NE 795 Assignment 4

Kyle Hansen

14 November 2025

Question 1 Differential Fourier Analysis

1(a) Iterative Error Equations Using the definitions of iterative error,

$$\delta\psi^{(s+1/2)} = \psi^\star - \psi^{(s+1/2)} \tag{1}$$
$$\delta F^{(s+1/2)} = F^\star - F^{(s+1/2)} \tag{2}$$
$$\delta J^{(s+1)} = J^\star - J^{(s+1)} \tag{3}$$
$$\delta\phi^{(s+1)} = \phi^\star - \phi^{(s+1)}, \tag{4}$$

The equations for iterative error of the transport equation can be found by subtracting the equation for the $(s+1/2)$ iterate from the equation for the true (converged) solution:

$$\mu\frac{d\psi^\star}{dx} - \mu\frac{d\psi^{(s+1/2)}}{dx} + \sigma_t\psi^\star - \sigma_t\psi^{(s+1/2)} = \frac{1}{2}\left(\sigma_s\phi^\star + q\right) - \frac{1}{2}\left(\sigma_s\phi^{(s)} + q\right) \tag{5}$$
$$\mu\frac{d\delta\psi^{(s+1/2)}}{dx} + \sigma_t\delta\psi^{(s+1/2)} = \frac{1}{2}\left(\sigma_s\delta\phi^{(s)}\right) \tag{6}$$

where

$$\delta\psi^{(s+1/2)}(0,\mu) = (\psi_{in}^+ - \psi_{in}^+) = 0 \quad \text{for } \mu > 0 \tag{7}$$
$$\delta\psi^{(s+1/2)}(X,\mu) = (\psi_{in}^- - \psi_{in}^-) = 0 \quad \text{for } \mu < 0 \tag{8}$$

and * denotes the true (converged) solution. The equations for error in the closure terms are found in a similar way:

$$\delta F^{(s+1/2)} = \int_{-1}^{1}\left(\frac{1}{3} - \mu^2\right)\delta\psi^{(s+1/2)}d\mu \tag{9}$$

$$\delta P_L^{(s+1/2)} = \int_{-1}^{1} \left(\frac{1}{2} - |\mu|\right) \delta\psi^{(s+1/2)}(0,\mu) d\mu \tag{10a}$$

$$\delta P_R^{(s+1/2)} = \int_{-1}^{1} \left(\frac{1}{2} - |\mu|\right) \delta\psi^{(s+1/2)}(X,\mu) d\mu. \tag{10b}$$

The equations for the errors in the SM equations are found the same way:

$$\frac{d\delta J^{(s+1)}}{dx} + (\sigma_t - \sigma_s)\,\delta\phi^{(s+1)} = 0 \tag{11}$$

$$\frac{1}{3}\frac{d\delta\phi^{(s+1)}}{dx} + \sigma_t \delta J^{(s+1)} = \frac{d\delta F^{(s+1/2)}}{dx} \tag{12}$$

$$\delta J^{(s+1)}(0) = -\frac{1}{2}\delta\phi^{(s+1)}(0) + \delta P_L^{(s+1/2)} \tag{13}$$

$$\delta J^{(s+1)}(X) = \frac{1}{2}\delta\phi^{(s+1)}(X) - \delta P_R^{(s+1/2)} \tag{14}$$

These equations resemble the original iterative equations, with source (and incoming flux boundary conditions) set to zero. By taking the derivative of Equation 12 with respect to x and plugging into equation Equation 11, these equations can be reduced to a single equation for $\delta\phi$ only:

$$\sigma_t \delta J^{(s+1)} = \frac{d\delta F^{(s+1/2)}}{dx} - \frac{1}{3}\frac{d\delta\phi^{(s+1)}}{dx} \tag{15}$$

$$\delta J^{(s+1)} = \frac{1}{\sigma_t}\frac{d\delta F^{(s+1/2)}}{dx} - \frac{1}{3\sigma_t}\frac{d\delta\phi^{(s+1)}}{dx} \tag{16}$$

$$\frac{d\delta J^{(s+1)}}{dx} = \frac{1}{\sigma_t}\frac{d^2\delta F^{(s+1/2)}}{dx^2} - \frac{1}{3\sigma_t}\frac{d^2\delta\phi^{(s+1)}}{dx^2}, \tag{17}$$

evaluating Equation 11 using this equation for $(\delta J)_x$,

$$(\sigma_t - \sigma_s)\,\delta\phi^{(s+1)} + \frac{d\delta J^{(s+1)}}{dx} = 0 \tag{18}$$

$$(\sigma_t - \sigma_s)\,\delta\phi^{(s+1)} + \frac{1}{\sigma_t}\frac{d^2\delta F^{(s+1/2)}}{dx^2} - \frac{1}{3\sigma_t}\frac{d^2\delta\phi^{(s+1)}}{dx^2} = 0. \tag{19}$$

The boundary conditions $J(0)$ and $J(X)$ can be used to formulate axiliary conditions for the equation for ϕ. Evaluating Equation 16 using Equations 13 and 14 yields the auxiliary conditions:

$$\frac{1}{\sigma_t(0)}\left.\frac{d\delta F^{(s+1/2)}}{dx}\right|_{x=0} - \frac{1}{3\sigma_t}\left.\frac{d\delta\phi^{(s+1)}}{dx}\right|_{x=0} = -\frac{1}{2}\delta\phi^{(s+1)}(0) + \delta P_L^{(s+1/2)} \tag{20a}$$

$$\frac{1}{\sigma_t(X)}\left.\frac{d\delta F^{(s+1/2)}}{dx}\right|_{x=X} - \frac{1}{3\sigma_t}\left.\frac{d\delta\phi^{(s+1)}}{dx}\right|_{x=X} = -\frac{1}{2}\delta\phi^{(s+1)}(X) + \delta P_R^{(s+1/2)} \tag{20b}$$

Then the full set of equations for iterative error for the SM iterative algorithm is:

$$\mu\frac{d\delta\psi^{(s+1/2)}}{dx} + \sigma_t\delta\psi^{(s+1/2)} = \frac{1}{2}\left(\sigma_s\delta\phi^{(s)}\right) \tag{21a}$$

$$\delta\psi^{(s+1/2)}(0,\mu) = 0 \quad \text{for } \mu > 0 \qquad \text{and} \qquad \delta\psi^{(s+1/2)}(X,\mu) = 0 \quad \text{for } \mu < 0 \tag{21b}$$

$$\delta F^{(s+1/2)} = \int_{-1}^{1}\left(\frac{1}{3} - \mu^2\right)\delta\psi^{(s+1/2)}d\mu \tag{21c}$$

$$\delta P_L^{(s+1/2)} = \int_{-1}^{1}\left(\frac{1}{2} - |\mu|\right)\delta\psi^{(s+1/2)}(0,\mu)d\mu \tag{21d}$$

$$\delta P_R^{(s+1/2)} = \int_{-1}^{1}\left(\frac{1}{2} - |\mu|\right)\delta\psi^{(s+1/2)}(X,\mu)d\mu \tag{21e}$$

$$\left(\sigma_t - \sigma_s\right)\delta\phi^{(s+1)} + \frac{1}{\sigma_t}\frac{d^2\delta F^{(s+1/2)}}{dx^2} - \frac{1}{3\sigma_t}\frac{d^2\delta\phi^{(s+1)}}{dx^2} = 0 \tag{21f}$$

$$\frac{1}{\sigma_t(0)}\left.\frac{d\delta F^{(s+1/2)}}{dx}\right|_{x=0} - \frac{1}{3\sigma_t}\left.\frac{d\delta\phi^{(s+1)}}{dx}\right|_{x=0} = -\frac{1}{2}\delta\phi^{(s+1)}(0) + \delta P_L^{(s+1/2)} \tag{21g}$$

$$\frac{1}{\sigma_t(X)}\left.\frac{d\delta F^{(s+1/2)}}{dx}\right|_{x=X} - \frac{1}{3\sigma_t}\left.\frac{d\delta\phi^{(s+1)}}{dx}\right|_{x=X} = -\frac{1}{2}\delta\phi^{(s+1)}(X) + \delta P_R^{(s+1/2)} \tag{21h}$$

1(b) Fourier Analysis Validity of Fourier analysis depends on the conditions:

- Infinite spatial domain
- Constant coefficients in all equations ($\sigma_t(x) = \sigma_t$ and $\sigma_s(x) = \sigma_s$).

If these conditions are true, the Fourier Ansatz is used to formulate an assumed form of the solutions to the system defined in 21:

$$\delta\psi^{(s)} = \int_{-\infty}^{\infty} f^{(s)}(\lambda,\mu)e^{i\lambda\sigma_t x}d\lambda \tag{22a}$$

$$\delta F^{(s)} = \int_{-\infty}^{\infty} g^{(s)}(\lambda)e^{i\lambda\sigma_t x}d\lambda \tag{22b}$$

$$\delta\phi^{(s)} = \int_{-\infty}^{\infty} h^{(s)}(\lambda)e^{i\lambda\sigma_t x}d\lambda \tag{22c}$$

where λ is the Fourier mode wavenumber. Substituting the Fourier Ansatz into the system 21:

$$\sigma_t(i\mu\lambda + 1)f^{(s+1/2)}(\lambda, \mu) = \frac{\sigma_s}{2}h^{(s)}(\lambda) \tag{23a}$$

$$g^{(s+1/2)}(\lambda) = \int_{-1}^{1}\left(\frac{1}{3} - \mu^2\right)f^{(s+1/2)}(\lambda, \mu)d\mu \tag{23b}$$

$$(\sigma_t - \sigma_s)\,h^{(s+1)}(\lambda) - \lambda^2\sigma_t g^{(s+1/2)}(\lambda) + \frac{1}{3}\lambda^2\sigma_t h^{(s+1)}(\lambda) = 0 \tag{23c}$$

Solving these equations yields the expression:

$$h^{(s+1)}(\lambda) = h^{(s)}(\lambda) \cdot \frac{\lambda^2}{1 - c + \frac{1}{3\sigma_t}\lambda^2}\int_{-1}^{1}\left(\frac{1}{3} - \mu^2\right)\frac{c}{2(1 + i\mu\lambda)}d\mu, \tag{24}$$

where c is the scattering ratio σ_s/σ_t. Then the iteration eigenvalue $\omega(\lambda)$ is

$$\omega(\lambda) = \frac{\lambda^2}{1 - c + \frac{1}{3\sigma_t}\lambda^2}\left[\frac{c}{2}\int_{-1}^{1}\left(\frac{1}{3} - \mu^2\right)\frac{1}{(1 + i\mu\lambda)}d\mu\right] \tag{25}$$

where the integral is equal to

$$\left[\frac{c}{2}\int_{-1}^{1}\left(\frac{1}{3} - \mu^2\right)\frac{1}{(1 + i\mu\lambda)}d\mu\right] = \frac{\left(2\left(\lambda^2 + 3\right)\tan^{-1}\lambda\right) - 6\lambda}{3\lambda^3} \tag{26}$$

1(c) Plots

1(d) Spectral Radius

Question 2 Discretized Fourier Analysis

2(a) Iterative Error Equations Subtracting the $(s + 1/2)$ transport iteration equation from the equation for the true solution $\psi^\star$ yields

$$\mu_m \left(\psi^\star_{m,i+1/2} - \psi^\star_{m,i-1/2}\right) - \mu_m \left(\psi^{(s+1/2)}_{m,i+1/2} - \psi^{(s+1/2)}_{m,i-1/2}\right) + \sigma_{t,i}\Delta x_i (\psi^\star_{m,i} - \psi^{(s+1/2)}_{m,i}) = \frac{1}{2}\left(\sigma_{s,i}\phi^\star_i - \sigma_{s,i}\phi^{(s)}_i + \cancel{(q_i - q_i)}\right)\Delta x_i \quad (27)$$

$$\mu_m \left(\delta\psi^{(s+1/2)}_{m,i+1/2} - \delta\psi^{(s+1/2)}_{m,i-1/2}\right) + \sigma_{t,i}\Delta x_i (\delta\psi^{(s+1/2)}_{m,i}) = \frac{1}{2}\left(\sigma_{s,i}\delta\phi^{(s)}_i\right)\Delta x_i, \quad (28)$$

with boundary conditions $\delta\psi^{(s+1/2)}_{m,1/2} = 0$ for $\mu_m > 0$ and $\delta\psi^{(s+1/2)}_{m,N+1/2} = 0$ for $\mu_m < 0$. The error in scalar flux is

$$\delta\phi^{(s)}_{i+1/2} = \sum_{m=1}^{M} \delta\psi^{(s)}_{m,i+1/2} w_m, \quad (29)$$

and the SC cell-center values are

$$\psi^\star_{m,i} - \psi^{(s+1/2)}_{m,i} = \alpha_{m,i}\left(\psi^\star_{m,i-1/2} - \psi^{(s+1/2)}_{m,i-1/2}\right) + (1-\alpha_{m,i})\left(\psi^\star_{m,i+1/2} - \psi^{(s+1/2)}_{m,i+1/2}\right) \quad (30)$$

$$\delta\psi^{(s+1/2)}_{m,i} = \alpha_{m,i}\left(\delta\psi^{(s+1/2)}_{m,i-1/2}\right) + (1-\alpha_{m,i})\left(\delta\psi^{(s+1/2)}_{m,i+1/2}\right). \quad (31)$$

The error in the second-moment closure term is

$$\delta F^{(s+1/2)}_i = \sum_{m=1}^{M} \left(\frac{1}{3} - \mu_m^2\right)\delta\psi^{(s+1/2)}_{m,i} w_m \quad (32)$$

$$\delta F^{(s+1/2)}_{i+1/2} = \sum_{m=1}^{M} \left(\frac{1}{3} - \mu_m^2\right)\delta\psi^{(s+1/2)}_{m,i+1/2} w_m, \quad (33)$$

and the error in d_i is

$$\delta d^{(s+1/2)}_i = \frac{1}{\sigma_{t,i-1/2}\Delta x_{i-1/2}}\left(F^\star_i - F^{(s+1/2)}_i - F^\star_{i-1} + F^{(s+1/2)}_{i-1}\right) \quad (34)$$

$$\delta d^{(s+1/2)}_i = \frac{1}{\sigma_{t,i-1/2}\Delta x_{i-1/2}}\left(\delta F^{(s+1/2)}_i - \delta F^{(s+1/2)}_{i-1}\right). \quad (35)$$

The boundary term error is

$$\delta P_L = \sum_{m=1}^{M} \left(\frac{1}{2} - |\mu_m|\right)\delta\psi^{(s+1/2)}_{m,1/2} w_m \quad (36)$$

$$\delta P_R = \sum_{m=1}^{M} \left(\frac{1}{2} - |\mu_m|\right)\delta\psi^{(s+1/2)}_{m,N+1/2} w_m. \quad (37)$$

The error in the SM equation is given by

$$\begin{aligned}-a_i(\delta\phi_{i-1}^{(s+1)}) + (a_{i+1} + a_i + (\sigma_{a,i})\Delta x_i)\delta\phi_i^{(s+1)} - a_{i+1}\delta\phi_{i+1}^{(s+1)} \\ = \delta d_i^{(s+1/2)} - \delta d_{i+1}^{(s+1/2)} + \Delta x_i \cancel{q_i - q_i},\end{aligned} \tag{38}$$

where $\sigma_{a,i} = \sigma_{t,i} - \sigma_{s,i}$. The errors for the boundary conditions are given by

$$\left(\frac{1}{2} + a_1\right)\delta\phi_0^{(s+1)} - a_1\delta\phi_1^{(s+1)} = \delta P_L^{(s+1/2)} - \delta d_1^{(s+1/2)} + \cancel{2J_{in}^+ - 2J_{in}^+} \tag{39a}$$

$$\left(\frac{1}{2} + a_{N+1}\right)\delta\phi_{N+1}^{(s+1)} - a_{N+1}\delta\phi_{N+1}^{(s+1)} = \delta P_R^{(s+1/2)} + \delta d_{N+1}^{(s+1/2)} - \cancel{2J_{in}^- + 2J_{in}^-}. \tag{39b}$$

Then the full set of equations for iterative error for the discretized SM Method is

$$\mu_m\left(\delta\psi_{m,i+1/2}^{(s+1/2)} - \delta\psi_{m,i-1/2}^{(s+1/2)}\right) + \sigma_{t,i}\Delta x_i(\delta\psi_{m,i}^{(s+1/2)}) = \frac{1}{2}\left(\sigma_{s,i}\delta\phi_i^{(s)}\right)\Delta x_i \tag{40a}$$

$$\delta\psi_{m,1/2}^{(s+1/2)} = 0 \text{ for } \mu_m > 0, \qquad \delta\psi_{m,N+1/2}^{(s+1/2)} = 0 \text{ for } \mu_m < 0 \tag{40b}$$

$$\delta\psi_{m,i}^{(s+1/2)} = \alpha_{m,i}\left(\delta\psi_{m,i-1/2}^{(s+1/2)}\right) + (1-\alpha_{m,i})\left(\delta\psi_{m,i+1/2}^{(s+1/2)}\right) \tag{40c}$$

$$\delta d_i^{(s+1/2)} = \frac{1}{\sigma_{t,i-1/2}\Delta x_{i-1/2}}\left[\sum_{m=1}^{M}\left(\frac{1}{3} - \mu_m^2\right)\left(\delta\psi_{m,i}^{(s+1/2)} - \delta\psi_{m,i-1}^{(s+1/2)}\right)w_m\right] \tag{40d}$$

$$\delta P_L = \sum_{m=1}^{M}\left(\frac{1}{2} - |\mu_m|\right)\delta\psi_{m,1/2}^{(s+1/2)}w_m \tag{40e}$$

$$\delta P_R = \sum_{m=1}^{M}\left(\frac{1}{2} - |\mu_m|\right)\delta\psi_{m,N+1/2}^{(s+1/2)}w_m \tag{40f}$$

$$-a_i(\delta\phi_{i-1}^{(s+1)}) + (a_{i+1} + a_i + (\sigma_{a,i})\Delta x_i)\delta\phi_i^{(s+1)} - a_{i+1}\delta\phi_{i+1}^{(s+1)} = \delta d_i^{(s+1/2)} - \delta d_{i+1}^{(s+1/2)} \tag{40g}$$

$$\left(\frac{1}{2} + a_1\right)\delta\phi_0^{(s+1)} - a_1\delta\phi_1^{(s+1)} = \delta P_L^{(s+1/2)} - \delta d_1^{(s+1/2)} \tag{40h}$$

$$\left(\frac{1}{2} + a_{N+1}\right)\delta\phi_{N+1}^{(s+1)} - a_{N+1}\delta\phi_{N+1}^{(s+1)} = \delta P_R^{(s+1/2)} + \delta d_{N+1}^{(s+1/2)} \tag{40i}$$

where

$$a_i = \frac{1}{3\sigma_{t,i-1/2}\Delta x_{i-1/2}} \tag{41}$$

$$\phi_0 = \phi_{1/2}, \qquad \phi_{N+1} = \phi_{N+1/2} \tag{42}$$

$$\psi_{m,0} = \psi_{m,1/2}, \qquad \psi_{m,N+1} = \psi_{m,N+1/2}. \tag{43}$$

2(b) Fourier Analysis Like the continuous form of the SM method, Fourier Analysis of the discretized form requires that these assumptions are satisfied:

- Infinite spatial domain
- Constant coefficients $\sigma_a, \sigma_t, \theta, \tau$
- Uniform spatial grid Δx

First, the iterative equations are rewritten to apply the assumptions, and eliminating the variable δd_i for simplicity. The new iterative system is

$$\mu_m \left(\delta\psi_{m,i+1/2}^{(s+1/2)} - \delta\psi_{m,i-1/2}^{(s+1/2)}\right) + \sigma_t \Delta x (\delta\psi_{m,i}^{(s+1/2)}) = \frac{1}{2}\left(\sigma_s \delta\phi_i^{(s)}\right)\Delta x \tag{44}$$

$$\delta\psi_{m,i}^{(s+1/2)} = \alpha_{m,i}\left(\delta\psi_{m,i-1/2}^{(s+1/2)}\right) + (1-\alpha_{m,i})\left(\delta\psi_{m,i+1/2}^{(s+1/2)}\right) \tag{45}$$

$$(2a+\sigma_a\Delta x)\delta\phi_i^{(s+1)} - a(\delta\phi_{i-1}^{(s+1)} + \delta\phi_{i+1}^{(s+1)}) = \frac{1}{\sigma_t \Delta x}\left[\sum_{m=1}^{M}\left(\frac{1}{3}-\mu_m^2\right)\left(2\delta\psi_{m,i}^{(s+1/2)} - \delta\psi_{m,i+1/2}^{(s+1/2)} - \delta\psi_{m,i-1/2}^{(s+1/2)}\right)\right]. \tag{46}$$

The SM equation can also be rewritten using the SC differencing equation to eliminate ψ_i dependence, reducing the system to two equations.

$$\begin{aligned}(2a+\sigma_a\Delta x)\delta\phi_i^{(s+1)} - a(\delta\phi_{i-1}^{(s+1)} + \delta\phi_{i+1}^{(s+1)}) = \\ \frac{1}{\sigma_t\Delta x}\left\{\sum_{m=1}^{M}\left(\frac{1}{3}-\mu_m^2\right)\left(2\left[\alpha_m\left(\delta\psi_{m,i-1/2}^{(s+1/2)}\right) + (1-\alpha_m)\left(\delta\psi_{m,i+1/2}^{(s+1/2)}\right)\right]\right.\right. \\ \left.\left. -\delta\psi_{m,i+1/2}^{(s+1/2)} - \delta\psi_{m,i-1/2}^{(s+1/2)}\right)\right\}\end{aligned} \tag{47}$$

Then, applying the Fourier Ansatz, the asssumed form of the two solution functions are

$$\delta\psi_{m,i+1/2}^{(s+1/2)} = \int_{-\infty}^{\infty} f_m^{(s+1/2)}(\lambda) e^{i\lambda\sigma_t x_{i+1/2}} d\lambda \tag{48}$$

$$\delta\phi_i^{(s)} = \int_{-\infty}^{\infty} h^{(s)}(\lambda) e^{i\lambda\sigma_t x_i} d\lambda \tag{49}$$

where $f_m(\lambda)$ denotes $f(\lambda, \mu_m)$. The cell-edge and neighboring cell terms can be expanded in the assumed form to simplify:

$$\begin{aligned}
\delta\psi_{m,i+1/2}^{(s+1/2)} - \delta\psi_{m,i-1/2}^{(s+1/2)} &= f_m^{(s+1/2)}\left(e^{i\lambda\sigma_t x_{i+1/2}} - e^{i\lambda\sigma_t x_{i+1/2}}\right) \\
&= f_m^{(s+1/2)}\left(e^{i\lambda\sigma_t (x_i + \frac{1}{2}\Delta x)} - e^{i\lambda\sigma_t (x_i - \frac{1}{2}\Delta x)}\right) \\
&= f_m^{(s+1/2)} e^{i\lambda\sigma_t x_i}\left(e^{\frac{1}{2}i\lambda\sigma_t\Delta x} - e^{-\frac{1}{2}i\lambda\sigma_t\Delta x}\right) \\
&= f_m^{(s+1/2)} e^{i\lambda\sigma_t x_i}\left(\cos\frac{1}{2}i\lambda\sigma_t\Delta x + i\sin\frac{1}{2}i\lambda\sigma_t\Delta x - \cos\frac{-1}{2}i\lambda\sigma_t\Delta x - i\sin\frac{-1}{2}i\lambda\sigma_t\Delta x\right) \\
&= f_m^{(s+1/2)}(\lambda) e^{i\lambda\sigma_t x_i}\left(2i\sin\left(-\frac{1}{2}i\lambda\sigma_t\Delta x\right)\right)
\end{aligned} \tag{50}$$

which uses the identities $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$. The SC differencing scheme is similarly rewritten:

$$\begin{aligned}
(\alpha)\delta\psi_{m,i+1/2}^{(s+1/2)} + (1-\alpha)\delta\psi_{m,i-1/2}^{(s+1/2)} &= f_m^{(s+1/2)}\left(\alpha e^{i\lambda\sigma_t x_{i-1/2}} + (1-\alpha)\, e^{i\lambda\sigma_t x_{i+1/2}}\right) \\
&= f_m^{(s+1/2)}\left(\alpha\left(e^{i\lambda\sigma_t (x_i - \frac{1}{2}\Delta x)} - e^{i\lambda\sigma_t (x_i + \frac{1}{2}\Delta x)}\right) + e^{i\lambda\sigma_t (x_i + \frac{1}{2}\Delta x)}\right) \\
&= f_m^{(s+1/2)} e^{i\lambda\sigma_t x_i}\left(\alpha\left(e^{-\frac{1}{2}i\lambda\sigma_t\Delta x} - e^{\frac{1}{2}i\lambda\sigma_t\Delta x}\right) + e^{\frac{1}{2}i\lambda\sigma_t\Delta x}\right) \\
&= f_m^{(s+1/2)} e^{i\lambda\sigma_t x_i}\left(-2i\alpha\sin\left(\frac{1}{2}\lambda\sigma_t\Delta x\right) + e^{\frac{1}{2}i\lambda\sigma_t\Delta x}\right)
\end{aligned} \tag{51}$$

The cell-centered scalar flux term from the SM equation becomes

$$\begin{aligned}
\delta\phi_{i+1}^{(s+1)} + \delta\phi_{i-1}^{(s+1)} &= h^{(s+1)}\left[e^{i\lambda\sigma_t x_{i+1}} + e^{i\lambda\sigma_t x_{i-1}}\right] \\
&= h^{(s+1)} e^{i\lambda\sigma_t x_i}\left[e^{i\lambda\sigma_t\Delta x} + e^{-i\lambda\sigma_t\Delta x}\right] \\
&= h^{(s+1)} e^{i\lambda\sigma_t x_i}\left[\cos(\lambda\sigma_t\Delta x) + i\sin(\lambda\sigma_t\Delta x) + \cos(-\lambda\sigma_t\Delta x) + i\sin(-\lambda\sigma_t\Delta x)\right] \\
&= h^{(s+1)} e^{i\lambda\sigma_t x_i}\left[2\cos(\lambda\sigma_t\Delta x)\right]
\end{aligned} \tag{52}$$

and the cell-edge scalar flux term in the SM equation is similarly

$$\begin{aligned}
\delta\psi_{m,i+1/2}^{(s+1/2)} + \delta\psi_{m,i-1/2}^{(s+1/2)} &= f_m^{(s+1/2)}\left[e^{i\lambda\sigma_t (x_i + \frac{1}{2}\Delta x)} + e^{i\lambda\sigma_t (x_i - \frac{1}{2}\Delta x)}\right] \\
&= \cdots \\
&= f_m^{(s+1/2)} e^{i\lambda\sigma_t x_i}\left[2\cos\left(\frac{1}{2}\lambda\sigma_t\Delta x\right)\right].
\end{aligned} \tag{53}$$

Next, Equation 44 is evaluated using using the Fourier Ansatz forms, and plugging in Equation 45 for $\delta\psi_{m,i}$ and using 50 and 51 for the cell-edge terms:

$$f \tag{54}$$

2(c) Plots

2(d) Spectral Radius