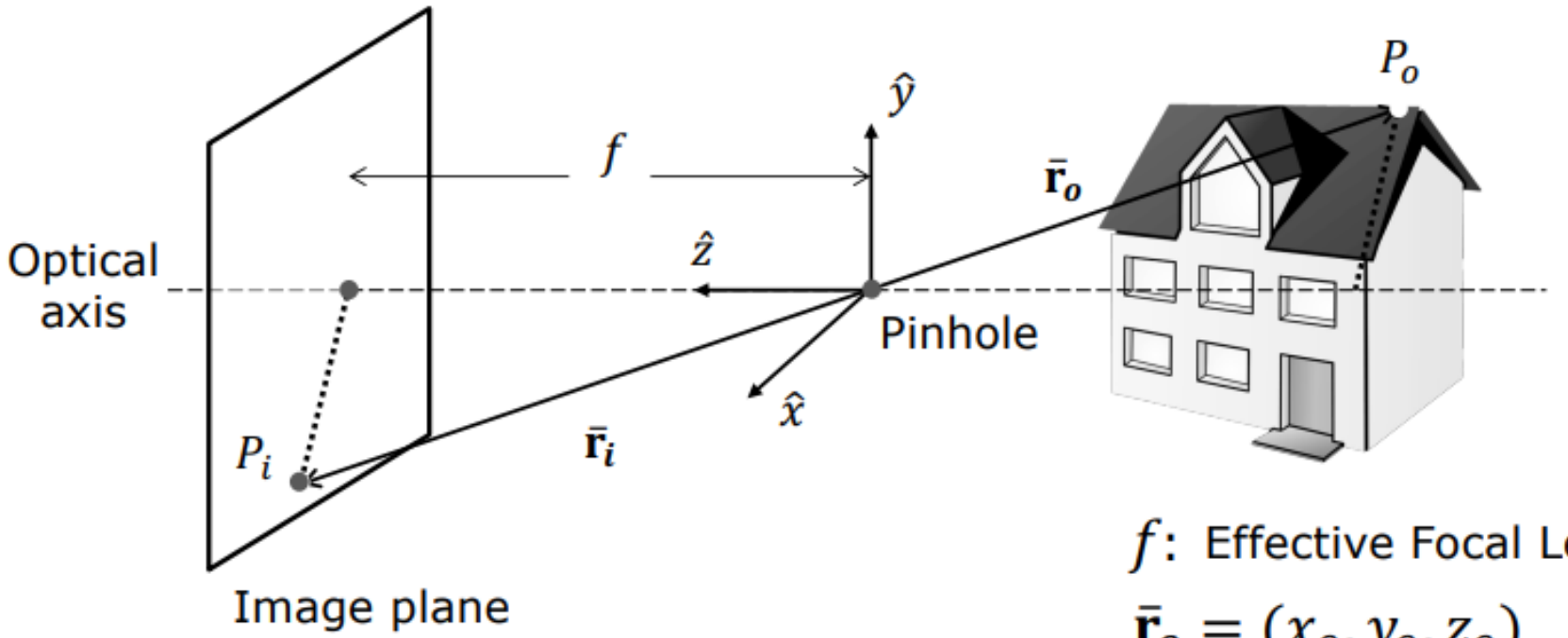
Image Formation
Pinhole and Perspective Projection

Perspective Imaging with Pinhole



f: Effective Focal Length

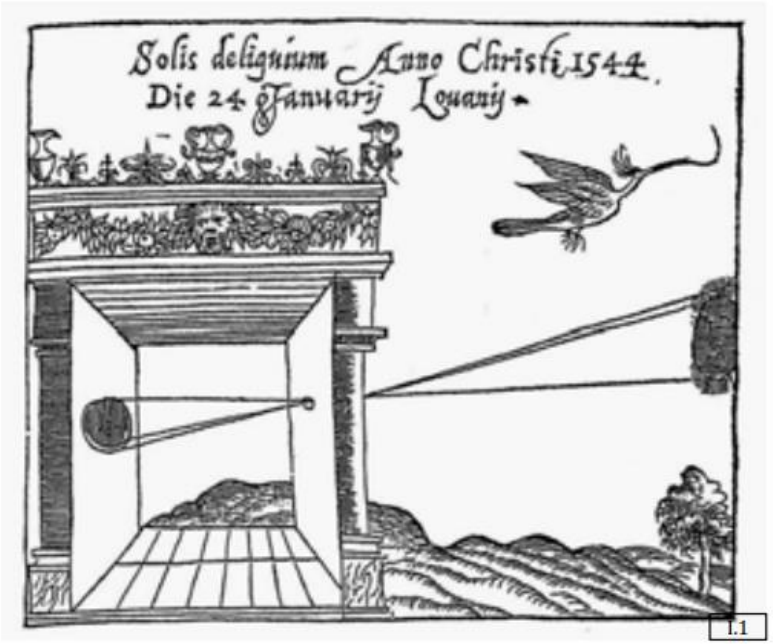
$\bar{\mathbf{r}}_o = (x_o, y_o, z_o)$

$\bar{\mathbf{r}}_i = (x_i, y_i, f)$

Using similar triangles:

$$\frac{\bar{\mathbf{r}}_i}{f} = \frac{\bar{\mathbf{r}}_o}{z_o} \rightarrow \frac{x_i}{f} = \frac{x_o}{z_o}, \frac{y_i}{f} = \frac{y_o}{z_o}$$

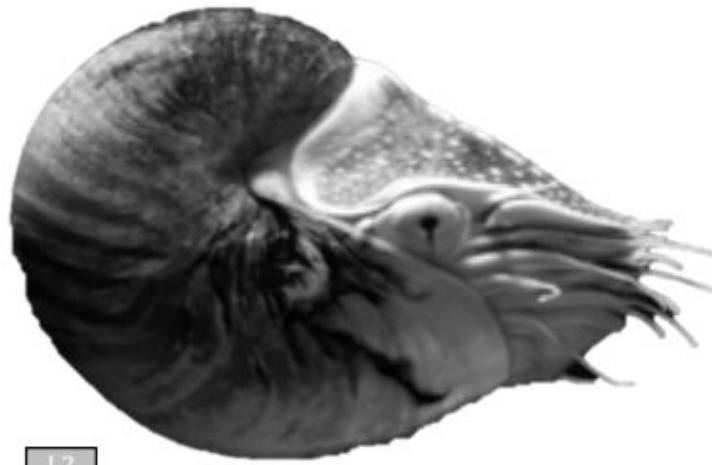
Camera Obscura



I.1

"Dark Chamber"

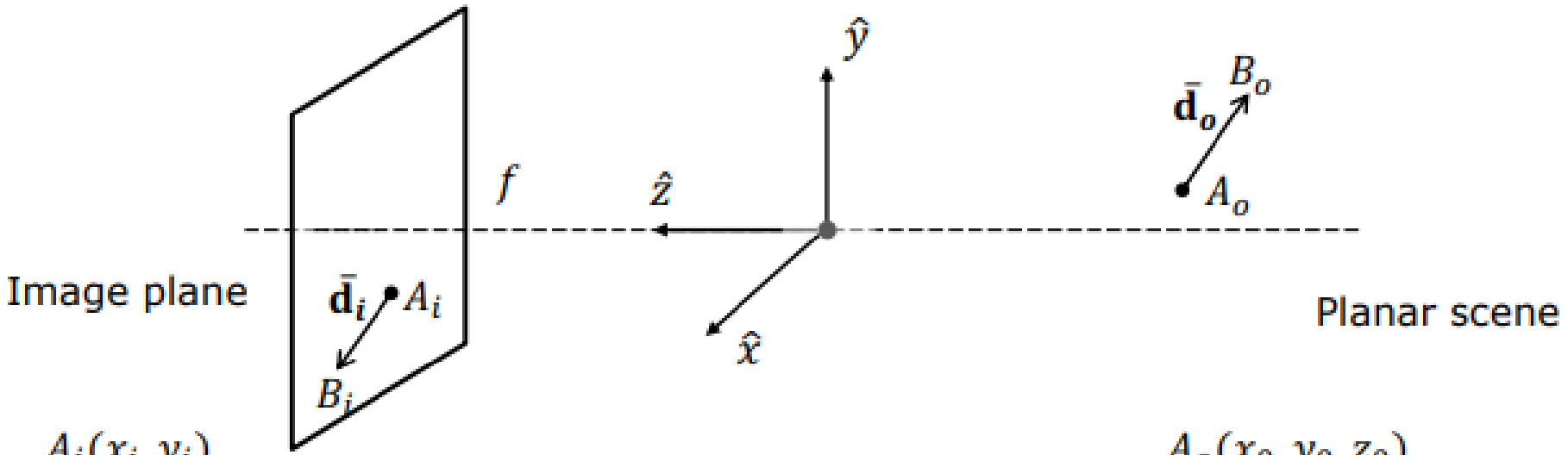
Pinhole Eye of *Nautilus pompilius*



I.2

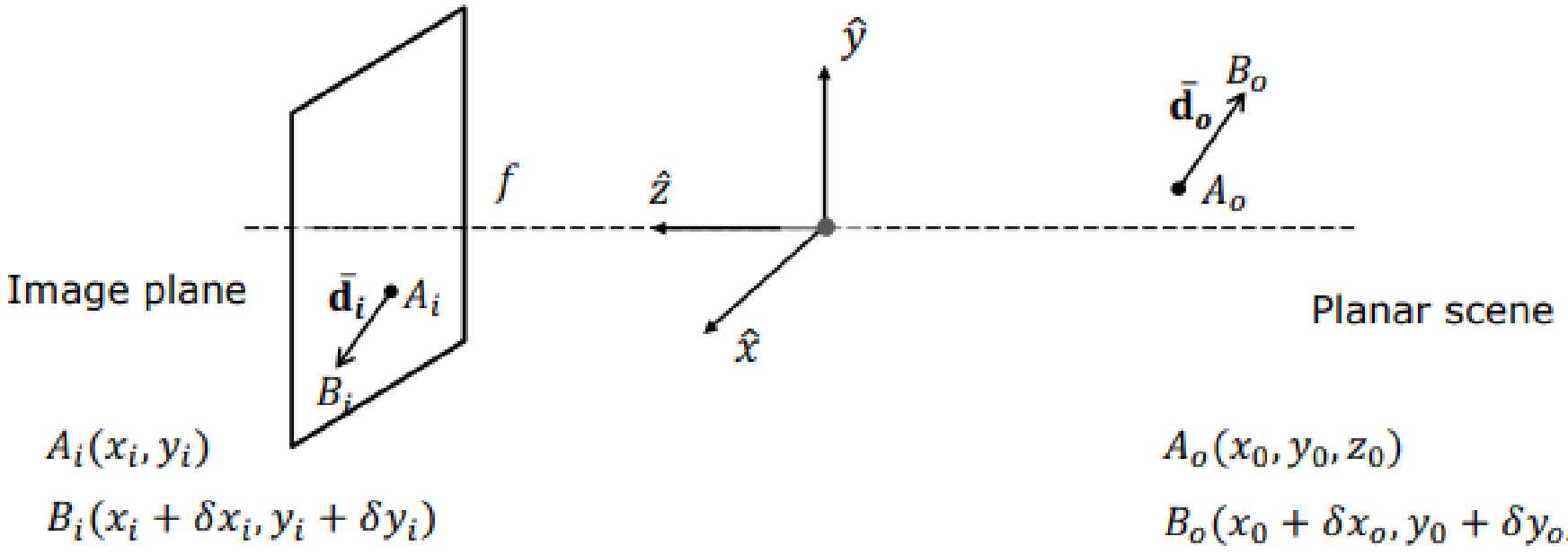
I.3

Image Magnification



Magnification: $|m| = \frac{\|\bar{\mathbf{d}}_i\|}{\|\bar{\mathbf{d}}_o\|} = \frac{\sqrt{\delta x_i^2 + \delta y_i^2}}{\sqrt{\delta x_o^2 + \delta y_o^2}} \Big/ \boxed{1}$

Image Magnification



From Perspective Projection:

$$\frac{x_i}{f} = \frac{x_o}{z_o} \quad \text{and} \quad \frac{y_i}{f} = \frac{y_o}{z_o} \qquad \text{(A)}$$

$$\frac{x_i + \delta x_i}{f} = \frac{x_o + \delta x_o}{z_o} \quad \text{and} \quad \frac{y_i + \delta y_i}{f} = \frac{y_o + \delta y_o}{z_o} \qquad \text{(B)}$$

Image Magnification

From (A) and (B) we get:

$$\frac{\delta x_i}{f} = \frac{\delta x_o}{z_o} \quad \text{and} \quad \frac{\delta y_i}{f} = \frac{\delta y_o}{z_o} \qquad \boxed{1}$$

Magnification:

$$|m| = \frac{\|\bar{\mathbf{d}}_i\|}{\|\bar{\mathbf{d}}_o\|} = \sqrt{\delta x_i{}^2 + \delta y_i{}^2} \Big/ \sqrt{\delta x_o{}^2 + \delta y_o{}^2} = \left|\frac{f}{z_o}\right|$$

$$\boxed{m = \frac{f}{z_o}}$$ m is negative when image is inverted

Image Magnification

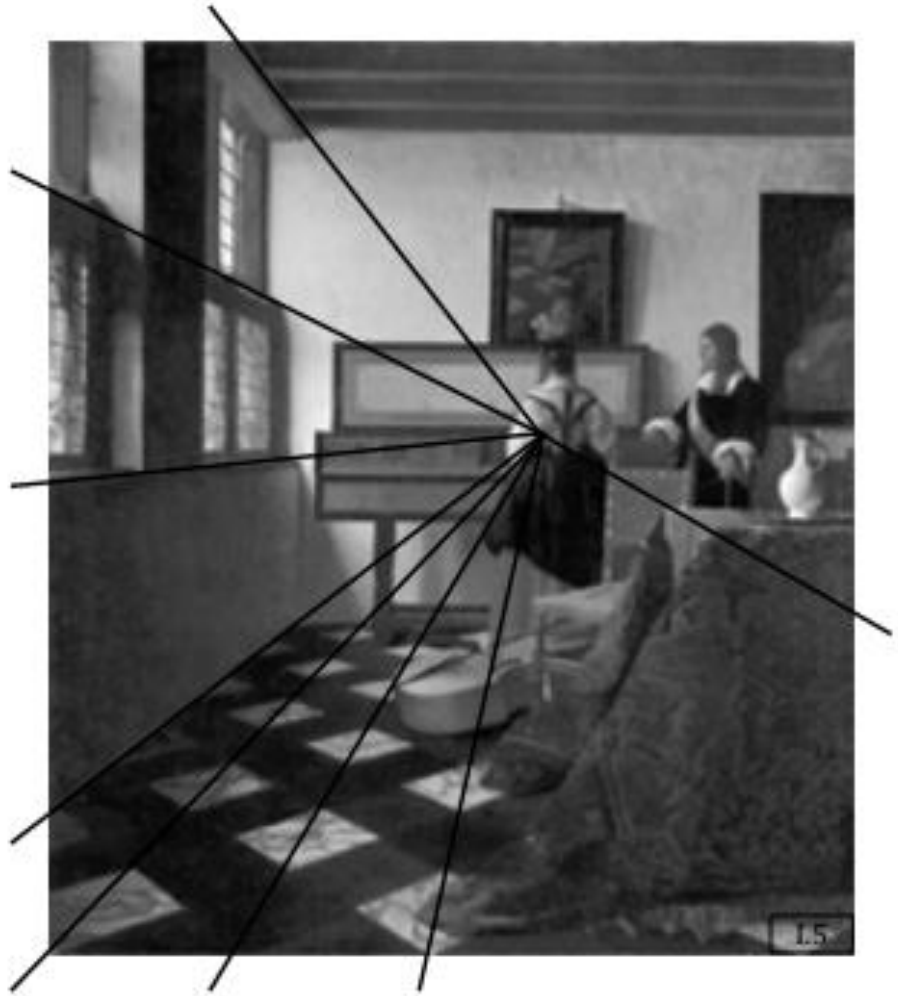
$$m = \frac{f}{z_o}$$

Image size inversely proportional to depth

Vanishing Point

Its location depends on orientation of parallel scene lines

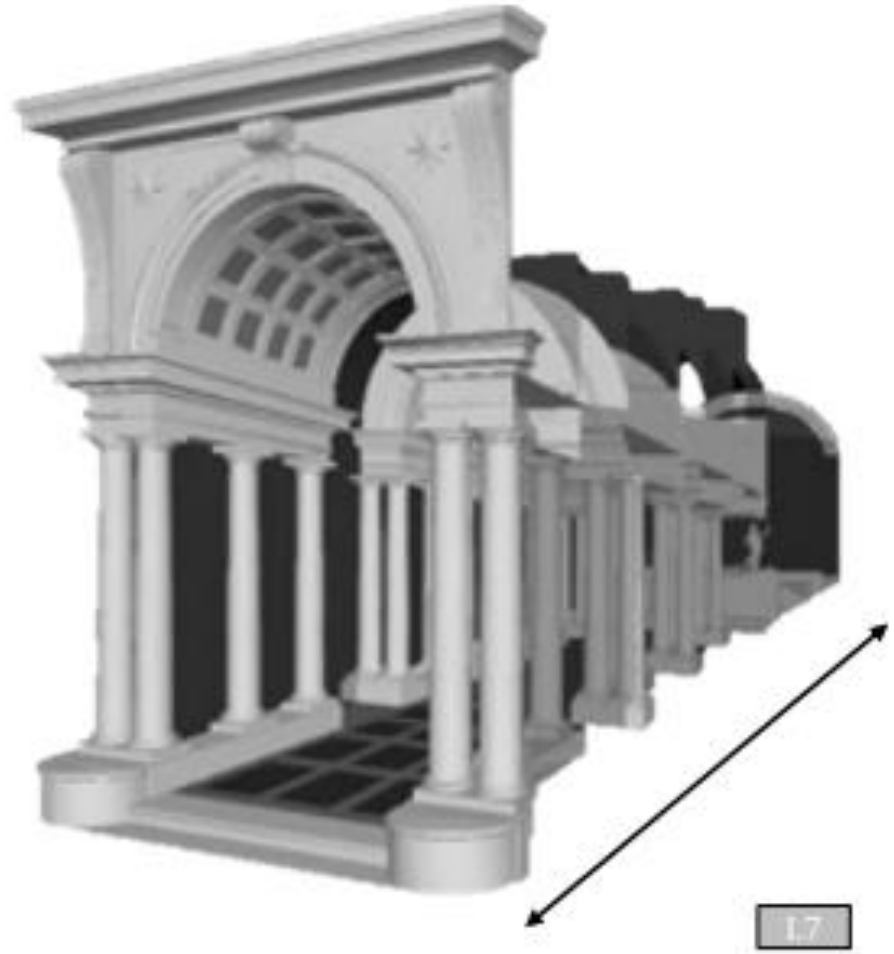
Use of Vanishing Point in Art



The Music Lesson, Johannes Vermeer, c. 1662-1664

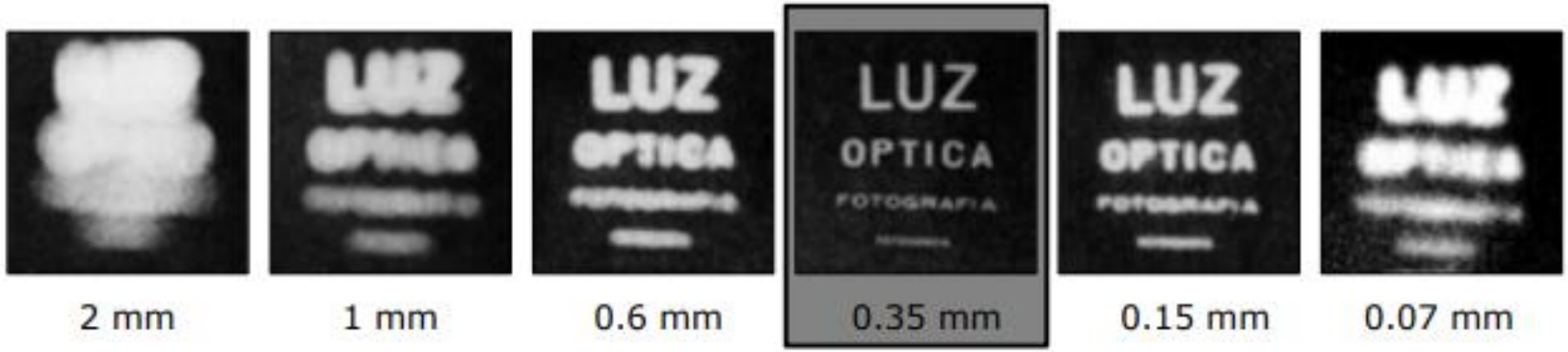
False Perspective

Depth appears to be ~155 feet



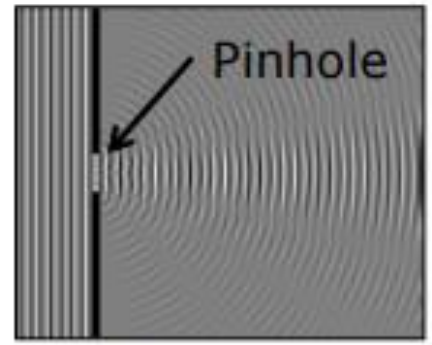
Depth is actually ~30 feet

What is the Ideal Pinhole Size?



2 mm | 1 mm | 0.6 mm | **0.35 mm** | 0.15 mm | 0.07 mm

The pinhole must be tiny,
but if it's too tiny it will cause diffraction.



Diffraction

Ideal pinhole diameter: $d \approx 2\sqrt{f\lambda}$ (1)

f: effective focal length
λ: wavelength

What about Exposure Time?

Pinholes pass less light and hence require long exposures to capture bright images.

$f = 73\ mm,\ d = 0.2\ mm,$
Exposure, $T = 12\ s$