NE 795 Assignment 2

Kyle Hansen

1 October 2025

Question 1

Show that

$$I_\nu = ch\nu\psi_\nu.$$

Intensity I is defined as the power (rate of energy delivery) by radiation per unit solid angle, equal to:

$$\text{Intensity} = \frac{\text{Energy}}{\text{particle}} \times \text{particle speed} \times \text{angular density} \tag{1}$$

Where, for photons, speed is always c and energy $E = h\nu$. Then

$$\text{Intensity} = I_\nu = h\nu \times c \times \text{angular density} \tag{2}$$

and the density is $\psi_\nu = \text{particles} \cdot \text{m}^{-3} \cdot \text{Sr}^{-1}$, and thus $I_\nu = ch\nu\psi_\nu$.

Question 2

Show that the equilibrium intensity is equal to

$$I_\nu = B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}.$$

Question 3

Derive the form of the following moments of the specific intensity:

$$E_\nu = \frac{1}{c}\int_{4\pi} I_\nu d\Omega, \qquad E = \int_0^\infty E_\nu d\nu,$$
$$F_\nu = \int_{4\pi} \Omega I_\nu d\Omega,$$
$$\mathbb{P}_\nu$$

Given the Radiative Transfer equation:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \vec{\Omega}\cdot\vec{\nabla} I_\nu + \varkappa_\nu I\nu = \eta_\nu \tag{3}$$

integrating w.r.t. angle:

$$\frac{1}{c}\int_{4\pi} \left\{ \frac{1}{c}\frac{\partial I_\nu}{\partial t} + \vec{\Omega}\cdot\vec{\nabla} I_\nu + \varkappa_\nu I\nu = \eta_\nu \right\} d\Omega \tag{4}$$

Question 4

Derive the speed of radiation wave in vauum in the radiative transfer (RT) model defined by

- The grey time-dependent P_1 equations
- The grey time-dependent $P_{1/3}$ equations

Question 5

Derive the system of the time-dependent P_1 and MEB equations in multigroup form from the spectral P_1 and MEB equations given by

$$\frac{\partial E_\nu}{\partial t} + \nabla F_\nu = c\varkappa_\nu E\nu = 4\pi\varkappa_\nu B_\nu$$
$$\frac{\partial F_\nu}{\partial t} + \frac{1}{3}\nabla E_\nu + \varkappa_\nu F_\nu = 0$$
$$\frac{\partial \varepsilon(T)}{\partial t} = \varkappa_\nu (cE_\nu - 4\pi B_\nu)$$

Note that as part of this derivation you will need to define group opacities in the multigroup photon balance, first moment, and MEB equations.