**NE 795**
**Spring 2025**

# Homework Assignment No. 2

1. (5 points) Show that
$$I_\nu = ch\nu\psi_\nu \, . \tag{1}$$

2. (5 points) Show that the equilibrium intensity is equal to
$$I_\nu = B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1} \, . \tag{2}$$

3. (5 points) Derive the form of the following moments of the specific intensity:
$$E_\nu = \frac{1}{c}\int_{4\pi} I_\nu d\Omega \, , \quad E = \int_0^\infty E_\nu d\nu \, , \tag{3}$$
$$\mathbf{F}_\nu = \int_{4\pi} \boldsymbol{\Omega} I_\nu d\Omega \, , \tag{4}$$
$$\mathbb{P}_\nu = \frac{1}{c}\int_{4\pi} \boldsymbol{\Omega}\otimes\boldsymbol{\Omega} I_\nu d\Omega \, , \quad \mathbb{P} = \int_0^\infty \mathbb{P}_\nu d\nu \tag{5}$$
when the radiation field is in the thermal equilibrium.

4. Derive the speed of radiation wave in vacuum in the radiative transfer (RT) model defined by
   - (5 points) the grey time-dependent $P_1$ equations,
   - (5 points) the grey time-dependent $P_{1/3}$ equations.

5. (10 points) Derive the system of the time-dependent $P_1$ and MEB equations in multigroup form from the spectral $P_1$ and MEB equations given by
$$\frac{\partial E_\nu}{\partial t} + \boldsymbol{\nabla}\mathbf{F}_\nu + c\varkappa_\nu E_\nu = 4\pi\varkappa_\nu B_\nu \, , \tag{6}$$
$$\frac{\partial \mathbf{F}_\nu}{\partial t} + \frac{1}{3}\boldsymbol{\nabla}E_\nu + \varkappa_\nu \mathbf{F}_\nu = 0 \, , \tag{7}$$
$$\frac{\partial \varepsilon(T)}{\partial t} = \varkappa_\nu(cE_\nu - 4\pi B_\nu) \, . \tag{8}$$
Note that as a part of this derivation you will need to define group opacities in the multigroup photon balance, first moment, and MEB equations.

| Problem | 1 | 2 | 3 | 4a | 4b | 5 | total | % |
|---|---|---|---|---|---|---|---|---|
| Points | | | | | | | | |
| Maximum | 5 | 5 | 5 | 5 | 5 | 10 | 35 | |