Iteration Bound

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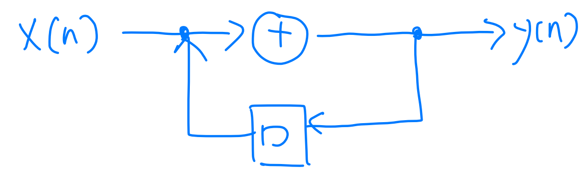
Iteration Bound T_∞



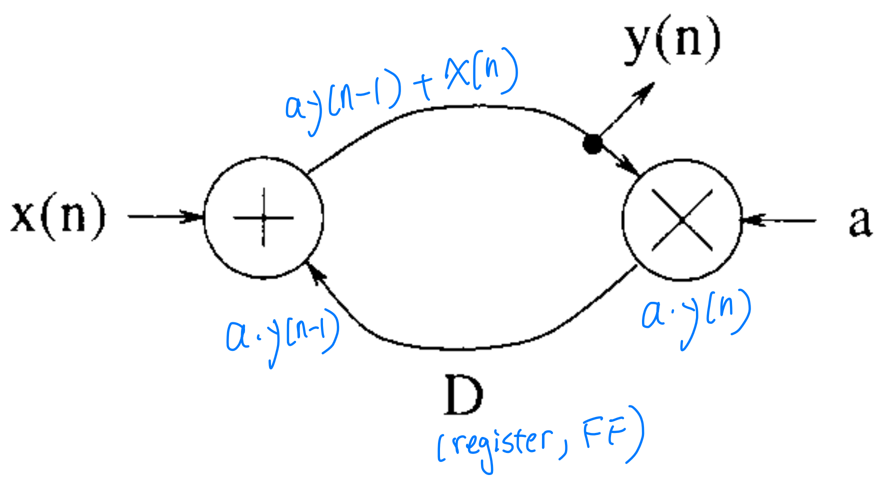
- Only for recursive algorithms which have feedback loops
- Impose an inherent fundamental lower bound on the achievable iteration or sample period
- A characteristic of data-flow graph (DFG)
- Two methods to calculate iteration bound
  - Longest path matrix (LPM)
  - Minimum cycle mean (MCM)

Data-Flow Graph Representations (1/2)

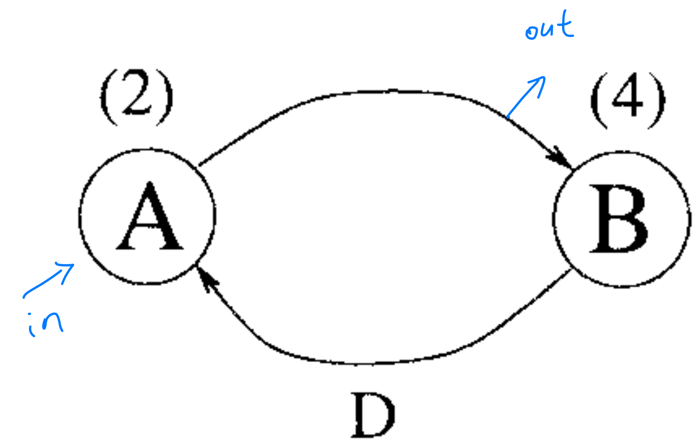
// IIR Filter

for $n = 0$ to ∞

$$y(n) = ay(n-1) + x(n)$$



Block diagram



Data-flow graph (DFG)

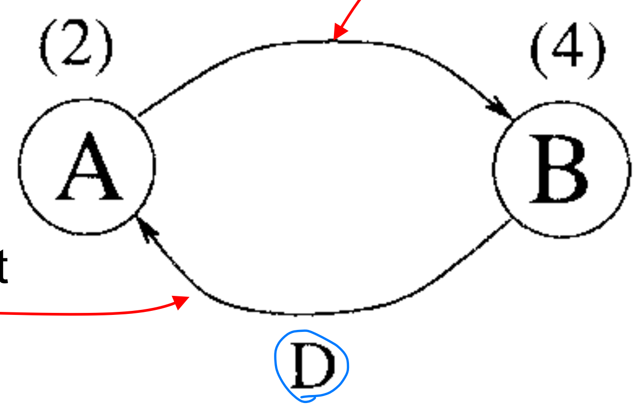
Data-Flow Graph Representations (2/2)

- Iteration
  - Execution of each node in the DFG exactly once
  - X_k: k-th iteration of node X
- Precedence constraints

In the same iteration

Intra-iteration precedence constraint

$$A_k \rightarrow B_k$$



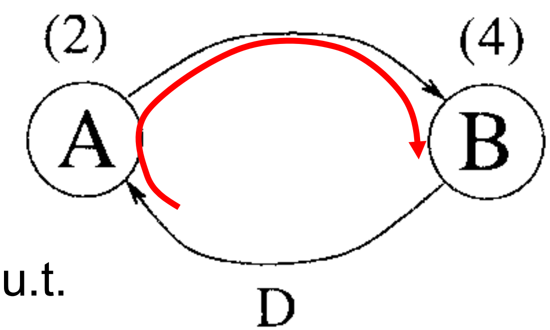
a FF is in between
=> next iteration

Inter-iteration precedence constraint

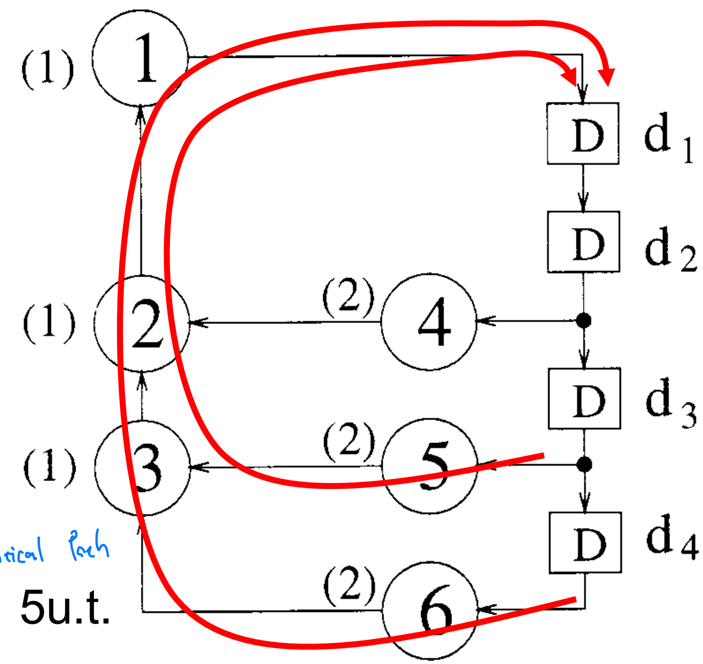
$$B_k \Rightarrow A_{k+1}$$

Critical Path

- The path with the longest computation time among all paths that contain zero delays



6u.t.

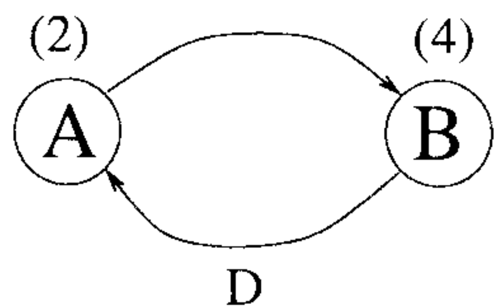


Critical Path

5u.t.

Loop Bound (1/2)

- Loop (cycle)
  - A directed path that begins and ends at the same node



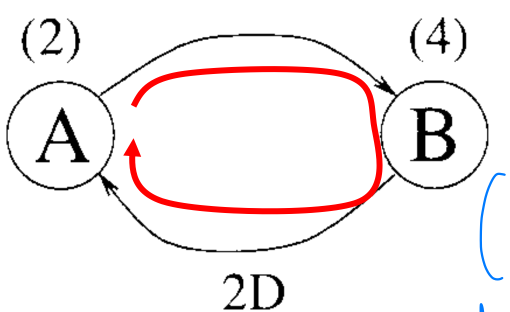
$$A \rightarrow B \rightarrow A$$

- Precedence constraints

$$A_0 \rightarrow B_0 \Rightarrow A_1 \rightarrow B_1 \Rightarrow A_2 \rightarrow B_2 \Rightarrow A_3 \rightarrow \cdots$$

Loop Bound (2/2)

- Loop bound
  - The lower bound on the loop computation time
  - Loop bound of l-th loop: t_l/w_l
  - t_l: loop computation time
  - w_l: the number of delays in the loop



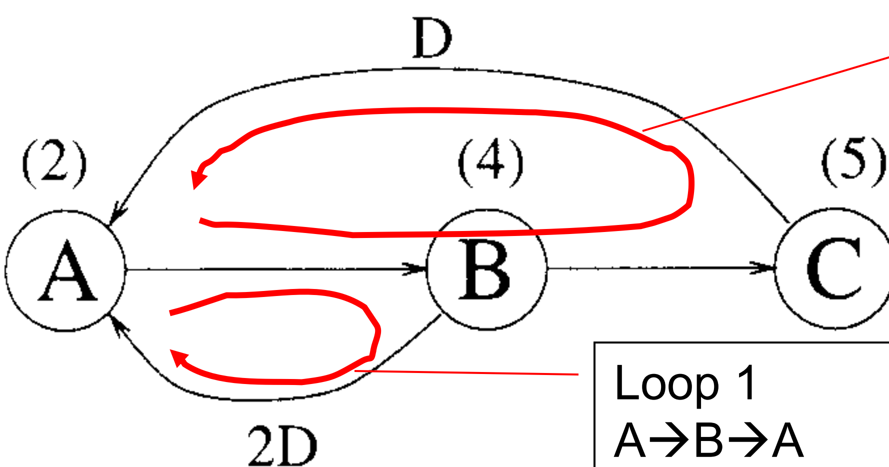
6/2=3 u.t.

$$\begin{cases} A_0 \rightarrow B_0 \Rightarrow A_2 \rightarrow B_2 \Rightarrow A_4 \rightarrow B_4 \Rightarrow A_6 \rightarrow \cdots \\ A_1 \rightarrow B_1 \Rightarrow A_3 \rightarrow B_3 \Rightarrow A_5 \rightarrow B_5 \Rightarrow A_7 \rightarrow \cdots \end{cases}$$

independent

Iteration Bound (1/3)

- Critical loop
  - The loop with the maximum loop bound
- Iteration bound
  - Loop bound of the critical loop $T_\infty = \max_{l \in L} \left\{ \frac{t_l}{w_l} \right\}$



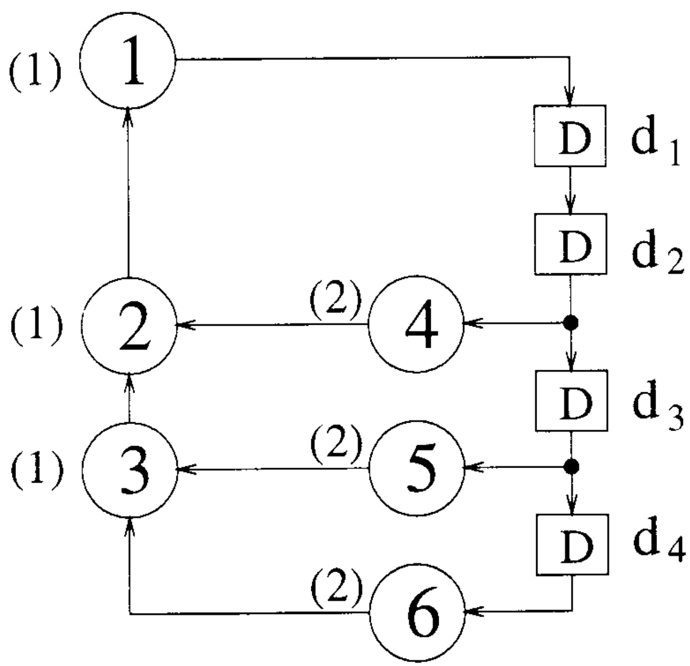
Loop 2
A→B→C→A
Loop bound: 11/1=11 u.t.

Loop 1
A→B→A
Loop bound: 6/2=3 u.t.

$$T_\infty = \max \left\{ \frac{6}{2}, \frac{11}{1} \right\} = 11 \text{ u.t.}$$

Iteration Bound (2/3)

- An other example



L1: 1→4→2→1

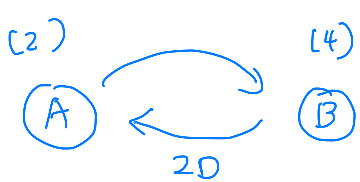
L2: 1→5→3→2→1

L3: 1→6→3→2→1

$$T_\infty = \max\left\{\frac{4}{2}, \frac{5}{3}, \frac{5}{4}\right\} = 2 \text{ u.t.}$$

Iteration Bound (3/3)

- **Iteration bound** is the lower bound on the iteration or sample period of the DSP program regardless of the amount of computing resources available



For example, the left circuit has its iteration bound $\frac{4+2}{2} = 3$ u.t.

However, the architecture on the left has critical path 6.u.t.

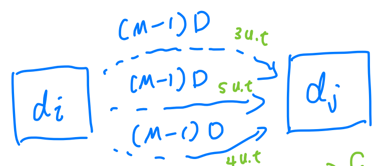
We may re-structure to retiming the circuit to approach its iteration bound,

Algorithms for Computing Iteration Bound

- Longest path matrix algorithm
  - We only introduce this one
- Minimum cycle mean algorithm
- Negative cycle detection algorithm

Longest Path Matrix Algorithm (LPM) (1/8)

- There are d delay elements in the DFG
- First, construct a series of matrices $L^{(m)}$, $m=1,2,\ldots,d$



⇒ find the longest delay (candidate for critical path)
$L_{i,j}^{(m)} = 5$ u.t.

- $l_{i,j}^{(m)}$
  - Longest computation time of all paths from delay element d_i to d_j that pass through exactly m-1 delays
  - If no such path exists, then $l_{i,j}^{(m)}=-1$

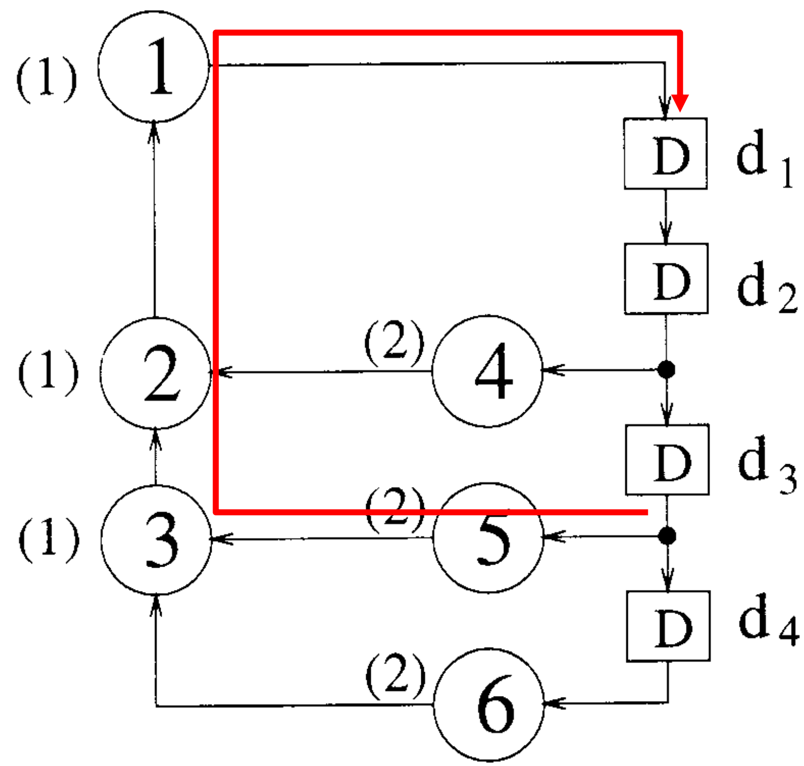
LPM (2/8)

- Ex:

(1-1) D

$\boxed{d_3}$ - - - $\boxed{d_1}$

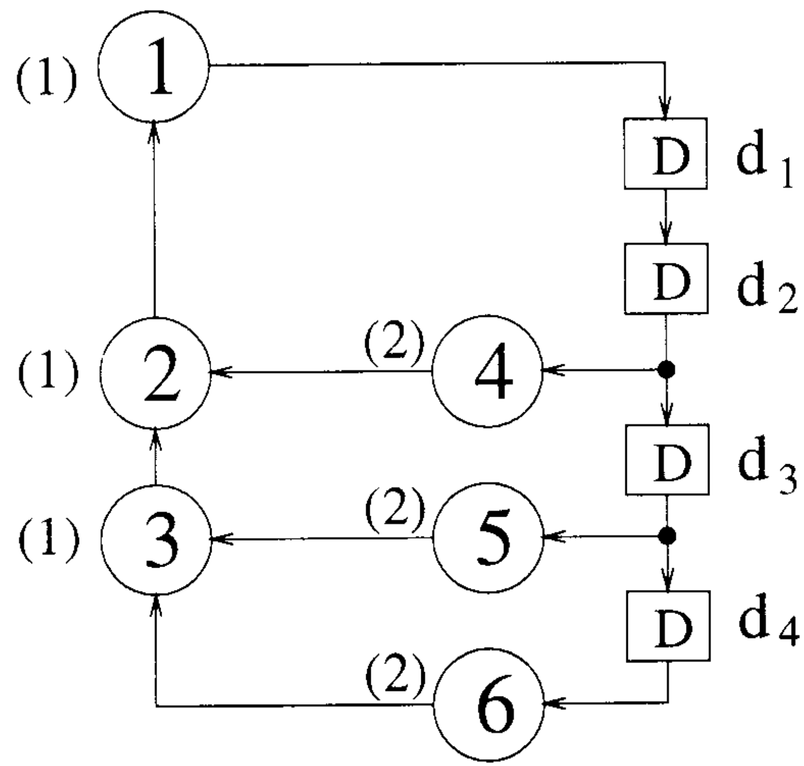
- $l_{3,1}^{(1)}$
  - d3→n5→n3→n2→n1→d1
  - So $l_{3,1}^{(1)}=5$
- $l_{4,3}^{(1)}$
  - No such path (2 delays, at least)
  - So $l_{4,3}^{(1)}=-1$



LPM (3/8)

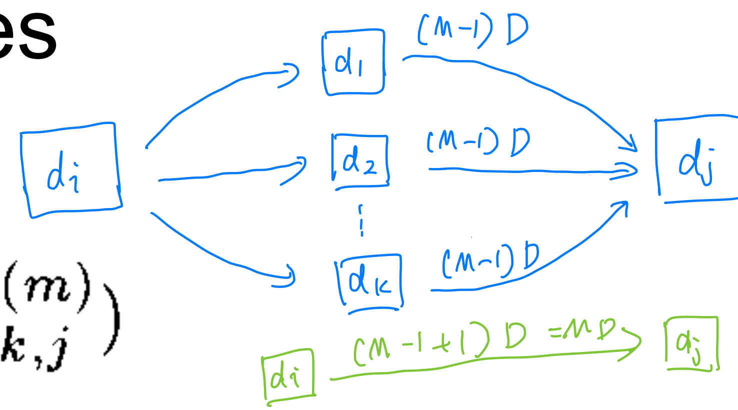
$$
\mathbf{L}^{(1)} = \begin{array}{c|cccc} & \text{O/P } d1 & d2 & d3 & d4 \\ \hline \text{I/P } d1 & -1 & 0 & -1 & -1 \\ d2 & 4 & -1 & 0 & -1 \\ d3 & 5 & -1 & -1 & 0 \\ d4 & 5 & -1 & -1 & -1 \end{array}
$$

pass through (1-1) FFs.



LPM (4/8)

- The higher order matrices
  - Can be derived from $L^{(1)}$
  - $$l_{i,j}^{(m+1)} = \max_{k \in K}(-1, l_{i,k}^{(1)} + l_{k,j}^{(m)})$$
  - K is the set of integers k in the interval [1,d] such that neither $l_{i,k}^{(1)}=-1$ nor $l_{k,j}^{(m)}=-1$ holds



LPM (5/8)

- Ex:
- $l_{2,1}^{(2)}$

$l_{k,j}^{(m)}$

$$\mathbf{L}^{(1)} = \begin{array}{cc} \text{O/P} & \begin{array}{cccc}\text{d1} & \text{d2} & \text{d3} & \text{d4}\end{array} \\ \begin{array}{r}\text{I/P d1} \\ \text{d2} \\ \text{d3} \\ \text{d4}\end{array} & \begin{bmatrix} -1 & 0 & -1 & -1 \\ 4 & -1 & 0 & -1 \\ 5 & -1 & -1 & 0 \\ 5 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

$l_{i,k}^{(1)}$

$$\begin{array}{cc} \cancel{4} & \cancel{-1} \\ \cancel{-1} & \cancel{4} \\ 0 & 5 \\ \cancel{-1} & \cancel{5} \end{array}$$

$$\begin{aligned} l_{2,1}^{(2)} &= \max_{k\in\{3\}} (-1, l_{2,k}^{(1)} + l_{k,1}^{(1)}) \\ &= \max(-1, 0+5) = 5. \end{aligned}$$

LPM (6/8)

$$
\mathbf{L}^{(1)} = \begin{array}{c|cccc} \text{I/P} \backslash \text{O/P} & d1 & d2 & d3 & d4 \\ \hline d1 & -1 & 0 & -1 & -1 \\ d2 & 4 & -1 & 0 & -1 \\ d3 & 5 & -1 & -1 & 0 \\ d4 & 5 & -1 & -1 & -1 \end{array}
$$

- L1, L1 ➔ L2
- L1, L2 ➔ L3 (L2, L1 → L3)
- L1, L3 ➔ L4 (L2, L2 → L4)

$$
\mathbf{L}^{(2)} = \begin{bmatrix} 4 & -1 & 0 & -1 \\ 5 & 4 & -1 & 0 \\ 5 & 5 & -1 & -1 \\ -1 & 5 & -1 & -1 \end{bmatrix}
$$

$$
\mathbf{L}^{(3)} = \begin{bmatrix} 5 & 4 & -1 & 0 \\ 8 & 5 & 4 & -1 \\ 9 & 5 & 5 & -1 \\ 9 & -1 & 5 & -1 \end{bmatrix}
$$

$$
\mathbf{L}^{(4)} = \begin{array}{c|cccc} \text{I/P} \backslash \text{O/P} & d_1 & d_2 & d_3 & d_4 \\ \hline d_1 & 8 & 5 & 4 & -1 \\ d_2 & 9 & 8 & 5 & 4 \\ d_3 & 10 & 9 & 5 & 5 \\ d_4 & 10 & 9 & -1 & 5 \end{array}
$$

$$
l^{(4)}_{3,1} = \overset{L_1 \quad 2}{[5 \ -1 \ -1 \ 0]} \quad L_3 \rightarrow \begin{bmatrix} 5 \\ 8 \\ 9 \\ 9 \end{bmatrix} \rightarrow 10.
$$

LPM (7/8)

- Iteration bound:

diagonal

$\boxed{d_i}$

total: mD

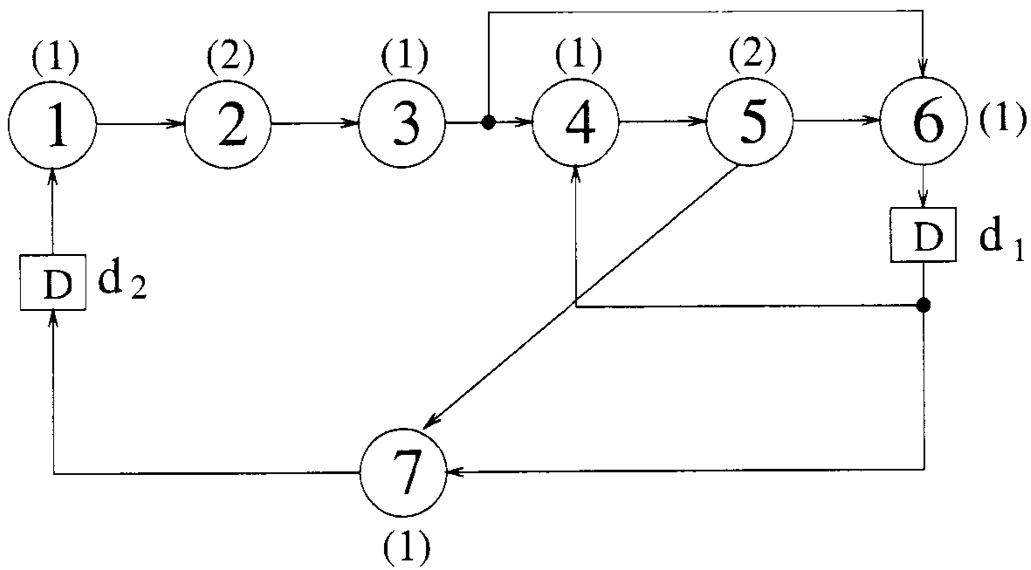
$(m-1)D$

$$T_\infty = \max_{i,m\in\{1,2,\ldots d\}} \left\{ \frac{l_{i,i}^{(m)}}{m} \right\}$$

$$T_\infty = \max\left\{\frac{4}{2},\frac{4}{2},\frac{5}{3},\frac{5}{3},\frac{5}{3},\frac{8}{4},\frac{8}{4},\frac{5}{4},\frac{5}{4}\right\} = 2.$$

LPM (8/8)

- Another example



$$\mathbf{L}^{(1)} = \begin{bmatrix} 4 & 4 \\ 8 & 8 \end{bmatrix}$$

$$\mathbf{L}^{(2)} = \begin{bmatrix} 12 & 12 \\ 16 & 16 \end{bmatrix}$$

$$T_\infty = \max\left\{\frac{4}{1}, \frac{8}{1}, \frac{12}{2}, \frac{16}{2}\right\} = 8.$$