# Synthetic Acceleration for Radiative Diffusion Calculations

## Notes on Morel, Larsen, Matzen 1985 [1]

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## Section 1 Radiative Diffusion Equations

The radiative diffusion equations are:

$$\rho C_v \frac{\partial T}{\partial t} = \nabla \cdot K \nabla T - \nabla \mathbb{P} u + \rho \int_0^\infty \kappa(E') \left[ I(E') - \beta(E', T) \right] dE' + Q \tag{1}$$

$$\frac{1}{c} \frac{\partial I}{\partial t} - \nabla \cdot D(E) \nabla I(E) + \rho \kappa(E) I(E) = \rho \kappa(E) \beta(E, T), \tag{2}$$

where $t$ is time, $\vec{r}$ is position, $E = h\nu$ is photon energy, $C_v$ is material specific heat, $K$ is thermal conductivity, $\mathbb{P}$ is the pressure tensor (from both material and radiation pressure), $u$ is material velocity, $\kappa$ is specific absorption opacity in units of [area/mass], $\rho$ is material density, $D$ is the photon diffusion coefficient, $T$ is material temperature, $I(\vec{r}, E) = \int d\Omega \, I(x, \Omega, E)$ is photon intensity in units of [energy/area $\cdot$ time], and $Q$ is material energy source.

Integrating Equation 1 and Equation 2 over $N_g$ energy groups yields the multigroup radiative diffusion equations:

$$\rho C_v \frac{\partial T}{\partial t} = \nabla \cdot K \nabla T - \nabla \mathbb{P} u + \rho \sum_{g=1}^{N_g} \kappa_g \left[ I_g - \beta_g(T) \right] + Q \tag{3}$$

$$\frac{1}{c} \frac{\partial I_k}{\partial t} - \nabla \cdot D_k \nabla I_k + \rho \kappa_k I_k = \rho \kappa_k \beta_k(T); \qquad k = 1 \ldots N_g \tag{4}$$

where

$$\int_k \cdot \, dE = \int_{E_k}^{E_{k+1}} \cdot \, dE \tag{5a}$$

$$I_k = \int_k I(E) dE \tag{5b}$$

$$\beta_k(T) = \int_k \beta(E,T) dE \tag{5c}$$

$$D_k \nabla I_k = \int_k D(E) \nabla I(E) dE \tag{5d}$$

$$\kappa_k = \frac{\int_g \kappa(E) \left[ \beta(E) - I(E) \right] dE}{\int_g \left[ \beta(E) - I(E) \right] dE} \tag{5e}$$

where the multigroup equations are only exact if the $E$-spectrum is known exactly within each energy group, though the approximation is close to exact when the number of energy groups is high. Rosseland- or Planck-weighted group constants are used to approximate the spectrum. Pressure $\mathbb{P}$, material velocity $\vec{u}$, thermal conductivity $K$, and density $\rho$ are taken to be known exactly, since they come from hydrodynamic equations.

Discretizing in time with Backward Euler and using lagged opacity, thermal conductivity, diffusion coefficient, and heat capacity:

$$\rho C_v^n \frac{T^{n+1} - T^n}{\Delta t^n} = \nabla \cdot K^n \nabla T^{n+1} - \nabla \mathbb{P} u + \rho \sum_{g=1}^{N_g} \kappa_g^n \left[ I_g^{n+1} - \beta_g^{n+1}(T) \right] + Q^{n+1} \tag{6}$$

$$\frac{1}{c} \frac{I^{n+1} - I^n}{\Delta t^n} - \nabla \cdot D_k^n \nabla I_k^{n+1} + \rho \kappa_k^n I_k^{n+1} = \rho \kappa_k^n \beta_k(T(t)); \qquad k = 1 \dots N_g \tag{7}$$

Then, the time advancement of $\beta$ can be approximated using a first-order Taylor expansion, $\beta_k^{n+1} \approx \beta_k^n + \frac{\partial \beta_k^n}{\partial T} \Delta T^{n+1}$, and the temperature operator is split into two parts– radiative/inhomogenous source and thermal conductivity/pressure. The linearized, time-discretized equations are:

$$\rho C_v^n \frac{\Delta T^{n+1/2}}{\Delta t^n} = \rho \sum_{g=1}^{N_g} \kappa_g^n \left[ I_g^{n+1} - \left( \beta_k^n + \frac{\partial \beta_k^n}{\partial T} \Delta T^{n+1} \right) \right] + Q^{n+1} \tag{8}$$

$$\frac{1}{c} \frac{I^{n+1} - I^n}{\Delta t^n} - \nabla \cdot D_k^n \nabla I_k^{n+1} + \rho \kappa_k^n I_k^{n+1} = \rho \kappa_k^n \left( \beta_k^n + \frac{\partial \beta_k^n}{\partial T} \Delta T^{n+1} \right); \qquad k = 1 \dots N_g \tag{9}$$

$$\rho C_v^n \frac{\Delta T^{n+1}}{\Delta t^n} = \nabla \cdot K^n \nabla T^{n+1} - \nabla \cdot \mathbb{P} u \tag{10}$$

Where $\Delta T^{n+1/2}$ is an intermediate value for the temperature at $t^{n+1}$, not at an intermediate time step, and $T^{n+1/2} = T^n + \Delta T^{n+1/2}$ and $T^{n+1} = T^{n+1/2} + \Delta T^{n+1}$. $\Delta T^{n+1/2}$ can be easily evaluated algebraically from Equation 8:

$$\Delta T^{n+1/2} = \left[ \sum_{g=1}^{N_g} \kappa_g^n \left( I_g^{n+1} - \beta_k^n \right) + \frac{Q^{n+1}}{\rho} \right] \cdot \left[ \frac{C_v^n}{\Delta t^n} + \sum_{g=1}^{N_g} \kappa_g^n \frac{\partial \beta_g^n}{\partial T} \right]^{-1} \tag{11}$$

Evaluating Equation 9 using Equation 11 yields

$$\frac{1}{c\Delta t^n} \left( I^{n+1} - I^n \right) - \nabla \cdot D_k^n \nabla I_k^{n+1} + \rho \kappa_k^n I_k^{n+1} = \dots TODO \tag{12}$$

which reduces to the final form of the multigroup diffusion equation,

$$\frac{1}{c\Delta t^n} \left( I^{n+1} - I^n \right) - \nabla \cdot D_k^n \nabla I_k^{n+1} + \rho \kappa_k^n I_k^{n+1} = \eta \chi_k \rho \sum_{g=1}^{N_g} \kappa_g I_g^{n+1} + q_k \tag{13}$$

where

$$\eta = \left[ \sum_{g=1}^{N_g} \kappa_g \frac{\partial \beta_g}{\partial T} \right] \cdot \left[ \frac{C_v}{\Delta t^n} + \sum_{g=1}^{N_g} \kappa_g \frac{\partial \beta_g}{\partial T} \right]^{-1} \tag{14a}$$

$$\chi_k = \left[ \kappa_k \frac{\partial \beta_k}{\partial T} \right] \cdot \left[ \sum_{g=1}^{N_g} \kappa_g \frac{\partial \beta_g}{\partial T} \right]^{-1} \tag{14b}$$

$$q_k = \rho \kappa_k \beta_k + \eta \chi_k \left[ Q - \rho \sum_{g=1}^{N_g} \kappa_g \frac{\partial \beta_g}{\partial T} \right] \tag{14c}$$

Equation 13 resembles the steady-state neutron diffusion equation, which can be made clear by further rearranging the terms:

$$-\nabla \cdot D_k \nabla I_k^{n+1} + \left( \sigma_a + \sigma_{f,k} \right) I_k^{n+1} = \eta \chi_k \sum_{g=1}^{N_g} \sigma_{f,g} I_g^{n+1} + S_k \tag{15}$$

where

$$\sigma_a = \frac{1}{c\Delta t^n} \tag{16a}$$

$$\sigma_{f,k} = \rho \kappa_k \tag{16b}$$

$$S_k = q_k + \frac{I_k^n}{c\Delta t^n} \tag{16c}$$

with "fission" and "absorption" cross sections, and fission emission spectrum $\chi_k$. The fission term will be known as the reimission term.

**1(a) Iterative solution**  Equation 15 can be solved by source iteration, starting from some initial guess $I_k^{(0)}$ and using to compute the next iterate $I^{(1)}$, or using each iterate $I_k^l$ to compute $I_k^{l+1}$. Surpressing the $n+1$ index, the iterative equation is

$$-\nabla \cdot D_k \nabla I_k^{l+1} + (\sigma_a + \sigma_{f,k}) I_k^{l+1} = \eta \chi_k \sum_{g=1}^{N_g} \sigma_{f,g} I_g^l + S_k \tag{17}$$

# Section 2  Acceleration Scheme

The iterative method described in subsection 1(a) can be written in operator notation as the solution to the linear system

$$\mathcal{M}x = y, \tag{18}$$

where the exact solution is denoted as $x^\star$. The solution is solved iteratively by splitting the operator $\mathcal{M} = \mathcal{A} - \mathcal{B}$, and $\mathcal{A}$ is easily inverted. The iterative method is then

$$\mathcal{A}x^{l+1} = \mathcal{B}x^l + y \tag{19}$$

$$x^{l+1} = \mathcal{A}^{-1}\mathcal{B}x^l + \mathcal{A}^{-1}y. \tag{20}$$

For the radiative diffusion equation (Equation 15), $\mathcal{A}$ is the streaming and absorption operator, and $\mathcal{B}$ is the reemission ("fission") source. The error of the $(l+1)$th iterate is

$$\epsilon^{l+1} = x^\star - x^{l+1} \tag{21}$$

Then the exact solution can be found from the $(l+1)$th iterate using

$$\mathcal{B}x^l + y = \mathcal{A}x^{l+1} \tag{22a}$$

$$y = \mathcal{A}x^{l+1} - \mathcal{B}x^l \tag{22b}$$

$$\mathcal{M}x^\star = \mathcal{A}x^{l+1} - \mathcal{B}x^l \tag{22c}$$

$$x^\star = \mathcal{M}^{-1}\left(\mathcal{A}x^{l+1} - \mathcal{B}x^l\right) \tag{22d}$$

$$= \mathcal{M}^{-1}\left(\mathcal{A}x^{l+1} - \mathcal{B}x^{l+1} + \mathcal{B}x^{l+1} - \mathcal{B}x^l\right) \tag{22e}$$

$$= \mathcal{M}^{-1}\left(\mathcal{M}x^{l+1} + \mathcal{B}(x^{l+1} - x^l)\right) \tag{22f}$$

$$x^\star = x^{l+1} + \mathcal{M}^{-1}\mathcal{B}(x^{l+1} - x^l) = x^{l+1} + \epsilon^{l+1} \tag{22g}$$

The iteration can be accelerated by evaluating Equation 22g using an approximation $\mathcal{W} \approx \mathcal{M}$, which can be more easily inverted. Here $\mathcal{M}$ is called the high-order operator, and $\mathcal{W}$ is the low-order operator.

The accelerated iteration scheme is

$$x^{l+1/2} = \mathcal{A}^{-1}\mathcal{B}x^l + \mathcal{A}^{-1}y \tag{23}$$

$$x^{l+1} = x^{l+1/2} + \mathcal{W}^{-1}\mathcal{B}\left(x^{l+1/2} - x^l\right) \tag{24}$$

when solving the radiative diffusion equation, the high-order operator $\mathcal{M}$ is the system of multigroup diffusion equations. The acceleerated scheme presented in [1] uses a one-group diffusion operator as the low-order operator $\mathcal{W}$.

Fourier analyis of the high-order equation shows that the slowest-converging mode is $\omega(\lambda = 0)$, or

$$\omega_{\mathrm{max}} = \omega(0) = \eta \sum_{g=1}^{N_g} \frac{\chi_g \sigma_{f,g}}{\sigma_a + \sigma_{f,g}} \tag{25}$$

Where $\omega_{\mathrm{max}}$ approaches 1 as the $\Delta t^n$ is increased. For small time steps, $\omega_{\mathrm{max}} \approx \eta$, which approaches unity for strongly coupled problems, where $C_v$ is small or $\kappa$ is large.

The most effective acceleration schemes should approximate the high-order equations accurately about $\lambda = 0$. This solution corresponds with an equilibrium solution, which will be the focus of the low-order equation. section 3 details the derivation of this result.

The multigroup equation for the error of the $l$th iterate of Equation 17 is

$$-\nabla \cdot D_k \nabla \epsilon_k^l + (\sigma_a + \sigma_{f,k})\,\epsilon_k^l = \eta \chi_k \sum_{g=1}^{N_g} \sigma_{f,g} \left[\epsilon_g^l + I_g^l - I_g^{l-1}\right]; \ k = 1, 2, \dots, N_g \tag{26}$$

In the equilibrium limit, where the gradient term becomes zero, this equation becomes

$$(\sigma_a + \sigma_{f,k})\,\epsilon_k^l = \eta \chi_k \sum_{g=1}^{N_G} \sigma_{f,g}\epsilon_g^l + \eta \chi_k \sum_{g=1}^{N_G} \sigma_{f,g}\left(I_g^l - I_g^{l-1}\right) \tag{27}$$

$$\sigma_{t,k}\epsilon_k^l = \eta \chi_k \left(\xi^l + r^l\right) \tag{28}$$

$$\epsilon_k^l = \frac{\eta \chi_k}{\sigma_{t,k}}\left(\xi^l + r^l\right) \tag{29}$$

where

$$\sigma_{t,k} = \sigma_a + \sigma_{f,k} \tag{30}$$

$$r^l = \sum_{g=1}^{N_G} \sigma_{f,g}\left(I_g^l - I_g^{l-1}\right) \tag{31}$$

$$\xi^l = \sum_{g=1}^{N_G} \sigma_{f,g}\epsilon_g^l \tag{32}$$

then, to solve for $\xi^i$, Equation 29 is multiplied by $\sigma_{f,k}$ and summed over all $k$:

$$\sum_{k=1}^{N_g} \sigma_{f,k} \epsilon_k^l = \sum_{k=1}^{N_g} \sigma_{f,k} \frac{\eta \chi_k}{\sigma_{t,k}} \left( \xi^l + r^l \right) \tag{33}$$

$$\xi^l = \eta \left( \xi^l + r^l \right) \sum_{k=1}^{N_g} \frac{\chi_k \sigma_{f,k}}{\sigma_{t,k}} \tag{34}$$

$$\xi^l = \eta \left( \xi^l + r^l \right) \frac{\langle \sigma_f \rangle}{\langle \sigma_t \rangle} \tag{35}$$

where

$$\langle \sigma_f \rangle = \langle \sigma_t \rangle \sum_{g=1}^{N_g} \frac{\chi_g \sigma_{f,g}}{\sigma t, g} \tag{36}$$

$$\langle \sigma_t \rangle = \left[ \sum_{g=1}^{N_g} \frac{\chi_g}{\sigma_{t,g}} \right]^{-1} \tag{37}$$

then Equation 35 can be solved for $\xi^l$,

$$\xi^l = \eta \left( \xi^l + r^l \right) \frac{\langle \sigma_f \rangle}{\langle \sigma_t \rangle} \tag{38a}$$

$$\frac{\xi^l \langle \sigma_t \rangle}{\eta \langle \sigma_f \rangle} = \left( \xi^l + r^l \right) \tag{38b}$$

$$\xi^l = \frac{r^l}{\frac{\langle \sigma_t \rangle}{\eta \langle \sigma_f \rangle} - 1} \tag{38c}$$

$$\xi^l = \frac{\eta \langle \sigma_f \rangle r^l}{\langle \sigma_t \rangle - \eta \langle \sigma_f \rangle} \tag{38d}$$

yielding the final equilibrium solution of the $l$th iterate error,

$$\epsilon_k^l = \frac{\eta \chi_k}{\sigma_{t,k}} \left( \xi^l + r^l \right) \tag{39a}$$

$$\sigma_{t,k} = \sigma_a + \sigma_{f,k} \tag{39b}$$

$$r^l = \sum_{g=1}^{N_G} \sigma_{f,g} \left( I_g^l - I_g^{l-1} \right) \tag{39c}$$

$$\xi^l = \frac{\eta \langle \sigma_f \rangle r^l}{\langle \sigma_t \rangle - \eta \langle \sigma_f \rangle} \tag{39d}$$

$$\langle \sigma_f \rangle = \langle \sigma_t \rangle \sum_{g=1}^{N_g} \frac{\chi_g \sigma_{f,g}}{\sigma t, g} \tag{39e}$$

$$\langle \sigma_t \rangle = \left[ \sum_{g=1}^{N_g} \frac{\chi_g}{\sigma_{t,g}} \right]^{-1} \tag{39f}$$

Then the specral shape (normalized) of this solution, where $E^l = \sum_{g=1}^{N_g} \epsilon_k^l$, is

$$\frac{\epsilon_k^l}{E^l} = \frac{\frac{\eta \chi_k}{\sigma_{t,k}} \left( \xi^l + r^l \right)}{\sum_{g=1}^{N_g} \frac{\eta \chi_g}{\sigma_{t,g}} \left( \xi^l + r^l \right)} \tag{40a}$$

$$= \frac{\eta \left( \xi^l + r^l \right) \frac{\chi_k}{\sigma_{t,k}}}{\eta \left( \xi^l + r^l \right) \sum_{g=1}^{N_g} \frac{\chi_g}{\sigma_{t,g}}} \tag{40b}$$

$$= \frac{\frac{\chi_k}{\sigma_{t,k}}}{\sum_{g=1}^{N_g} \frac{\chi_g}{\sigma_{t,g}}} \tag{40c}$$

$$\frac{\epsilon_k^l}{E^l} = \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}}. \tag{40d}$$

Which is exact as the time step $\Delta t$ approaches infinity (in the equilibrium limit), and is equivalent to the spectral shape used to evaluate Rosseland group opacities. Then, evaluating Equation 26 using the assumed spectral shape,

$$-\nabla \cdot D_k \nabla E^l \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} + \left( \sigma_a + \sigma_{f,k} \right) E^l \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} = \eta \chi_k r^l + \eta \chi_k \sum_{g=1}^{N_g} \sigma_{f,g} E^l \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}}. \tag{41}$$

Expanding the diffusion term using the product rule,

$$-\nabla \cdot \left[ D_k \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \nabla E^l + D_k E^l \nabla \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \right] + (\sigma_a + \sigma_{f,k}) E^l \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} =$$

$$\eta \chi_k r^l + \eta \chi_k \sum_{g=1}^{N_g} \sigma_{f,g} E^l \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}}. \quad (42)$$

Summing over all groups,

$$-\nabla \cdot \left[ \left( \sum_{k=1}^{N_g} D_k \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \right) \nabla E^l + E^l \left( \sum_{k=1}^{N_g} D_k \nabla \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \right) \right] + E^l \sum_{k=1}^{N_g} (\sigma_a + \sigma_{f,k}) \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} =$$

$$\eta r^l + \sum_{k=1}^{N_g} \eta E^l \chi_k \langle \sigma_t \rangle \sum_{g=1}^{N_g} \sigma_{f,g} \frac{\chi_g}{\sigma_{t,g}} \quad (43)$$

$$-\nabla \cdot \left[ \left( \sum_{k=1}^{N_g} D_k \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \right) \nabla E^l + E^l \left( \sum_{k=1}^{N_g} D_k \nabla \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \right) \right] + E^l \sum_{k=1}^{N_g} (\sigma_a + \sigma_{f,k}) \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} =$$

$$\eta r^l + \eta E^l \langle \sigma_f \rangle, \quad (44)$$

which can be written as

$$-\nabla \cdot \left[ \langle D \rangle \nabla E^l + \langle D' \rangle E^l \right] + E^l \sum_{k=1}^{N_g} (\sigma_a + \sigma_{f,k}) \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} = \eta r^l + \eta E^l \langle \sigma_f \rangle, \quad (45)$$

where

$$\langle D \rangle = \sum_{k=1}^{N_g} D_k \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \quad (46a)$$

$$\langle D' \rangle = \sum_{k=1}^{N_g} D_k \nabla \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}}. \quad (46b)$$

Expanding the removal term,

$$E^l \sum_{k=1}^{N_g} (\sigma_a + \sigma_{f,k}) \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} = \sigma_a \sum_{k=1}^{N_g} E^l \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} + \sum_{k=1}^{N_g} E^l \frac{\langle \sigma_t \rangle \chi_k \sigma_{f,k}}{\sigma_{t,k}} \quad (47a)$$

$$= \sigma_a E^l + \langle \sigma_f \rangle E^l, \quad (47b)$$

so the one-group error diffusion equation becomes

$$-\nabla \cdot \left[\langle D \rangle \nabla E^l + \langle D' \rangle E^l\right] + E^l(\sigma_a + \langle \sigma_f \rangle) = \eta r^l + \eta E^l \langle \sigma_f \rangle \tag{48}$$

$$-\nabla \cdot \left[\langle D \rangle \nabla E^l + \langle D' \rangle E^l\right] + E^l \left[\sigma_a + \langle \sigma_f \rangle (1 - \eta)\right] = \eta r^l. \tag{49}$$

This provides an equation for the one-group iterative error of a diffusion iteration, which can be used to calculate group errors using Equation 40d. Then the set of equations for the accelerated radiative diffusion equation becomes:

$$-\nabla \cdot D_k \nabla I_k^{l+1} + (\sigma_a + \sigma_{f,k}) I_k^{l+1} = \eta \chi_k \sum_{g=1}^{N_g} \sigma_{f,g} I_g^l + S_k \tag{50}$$

$$-\nabla \cdot \left[\langle D \rangle \nabla E^{l+1/2} + \langle D' \rangle E^{l+1/2}\right] + E^{l+1/2} \left[\sigma_a + \langle \sigma_f \rangle (1 - \eta)\right] = \eta r^{l+1/2} \tag{51}$$

$$I^{l+1} = I_k^{l+1} + E^{l+1/2} \frac{\langle \sigma_t \rangle \chi_k}{\sigma_{t,k}} \tag{52}$$

where

$$r^{l+1/2} = \sum_{g=1}^{N_g} \sigma f, g \left(I_g^{l+1/2} - I_g^l\right). \tag{53}$$

The full algorithm for the time-dependent problem then becomes:

# Section 3 Fourier Analysis

[2]

# References

[1] J. E. Morel, E. W. Larsen, and M. Matzen, "A synthetic acceleration scheme for radiative diffusion calculations," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 34, no. 3, pp. 243–261, 1985.

[2] J. A. Fleck Jr and J. Cummings Jr, "An implicit monte carlo scheme for calculating time and frequency dependent nonlinear radiation transport," *Journal of Computational Physics*, vol. 8, no. 3, pp. 313–342, 1971.

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**Algorithm 1** Morel, Larsen, Matzen Diffusion Acceleration Scheme [1]

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**while** $t^j \leq t^{end}$ **do**
    $j \leftarrow j + 1$
    Compute temperature-dependent coefficients using $T|_{t=t^{j-1}}$
    **while** $\left\|I^{(s)} - I^{(s-1)}\right\| > \epsilon \left\|I^{(s-1)}\right\|$ **do**
        $s \leftarrow s + 1$
        Solve multigroup diffusion equation for $I_k^{(s+1/2)}$, using $I_k^{(s)}$ for the reemission source
        Compute $r^{(s+1/2)}$
        Solve one-group error diffusion equation for $E^{(s+1/2)}$
        Compute $I_k^{(s+1)} = I_k^{(s+1/2)} + E^{(s+1/2)} \langle \sigma_t \rangle \chi_k / \sigma_{t,k}$
    **end while**
    $I_k^j \leftarrow I_k^{(s+1)}$
    Compute $\Delta T^{j+1/2}$ from $I_k^j$
    **if** $K \neq 0 \wedge u \neq 0$ **then**
        Solve thermal diffusion equation for $\Delta T^{j+1}$
    **end if**
    $T^{j+1} \leftarrow T^j + \Delta T^{j+1/2} + \Delta T^{j+1}$
**end while**

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