregate implications of reallocation has been strongly motivated by the literature on the worker level costs of reallocation. Starting from Neal (1995) who showed that among workers who switched industries those with higher tenure suffered larger earnings losses. One strand of the literature focuses on the effect of specific shocks driving reallocation such as Autor, Dorn and Hanson (2016), Walker (2013), Ferriere, Navarro and Reyes-Heroles (2023), Traiberman (2019) and Braxton and Taska (2023). Then there are papers motivated by the earnings effects in recessions such as Davis and Von Wachter (2011), Huckfeldt (2022) and Jarosch (2023). In general these papers find large persistent earnings losses for workers who are displaced from their occupation or industry. This paper builds on elements argued to be important in these papers such as specific skills but focuses on the aggregate unemployment effects rather than effects at the worker level.

The study of sectoral reallocation and unemployment from a general macro perspective goes back to the seminal contributions of Lilien (1982) ,Abraham and Katz (1986) Rogerson (1987). There has also been a wide array of recent work Dvorkin (2014), Baley, Figueiredo and Ulbricht (2022), Pilossoph (2012), Chodorow-Reich and Wieland (2020), Carrillo-Tudela and Visschers (2023) and Bocquet (2025). These papers all use search and matching models with multiple sectors to study the impact of reallocation on unemployment.

I contribute to this literature in two ways, the first contribution concerns the substitutability between workers of different skill levels. While the previous literature has assumed perfect substitutability, in this paper I allow for imperfect substitutability. Kambourov and

Manovskii (2009) contains a model with occupational-specific skill and a CES sectoral production function but does not consider the impact of reallocation on unemployment. Instead, it focuses on the link between occupational mobility and wage inequality. I show that relaxing this assumption has a large impact on the effect of reallocation on unemployment. For estimates of this substitutability in the range of the empirical literature, the effect of reallocation on unemployment is quantitatively sizable. This is unlike the null results found in Pilossoph (2012), Carrillo-Tudela and Visschers (2023) and Chodorow-Reich and Wieland (2020)[2].

The second contribution is I allow for heterogeneity in the importance of industry-specific skill across industries. Both Carrillo-Tudela and Visschers (2023) and Kambourov (2009) allow for occupational-specific skills but don't allow the accumulation process to differ across sectors. Wiczer (2015) allows the skill level that workers who have just entered an occupation have relative to higher tenure workers to differ across occupations. The speed of accumulation is fixed, however, at one model period limiting the degree of heterogeneity. By allowing for heterogeneity in the importance of industry-specific skill I find reallocations of the same magnitude can have very different effects on unemployment depending on the industries affected.

The assumption of imperfect substitutability between workers of different skill levels is related to Michaillat (2012) and Mercan, Schoefer and Sedláček (2024) in which the marginal product of workers varies endogenously. Mercan, Schoefer and Sedláček (2024) has related setup to this paper in which they allow for imperfect substitutability between workers of different employment tenure in a one sector model. These papers focus on the role this style of mechanism in a business cycle context while this paper studies the impact of longer run reallocations of labor demand.

This paper is also related to the literature on mismatch unemployment. Shimer (2007)

[2] Chodorow-Reich and Wieland (2020) argue that reallocation only causes a rise in unemployment during recessions. This is because reallocation exacerbates the binding of nominal wage rigidity. They find however no effect outside of a recession when nominal wage rigidity isn't binding.

and Şahin et al. (2014) study how mismatch between workers and the industry they are searching in can lead to unemployment. This paper highlights how there can be mismatch between skills demanded and supplied within an industry not just across industries.

This paper also relates to the literature on the impact of trade shocks when there are costs to switching sectors of which Artuç, Chaudhuri and McLaren (2010) is a seminal paper. This paper builds on the emphasis on specific skills of Traiberman (2019) and Dix-Carneiro (2014) but focusses on the role they can play for unemployment rather than earnings. This empahsis is shared by Caliendo, Dvorkin and Parro (2019), Kim and Vogel (2020) and Galle, Rodríguez-Clare and Yi (2023) but they model unemployment as either a choice in a Roy model or due to nominal wage rigidities. Dix-Carneiro et al. (2023) label some changes in unemployment due to trade shocks as "reallocation unemployment". However, their concept of "reallocation unemployment" is distinct from this paper's, theirs arises mechanically from shifts in industry composition. [3].

Another place the analysis in the paper applies to is the case of automation. Eden and Gaggl (2018) and Vom Lehn (2020) Humlum (2019) study the effects of automation with substitutability with multiple occupations but focus on earnings responses not unemployment. Jaimovich et al. (2021) extends these type of analyses to allow for unemployment but only as a form of occupational choice. Closest to this paper is Restrepo (2015) which studies unemployment due to automation in a search model with frictional unemployment with skilled and unskilled workers. The driving force of unemployment in their model is search being undirected. When there are more unskilled workers firms post fewer vacancies as an expected match is less productive. In this paper, by contrast, the search is directed but the marginal product of unskilled workers is lower when the relative supply of them is higher.

[3]The changes in unemployment in their model are driven by two main forces. First, unemployment falls to the overall gains from trade. Second, they calibrate different industries to have different unemployment rates so as the relative size of industries fluctuates the unemployment rate changes. Unemployment in the US rises initially as production shifts towards manufacturing temporarily but then declines production shifts away from manufacturing. Thus a reallocation shock as defined in this paper would have small if not zero effects in their model.

2 Illustrative Framework

I first consider a simple discrete time one industry model to illustrate how imperfect substitutability combined with industry specific skills can lead to short run unemployment in response to reallocation. The one industry being modelled is the one whose labor demand increases in response to reallocation. This is because the effect of the decline in labor demand on unemployment in the shrinking industry is consistent with many models of labor markets. What will determine the aggregate impact is the extent to which this is offset by the unemployment response in the expanding industry.

I assume that aggregate output is a constant returns to scale function of two labor inputs, one produced by workers with low industry specific skill (N) and one produced by workers with high industry specific skill (S). I will refer to these workers as unskilled or skilled respectively. There is also a Hicks neutral productivity term A and so the production function can be written

$$Y = AF(N, S)$$

To simplify the analysis I assume that all the skilled workers are always employed. On the other hand the employment of unskilled workers is determined by search and matching[4]. Firms must post vacancies v at cost κ in order to hire workers. A worker firm match is only productive the period after the match forms. The number of matches is determined by a matching function $m(v, u)$ that is constant returns to scale in the number of unemployed workers u and the number of vacancies. Then the vacancy fill rate $q(\frac{v}{u}) = \frac{m(v,u)}{v}$ and the job finding rate $f(\frac{v}{u}) = \frac{m(v,u)}{u}$ can be defined. The matches that workers form with firms

[4] Given these simplifying assumptions the analysis is closely related to Michaillat (2012), Michaillat (2024). An inelastic supply of skilled workers provides a justification for the assumption of decreasing returns to scale made in those papers. Additionally the quantitative model with skill acquisition allows for long run constant returns to scale in the size of the labor force. Thus avoiding long run unemployment rates being a function of labour force size and allowing the study of medium run dynamics.

exogenously separate after production each period with probability δ. In steady state the number of separations must equal the number of new matches formed. Denoting the size of the unskilled labor force as L

$$\delta(L-u) = f(\frac{v}{u})u$$

This equation then pins down the level of unemployment as a function of market tightness $\theta = \frac{v}{u}$ the ratio between vacancies and unemployed workers.

$$\frac{u}{L} = \frac{\delta}{\delta + f(\theta)} \tag{1}$$

Or in terms of employment of unskilled workers e.

$$\frac{e}{L} = \frac{f(\theta)}{\delta + f(\theta)} \tag{2}$$

The level of vacancy posting is determined by a free entry condition. Vacancies are posted until the value of a posted vacancy is equal to the cost of posting it. The value of a vacancy is equal to the vacancy fill rate times the expected value of a match to the firm. The expected value of a match to the firm is the expected discounted profit. For simplicity, I assume the wage is constant, and the output of the match is the marginal product of the worker F_N. Thus the expected discounted profit is

$$\frac{\beta(F_N - w)}{1 - \beta\delta}$$

Firms discount at rate $\beta\delta$ as matches may break up with probability δ and β is the discount rate. Thus the equilibrium is determined by the system of two equations in θ and e

$$\frac{e}{L} = \frac{f(\theta)}{\delta + f(\theta)} \tag{3}$$

$$\frac{\kappa}{q(\theta)} = \frac{\beta(F_N - w)}{1 - \beta\delta} \tag{4}$$

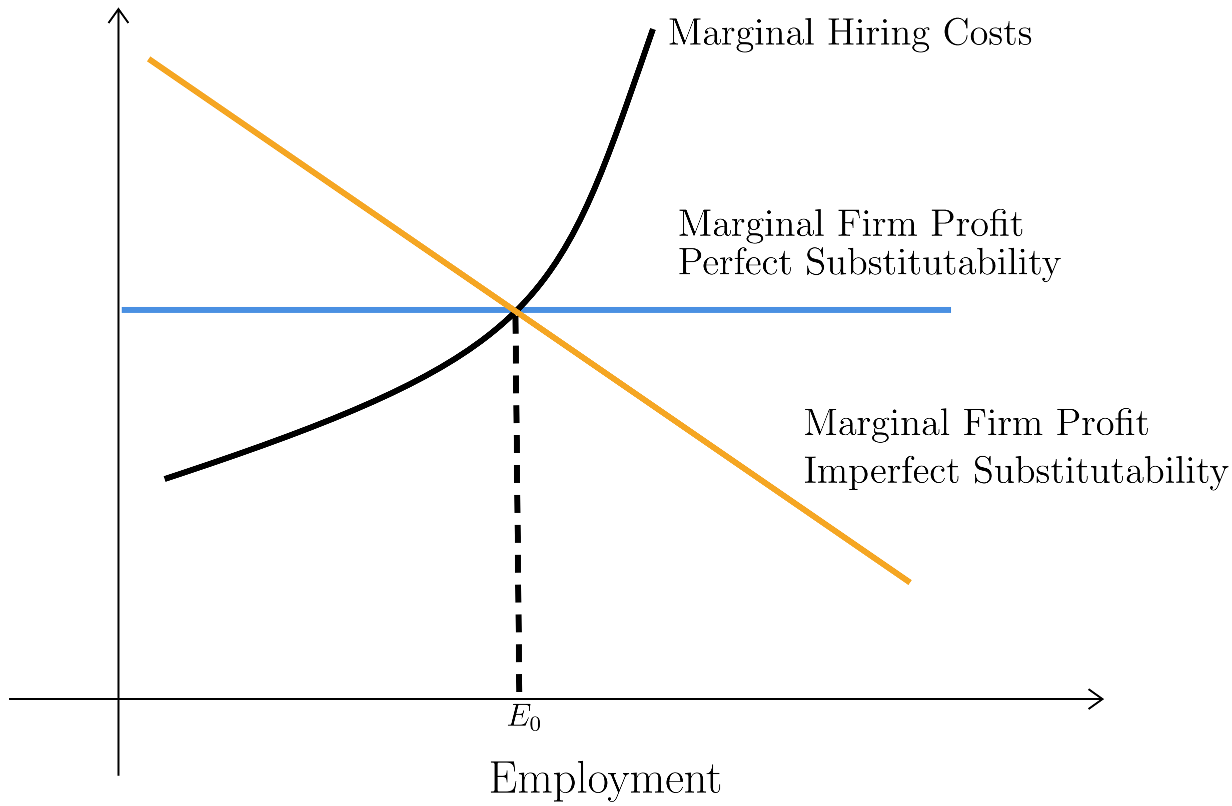
Where the left hand side of Equation 4 is the marginal cost of hiring a worker and the right hand side is the marginal profit.

I will consider two cases, first the case of perfect substitutability between unskilled and skilled workers i.e. $F_{NN} = 0$. Second the case of imperfect substitutability i.e. $F_{NN} < 0$. In Figure 1 I plot the initial steady state equilibrium for both cases where I choose the equilibrium level of employment to be the same. The marginal hiring cost curve is upwards sloping because $\frac{1}{q(\theta)}$ is increasing in θ and the higher e is the higher θ must be to satisfy Equation 2. For perfect substitutability the marginal profit curve is flat while in the case of imperfect substitutability it is downwards sloping.

I then add two shocks to the model to mimic the effect of reallocation on the expanding industry. The first shock is a positive shock to A. This is the direct effect of the shock causing reallocation on the expanding industry. It raises the marginal products of workers, thereby increasing demand for them. The second shock is an entry of unskilled workers so that L goes up. This mimics the indirect effects of reallocation on the expanding industry. As workers in the shrinking industry face lower wages and higher unemployment, they move to the expanding industry. These entering workers are unskilled because the skill under consideration here is industry specific. Even if they were skilled in their previous industry, they become unskilled when entering the expanding industry.

I plot the impact of these shocks in Figure 2. The positive shock to A raises the marginal profit curve in both cases. While the entry shock shifts the marginal hiring cost curve

Figure 1: Initial Equilibrium of One Industry Model

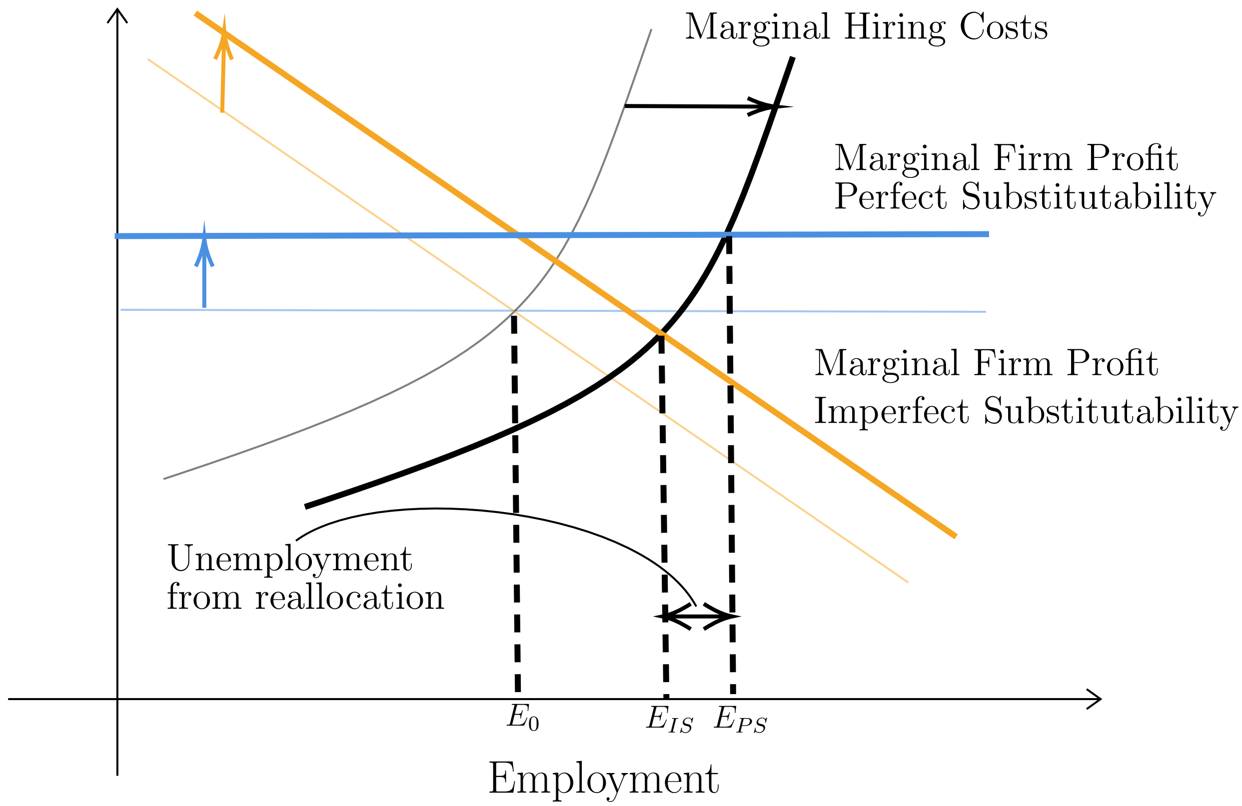


down. This is because the larger pool of workers means firms have to post fewer vacancies to maintain the same level of employment.

As can be seen in Figure 2, the new steady state equilibrium under imperfect substitutability E_{IS} features lower employment than under perfect substitutability E_{PS}. This is because as the entering workers become hired the marginal product of unskilled workers declines as the number of skilled workers is fixed. Thus the marginal profit of new workers declines leading firms to let the market tightness slacken and therefore a smaller employment response. Since there is a negative employment effect in the unmodelled shrinking industry, this smaller positive employment response leads to an aggregate rise in unemployment.

Not only is the employment response smaller, but depending on the degree of imperfect substitutability the unemployment rate in the expanding industry can rise in response to reallocation. In Figure 2 the new equilibrium under imperfect substitutability occurs at a lower value of θ than in the initial equilibrium. This can be seen from the marginal hiring cost curve being intersected at a lower level on the y axis. As discussed above in Equation 1 the unemployment rate is a function of θ and a lower θ implies a higher unemployment rate.

Figure 2: Response of One Industry Model to Reallocation



This comparison of steady state equilibria is short run in the sense that the supply of skilled workers is fixed. In the long run this supply adjusts such that the employment level in both substitutability cases is equal. So in the long run there is no unemployment effect from the shocks.

In the rest of the paper I will build and calibrate a dynamic general equilibrium model featuring industry specific skills and imperfect substitutability between workers with different levels of these skills. The model will allow me to quantify the magnitude of the mechanism discussed in this section. Additionally, I will use the model to study the dynamic response of unemployment to reallocation.

3 Quantitative Model

I build a search and matching model in which workers accumulate industry specific skills on the job. Workers can switch industries if separated however they give up their industry specific skill if they decide to switch industry. Following Artuç, Chaudhuri and McLaren (2010) I model this switching decision as a discrete choice subject to taste shocks. The main contrast of this model from the literature is allowing for the marginal product of a worker to depend on the distribution of skill within the industry.

3.1 Labor Market

There is a separate labor market for each industry (k) - skill level(s) pair. Firms can post vacancies $v(k, s)$ in the market of their choosing. There is also a pool of unemployed workers $u(k, s)$ for each worker skill - industry pair. The market tightness for a given labor market is defined as usual as vacancies divided by unemployed workers $\theta(k, s) = \frac{v(k,s)}{u(k,s)}$. The labor markets have a matching friction in the form of the standard cobb-douglas matching function

$$m(u, v) = \mu u^{\rho} v^{1-\rho}$$

Where ρ is the elasticity of matching and μ is a matching efficiency parameter. The cobb-douglas matching function can produce more matches than either the number of unemployed workers or vacancies. In these cases, I truncate the number of matches to the minimum of the number of unemployed workers or vacancies. Given this matching function and the definition of labor market tightness, the job finding rate can be written $f(\theta(k, s)) = \frac{m(u(k,s),v(k,s))}{u(k,s)} = \theta(k, s)^{1-\rho}$. The vacancy fill rate can similarly be written as $q(\theta(k, s)) = \theta^{-\rho}$

3.2 Workers

There is a unit mass of workers, who are risk neutral and discount at rate β. They can either be employed or unemployed and are at all times attached to an industry. Workers employed in an industry at the start of a period keep their job with a fixed probability $(1-\delta)$ and lose it with probability δ. This timing is based on Christiano, Eichenbaum and Trabandt (2016). It allows for the possibility of workers switching jobs without a period of unemployment consistent with the large numbers of job-to-job transitions observed in the data. Those that lose their job at the beginning of the period or who were unemployed at the start of the period face a choice over whether to change industries. I model this as a discrete choice where workers choose the sector k' that maximises their utility. The utility of a worker making a industry choice can be written as.

$$S_t(k, s, \boldsymbol{\zeta}) = \max\{U_t(k, s) + \zeta_{i,k}, \max_{k' != k} U_t(k', 0) - \alpha_{k,k'} + \zeta_{i,k'}\}$$

Where $U(k', 0)$ is the expected utility from being in sector k' with no industry-specific skill, $\alpha_{k,k'}$ is a utility cost of switching from sector k, k' and $\zeta_{i,k'}$ is the type 1 extreme value taste shock for sector k' which is iid across sectors and time and has variance σ_ζ. There are three factors that determine if a worker will move. First, $U_t(k, s) - U_t(k', 0)$ the difference in utility in being unemployed across industries. This term captures the fact that workers can't transfer their industry-specific skill when they switch industries. Thus workers who have accumulated industry specific skill will be less inclined to switch industries. Second, the taste shocks $\zeta_{i,k}$ and $\zeta_{i,k'}$ generates a motive for workers to switch in both directions Some workers in industry k will draw a high taste shock for industry k' and so will want to switch to that industry even if $U(k', 0) < U(k, s)$. Finally, $\alpha_{k,k'}$ creates a bias towards staying. This helps match empirical gross flow rates as otherwise if $U(k, 0)$ and $U(k', 0)$ are close then workers will be equally likely to stay in k versus switch to k'.

Given this structure the probability of switching from k to k' can be written tractably as

$$P(k \to k'|s) = \frac{e^{(U_t(k',0)-\alpha_{k,k'})/\sigma_\zeta}}{e^{U_t(k,s)/\sigma_\zeta} \sum_{\hat{k}!=k} e^{(U_t(\hat{k},0)-\alpha_{k,\hat{k}})/\sigma_\zeta}}$$

And the expected value function when making the choice has the following form

$$S_t(k,s) = E_{\boldsymbol{\zeta}}[S_t(k,s,\boldsymbol{\zeta})] = \sigma_\zeta(\gamma + \log(e^{U_t(k,s)/\sigma_\zeta} \sum_{\hat{k}!=k} + e^{(U_t(\hat{k},0)-\alpha_{k,\hat{k}})/\sigma_\zeta}) - \sigma_\zeta\gamma + \sigma_\zeta \log(n_k))$$

I add a search cost of $\sigma_\zeta\gamma + \sigma_\zeta \log(n_k)$ to eliminate most of the gains in utility from search due to the type 1 extreme value shocks. The first part $\sigma_\zeta\gamma$ reflects the mean type I extreme value while $\sigma_\zeta \log(n_k)$ eliminates the gains due to more alternatives which increase the expected value of the maximum of the shocks. This ensures workers don't prefer to be unemployed in order to be exposed to the taste shocks.

Once industry switching decisions are made, unemployed workers search for a job in the job market associated with their current industry and level of skill human capital. They thus find a job with probability $f(\theta(k,s))$ and remain unemployed with probability $1 - f(\theta(k,s))$. After this production occurs, the employed receive a wage $w(k,s)$ and the unemployed receive unemployment benefits b.

Finally, at the end of the period, two events can occur. First employed workers potentially gain human capital in their current industry with probability ψ_k. On the other hand, unemployed workers lose their industry-specific human capital with probability ρ. The second event is that a proportion d of workers die and are replaced by unemployed workers in the same industry with no industry-specific skill. I add death to the model as I will target wage growth in calibrating the human capital parameters. As workers experience wage growth over the lifecycle, not adding death will lead to too many workers with high levels of human capital in the steady state distribution.

Given this the values of employment V and unemployment U are

$$
\begin{aligned}
V_t(k,s) =& \delta S_t(k,s) + (1-\delta)(w_t(k,s) \\
& + \mathrm{m}_t\,(1-d)((1-\psi_k)V_{t+1}(k,s) + \psi_k V_{t+1}(k,s+1))) \\
U_t(k,s) =& f(\theta_t(k))(w_t(k,s) + \mathrm{m}_t\,(1-d)((1-\psi_k)V_{t+1}(k,s) + \psi_k V_{t+1}(k,s+1))) \\
& + (1-f(\theta_t(k)))(b + \mathrm{m}_t\,(1-d)((1-\rho(k))S_{t+1}(k,s) + \rho(k)S_{t+1}(k,s-1)))
\end{aligned}
$$

This model focuses on the industry-specific component of skill. It thus does not contain ex-ante heterogeneity (e.g., college versus non-college workers) as in papers like Dix-Carneiro (2014).

3.3 Firms

There is a continuum of firms in each industry which each employs one worker. A firm must post a vacancy in order to hire a worker. The cost of posting a vacancy for a worker of skill s is denoted $\kappa(k,s)$ and there is free entry into the market for vacancies. This implies the free entry condition for firms

$$
\kappa_{k,s} = q(\theta)E[J_t(k,s)]
$$

Where $J_t(k,s)$ is the value of a filled vacancy, which solves the following Bellman equation.

$$
J_t(k,s) = (y(k,s) - w(k,s)) + \beta(1-d)(1-\delta)[(1-\psi_k)J_{t+1}(k,s) + \psi_k J_{t+1}(k,s+1)]
$$

Where $y(k,s)$ is the revenue generated by a match of worker with skill s in industry k. I assume wages are set by Nash bargaining between the firm and worker with equal bargaining weights. So in steady-state the wage can be calculated using the equation

$$J(k,s) = w(k,s) - b + \beta * (1-d)([(1-\psi_k)V(k,s) + \psi_k V(k,s+1)] - [(1-\rho)S(k,s) + \rho S(k,s-1)])$$

$$w(k,s) = J(k,s) + b - \beta * (1-d)([(1-\psi_k)V(k,s) + \psi_k V(k,s+1)] - [(1-\rho)S(k,s) + \rho S(k,s-1)])$$

I assume for each industry there is a Constant Elasticity of Substition (CES) aggregator of the output of different skill types with each worker employed in an industry producing one unit of industry-skill-specific output.

$$Y_k = A_k \left(\sum_s \tau_{k,s} e[k,s]^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

I assume the production function is constant returns to scale implying $\sum_s \tau_{k,s} = 1$. The industry-skill CES parameters $\tau_{k,s}$ determine the relative marginal product of different skill levels in each industry and thus influence the relative wages. This combined with the probability of gaining skill ψ_k determines the expected returns to staying in a given industry for a long time. The industrial productivity A_k affects the relative wage across industries and therefore the relative size of different industries. Later in the quantitative exercise I will shock these productivities to induce reallocation of workers across industries. The elasticity of substitution across skills η is an important parameter governing the response of unemployment in the model to reallocation as controls how the relative marginal products of workers of different skill levels respond to changes in the relative supply of workers of different skill levels within an industry. So if workers move into an industry this will increase the relative supply of unskilled workers and thus decrease the marginal product of unskilled workers. If η is large the change in marginal product will be small but if η is close to 0 then the change in marginal product will be large and thus the value to firms of posting vacancies for these workers will fall greatly.

3.4 Household

All workers are members of the representative household. The household's preferences over the output of each industry are given by a constant elasticity of substitution (CES) aggregator over industry output.

$$U(\{c_k\}_{k\in\{1,\dots,n_k\}}) = \left(\sum_k \omega_k^{\frac{1}{\sigma}} c_k^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

Where the ω_k are the CES weights and σ is the elasticity of substitution over industry output. The profits of the firms are paid out as dividends to the household as well as the wages of the workers.

4 Calibration

I now calibrate the model to match relevant moments of data on the US labor market in order to study the quantitative importance of reallocation. I take the model period to be a month. This allows for a reasonable frequency of churn across jobs, skill levels, employment and industries while not being so divorced from some of the data which is only available at the annual level.

Then I fix some parameters to values that are standard in the literature. I set the discount factor β to 0.996 which implies an annual discount rate of approximately 5%. The parameter for the probability of losing skill while unemployed ρ I take from Carrillo-Tudela and Visschers (2023) as 0.02. The model in this paper does not have the idiosyncratic heterogeneity in productivity which enables Carrillo-Tudela and Visschers (2023) to explain duration dependence of unemployment which they use to calibrate this parameter. However, for the aggregates of interest in this paper, the results will not be sensitive to reasonable choices of this parameter. This is because only a small percentage of workers are unemployed each period and reasonable estimates of ρ are of a similar magnitude so the changes in skill driven by skill loss while unemployed are small compared to other sources of skill change. I set the probability of death d to $\frac{1}{480}$ which implies an average working life of 40 years. I set the elasticity of substitution across industry output in the household's utility function to 4 which is within the range estimated by Broda and Weinstein (2006). I set the productivity of the matching function μ to be 0.1. As I will calibrate the vacancy posting cost κ to match the unemployment rate, this is a normalisation for all outcomes except vacancies which is not the focus of this paper. I discuss the details of this further in A.1. Finally, I set the value of the unemployment benefit b to 0.55 of the average wage to match the estimates from Chodorow-Reich and Karabarbounis (2016).

For η, the elasticity of substitution across skills in the production function, I use the Cobb-Douglas benchmark of 1. This parameter is difficult to identify as differences in the relative wages of more skilled workers across industries could be driven by other factors such as differences in the direct returns to skill captured in the model by the CES production weights $\tau_{k,s}$. However, the value of 1 lies in the range that has been estimated by the prior literature. Dustmann, Frattini and Preston (2013) find a value of 0.6 using the response of the wages of natives to immigration in the UK. Heathcote, Storesletten and Violante (2017) find a value of 3.1 using a structural model of wages earnings and hours applied to the US. Mercan, Schoefer and Sedláček (2024) estimate an elasticity between newly hired workers at a firm and incumbent workers of 1.3 by minimizing the distance between their model and responses in the data to separation shocks. While the elasticities estimated by these papers are not directly comparable to the one in this paper as they do not focus on industry specific skill, they are all in the range of 1. More importantly, these estimates are all far from ∞ the case of perfect substitutability. After the main analysis, I will study the sensitivity of the results to a reasonable range of values of η.

I then calibrate the rest of the parameters, κ the vacancy posting cost, the parameters relating to skill and production A_k $\tau_{k,s}$ and ψ_k, and parameters relating to industry choice σ_ζ and α. The targets fall primarily into two sets of moments, wage returns to industry tenure and rates of worker mobility across industries. There are three further targets, an unemployment rate of 4%, an average wage of 2 and that all industries are the same size.

Before discussing the moments in detail, I will first go over choices I make regarding the calibrated parameters. For the cost of vacancy posting κ, I assume it to be constant across industries and skill levels. The other major choice is regarding the number of skill levels and industries.

The number of skill levels is set to 2 which I will refer to as skilled and unskilled. There are two reasons for this choice. The first is that the computational complexity of the model scales in the product of the number of industries and the number of skill levels. Thus a low number of skill levels keeps the model computationally tractable. The second reason is that additional skill levels would require additional moments to calibrate. The most relevant targets are the wage returns to industry tenure which could be estimated for every year of tenure. However, given the noise inherent in estimating returns to tenure by industry these additional moments are unlikely to be informative.

Then I set the number of industries to 4 categorized into two distinct types, high skill specificity and low skill specificity. The types can differ in their productivity A_k, CES production weights $\tau_{k,s}$[5] and skilling rate ψ_k. Having two types of industry will allow me to study heterogeneity in the impact of a reallocation shock depending on the skill specificity of the positively and/or negatively affected industry. I assume that high skill specificity industries will have higher wage returns to tenure and lower mobility than low skill specificity industries. In high skill specificity industries, the wage premium for industry-specific skill is higher. This generates both higher returns to tenure—since tenure proxies for skill

[5] Due to there only being two skill levels only $\tau_{k,0}$ needs to be calibrated as $\tau_{k,1}$ is then determined by $\tau_{k,0} + \tau_{k,1} = 1$

accumulation—and lower mobility, as workers face greater opportunity costs when leaving and losing their valuable specific skills.

That two industries are of the same type should not be interpreted as them being similar in the type of skills they require. Instead type solely captures the importance of industry specific skills. For concreteness think of medical services and legal services. These industries share characteristics such as long training periods suggestive of high skill specificity. However many of the skills such as detailed knowledge of anatomy or case law are not transferable between the two industries. Thus workers who move between these industries would find that much of their accumulated skill is not transferable and for computational tractability in the model this is modelled as skill loss.

The choice of two industries of each type is to allow for a separation of worker mobility rates from the relative size of the industries. With just two industries, a steady state requires that gross flows of workers between them be equal for their sizes to remain constant. This directly implies that industry size is determined by relative mobility; an industry with a higher exit rate must be smaller to maintain the balance. This would then contaminate the role of industry skill specificity in the effect of reallocation with potential effects due to size.

Allowing for two industries of each type breaks this link. Workers can move more frequently between the lower skill specificity industries. Thus allowing them to have higher gross flow rates even when all industries are the same size. Then in the calibration size is determined by the relative attractiveness of the industries. Thus targeting equal size pins down the relative value of the wages and wage growth of each type of industry.

The second non data moment is the average wage which I target to be 2[6]. This choice of target sets the scale of the model. An alternative approach would be to fix the productivity of one of the industries. This would then require replacing the average wage target with the ratio of b to the average wage of 0.55 taken from Chodorow-Reich and Karabarbounis (2016). As well as the industry productivity that is fixed being replaced by b in the parameters

[6] 2 is chosen over the more natural 1 for numerical reasons relating to the stability of the optimisation routine when calibrating.

calibrated. This is not a substantive difference and indeed the target of an average wage of 2 with a fixed b of 1.1 can be thought of in terms of the Chodorow-Reich and Karabarbounis (2016) moment. If the average wage is 2 then the ratio matches 0.55 while if the wage is higher then the ratio will miss the target below[7].

Now moving onto the moments based on data. I estimate the returns to career tenure using the NLSY79 data following Pavan (2011). I make two major changes to the specification, first I use OLS estimates rather than IV. This is because the selection effect that biases OLS is present in the model and is informative about industry choice parameters, as I will discuss further shortly. Secondly, I allow the returns to vary depending on 1 digit industry and I take the 25th and 75th percentiles of the estimated returns as my targets.

For industry mobility I take the estimates of annual mobility from Dvorkin (2021). I again take the 25th and 75th percentiles of the industry transition probabilities as my targets. To do this I consider the estimated transition probability for each industry period pair as an independent data point. In the model I consider the proportion of workers who started employed in an industry in a month and in 12 months time are employed in a different industry. Finally I choose as a target an aggregate unemployment rate of 4% inline with the level of unemployment seen typically outside of a recession in the US.

Table 1

Calibrated parameters

	Parameter	Value
Sectoral Productivity	A_k	[12.7, 15.5]
CES Production Weights	$\tau_{k,0}$	[0.11,0.25]
Skilling Rate	ψ_k	[0.024, 0.022]
Utility Cost of Switching	α	6.0
Variance of Taste Shocks	σ_ζ	3.0
Vacancy Posting Cost	κ	0.003

While intuitively one might think that the returns to career tenure will primarily inform the skill parameters and the industry mobility data will primarily inform the industry choice

[7] It is not possible to directly force the average wage to be 2 as it depends on the equilibrium distribution of workers across skill levels which changes as other parameters change

parameters, these parameters and moments are heavily interrelated. In the case of industry returns to tenure, the OLS estimates in the data are contaminated by selection bias as workers who experience lower returns may be more likely to leave the industry. The model also has this selection bias as workers who have accumulated skill in an industry are less likely to leave. The degree to which mobility is selective is partially determined by σ_ζ the variance of the taste shocks. If σ_ζ is low, the staying probability will be more sensitive to the value of staying. Thus skilled workers will be much less likely to move than skilled workers, so selection bias will be high. Similarly, fixing mobility parameters, the skill and production parameters will affect the degree of mobility. The CES production weights $\tau_{k,s}$ determine the wage premium to skill and therefore the returns to staying in an industry relative to moving. The skilling rate ψ_k will play two roles, first it changes the proportion of workers who are skilled for a given mobility rate and skilled workers will move less. Secondly, it lowers the cost of moving to a new industry as workers will accumulate skill faster and so the earnings loss from moving is lower.

Another subtlety of the identification of the parameters is in the relative magnitude of ψ_k, the probability of gaining skill, between the high and low skill specificity industries. A higher ψ_k, all else held equal, will lead to higher returns to tenure as workers will gain skill faster and therefore it might be expected that the high-skill specific industry will have a higher ψ. This need not be the case, however, as the calibration must also match the lower mobility in the high-specificity industry. In steady state, net flows must be zero and thus in-migration must be lower. A low initial wage plus slow skill accumulation would make the industry too unattractive to workers who would enter as unskilled. Also too high a ψ_k would lead to many workers being skilled in the high specificity industry and given the high returns these workers would be unlikely to leave leading to excessively low outmigration.

Table 2

Model and Data Moments		
Moment	Model	Data
2 year returns to industry tenure high specificity	9.0%	8.6 %
2 year returns to industry tenure low specificity	3.5%	3.3 %
5 year returns to industry tenure high specificity	14.9%	16.4 %
5 year returns to industry tenure low specificity	6.4%	7.2 %
Average wage	2.17	2
Transition probability away high specificity	5.3%	5.4 %
Transition probability away low specificity	10.3%	11.2%
Unemployment rate	4.1%	4%
Industry size high and low Specificity	25.06%, 24.94%	25%, 25%

4.1 Calibrated Parameters

As can be seen in Table 2 the model in general does a good job matching the moments of the data. There is some overshooting of the 2 year returns to tenure followed by an undershooting of the 5 year returns. That this is consistent across both high and low skill specificity industries suggests it may be due to the restrictiveness of having only two skill levels. Additionally, the average wage is 8.5% higher than the target. As discussed previously this implies a lower ratio of flow benefits to average wage compared to the estimates of Chodorow-Reich and Karabarbounis (2016). As this implies workers are getting more surplus from matches and I assume the total surplus is split by nash bargaining, this leads to overall a larger fundamental surplus which Ljungqvist and Sargent (2017) argues dampens unemployment fluctuations. Thus this calibration error will if anything lead to smaller responses of unemployment in the quantitative experiment.

The estimated rates of skill accumulation of 2.4% and 2.2% per month are in line with the values assumed in Carrillo-Tudela and Visschers (2023) of 1.7% per month. The low value of the κ the vacancy posting cost, can only be understood when taking into account the productivity of the matching function which I take to be 0.1. Given the steady state vacancy fill rates, the cost per match for a firm ranges between 0.11 and 0.73. Compared to a marginal product of a match ranging between 1.9 and 2.26.

In order to compare the estimates of the variance of the taste shocks σ_ζ and utility costs of moving to those from Artuç, Chaudhuri and McLaren (2010) (ACM) and Dix-Carneiro et al. (2023) (DCPRHT) respectively, a couple adjustments must be made. First, an adjustment must be made for the timing of the model as ACM is estimated at the annual level. DCPRHT propose a conversion from annual to quarterly of $\frac{\beta^4}{1-\beta^4}\frac{\beta}{1-\beta}$. Using the same formula but to go from annual to monthly would be $\frac{\beta^{12}}{1-\beta^{12}}\frac{\beta}{1-\beta}$ which equals 4.6 when using the value of β used by ACM. Additionally, as the average wage is 2.17 rather than 1 this implies that the taste shocks should be twice as large to be the same relative size to worker value functions. Combining these two adjustments with the estimate of σ_ζ from ACM of 1.61 gives an equivalent estimate of 16.0. This is five times as large as the estimate in this paper. On the other hand, the utility cost of moving relative to the variance of the taste shocks and the wage is $\frac{6.03}{2.97\times 2.17} = 0.94$. This is low compared to the costs estimated in DCPRHT who allows the cost to vary for every source-destination pair and finds values ranging between 0 and 3.43.

A potential driver of this difference is the presence of industry-specific skill. This acts as an incentive to stay within an industry that is absent from ACM and DCPRHT. Thus it is unsurprising that the mobility cost estimate is lower and thus a smaller variance in the taste shock is required. Secondly, in ACM the estimates of σ_ζ varies depending on the value chosen for β with lower β implying a lower value. As the β I chose is lower, this may also be driving some of the difference.

4.2 Earnings Losses from Displacement

In order to validate the model, I compare the earnings losses from displacement in the model to the data by industry stayers and leavers. This is the industry version of the moment that Huckfeldt (2022) targets in the calibration of his model[8]. This is a useful check in two senses. First, it alleviates concerns that the results might be driven by specific features of

[8] The difference between Huckfeldt (2022) results when using industry or occupation are very small

the industry tenure moments that I target. One potential concern is that the selection in the model may not be of a similar magnitude to the selection in the data and thus the calibration may over or understate the underlying returns to tenure. Secondly, this is a moment directly informative about the micro costs of reallocation. For the results of the model for the macro costs of reallocation to be credible, the micro costs must be realistic.

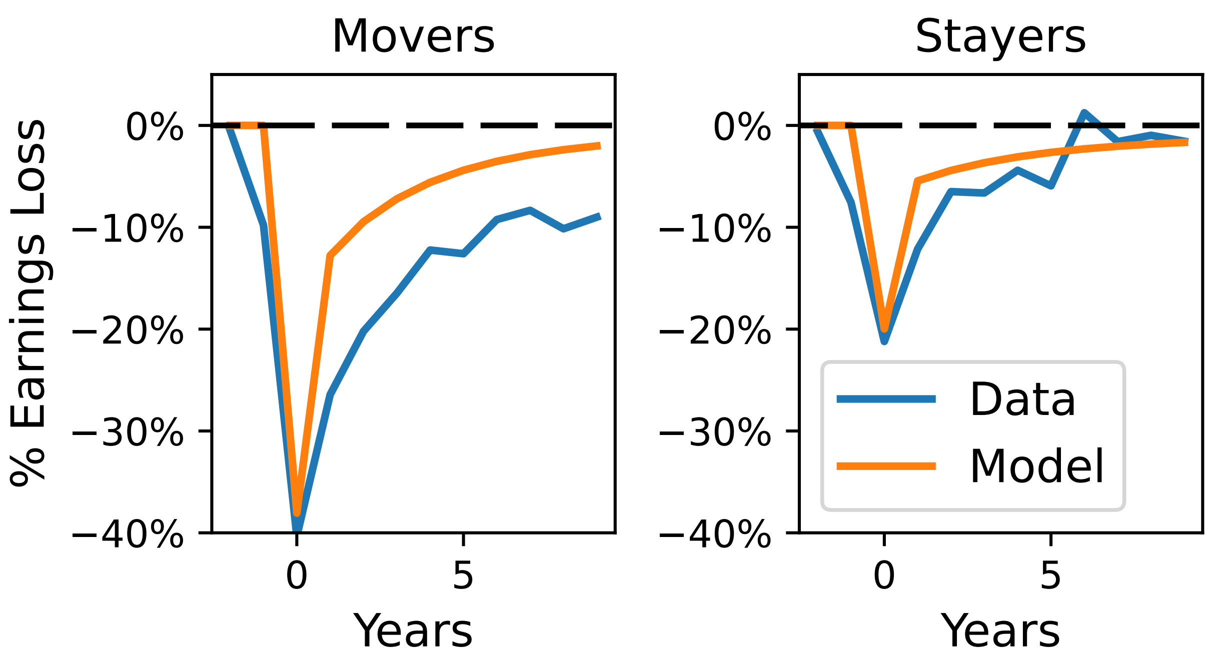
To calculate the earnings losses from displacement I need to take a stand on what displacement is in the model. In the data, workers are considered displaced if they lose their job for reasons of slack work, plant closings, and abolished jobs which are considered exogenous to the worker. In the model all job destruction is considered exogenous to the worker, however the level of job destruction is high to allow for 'job-to-job' transitions in the model without directly modelling them. For this reason, a large number of workers who lose their job in a period will find a new job in the same period. So if I were to label all workers who separate at the beginning of a period as displaced, the earnings losses of stayers would be small as it would be dominated by these 'job-to-job' transitions. Therefore I define displacement as a worker who separates from their job and does not find a new job in the same month. I then define an industry stayer as a worker who is next employed in the same industry as the job from which they are displaced and an industry leaver as a worker who is next employed in a different industry.

To construct the comparison group I use workers who do not separate from their job in the period. I start with the steady state distribution of workers across industries and skill levels. I then iterate forward the distributions of workers who are both displaced and not displaced. This gives me the full time path of these distributions without simulation error. I then use this distribution and the wages to calculate the average monthly earnings of all three groups. Then to compare to the data I aggregate up to an annual frequency.

I plot the results in Figure 3. Comparing this to the data on earnings losses from Huckfeldt (2022) the model does a good job of matching the earnings losses of industry stayers. The losses on impact are very close to those of the data, with around 20% for stayers and 40%

for movers. Additionally, the model also does a reasonable job of matching the dynamics of earnings losses over time. The earnings jump up in the first year after the displacement and then slowly recover over time. Given both the initial impact and the dynamics are untargeted in the calibration, this is a validation of the model's ability to capture the micro level costs to workers of reallocation.

Figure 3: Comparison of Earnings Losses From Unemployment



5 Quantitative Experiment

In order to understand the effect of reallocation on aggregate employment in the model, I study the response to a shock to the productivity of two industries in the economy. One industry receives a positive productivity shock and the other a negative productivity shock. The shock takes the form of an unanticipated MIT shock which takes effect in a linear manner over a decade. I determine the shock size by finding the negative shock to a high-skill specific industry and positive shock to the other high-skill specific industry that leads to the same steady state employment as the initial steady state and a change in industry shares in line with decadal changes in industry shares. This ensures that changes in employment in the model are driven by temporary transitional dynamics rather than permanent effects due to the shock. I then take the same-sized negative shock and solve for the positive shock that leads to the same steady state employment for all other combinations of industry types getting shocks.

I plot the results for employment in Figure 4. Each line is the employment response for a given combination of industry types receiving the positive and negative productivity shocks. So the line labelled "Low to High" is the employment response when one of the low skill specificity industries receives the negative shock and one of the high skill specificity industries receives the positive shock. Therefore workers will move on net to the high skill specificity industry hit by the positive shock.

There are two main takeaways from this figure. First is that the reallocation shock can lead to a substantial decrease in employment in this model. The lowest trough in employment is 0.5 percentage points below steady state, which implies a 12% increase in unemployment from steady state. Additionally, the employment response is highly persistent, with the recovery taking a decade to complete. This unemployment effect occurs without rigid wages and a coincidental negative aggregate demand shock such as in Chodorow-Reich and Wieland (2020), Michaillat (2024) and Kim and Vogel (2020).

Secondly, the impact of the reallocation shock is heterogeneous in both magnitude and dynamics with reallocation involving high skill specificity industries having larger effects. In particular, if the growing industry is highly skill specific, this leads to a persistent fall in employment. While if the expanding industry is low skill specificity the decline recovers quickly and the cumulative employment loss is close to 0.

These effects are driven by the dynamics of marginal productivity of different workers. To show this I plot in Figure 5 the response of workers' marginal product to the shock that reallocates between the high skill specificity industries. As workers move to the growing industry in response, they enter as unskilled workers due to the lack of skill transferability. As this inflow is hired this drives up the relative employment of unskilled workers in the growing industry. Due to imperfect substitutability this drives down the marginal product of unskilled workers. This effect is large enough to outweigh the direct effect of the productivity shock leading to a maximum decline of 7% relative to steady state. Due to this lower marginal product firms are not willing to post enough vacancies for unskilled workers to absorb the inflow leading to higher unemployment in the aggregate.

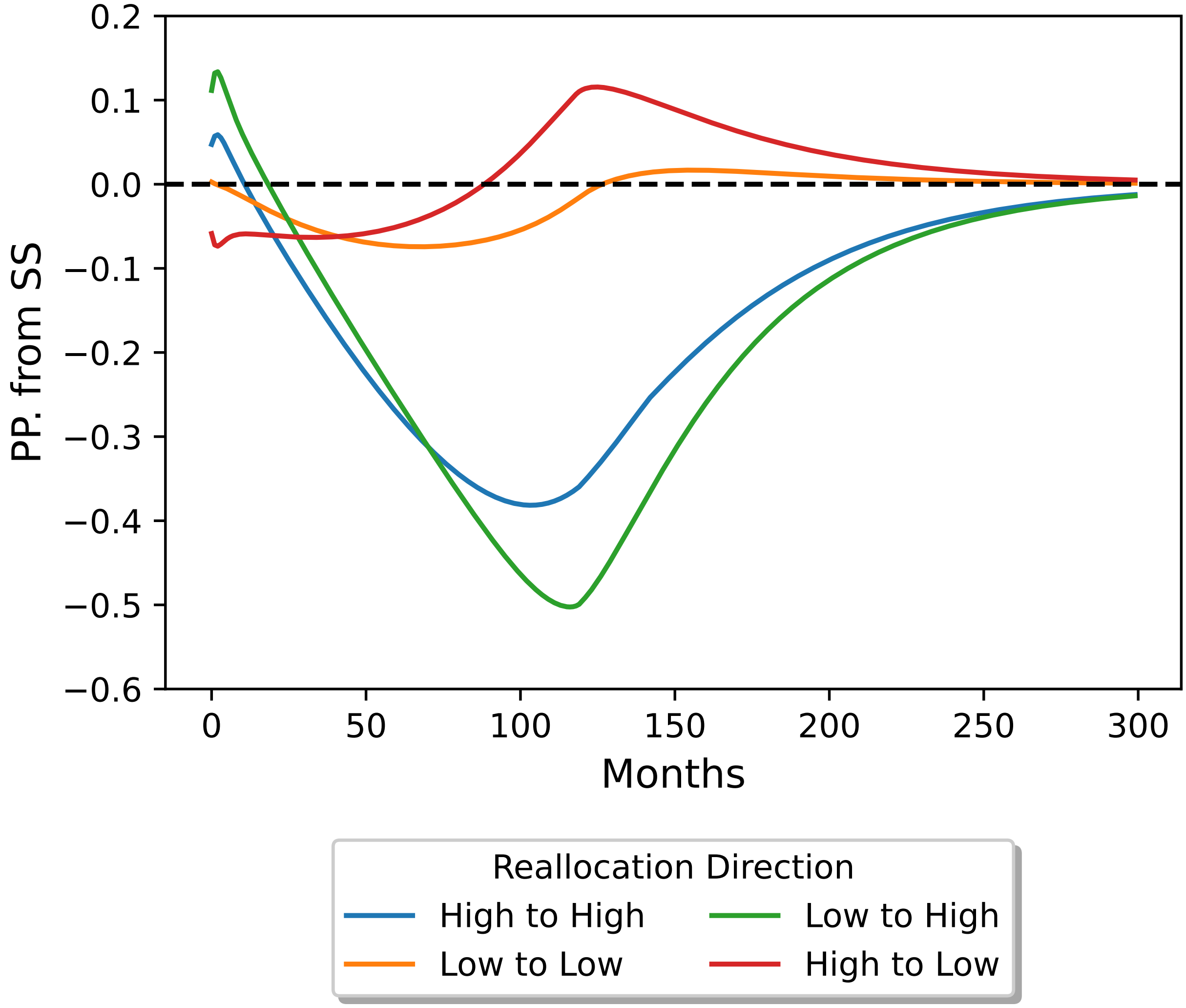
In the long run the share of unskilled workers in the growing industry declines as workers accumulate skill on the job. This brings unskilled marginal products back up. Thus firms are willing to post more vacancies for these workers returning employment back to its steady state level.

This drives the heterogeneity as high skill specificity industries have both a smaller steady state employment share of unskilled workers and lower unskilled marginal product. Thus the inflow of unskilled workers causes a larger change in relative employment shares and thus a larger proportional decline in marginal products. This larger proportional decline is then applied to a lower initial marginal product leading to a larger decline in job surplus for unskilled workers.

5.1 The Role of Substitutability Between Skills

The results above highlight the importance of changing relative supplies of different skills. This points to η, the elasticity of substitution between workers of different skill levels, as a key parameter in the model to generate unemployment in response to reallocation. In this subsection, I conduct a sensitivity analysis for a broad range of values of η. While changing η does affect the steady state of the model, it is also possible to change A_k and $\tau_{k,s}$ simultaneously such that for the steady state distribution of workers marginal products are unchanged. Thus the original steady state is preserved while changing η allowing for a clean comparison. I plot the results in Figure 6.

Figure 4: Employment Responses by Importance of Skill Specificity

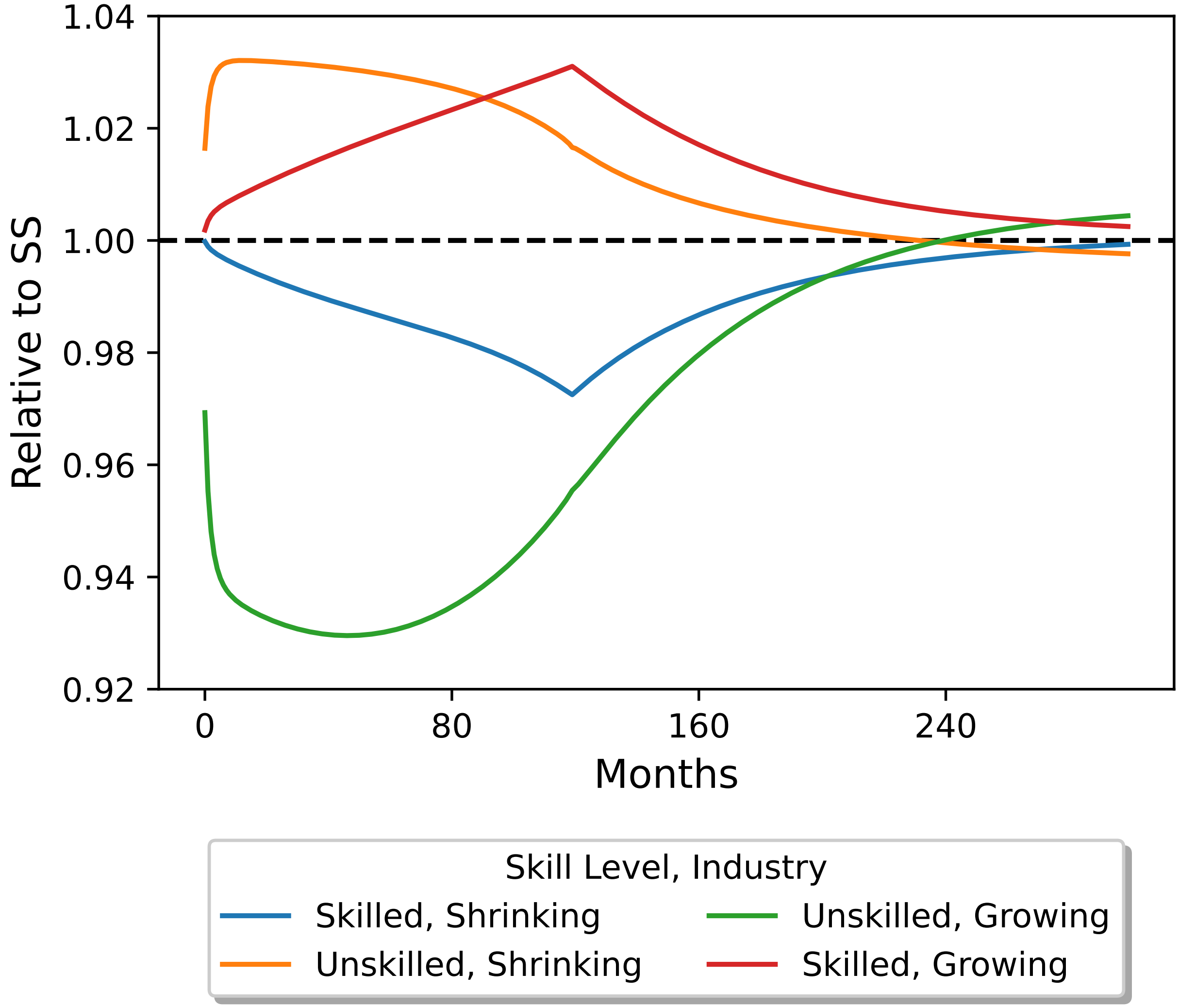


All lines are in response to productivity shock to two industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

As can be seen in the figure, the impact of the demand shock on employment is decreasing in η. Going from an η of 0.5 to an η of 10 reduces the size of the unemployment response by 45%. While this reduction is substantial the unemployment response is still economically significant for η of 10 peaking at decline of 0.25 percentage points.

The cause of this decline is that the higher the elasticity of substitution the less responsive the relative marginal products of different skill levels are to changes in the ratio of workers of different skill levels. So when unskilled workers leave the industry with the negative

Figure 5: Dynamics of Workers' Marginal Products

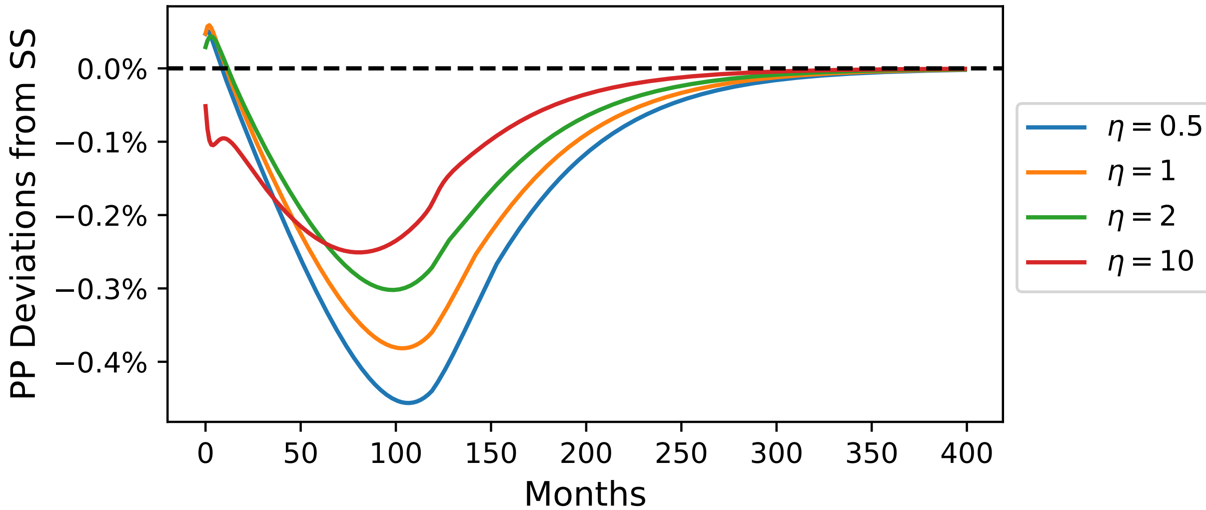


The shock is to the two high-specificity industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

productivity shock, the relative marginal product of skilled workers falls by less the higher is η. Similarly the relative marginal product of unskilled workers in the industry with the positive productivity shock fall by less the higher η as workers enter the industry. As these are the locations where most of the unemployment occurs, the more their marginal products fall the more unemployment there is.

This is the important difference from previous work such as Carrillo-Tudela and Visschers

Figure 6: Comparison of Employment Depending on η



All lines are in response to productivity shock to the two high-specificity industries. One hit positively and one negatively. I resolve the steady state for each η. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

(2023). In their model workers have idiosyncratic productivity for the sector they are in which accumulates as well as having stochastic variation. However, in their model workers with different productivities are perfect substitutes so when there is a demand shock to a sector the marginal product of workers of all skills will rise no matter the skill distribution. Thus the value to a firm of the low productivity workers still increases when workers without industry specific skill enter. So firms post enough vacancies to absorb these incoming workers. So despite their model having sector specific skill, reallocation does not increase unemployment.

5.2 Mismatch Unemployment

Previous work has considered a particular channel through which reallocation can lead to unemployment. The idea is that there may be a mismatch between the industries that unemployed workers are searching in and the industries which are posting more vacancies. This leads to fewer total matches due to the concavity of the matching function and thus higher unemployment. In this section I study the importance of this channel to the results of this paper.

Şahin et al. (2014) proposed a measure of mismatch $\mathcal{M}_t$ defined below.

$$\mathcal{M}_t = 1 - \sum_k \left(\frac{\phi_{k,t}}{\bar{\phi}_t}\right)\left(\frac{v_{k,t}}{v_t}\right)^{\eta}\left(\frac{u_{k,t}}{u_t}\right)^{1-\eta} \tag{5}$$

Where $\phi_{k,t}$ is the industry-level matching efficiency, which in this paper is a constant μ. $\bar{\phi}_t$ is the economy-wide matching efficiency, which is also constant at μ. v_t and u_t are the aggregate number of vacancies and unemployed workers. This measure captures the proportion of unemployment that could be eliminated if vacancies were moved across labor markets in order equalise the labor market tightness.

Şahin et al. (2014) found in the context of the great recession that this index explained one-third of the rise in unemployment. I calculate this index for the model economy in response to reallocation between the two high-skill specificity industries and plot it in Figure 7. In the left hand side of the figure is the dynamics of the Şahin et al. (2014) index. Mismatch rises in response to the shock however it remains small peaking at 0.6% of total unemployment being ascribed to mismatch. As the shock generates additional unemployment of around 12% the index finds that the vast majority of unemployment from reallocation is not due to mismatch across industries.

This discrepancy may be due to two features of the reallocation shock considered compared to the great recession. First, the shock to is less severe than the shock in the great recession, playing out over a decade rather than the sharp decline in employment in the great recession. Second, the shock considered here is permanent while the great recession feature in large part a temporary aggregate shock. This dampens the value of leaving an industry that is currently posting fewer vacancies as workers anticipate that employment opportunities will recover in the future.

More importantly, however, the Şahin et al. (2014) measure ignores potential mismatch across skills within industries. As the key mechanism occurs through the marginal product of unskilled workers falling while that of skilled increases, there is the potential for large mismatch between workers and vacancies across skill. Therefore I augment the measure of Şahin et al. (2014) to allow for mismatch across skills.

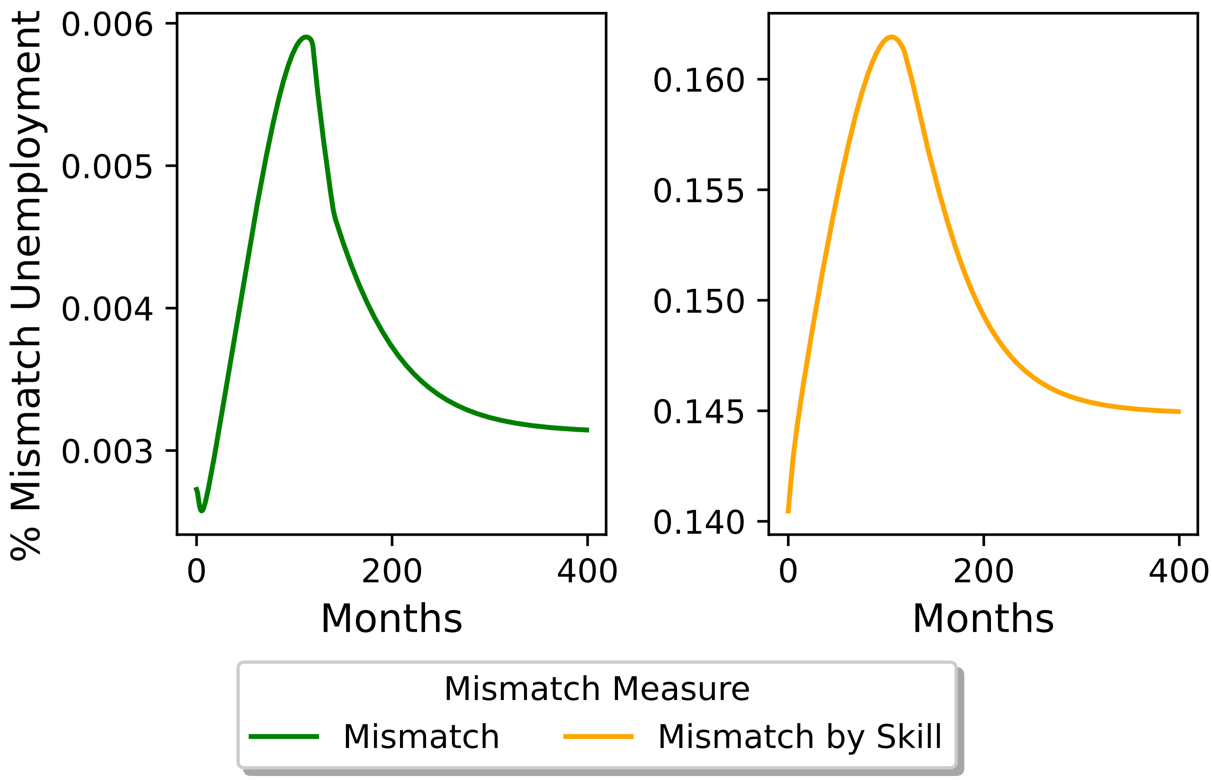
$$\mathcal{M}_t = 1 - \sum_k \sum_s \left(\frac{\phi_{k,s,t}}{\bar{\phi}_t}\right)\left(\frac{v_{k,s,t}}{v_t}\right)^{\eta}\left(\frac{u_{k,s,t}}{u_t}\right)^{1-\eta} \tag{6}$$

I plot the dynamics of this index in response to the shock in the right hand side of Figure 7. Here mismatch increases by much more than in the case without skill, peaking at over 16% of total unemployment. This large difference suggests that allowing for mismatch across skill is an important avenue for future empirical research on skill mismatch.

To calculate the contribution of mismatch to the rise in unemployment, I calculate the amount of unemployment due to mismatch before the shock and at the peak. As the share of unemployment due to mismatch starts in steady state at 14.0%, this implies the amount of unemployment in percentage point terms starts at $0.140 \times 4.13 = 0.58$. Repeating this calculation at the peak of unemployment gives $0.162 \times 4.52 = 0.73$, an increase in unemployment of 0.15 percentage points. So given a total increase in unemployment of 0.38 percentage points, mismatch explains around 2/5 of the total rise in unemployment, a large contribution but not the majority.

Since mismatch across skill does not drive a majority of unemployment in response to reallocation, it must be that there is a decline in aggregate vacancies and thus market tightness. This occurs because the match surplus of unskilled workers is smaller than that of skilled workers. Therefore declining marginal products of unskilled workers causes a larger proportional decline in match surplus than rising marginal products of skilled workers causes an increase in match surplus. This leads to a larger response of vacancies for unskilled workers than for skilled workers, resulting in a decline in average market tightness and higher unemployment.

Figure 7: Evolution of Mismatch in Response to Reallocation



All lines are in response to productivity shock to the two high-specificity industries. One hit positively and one negatively. The mismatch indexes are as described in the text

5.3 The Dynamics of Industry Transitions

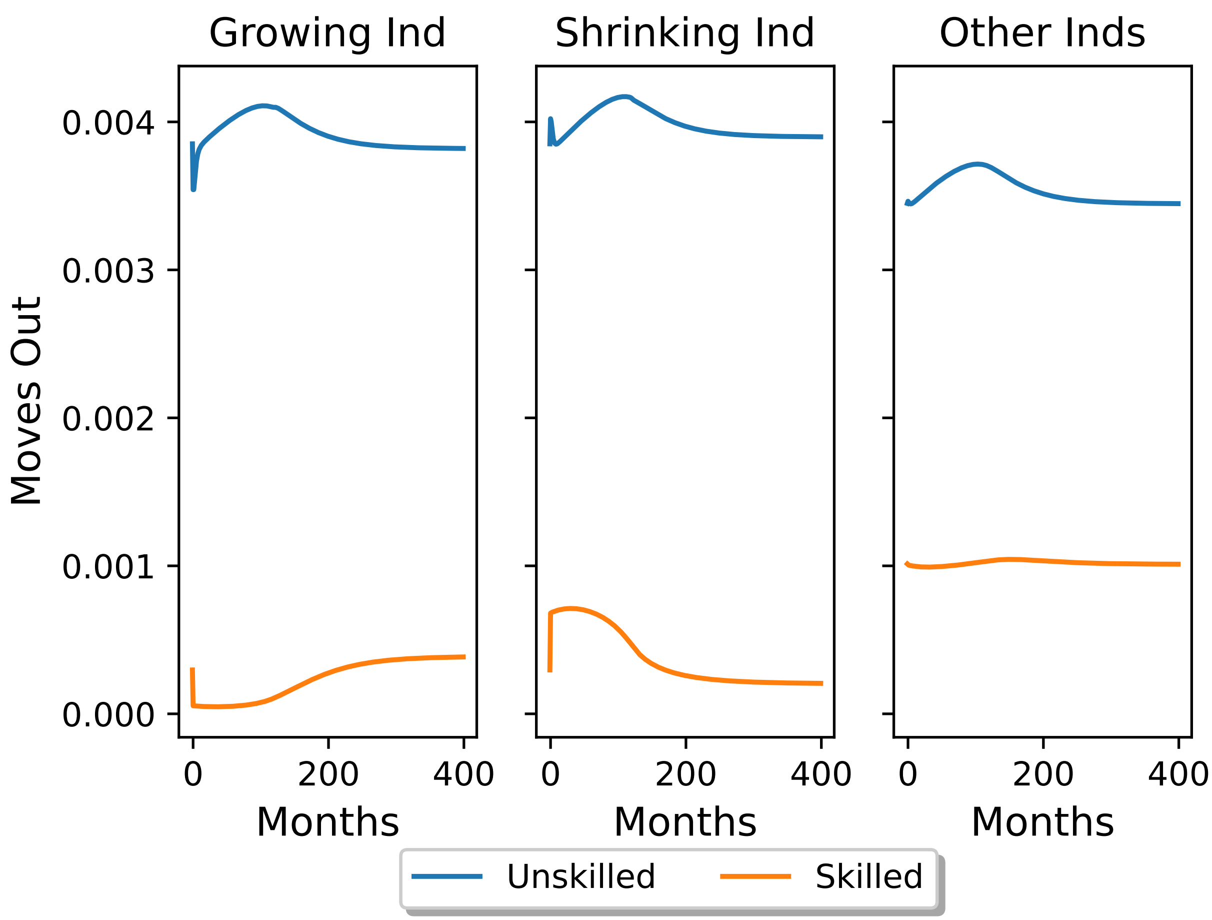
An important feature of the model is that workers move across industries. In Figure 8 I plot how the absolute flows of workers out of industries evolve in response to the shock discussed above which reallocates from a high skill specificity industry to the other high skill specificity industry. In order to illustrate the change from the initial steady state, the graph begins the month before the shock is realised. I label this month as month -1. The first thing to notice is that the absolute flows of unskilled workers are always higher than the flows of skilled workers. This is because skilled workers face a larger opportunity cost of leaving in the form of losing their accumulated skill.

In the first panel are the dynamics of moves out of the industry receiving the positive shock. For both skilled and unskilled workers, the number of flows drops on impact as the wages of this industry respond to the shock. For unskilled workers, this quickly reverses as wages decline due to the entry of unskilled workers and the substitutability across skills in production as described previously. The flow of skilled workers declines due to the rise in the opportunity cost driven by the increase in the wages paid to skilled workers in this industry. In the longer run the number of flows of skilled workers increases as the skill premium declines and the number of workers in the industry rises.

In the middle panel are the dynamics for the industry receiving the negative shock. In the short run the results are exactly the opposite of the case of the growing industry. The number of flows of both skilled and unskilled rises due to the direct effect of the shock on wages. The outflow of skilled workers then slowly declines as the skilled to unskilled ratio in the industry returns to steady state and the absolute employment in the industry declines. The outflows of unskilled workers, similar to before, quickly rebound due to the change in the skill ratio. After this it begins to rise again as the value of being an unskilled worker in any industry falls due to the increased number of unskilled workers as skilled workers leave the shrinking industry. Thus it begins falling again as the ratio converges to the new steady state. These dynamics are a consequence of modelling industry choice as a discrete choice subject to idiosyncratic type I extreme value taste shocks. When the value of all options falls, the taste shocks become more important pushing towards increased moves across industries. For the low skill specificity industries the outflows of unskilled workers rise and then fall along with the general rise and fall of being unskilled in any sector. The outflow of skilled workers moves little.

Carrillo-Tudela and Visschers (2023) argue that this increase in gross moves is inconsistent with the data. They proposed an alternative formulation of search across industries in which search is costly. Thus when the values of being unskilled fall, gross moves also decline. However, the size of gross flows is not a major driver of the results in this paper as workers still experience unemployment even if they make it to the growing industry.

Figure 8: Dynamics of Industry Transitions



All lines are in response to productivity shocks to the two high-specificity industries. One hit positively and one negatively. Thus both the growing industry and shrinking industry are high skill specificity. The other industries are both low skill specificity and since neither receives a shock they respond identically and thus only one panel is shown. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state. The y axis is the absolute number of workers of that skill level leaving that industry in a given month.

6 Random Search

The assumption of directed search is strong, implicitly assuming full information on the skills of workers. In this section, I consider the other extreme of random search where firms meet workers at random. To avoid adding additional complications from learning like those considered in Baley, Figueiredo and Ulbricht (2022) I assume that workers know their own skills and it is revealed to firms upon matching. This changes the free entry condition in the model to be

$$\kappa = q(\theta_k)\mathbb{E}_s\left[J(k,s)\right]$$

Where θ_k is the labor market tightness of industry k defined as the vacancies posted by firms in that industry divided by the number of unemployed workers summing over all skill levels. The expectation of $J(k,s)$ is taken with respect to the distribution of skill levels among unemployed workers. I assume that workers of all skill levels are equally likely to find a match. Thus in the case of two skill levels where the share of workers who are unskilled is denoted χ_k this can be expressed as

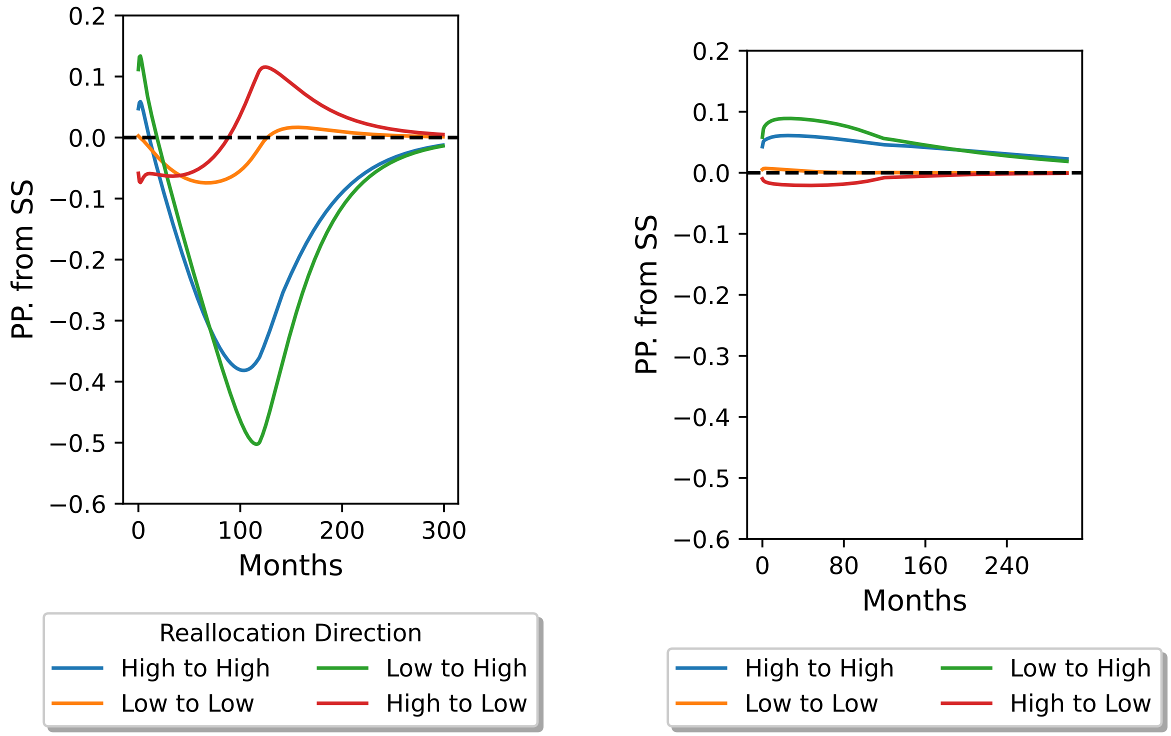
$$\mathbb{E}_s\left[J(k,s)\right] = \chi_k J(k,0) + (1-\chi_k)J(k,1)$$

So as in equilibrium $J(k,1) > J(k,0)$ due to calibration targeting wage growth, an increase in the χ_k will decrease $\mathbb{E}_s\left[J(k,s)\right]$. Thus as more workers in the unemployed pool are unskilled the benefit of posting a vacancy decreases.

I then recalibrate the model to match the same moments as in the directed search model. Then I feed in the same productivity shocks as in the directed search model and compare the results in Figure 9. In the random search model there is no large decrease in employment for any of the shocks. In fact for a couple of the shocks employment goes above the steady state level and converges back to steady state from above.

The key reason for this is that the effect on relative marginal products cancel out in the vacancy posting decision. As the relative supply of unskilled workers increases, their marginal product decreases while the marginal product of skilled workers increases. However, due to

Figure 9: Comparison of Employment Response



(a) Directed Search (b) Random Search

All lines are in response to productivity shock to the two industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

random search firms can't direct their vacancies by skill and so the first part of the change in marginal products decreases $\mathbb{E}_s [J(k, s)]$ but the second increases it.

There is also an channel from changes in the distribution of skill among the unemployed. Workers who enter from other industries enter as unskilled and unemployed. This decreases $\mathbb{E}_s [J(k, s)]$ potentially leading to lower vacancy posting. However, this effect on the distribution of skill among the unemployed will not be large of persistent. This is because the share of skilled workers is large and the separation rate δ is high. Thus while initially there may be an increase in χ_k due to newly arrived unskilled and unemployed workers, this will quickly revert to the total share of unskilled workers.

This section shows that the ability of firms to distinguish between skilled and unskilled workers is key to the finding on a negative effect of reallocation on unemployment. Given that firms do observe industry tenure as well as job titles and responsibilities as well as the slow rate of skill accumulation from the calibration, directed search may well describe the labor market better in this particular setting.

7 Conclusion

This paper argues that the reallocation of labor demand can have consequences for aggregate unemployment. This result comes from allowing for a realistic structure of substitutability between workers of different skill levels. When different skill levels are not perfect substitutes the demand for unskilled workers will be lower in the transition than in steady state. As this is also where there is a greater supply of workers during the transition this can lead to transitory unemployment.

That substitutability between workers is important for the response to shocks may also apply to other cases. Many modern macro models assume the marginal product of a match is independent of the distribution of matches in the economy for tractability. So there is a need to better understand when this powerful assumption is a good approximation to the real world.

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A Appendix

A.1 Normalisation of μ

In this section I will discuss the irrelevance of the level of μ the productivity of the matching function for the results of the paper. First consider the dynamics of the model conditional of steady state values of market tightness θ and value of a job J as well as κ and μ. The identity of the industry k and skill level s do not matter and thus I supress the notation here. The two relevant equations are the free entry condition and the job finding rate as other equations only depend on μ via its impact on J and $f(\theta)$.

$$\kappa = \mu\theta^{-\rho}J \tag{7}$$

$$f(\theta) = \mu\theta^{1-\rho} \tag{8}$$

Taking logs, totally differentiating and denoting $\frac{dx}{x} = \hat{x}$

$$\begin{aligned} \rho\hat{\theta} =& \hat{J} \\ \hat{f} =& (1-\rho)\hat{\theta} \\ \implies \hat{f} =& \frac{1-\rho}{\rho}\hat{J} \end{aligned}$$

Thus conditional on the steady state values of f and J the dynamics of the model are independent of μ. So all that remains is to show that any value of μ, the value of κ can be chosen such that the same steady state values of f and J are obtained. From the job finding rate equation we can substitute out θ in the free entry condition.

$$
\begin{aligned}
f =& \mu\theta^{1-\rho} && (9)\\
\theta =& \left(\frac{f}{\mu}\right)^{\frac{1}{1-\rho}} && (10)\\
\kappa =& \mu\left(\frac{f}{\mu}\right)^{-\frac{\rho}{1-\rho}} J && (11)\\
\frac{\kappa}{\mu^{\frac{1-2\rho}{1-\rho}}} =& f^{\frac{\rho}{1-\rho}} J && (12)
\end{aligned}
$$

So for any value of μ I can choose κ such that it is consistent with the same steady state values of f and J and thus the rest of the model steady state. Thus the choice of μ only matters for the level of vacancies, and thus θ and vacancy fill rate. As vacancies are not the focus of this paper, this choice is irrelevant for the results.

The only caveat is that the Cobb-Douglas matching function should be truncated such that the number of matches is not larger than the number of vacancies or the number of workers or else the job finding rate or vacancy fill rate would be greater than one. If the values of θ lie in the region where truncation is required then the above derivations will not hold as I have used the untruncated value of $f(\theta)$ and $q(\theta)$. However for the value of μ chosen this is not an issue in either the steady state or the dynamics.

B Constructing the Reallocation Shock

To calculate the reallocation shock I first calculate relative absolute changes in industry shares using data from the Quarterly Census of Employment and Wages. I take NAICS 3 digit at the national level from January for the years 1990,2000,2010 and 2020. I calculate the industry share's of total employment at each of these dates and then take the decadal difference. I first take the absolute value and then convert into relative changes by dividing by initial industry share of employment. Finally I take the mean across all industries and all decades which gives a value of 15.5%.

Then in the model I will shock one industry with a positive productivity shock and another with a negative productivity shock. I use a root finding algorithm to find the magnitudes of these shocks that result in no change in steady state aggregate employment and delivers the empirical change in relative absolute changes in industry shares. I do this separately for the different identities of the negatively and positively shocked industries such that the responses can be compared.

For reallocation between high skill specificity industries the size of the negative shock is 12.1% while the positive productivity shock is 10.1%. To give a sense of magnitude these numbers can be compared to Chodorow-Reich and Wieland (2020) who do a similar exercise, identifying 10 log linear productivity paths to match the deciles of the dispersion in industry employment share growth rates. This can be seen in Figure 4 in their paper. In percentage terms the shocks range from over 30% increases for the fastest growing industries and over 25% decreases for the declining industries. So the shocks are economically significant magnitude but well within a reasonable range observed in the data.

The shocks play out linearly over a decade though are fully anticipated starting in period 0. I plot the dynamics of the productivity shocks relative to initial productivity in Figure 10

B.1 Numerical Appendix

B.1.1 Solving For the Steady State

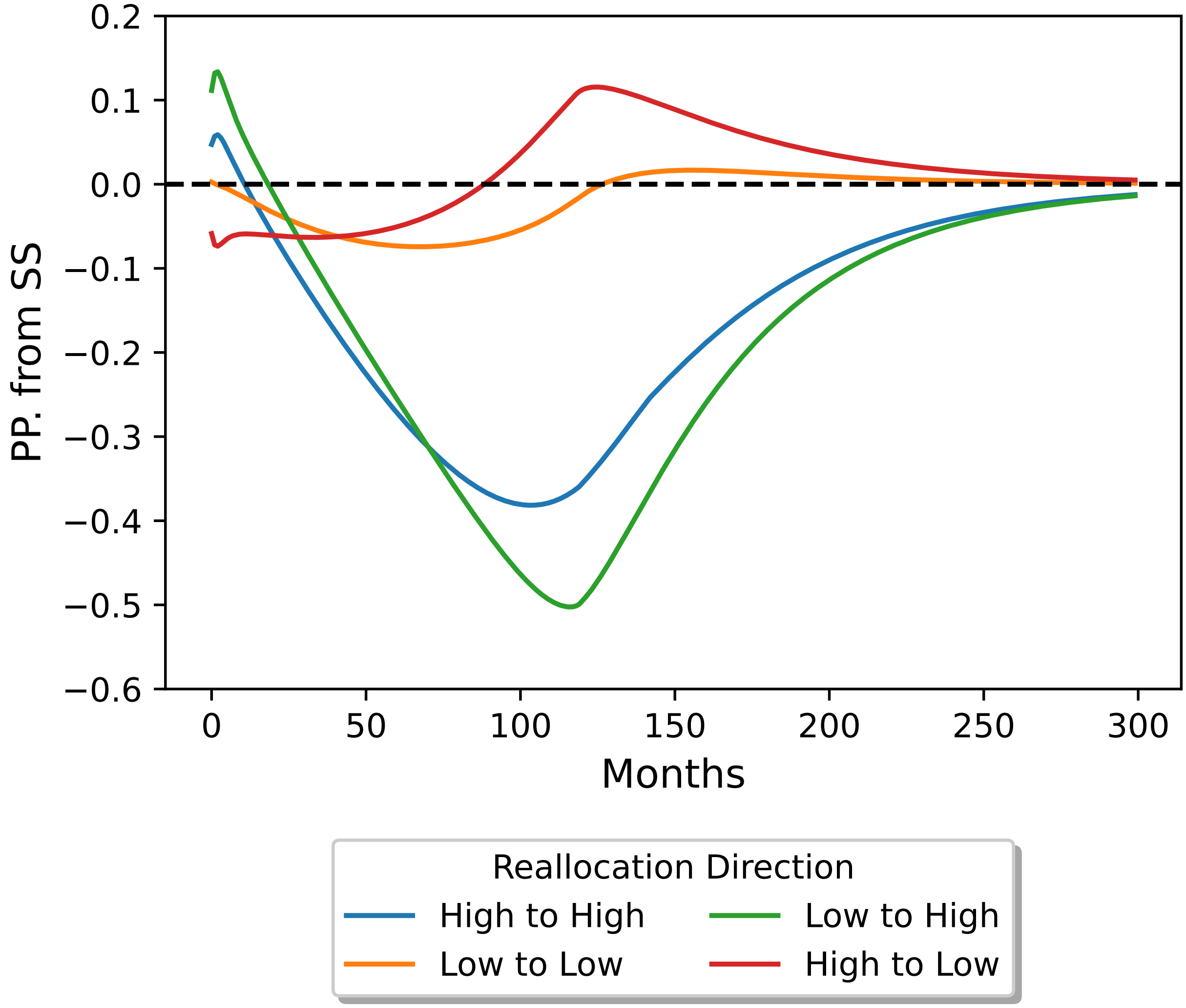
To solve for the steady state of the model I set up the following minimisation problem

1. Guess a value of a match to firms $J(k, s)$.

2. Given $J(k, s)$ invert the free entry condition to obtain the market tightness in each job market $\theta(k, s)$

3. Then calculate the surplus of a match to workers using $J(k, s)$ and the nash bargaining solution.

Figure 10: Employment Responses by Importance of Skill Specificity



Productivity path when considering reallocation between high skill specificity industries. The y axis is percent change from initial productivity.

4. Given this surplus and $\theta(k, s)$ iterate to find the value of unemployment $U(k, s)$, value of employment $V(k, s)$, value of working choosing across industries $S(k, s)$ and switching probabilities

5. Use the switching probabilities and $\theta(k, s)$ to calculate the steady state distribution of workers across industries and skill levels.

6. Given the distribution of workers calculate the output of each industry and thus the

marginal product of each skill level in each industry can be calculated.

7. Invert the equation for surplus to workers to find the wage $w(k,s)$ in each industry and skill level.

8. Given wage and marginal products solve for the value of a match to firms $\tilde{J}(k,s)$.

9. I then compute $\sum_{k,s}(J(k,s) - \tilde{J}(k,s))^2$

I then use an optimiser to find the guess of $J(k,s)$ that minimises the sum of squared differences between $J(k,s)$ and $\tilde{J}(k,s)$ computed at the end. The use of an optimiser rather than iteration of the guess is for stability reasons. The dependence of the marginal products on the distribution of workers can make iterating the guess of $J(k,s)$ unstable.

B.1.2 Calibration

When calibrating there are two moments of which the construction is not straightforward. The first is the proportion of workers who switch industries at an annual rate. To calculate this I initialise a distribution with all workers only in one industry at one skill level. I then iterate the distribution forward 12 periods (i.e. a year) omitting the death shocks[9]. I then calculate the proportion of workers who are attached to a different industry at the end of the year.

The second moment is the wage growth with tenure. For this I create a new distribution iteration function that keeps track of the tenure of workers in an industry and their last industry of employment. Industry tenure only resets when a worker is employed in a different industry to the one that they had been building up tenure in. I cap the tenure at 10 years and one month as I only calibrate to wage growth over the first 10 years of tenure. I then calculate the average wage by tenure after 2 years and after 10 years.

[9] Since the shocks are unrelated to states this is equivalent to conditioning on not having received a shock which is the relevant comparison to the data

I implement the calibration using a two step procedure. In the first step I use an optimiser to minimise the distance between the model and data moments. However I replace the production parameters (industry productivity and CES skill weights) with marginal products at the parameters to calibrate. This is as if I was assuming perfect substitutability and calibrating industry-skill level productivities. Then having found the value of the rest of the parameters and marginal products that minimises the distance between the model and data moments I then use the marginal products to find the production parameters that match the data moments conditional on a choice of the elasticity of substitution between workers of different skill levels.

B.1.3 Solving for Dynamics

In the quantitative exercise I solve for the response starting in a non stochastic steady steady state. Non stochastic referring to the lack of shocks to industry productivities and not to the idiosyncratic shocks to workers such as death and skill acquisition/loss. Then I feed a shock to industry productivities into the model. This shock increases the productivity of one industry and decreases the productivity of another. Though the change in industry productivities occurs in a linear fashion over a decade it is fully anticipated starting from date 0.

To solve for the dynamics, first I solve for the new steady state after the shock has played out. Then we setup a similar fixed problem to the steady state problem but over time.

1. Guess a value of a match to firms over time $J_t(k, s)$.

2. Given $J_t(k, s)$ invert the free entry condition to obtain the market tightness in each job market at each time $\theta_t(k, s)$

3. Then calculate the surplus of a match to workers using $J_t(k, s)$ and the nash bargaining solution.

4. Given this surplus and $\theta_t(k, s)$ iterate backwards from the new steady state values to

find the value of unemployment $U_t(k, s)$, value of employment $V_t(k, s)$, value of working choosing across industries $S(k, s)$ and switching probabilities

5. Use the switching probabilities and $\theta_(k, s)$ iterate the distribution forward starting at the original steady state distribution.

6. Given the distribution of workers and the path of industry productivity calculate the output of each industry and thus the marginal product of each skill level in each industry can be calculated.

7. Invert the equation for surplus to workers to find the wage $w_t(k, s)$ in each industry and skill level over time.

8. Given wage and marginal products solve for the value of a match to firms $\tilde{J}_t(k, s)$.

I then use Anderson's acceleration to find the fixed point of this problem.