NE 795 Assignment 3

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Question 1 Plasma Based Ion Thrusters

1(a) Exit velocity A particle of charge $1 \cdot q_e$ accelerated through a field with a potential difference of 500 V will gain an energy of 500 eV. Then assuming $v_0 = 0$ and Xenon mass is 131.3 amu $\cdot$ 931.5 MeV/c2 amu, the velocity gain at the end of the cylinder is

$$\boxed{v_E = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \cdot 500eV}{129.76 \text{ GeV/c2}}} = 8.18 \cdot 10^{-5}c = 2.711 \cdot 10^{4} \text{m s}^{-1}.} \tag{1}$$

The electric field strength can be found by dividing the potential difference by the length of the cylinder:

$$\vec{E} = \frac{500 \text{ V}}{0.15 \text{ m}} = 3.333 \cdot 10^{3} \text{ V m}^{-1} = 3.333 \cdot 10^{3} \text{ N C}^{-1} \tag{2}$$

Then the $E \times V$ velocity is:

$$\boxed{|v_{E\times B}| = \frac{|\vec{E}_{\perp} \times \vec{B}|}{|B|^2} = \frac{2.5 \times 10^{-3} \cdot 3.33 \times 10^{3}}{\left(2.5 \times 10^{-3}\right)^2} \frac{\text{m}}{\text{s}} = 1.333 \cdot 10^{6} \frac{\text{m}}{\text{s}}} \tag{3}$$

where the vectors are orthogonal, so $|\vec{E} \times \vec{B}| = |\vec{E}| \cdot |\vec{B}|$. The direction of the velocity, defining the E-field to be parallel with the z-axis, is clockwise motion (away from the page at the bottom of the cylinder, into the page at the top of the cylinder)– tangent to the cylinder.

1(b) Xe feed rate Since force can be expressed in a change in momentum over time dp/dt, the force a flow of Xe fuel can be calculated using the exit velocity:

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(m)}{\Delta t} = \Delta v \frac{\Delta m}{\Delta t} = v_{\text{exit}} \dot{m} \tag{4}$$

then, accounting for ionization efficiency (def. η), the mass consumption rate is:

$$\boxed{\dot{m} = \frac{F}{\eta \cdot v_{\text{exit}}} = \frac{0.08 \text{ N}}{0.9 \cdot 2.711 \cdot 10^4 \text{ m s}^{-1}} = 3.280 \cdot 10^{-6} \frac{\text{kg}}{\text{s}}} \tag{5}$$