NE 795 Assignment 3

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Question 1 Equilibrium Diffusion Limit

The asymptotic analysis of the full TRT problem (in intensity rather than energy density) finds that:

$$(C_v + 4a_R T^3)\frac{\partial T}{\partial t} - \nabla \frac{4a_R T^3}{3\varkappa}\nabla T = 0$$

To order of ϵ^1.

The $P_{1/3}$ equations are given by:

$$\begin{aligned}
\frac{\partial E}{\partial t} + \vec{\nabla}\cdot\vec{F} + c\varkappa E &= \varkappa ac T^4 \\
\frac{1}{3c}\frac{\partial F}{\partial t} + \frac{c}{3}\nabla E + \varkappa\vec{F} &= 0 \\
c_v\frac{\partial T}{\partial t} &= \varkappa c\left(E - aT^4\right)
\end{aligned}$$

where $a = a_R$. In the asymptotic limit, where $\frac{\partial}{\partial t} \to \epsilon^2\frac{\partial}{\partial t}$ and $\nabla \to \epsilon\nabla$, assuming the true distributions can be expanded as a power series in ϵ, the expanded equations become:

$$\epsilon^2\sum\epsilon^n\frac{\partial E^{(n)}}{\partial t} + \epsilon\vec{\nabla}\cdot\sum\epsilon^n\vec{F^{(n)}} + c\varkappa\sum\epsilon^n E^{(n)} = \varkappa ac\sum\epsilon^n\left(T^4\right)^{(n)} \tag{1}$$

$$\frac{\epsilon^2}{3c}\sum\epsilon^n\frac{\partial F^{(n)}}{\partial t} + \frac{\epsilon}{3}\sum\epsilon^n\nabla E^{(n)} + \varkappa\sum\epsilon^n\vec{F^{(n)}} = 0 \tag{2}$$

$$\epsilon^2 c_v\sum\epsilon^n\frac{\partial T^{(n)}}{\partial t} = \varkappa c\sum\epsilon^n\left(E^{(n)} - a\left(T^4\right)^{(n)}\right) \tag{3}$$

Then, the $O(\epsilon^0)$ equations are:

$$c\varkappa E^{(1)} = c\varkappa a\left(T^4\right)^{(1)} \tag{4}$$

$$\varkappa \vec{F^{(1)}} = 0 \tag{5}$$

$$0 = \varkappa c \left[E^{(0)} - a \left(T^4 \right)^{(0)} \right] \tag{6}$$

And the $O(\epsilon^1)$ equations are:

$$\vec{\nabla} \vec{F}^{(0)} + c\varkappa E^{(1)} = c\varkappa a \left(T^4 \right)^{(1)} \tag{7}$$

$$\frac{c}{3} \nabla E^{(0)} + \varkappa \vec{F^{(1)}} = 0 \tag{8}$$

$$0 = \varkappa c \left[E^{(1)} - a \left(T^4 \right)^{(1)} \right] \tag{9}$$

And the $O(\epsilon^2)$ equations are:

$$\frac{\partial E^{(0)}}{\partial t} + \vec{\nabla} \vec{F}^{(1)} + c\varkappa E^{(2)} = c\varkappa a \left(T^4 \right)^{(2)} \tag{10}$$

$$\frac{1}{3c} \frac{\partial F^{(0)}}{\partial t} + \frac{c}{3} \nabla E^{(1)} + \varkappa \vec{F^{(2)}} = 0 \tag{11}$$

$$c_v \frac{\partial T^{(0)}}{\partial t} = \varkappa c \left[E^{(2)} - a \left(T^4 \right)^{(2)} \right] \tag{12}$$

And the $O(\epsilon^3)$ equations are:

$$\frac{\partial E^{(1)}}{\partial t} + \vec{\nabla} \vec{F}^{(2)} + c\varkappa E^{(3)} = c\varkappa a \left(T^4 \right)^{(3)} \tag{13}$$

$$\frac{1}{3c} \frac{\partial F^{(1)}}{\partial t} + \frac{c}{3} \nabla E^{(2)} + \varkappa \vec{F^{(3)}} = 0 \tag{14}$$

$$c_v \frac{\partial T^{(1)}}{\partial t} = \varkappa c \left[E^{(3)} - a \left(T^4 \right)^{(3)} \right] \tag{15}$$

From these equations, some results are immediately apparant. From Equation 4 and Equation 5,

$$\boxed{E^{(0)} = a(T^4)^{(0)} = a \left(T^{(0)} \right)^4} \tag{16}$$

$$\boxed{\vec{F}^{(0)} = 0.} \tag{17}$$

From Equation 9,

$$0 = \varkappa c \left[E^{(1)} - a \left(T^4 \right)^{(1)} \right] \tag{18}$$

$$0 = \left[E^{(1)} - a \left(T^4 \right)^{(1)} \right] \tag{19}$$

$$\boxed{E^{(1)} = a \left(T^4 \right)^{(1)} = 4a \left(T^{(0)} \right)^3 T^{(1)}} \tag{20}$$

and from Equation 8:

$$\vec{F^{(1)}} = -\frac{c}{3\varkappa} \nabla E^{(0)} \tag{21}$$

$$\boxed{\vec{F^{(1)}} = -\frac{ca}{3\varkappa} \nabla \left(T^{(0)} \right)^4} \tag{22}$$

These results give the $O(\epsilon)$ and $O(1)$ asymptotic limits of energy density and flux. The temperature approximation can be found using Equation 10 and plugging in the results found above,

$$\frac{\partial E^{(0)}}{\partial t} + \vec{\nabla} \vec{F}^{(1)} + c\varkappa E^{(2)} = c\varkappa a \left(T^4 \right)^{(2)} \tag{23}$$

$$a \frac{\partial \left(T^{(0)} \right)^4}{\partial t} + \nabla \left(-\frac{ca}{3\varkappa} \nabla \left(T^{(0)} \right)^4 \right) + \cancelto{0}{c\varkappa E^{(0)}} = \cancelto{0}{c\varkappa E^{(0)}} - c_v \frac{\partial T^{(0)}}{\partial t} \tag{24}$$

$$c_v \frac{\partial T^{(0)}}{\partial t} + a \frac{\partial \left(T^{(0)} \right)^4}{\partial t} - \nabla \frac{ca}{3\varkappa} \nabla \left(T^{(0)} \right)^4 = 0 \tag{25}$$

and, since $\frac{\partial T^4}{\partial t} = 4T^3 \frac{\partial T}{\partial t}$ and $\nabla T^4 = 4T^3 \nabla T$, this result reduces to

$$\boxed{\left(c_v + 4a \left(T^{(0)} \right)^3 \right) \frac{\partial T^{(0)}}{\partial t} - \nabla \frac{4ca \left(T^{(0)} \right)^3}{3\varkappa} \nabla T^{(0)}.} \tag{26}$$

Then, similarly evaluating Equation 13, using terms from Equation 9 and Equation 11, and expanding T^4 terms as before,

$$\frac{\partial E^{(1)}}{\partial t} + \vec{\nabla}\vec{F}^{(2)} + c\varkappa E^{(3)} = c\varkappa a\left(T^4\right)^{(3)} \tag{27}$$

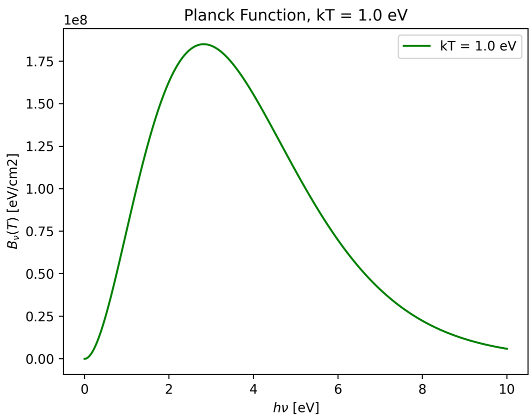
$$\frac{\partial}{\partial t}\left(a\left(T^4\right)^{(1)} = 4a\left(T^{(0)}\right)^3 T^{(1)}\right) + \vec{\nabla}\left(-\cancelto{0}{\frac{1}{3\varkappa c}\frac{\partial F^{(0)}}{\partial t}} - \frac{c}{3\varkappa}\nabla E^{(1)}\right) + \cancelto{0}{c\varkappa E^{(3)}} = \cancelto{0}{c\varkappa E^{(3)}} - c_v\frac{\partial T^{(1)}}{\partial t} \tag{28}$$

$$\boxed{c_v\frac{\partial T^{(1)}}{\partial t} + 4aT^{(0)3}\frac{\partial T^{(1)}}{\partial t} - \nabla\frac{4ca\left(T^{(0)}\right)^3}{3\varkappa}\nabla T^{(1)}} \tag{29}$$

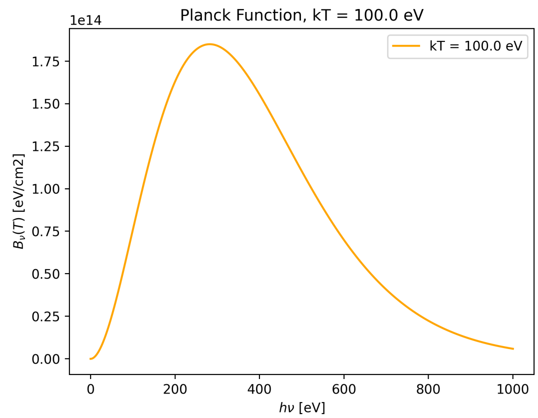
Then, the $O(\epsilon)$ approximation of temperature is $T = T^{(0)} + \epsilon T^{(1)}$. Then, multiplying the $T^{(1)}$ equation by ϵ and adding to the $T^{(0)}$ equation yields the equilibrium diffusion limit which is consistent with the tranpsort limit:

$$\boxed{\left(c_v + 4aT^3\right)\frac{\partial T}{\partial t} - \nabla\frac{4caT^3}{3\varkappa}\nabla T.} \tag{30}$$

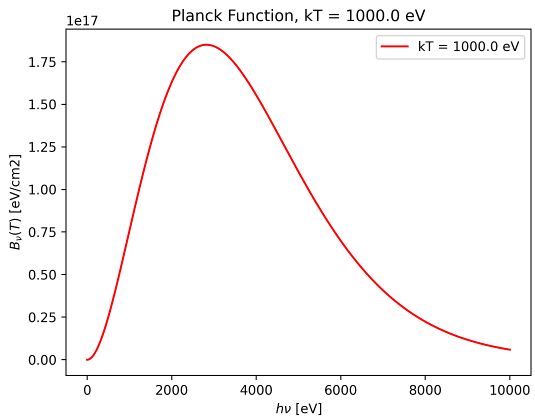
Question 2 Spectral Planck Function



(a) 1 eV



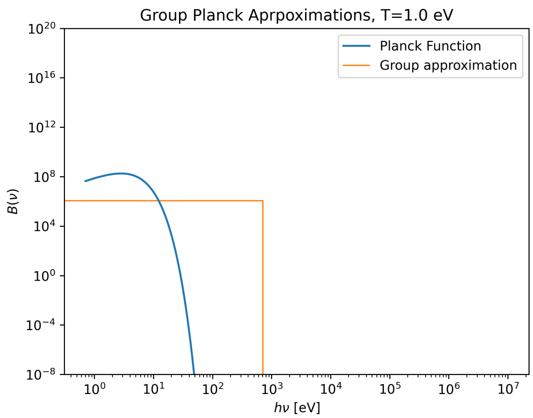
(b) 100 eV



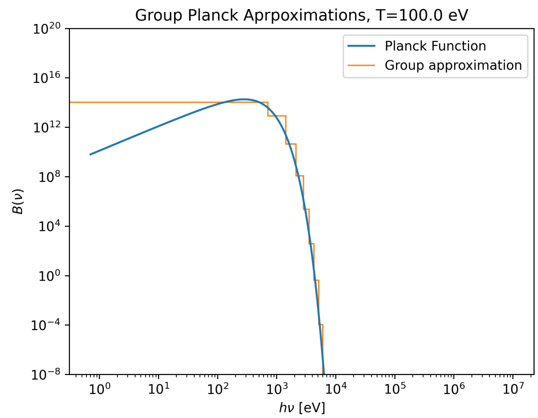
(c) 1000 eV

Figure 1: Planck function

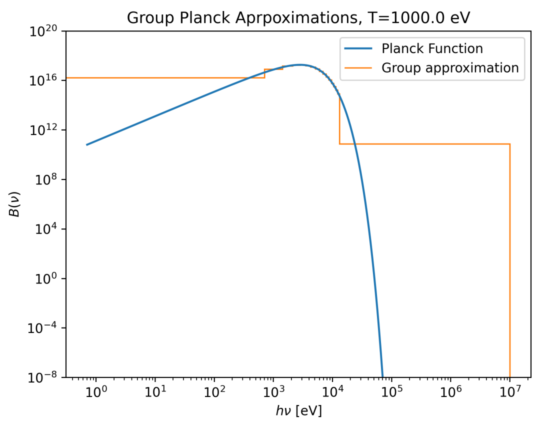
Question 3 Group Planck Function



(a) 1 eV



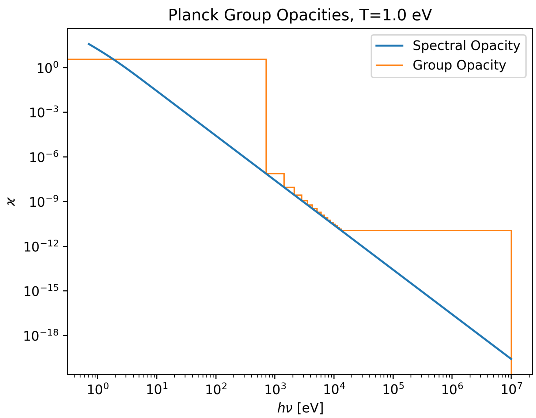
(b) 100 eV



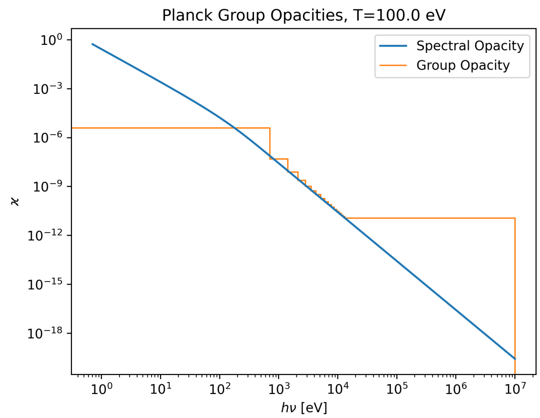
(c) 1000 eV

Figure 2: Grouped planck function

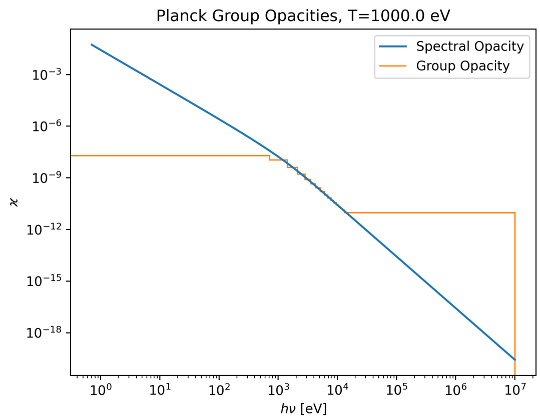
Question 4 Group Planck Opacities



(a) 1 eV



(b) 100 eV



(c) 1000 eV

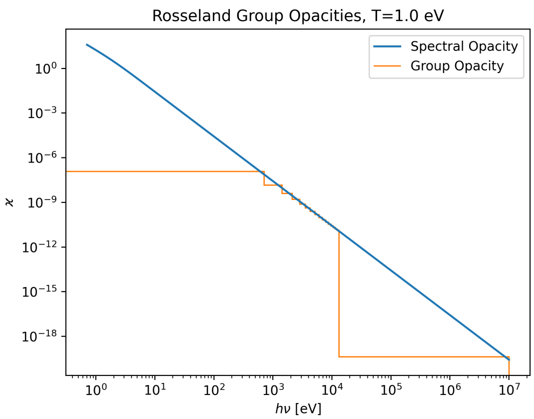
Figure 3: Group Planck Opacities

Question 5 Group Rosseland Opacities

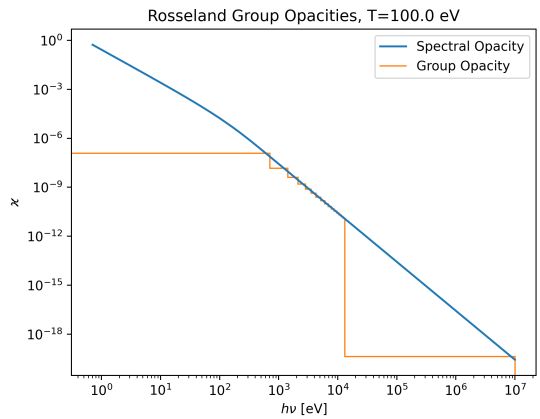
Algebraic manipulation yields a formula for Rosseland opacities when using Fleck-Cummings opacity:

$$\varkappa_{g,R} = \frac{\varkappa^\star}{h^3} \frac{\int_g d\nu\ \nu^4 \left(1 - e^{-h\nu/kT}\right)^{-2}}{\int_g d\nu\ \nu^7 \left(1 - e^{-h\nu/kT}\right)^{-3}} \tag{31}$$

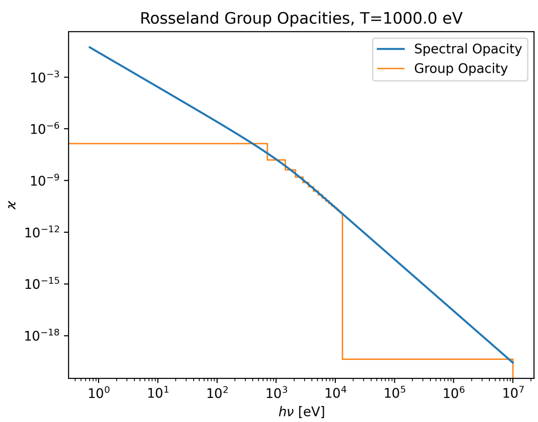
which I numerically integrated using `scipy.integrate.quad()`.



(a) 1 eV

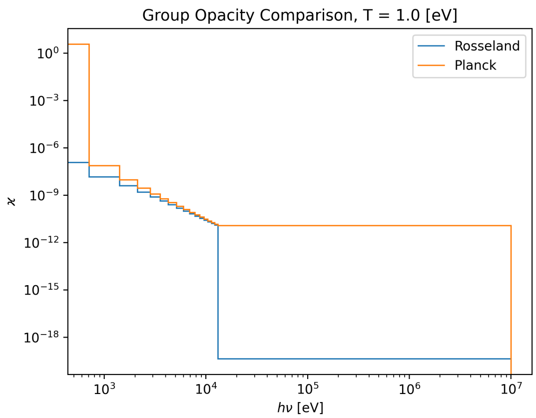


(b) 100 eV

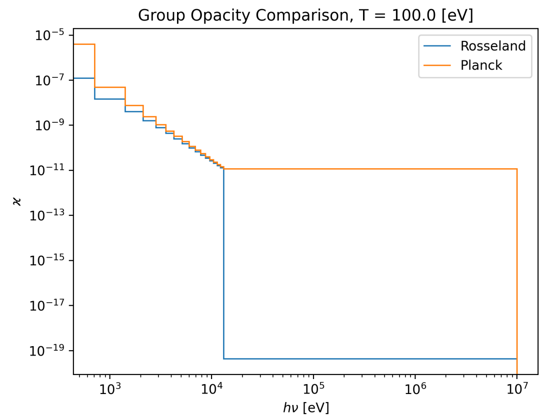


(c) 1000 eV

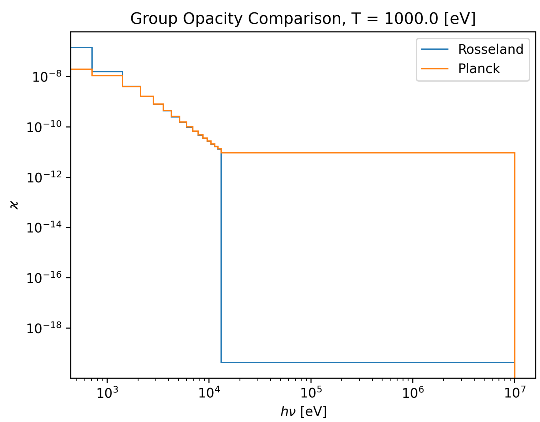
Figure 4: Group Rosseland Opacities



(a) 1 eV



(b) 100 eV



(c) 1000 eV

Figure 5: Grouped Opacities