# Notes on Bao et al. (2022)

October 9, 2025

**Definition 1** (Dense Associative Memory)**.** For a *Dense Associative Memory* (DAM), let $\Xi = [\xi^1, \xi^2, \dots, \xi^N]$ be the set of desired patterns to store in the associative memory, and let $M$ be the dimension of each $\xi^i$. We define the energy function:

$$E(\sigma^{(t)}) = -\sum_{\mu=1}^{N} F_n\left(\sum_{i=1}^{M} \xi_i^\mu \sigma_i^{(t)}\right), \tag{1}$$

where $\sigma^{(t)}$ is the current state of the network at time $t$, and $F_n$ is a polynomial "transmission" function:

$$F_n(x) = x^n,\ n \geq 2. \tag{2}$$

The update rule $T$ of the DAM is a sequence of functions $(T_1, T_2, \dots, T_M)$, where each $T_i$ updates the state vector $\sigma^{(t)}$ at index $i$ by,

$$T_i(\sigma^{(t)}) = \operatorname{sgn}\left[E_{i^-}(\sigma^{(t)}) - E_{i^+}(\sigma^{(t)})\right], \tag{3}$$

where,

$$E_{i^\pm}(\sigma^{(t)}) = -\sum_{\mu=1}^{N} F_n\left((\pm 1)\xi_i^\mu + \sum_{j\neq 1} \xi_j^\mu \sigma_j^{(t)}\right). \tag{4}$$

The full update rule is then,

$$T(\sigma^{(t)}) = [T_1(\sigma^{(t)}), \dots, T_N(\sigma^{(t)})] = \sigma^{(t+1)}. \tag{5}$$

We can show that the update rule always reduces the energy of the state vector, but we won't here.

**Lemma 1.** *The update rule in Def. 5 can be expressed for even $n$ (even polynomial function $F_n$):*

$$T_i(\sigma^{(t)}) = sgn\left[2\sum_{\mu=1}^{N}\sum_{k=1}^{M}\binom{n}{2k-1}(\xi_i^\mu)^{n-2k+1}\left(\sum_{j\neq i}\xi_j^\mu\sigma_j^{(t)}\right)^{2k-1}\right]. \tag{6}$$

*For odd $n$:*

$$T_i(\sigma^{(t)}) = sgn\left[2\sum_{\mu=1}^{N}\sum_{k=0}^{(n-1)/2}\binom{n}{2k}(\xi_i^\mu)^{n-2k}\left(\sum_{j\neq i}\xi_j^\mu\sigma_j^{(t)}\right)^{2k}\right]. \tag{7}$$

*Proof.* Recall that,

$$E_{i^\pm}(\sigma^{(t)}) = -\sum_{\mu=1}^{N}F_n\left((\pm 1)\xi_i^\mu + \sum_{j\neq 1}\xi_j^\mu\sigma_j^{(t)}\right). \tag{8}$$

For the positive case, this is:

$$E_{i^+}(\sigma^{(t)}) = -\sum_{\mu=1}^{N}\left(\xi_i^\mu + \sum_{j\neq 1}\xi_j^\mu\sigma_j^{(t)}\right)^n. \tag{9}$$

This is equivalent to:

$$-\sum_{\mu=1}^{N}\sum_{k=1}^{n}\binom{n}{k}(\xi_i^\mu)^{n-k}\left(\sum_{j\neq i}\xi_j^\mu\sigma_j\right) \tag{10}$$

By the binomial theorem, this is:

$$E_{i^+} \tag{11}$$

$\square$

## References

Bao, H., Zhang, R., & Mao, Y. (2022). The capacity of the dense associative memory networks. *Neurocomputing*, *469*, 198–208. https://doi.org/10.1016/j.neucom.2021.10.058