NE 795
Spring 2025

Homework Assignment No. 3

1. (10 points)
   Perform asymptotic analysis of the grey $P_{1/3}$ equations coupled with the material energy balance (MEB) equation to find out accuracy of this RT model in the equilibrium diffusion limit in terms of the small parameter ε.

2. (10 points)
   Plot the spectral Planck function

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1} \tag{1}$$

   for $T = \{1\,eV, 10^2\,eV, 10^3\,eV\}$ as a function of ν in [eV].

3. (10 points)
   Calculate the group Planck function

$$B_g = \int_{\nu_g}^{\nu_{g+1}} B_\nu(T)d\nu\,, \tag{2}$$

   where $B_\nu(T)$ is given by Eq. (1). Use 17 energy groups shown in Table 1. Plot B_ν and histogram of $\frac{1}{\nu_{g+1}-\nu_g}B_g$ for $T = \{1\,eV, 10^2\,eV, 10^3\,eV\}$. (See NE 795 notes on integration of the Planck function.)

Table 1: Upper boundaries for each frequency group

g	1	2	3	4	5	6	7	8	9
ν_g [KeV]	0	0.7075	1.415	2.123	2.830	3.538	4.245	5.129	6.014
g	10	11	12	13	14	15	16	17	18
ν_g [KeV]	6.898	7.783	8.667	9.551	10.44	11.32	12.20	13.09	1×10^4

4. (10 points)
   Given the spectral opacity defined by [1]

$$\varkappa_\nu = \frac{27}{(h\nu)^3}\left(1-e^{-\frac{h\nu}{kT}}\right) \tag{3}$$

   compute group Planck opacities given by

$$\varkappa_{B,g} = \frac{\int_{\nu_g}^{\nu_{g+1}} \varkappa_\nu(T)B_\nu(T)d\nu}{\int_{\nu_g}^{\nu_{g+1}} B_\nu(T)d\nu} \tag{4}$$

   for 17 energy groups shown in Table 1. For a given T, plot $\varkappa_\nu$ and the histogram of $\varkappa_{B,g}$ on the same graph. Make a set of plots for $T = \{1\,eV, 10^2\,eV, 10^3\,eV\}$. (See the NE 795 notes on calculation of group opacities in Fleck-Cummings test.)

5. (10 points)
Compute group Rosseland opacities

$$\frac{1}{\varkappa_{R,g}} = \frac{\int_{\nu_g}^{\nu_{g+1}} \frac{1}{\varkappa_\nu(T)} \frac{\partial B_\nu(T)}{\partial T} d\nu}{\int_{\nu_g}^{\nu_{g+1}} \frac{\partial B_\nu(T)}{\partial T} d\nu} \tag{5}$$

for spectral opacity given by Eq. (3) using 17 energy groups shown in Table 1. For a given T,

- plot $\varkappa_\nu$ and histogram of $\varkappa_{R,g}$ on the same graph,
- plot histograms of $\varkappa_{B,g}$ and $\varkappa_{R,g}$ on the same graph to compare them.

Make this collection of plots for $T = \{1\,eV, 10^2\,eV, 10^3\,eV\}$.

Problem	1	2	3	4	5	total	%
Points							
Maximum	10	10	10	10	10	10	

References

[1] J. A. Fleck, J. D. Cummings, "An implicit Monte Carlo scheme for calculating time and frequency dependent nonlinear radiation transport, " *Journal of Computational Physics*, **8**, 313–342 (1971).