

Optimal Control and Estimation

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Preface

This is the textbook for Harvard ES/AM 158: Introduction to Optimal Control and Estimation. Information about the offerings of the class is listed below.

2023 Fall

Time: Mon/Wed 2:15 - 3:30pm

Location: Science and Engineering Complex, Room TBD

Instructor: Heng Yang

Teaching Fellow: Weiyu Li

Syllabus

Acknowledgment

Chapter 1

The Optimal Control Formulation

1.1 The Basic Problem

Consider a discrete-time dynamical system

$$x_{k+1} = f_k(x_k, u_k, w_k), \quad k = 0, 1, \dots, N-1 \quad (1.1)$$

where

- $x_k \in \mathbb{X} \subseteq \mathbb{R}^n$ is the *state* of the system,
- $u_k \in \mathbb{U} \subseteq \mathbb{R}^m$ is the *control* we wish to design,
- $w_k \in \mathbb{W} \subseteq \mathbb{R}^p$ a random *disturbance* or noise (e.g., due to unmodelled dynamics) which is described by a probability distribution $P_k(\cdot \mid x_k, u_k)$ that may depend on x_k and u_k but not on prior disturbances w_0, \dots, w_{k-1} ,
- k indexes the discrete time,
- N denotes the horizon,
- f_k models the transition function of the system (typically $f_k \equiv f$ is time-invariant, especially for robotics systems; we use f_k here to keep full generality).

Remark (Deterministic v.s. Stochastic). When $w_k \equiv 0$ for all k , we say the system (1.1) is *deterministic*; otherwise we say the system is *stochastic*. In the following we will deal with the stochastic case, but most of the methodology should carry over to the deterministic setup.

We consider the class of *controllers* (also called *policies*) that consist of a sequence of functions

$$\pi = \{\mu_0, \dots, \mu_{N-1}\},$$

where $\mu_k(x_k) \in \mathbb{U}$ for all x_k , i.e., μ_k is a *feedback* controller that maps the state to an admissible control. Given an initial state x_0 and an admissible policy π , the state *trajectory* of the system is a sequence of random variables that evolve according to

$$x_{k+1} = f_k(x_k, \mu_k(x_k), w_k), \quad k = 0, \dots, N-1 \quad (1.2)$$

where the randomness comes from the disturbance w_k .

We assume the state-control trajectory $\{u_k\}_{k=0}^{N-1}$ and $\{x_k\}_{k=0}^N$ induce an *additive cost*

$$g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k) \quad (1.3)$$

where $g_k, k = 0, \dots, N$ are some user-designed functions.

With (1.2) and (1.3), for any admissible policy π , we denote its induced *expected cost* with initial state x_0 as

$$J_\pi(x_0) = \mathbb{E} \left\{ g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k)) \right\},$$

where the expectation is taken over the randomness of w_k .

Chapter 2

Literature

Here is a review of existing methods.

Chapter 3

Methods

We describe our methods in this chapter.

Math can be added in body using usual syntax like this

3.1 math example

p is unknown but expected to be around $1/3$. Standard error will be approximated

$$SE = \sqrt{\left(\frac{p(1-p)}{n}\right)} \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

You can also use math in footnotes like this¹.

We will approximate standard error to 0.027^2

¹where we mention $p = \frac{a}{b}$

² p is unknown but expected to be around $1/3$. Standard error will be approximated

$$SE = \sqrt{\left(\frac{p(1-p)}{n}\right)} \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

Chapter 4

Applications

Some *significant* applications are demonstrated in this chapter.

4.1 Example one

4.2 Example two

Chapter 5

Final Words

We have finished a nice book.