Monotonicity of the stochastic discount factor and expected option returns

Ranadeb Chaudhuri*

and

Mark Schroder†

June 9, 2010

Abstract

This paper introduces a simple nonparametric approach for testing whether the stochastic discount factor (SDF), or marginal rate of substitution, is decreasing in the return of an asset. We characterize the class of option strategies whose expected returns are increasing in the strike price under a strictly monotonic SDF. Call and put options are special cases, but the set also includes butterfly spreads, bullish call spreads, and binary options. Consistent with many papers, our empirical results do not support strict monotonicity of the SDF respect to S&P 500 returns, but do support strict monotonicity with respect to individual stocks.

JEL classification: G12, G13, C8, C50

Keywords: SDF; Risk Premium; Derivatives; Expected Returns

*School of Business Administration, Oakland University, Rochester, MI 48309. Tel.: (248) 370-3204, E-mail: chaudhur@oakland.edu.

†Department of Finance, The Eli Broad College of Business, Michigan State University, East Lansing, MI 48824-1121. Tel.: 517-432-0622, E-mail: schroder@bus.msu.edu. We are thankful for comments from seminar participants at Michigan State University and Nanyang Business School.

A fundamental issue in asset pricing is whether the marginal rate of substitution, or the stochastic discount factor (SDF), decreases as the price of an asset increases. In a simple representative agent model, the SDF is decreasing in the price of the market portfolio because of diminishing marginal utility of wealth. Under CAPM the SDF is decreasing and affine in the value of the market portfolio. If the asset is an individual stock, then some commonly assumed joint distributions of the stock and SDF imply that a positive risk premium on the stock is equivalent to an SDF decreasing in the stock price. The Black-Scholes model is one of many examples.

The shape of the SDF when graphed against the price or return of an asset therefore has important implications for asset pricing. A graph that is not monotonically decreasing against market returns contradicts CAPM and a class of representative-agent models. A graph that is not monotonically decreasing against individual stock returns contradicts a class of option pricing models that includes Black-Scholes. Furthermore, the shape of the graph has implications for not only the expected return of the asset, but expected returns of all contingent claims on the asset return.

Options provide a powerful tool for discerning the shape of the SDF. While the risk premium of an asset can identify only the covariance between the SDF and returns (or equivalently, the slope of the linear regression), Coval and Shumway (2001) show that call and put option expected returns can be used to identify conditional correlations (or slopes of truncated regressions). We show that there can be a large gap between the class of SDF's whose conditional correlations are all negative, and those that are in fact monotonically decreasing. For example, we show that the graph of the SDF against the S&P 500 returns passes the conditional correlation test, but not our monotonicity test. We provide a necessary and sufficient characterization of SDF monotonicity in terms of the expected returns of a class of option strategies.

We show that the SDF is strictly monotonic if and only if expected returns of a class of option trading strategies is increasing in the strike price. The class is uniquely identified by the property that the logarithm of the payout function is concave over the set of ending security prices where the payouts are positive. In addition to calls and puts, the class includes butterfly spreads, bullish call spreads, and binary call and put options. Intuitively, an increase in the strike price for any strategy in this class shifts the payoffs to lower utility states, requiring a higher return to compensate. More precisely, an increase in the strike shifts, in the first-order stochastic dominance sense, the probability weighted payoffs to lower values of the SDF, resulting in lower prices and higher expected returns.

We apply our monotonicity test first to the S&P 500. Consistent with Coval and Shumway (2001), we find that average returns of long call and put option strategies are increasing in strike; that is, the conditional correlations between the SDF and returns are all negative. However we find that average returns of long butterfly spreads, bullish call spreads and binary calls are not increasing in strike, which contradicts strict monotonicity. Our results, which imply an SDF-return graph that is downward sloping for high and low returns, but not downward sloping for intermediate returns is consistent with results of Ait-Sahalia and Lo (2000), Jackwerth (2000), Carr, Geman, Madan, and Yor (2002), Rosenberg and Engle (2002), Brown and Jackwerth (2004) and Shive and Shumway (2006). Theoretical explanations for the nonmonotonicity include state dependence in the representative agent's utility (Brown and Jackwerth (2004)), heterogeneous investor beliefs (Ziegler (2007) and Bakshi and Madan (2009)), the influence of a latent state variable (Chabi-Yo, Garcia, and Renault (2008)), and investor sentiment (Han (2004)).

The main empirical application of our test is to individual stocks. If there exists an expected-utility maximizing agent whose wealth distribution conditional on the stock return improves (in the first-order stochastic dominance sense) as the return increases, then the agent's marginal utility and therefore the SDF will be decreasing in the stock return.[1] This is true, for example, if the agent's wealth and the terminal stock price are positively correlated and distributed either bivariate normal or bivariate lognormal. In contrast to the S&P

[1]Note that this agent's beliefs are assumed unbiased in the sense that expectation is under the true measure.

500 results, our empirical analysis of option returns supports strict SDF monotonicity with respect to individual stocks. For each strategy within the log-concave class, we assign returns to strike groups and compute return differences between consecutive strike groups. We find that average return differences between strike groups are almost all positive. To assess the statistical significance, we adjust the usual t-statistics to account for the skewness inherent in option return distributions. This simple adjustment, developed in Chen (1995), has been shown to be accurate for small sample sizes and for distributions as asymmetric as the exponential distribution.

To compliment our pairwise tests, we apply the Page test for ordered alternatives (see Page (1963) and Pirie (1985)), which provides a single statistic to jointly test monotonicity of expected returns in the strike price. The null hypothesis of identical expected returns across strike groups is tested against the alternative hypothesis that expected returns are increasing across strike groups. The Page test statistics are positive and highly significant for all the considered option strategies, providing strong evidence in support of strict monotonicity.

Our work contributes to the large body of research on SDFs. Previous work can be broadly categorized into either parametric or non-parametric approaches to SDF estimation. Parametric approaches usually involve estimating the parameters of the SDF in a representative agent setting (eg., Hansen and Singleton (1982), Hansen and Singleton (1983), Hansen and Jagannathan (1997), Cambell and Cochrane (2000) and Giovannini and Weil (1989)). Some exceptions include Constantinides (1992) and Bakshi and Chen (2001) who explicitly specify stochastic processes for the SDF. Non-parametric approaches (eg., Brown and Jackwerth (2004)) typically use option prices to develop an estimator for the SDF that does not depend on a particular specification of consumer preferences or asset return processes. Rosenberg and Engle (2002) also use options data to investigate the characteristics of investor risk aversion over equity return states by estimating a time-varying SDF, and Ait-Sahalia and Lo (2000) use kernel estimation of the risk-neutral density to provide non-parametric estimate of the SDF. However, most of the empirical applications of the above approaches have been to stock indices or to aggregate consumption data. Options data is limited for most stocks (the median number of strikes available on each date in our sample is 3), making individual-stock SDF estimation difficult. Our paper contributes to the literature by providing a simple method to test monotonicity of the projected SDF without explicitly estimating its shape. Our approach is applicable to all underlying processes (whether or not they are traded assets) with traded options.

The only other study we are aware of that tests SDF monotonicity with respect to individual stocks is Ni (2007), who applies Coval and Shumway (2001) to test for weak SDF monotonicity by testing monotonicity of call expected returns with respect to strike price (put options are not tested). Inconsistent with weak monotonicity, she finds that deep out-of-the-money (OTM) calls earn significantly lower average returns than deep in-the-money (ITM) calls. She suggests that investors are sometimes risk-seeking and a preference for idiosyncratic skewness induces a premium for deep OTM options. We provide evidence that this anomaly is caused by a flaw in the test design. We show that heterogeneity among stocks (likely causing heterogeneity in expected call returns), combined with time variation in the representation of stocks in the various strike groups, biases her estimates. We show that her deep OTM strike group is overrepresented by stocks having lower elasticity options, resulting in an average elasticity for the deep OTM calls that is smaller than for the at-the-money group.[2]

The remainder of the paper is organized as follows. Section I characterizes weak and strict monotonicity in terms of risk premia and expected returns of trading strategies, and provides conditions for strict monotonicity. Section II describes the dataset and Section III presents the empirical results. Section IV concludes the paper.

[2]Recall that in the Black-Scholes model, for example, elasticities are increasing in strike and option risk premia are proportional to elasticities.

I. Characterization of SDF Monotonicity

In this section we consider three progressively stronger senses in which the SDF is decreasing in the price (or return) of an asset, and characterize these in terms of expected returns of trading strategies. We show that each form of SDF monotonicity is equivalent to the monotonicity of expected returns of a set of asset/option trading strategies in a shift parameter (typically a strike price). Our main contribution in this section is to characterize what we term "strict monotonicity" (a strictly decreasing SDF as a function of return). In particular, we characterize the set of trading strategies with expected return increasing in strike under strict monotonicity. In addition to long call and put option positions,[3] this set of trading strategies includes butterfly spreads and binary cash-or-nothing calls and puts. Because many mainstream asset pricing models predict strictly monotonicity of the SDF, this provides a useful complement to the test of Coval and Shumway (2001).

Fixing some payout date T, an SDF, or pricing kernel, m is a (time-T measurable) random variable such that the current price $p(X)$ of any time-T payout X satisfies $p(X) = E(Xm)$. If the number of states is finite, then the state-s value of the SDF, m_s, can be interpreted as the per-unit-probability price of a claim to one unit of state-s consumption. Technicalities aside, it is well known that a strictly positive SDF exists if and only if there is no arbitrage.[4] For example, if there exists an agent with utility over current and future consumption given by $U(c_0, c_T) = u_0(0, c_0) + u(T, c_T)$, then the first-order condition for optimality implies that there exists an SDF equal to the agent's marginal rate of substitution, $m = u_c(T, c_T) / u_c(0, c_0)$, and risk aversion is equivalent to m decreasing in c_T. With some preference and distributional assumptions, standard aggregation arguments imply monotonicity of m in aggregate time-T consumption and time-T wealth.[5]

[3]Coval and Shumway (2001) show that what we term "weak monotonicity" (negative slopes of all truncated regressions of the SDF on return) is equivalent to call and put option expected returns increasing in strike.

[4]In the finite-state setting, the result follows directly from the separating hyperplane theorem. Note that throughout we are considering the case of unconstrained trading (no short-selling constraints, for example).

[5]The conditions on preferences are generally strong. Gorman aggregation allows heterogeneity essentially only with respect to additive shifts consumption with homothetic preferences, and scalar multiples of

All asset-pricing models are essentially models of the SDF, and its monotonicity properties determine the risk premia properties of all traded assets. Below we define three forms of SDF monotonicity in terms of the slopes of various "regressions" of a terminal stock price on the SDF and then characterize each in terms of the monotonicity of expected option strategy returns with respect to shifts in the strike price. This characterization forms the basis of the empirical tests in Section III. Define the (gross) expected return function R by $R(X) = E(X)/E(mX)$ for any (time-T) payout X. Note that $R(1)$ is the riskless return. Fix throughout some stock S with time-T price (with dividends reinvested) S_T. The distribution function of S_T is denoted $F()$, and is assumed throughout (for simplicity) to have a density function which is strictly positive on $(0, \infty)$. We use the common notation $x^+ = \max(0, x)$ for any $x \in \mathbb{R}$. Finally, we define the risk-neutral measure Q as usual via

$$\frac{dQ}{dP} = \frac{m}{E(m)},$$

(that is, risk-neutral probabilities are obtained by scaling the SDF-weighted true probabilities).[6]

For comparison with our later results, the following proposition gives some well-known equivalent characterizations of a positive risk premium (expected return in excess of the riskless return).

Proposition 1 *The following are equivalent:*

i) $Cov(S_T, m) < 0$.

ii) $R(S_T) > R(1)$.

iii) $E(S_T) > E^Q(S_T)$.

iv) $R(S_T - K)$ *and* $R(K - S_T)$ *are strictly increasing in* K.

Proof. See the appendix. ■

consumption with translation-invariant preferences. A deterministic investment opportunity set and homothetic preferences imply a deterministic consumption to wealth ratio, from which we get monotonicity with respect to wealth. See Section C for more detailed conditions under which an SDF is monotonic in an asset return.

[6] In the finite-state setting, with state-s probability π_s, the state-s risk-neutral probability is $\pi_s m_s / \sum \pi_s m_s$, which represents the forward price per unit payout in state s.

The intuition for the equivalence of (i) and (ii) is that if higher stock payouts are associated with less valuable payout states, in the sense that a linear regression of S_T against m has a negative slope, then the current price is more heavily discounted, resulting in a higher expected return. The equivalence to (iii) follows because replacing the true probability for each state by the forward price per unit payout in that state reduces the expected payout if payouts are higher in low-value states. Finally, the equivalence to (iv) follows because increased leverage in a long stock position (and decreased leverage in a short-stock/lending position) increases expected returns if and only if the risk premium of the stock is positive.

A. Weak Monotonicity

A stronger condition than negative covariance, or a negative regression slope, is what we term weak monotonicity with respect to S_T, which we define by negative slopes for all regressions truncated to a half-line.

Definition 2 *The SDF m is* weakly monotonic *with respect to S_T if*

$$Cov\left(S_T, m \mid S_T \in [\alpha, \beta]\right) < 0 \quad \text{for all } 0 \leq \alpha < \beta \text{ with either } \alpha = 0 \text{ or } \beta = \infty.$$

The following proposition relates weak monotonicity to monotonicity of option expected returns, and, analogous to the unconditional case, characterizes weak monotonicity in terms of conditional risk premia and conditional risk-neutral expectations:

Proposition 3 (weak monotonicity) *The following are equivalent:*

i) m is weakly monotonic with respect to S_T.

ii) $R\left(S_T 1_{\{S_T \in [\alpha,\beta]\}}\right) > R\left(1_{\{S_T \in [\alpha,\beta]\}}\right)$ for all $0 \leq \alpha < \beta$ with either $\alpha = 0$ or $\beta = \infty$.

iii) $E\left(S_T \mid S_T \in [\alpha, \beta]\right) > E^Q\left(S_T \mid S_T \in [\alpha, \beta]\right)$ for all $0 \leq \alpha < \beta$ with either $\alpha = 0$ or $\beta = \infty$.

iv) (Coval and Shumway (2001)) $R\left((S_T - K)^+\right)$ *and* $R\left((K - S_T)^+\right)$ *are strictly increasing in* K*.*

Proof. See the appendix. ■

Part (ii) says that weak monotonicity is equivalent to strict positivity of conditional risk premia; that is, positive risk premia for all asset-or-nothing binary calls (puts) relative to cash-or-nothing binary calls (puts). Part (iv) is the Coval and Shumway (2001) characterization in terms of traded option strategies, which they apply to testing weak monotonicity of m with respect to the S&P 500 index. Note that (iv) holds regardless of the stock-price distribution function F.

B. Strict Monotonicity

We define strict monotonicity as monotonicity of the conditional expectation, or nonlinear regression of m on S_T:.

Definition 4 *The SDF* m *is* strictly monotonic *with respect to* S_T *if* $m(s) = E(m \; | S_T = s)$ *is strictly decreasing in* s*.*

While it might appear that weak monotonicity is close to strict monotonicity, the following simple example shows that weak monotonicity can hold despite the absence of strict monotonicity for intermediate stock-price values.[7] It is a highly stylized representation of the behavior of the SDF projected on S&P 500 returns.

Example 5 *Suppose a discrete setting with four equally-likely and equally-spaced possible terminal stock prices:* $s_1 < s_2 < s_3 < s_4$*. Then* m *is weakly monotonic if and only if* $m(s_1) > m(s_4)$ *and* $m(s_2), m(s_3) \in (m(s_1), m(s_4))$*. That is, the price of consumption in the lowest stock-price state exceeds that in the highest stock-price state, and the prices of*

[7]A derivation is provided in the appendix.

consumption in the two middle states are in between. Therefore weak monotonicity can hold even though $m(s_2) < m(s_3)$; furthermore, $m(s_3)$ can exceed $m(s_2)$ by almost $m(s_1) - m(s_4)$ (the range of $m()$).

Strict monotonicity obviously implies weak monotonicity, and therefore a violation of monotonicity of expected call or put returns with respect to the strike price would contradict strict monotonicity of m. However, a larger family of trading strategies must be considered to affirm strict monotonicity. We obtain such a set of trading strategies in terms of a class of payoff functions $G()$ which satisfy some (mostly technical) regularity conditions.

Definition 6 *A payoff function G is* regular *if $G(x) > 0$ for $x \in (x_1, x_2)$, where $-\infty \leq x_1 < x_2 \leq \infty$, and $G(x) = 0$ for $x \notin [x_1, x_2]$.*[8] *Furthermore, $G(x)$ is continuous for $x \in (x_1, x_2)$ and $G'(x)$ is piecewise continuous for $x \in (x_1, x_2)$.*

The following proposition characterizes the class of regular payoff functions G such that the expected return $R(G(S_T - K))$ is increasing in the shift parameter (or, strike price) K for all stock-price distribution functions and any strictly monotonic SDF.

Proposition 7 *Suppose m is strictly monotonic and the payoff function G is regular. Then the expected return $R(G(S_T - K))$ is increasing in K for all distribution functions F if and only if*

$$\ln(G(x)) \text{ is strictly concave in } x \text{ for all } x \in (x_1, x_2), \tag{1}$$

where (x_1, x_2) is the support of G.

Proof. See the appendix. ■

For any strictly monotonic m, the only class of regular payoffs with expected returns monotonic in strike (for any distribution function) is that characterized by log concavity; that

[8]That is, the support of G is (x_1, x_2).

is, $G'(x)/G(x)$ is strictly decreasing in x over the support of G.[9] Log concavity of the payoff implies that increases in the strike shifts (in the first-order dominance sense) the probability-weighted payoffs to higher stock prices; the lower values of the SDF corresponding to higher stock prices implies a lower price (per unit probability-weighted payoff) and therefore a higher expected return. Part (5) of the following proposition shows that monotonicity of expected returns in strike for this payoff class is equivalent to strict monotonicity of m.

Proposition 8 (strict monotonicity) *Suppose $m()$ is continuously differentiable. Then the following are equivalent:*

i) m is strictly monotonic with respect to S_T.

ii) $Cov(S_T, m \mid S_T \in [\alpha, \beta]) < 0$ for all $0 \le \alpha \le \beta$.

iii) $R\left(S_T 1_{\{S_T \in [\alpha,\beta]\}}\right) > R\left(1_{\{S_T \in [\alpha,\beta]\}}\right)$ for all $0 \le \alpha \le \beta$.

iv) $E(S_T \mid S_T \in [\alpha, \beta]) > E^Q(S_T \mid S_T \in [\alpha, \beta])$ for all $0 \le \alpha \le \beta$.

v) $R(G(S_T - K))$ is strictly increasing in K for all regular payout functions G satisfying the log-concavity condition (1).

Proof. See the appendix. ■

The proposition shows that strict monotonicity is also equivalent to the positivity of all conditional risk premia, with payouts confined to arbitrary intervals; or, equivalently, the positivity of risk premia of asset-or-nothing versus cash-or-nothing binary options with payouts contingent on the stock price falling within an arbitrary interval. Because binary options were not exchange traded in our sample period,[10] the characterization in (5) provides a better empirical framework for testing monotonicity because we can rely mostly on commonly traded option strategy returns.

[9] More precisely, the left and right-hand and derivatives of $\ln(G())$ exist and are strictly decreasing. Concave functions are differentiable (the left and right-hand derivatives are the same) almost everywhere.

[10] The CBOE introduced binary options on SPX and VIX indices in July 2008.

The first two examples below present exchange-traded strategies with payoffs that satisfy the log-concavity condition (1). Strict monotonicity of m therefore implies monotonicity of expected returns in the strike price. For each particular trading strategy, monotonicity of expected returns in the strike price implies a weaker form of SDF monotonicity.

Example 9 (Call and put options) *The call and put payoff functions are $G(x) = x^+$ and $G(x) = (-x)^+$, respectively. Proposition 3 shows the equivalence of weak monotonicity and the monotonicity of expected returns in the strike price for call and puts.*

The next example considers symmetric butterfly spreads. It is easy to confirm that asymmetric butterfly spreads also satisfy the log-concavity condition.

Example 10 (Butterfly spread) *A strategy of long one call each at strike prices $K - \Delta K$ and $K + \Delta K$, and short two calls at strike K, has the payoff function $G(x) = (\Delta K - |x|)^+$. The inverse of the expected return represents a weighted average of $m(s)$ with strictly positive weights only for $s \in (K - \Delta K, K + \Delta K)$:*

$$\frac{1}{R\left((\Delta K - |S_T - K|)^+\right)} = E\left(m(S_T)\frac{(\Delta K - |S_T - K|)^+}{E\{(\Delta K - |S_T - K|)^+\}}\right).$$

The following two examples also satisfy log concavity, but are not exchange-traded.

Example 11 (Binary cash-or-nothing option) *The call version pays \$1 if the stock price expires in the money (i.e, $S_T \geq K$) and zero otherwise. Therefore $G(x) = 1_{\{x \geq 0\}}$ and the expected return,*

$$R\left(1_{\{S_T \geq K\}}\right) = \frac{1}{E\left(m \mid S_T \geq K\right)},$$

is increasing in K if and only if $E(m \mid S_T > K)$ is decreasing in K, which is equivalent to

$$m(K) > E\left(m(S_T) \mid S_T \geq K\right) \text{ for all } K \geq 0.$$

The put version pays \$1 if $S_T \leq K$ and zero otherwise. Therefore $G(x) = 1_{\{x \leq 0\}}$, and the expected return is increasing in K if and only if

$$m(K) < E\left(m(S_T) \mid S_T \leq K\right) \text{ for all } K \geq 0.$$

If the expected returns of both vanilla call options and the binary cash-or-nothing call options are increasing in the strike price, then

$$m(K) > E\left(m(S_T) \mid S_T \geq K\right) > \frac{E\left(m(S_T)S_T \mid S_T \geq K\right)}{E\left(S_T \mid S_T \geq K\right)} \text{ for all } K \geq 0$$

(an analogous relationship holds for the put case). The call-option-return test puts more weight on the right tail of the stock-price distribution than the binary test. For example, if $m(s)$ is not strictly monotonic, but declines sharply as s becomes large, then call option returns may be increasing in the strike, but the binary calls returns may not.

Example 12 (Modified bullish call spread) *Consider a portfolio that is long a call with strike K, short a call with strike $K + \Delta K$, where $\Delta K > 0$, and short a cash-or-nothing binary call with payoff $\Delta K \cdot 1_{\{S_T > K + \Delta K\}}$ (all on the same stock and same expiration T).*[11] *The portfolio payoff is $(S_T - K)\, 1_{\{S_T \in [K, K+\Delta K]\}}$, and*

$$\frac{d}{dK} R\left((S_T - K)\, 1_{\{S_T \in [K, K+\Delta K]\}}\right) = -\kappa \frac{Cov\left(S_T, m \mid S_T \in [K, K+\Delta K]\right)}{Var\left(S_T \mid S_T \in [K, K+\Delta K]\right)},$$
$$\text{where } \kappa = Var\left(S_T \mid S_T \in [K, K+\Delta K]\right) \left(\frac{P\left(S_T \in [K, K+\Delta K]\right)}{E\left\{m \cdot (S_T - K)\, 1_{\{S_T \in [K, K+\Delta K]\}}\right\}}\right)^2.$$

As the increment ΔK converges to to zero, the derivative (scaled by ΔK) becomes propor-

[11] The standard bullish call spread (long a call with strike K and short a call with strike $K + \Delta K$) also satisfies the log-concavity condition. The bearish call spread and bullish put spread violate regularity (because the payoffs are nonpositive). The bearish put spread payoff is regular, but violates log concavity.

tional to the slope of $m()$, *as is easily seen from*

$$\lim_{\Delta k \downarrow 0} \frac{Cov\left(S_T, m \middle| \, S_T \in [K, K + \Delta K]\right)}{Var\left(S_T \middle| \, S_T \in [K, K + \Delta K]\right)} = m'(K)$$

(assuming continuous differentiability of $m()$*).*

C. Conditions for Strict Monotonicity

In this section we provide conditions for strict monotonicity of m. The first approach is to provide conditions on the joint distribution of the terminal stock price and terminal wealth so that monotonicity of m holds for any risk-averse agent maximizing expected utility of terminal wealth. Necessary and sufficient conditions are given in Lemma 13 below and illustrated in Example 14. The second approach is to impose conditions directly on the joint distribution of (m, S_T). Sufficient conditions are provided in the remaining examples.

In the following lemma, we can interpret the random variable W as the time-T wealth of a risk-averse agent choosing a portfolio to maximize $Eu(W)$.

Lemma 13 *Suppose an SDF* m *satisfies* $m = u'(W)$ *for some random variable* W *and strictly decreasing* $u'()$. *Then a necessary and sufficient condition for the projection* $m(s) = E\left(u'(W) \,|S_T = s\right)$ *to be strictly decreasing in* s *for all* $s \geq 0$ *and any strictly decreasing* $u'()$ *is that* $P\left(W \leq w \middle| \, S_T = s\right)$ *is strictly decreasing in* s *for all* w, s.

Proof. Follows directly from results on first-order stochastic dominance (see Huang and Litzenberger (1988)). ■

The Lemma says that strict monotonicity holds if and only if there is some expected-utility maximizing agent whose distribution of wealth conditional on a higher terminal stock price dominates, in the first-order sense, the distribution conditional on a lower price. This condition is generally stronger than positive correlation.

Example 14 *Let z be a random variable independent of S_T, and $W = g(S_T, z)$, where g is strictly increasing in its first argument for all z. Then strict monotonicity of m holds for any concave u.*

Example 14 includes the special cases when wealth and the terminal stock price are positively correlated and distributed either bivariate normal or bivariate lognormal.

The remaining examples obtain strict monotonicity of m by imposing assumptions directly on the joint distribution of (m, S_T).[12] If the distribution of (m, S_T) is bivariate elliptical.[13], then Vershik (1964) shows that the projection of m on S_T is linear:

$$E(m \mid S_T = s) = E(m) + \frac{\mathrm{Cov}(m, S_T)}{\mathrm{Var}(S_T)}(s - E(S_T)), \tag{2}$$

and therefore a positive risk premium (which is equivalent to $\mathrm{Cov}(m, S_T) < 0$) is equivalent to strict monotonicity of m. The bivariate normal distribution is a special case of the elliptical distribution. The same linear projection holds if (m, S_T) is bivariate Pareto.[14] That is,

$$m = \lambda_1 + \theta_1 \frac{X_1}{Z}, \quad S_T = \lambda_2 + \theta_2 \frac{X_2}{Z},$$

where X_1 and X_2 are independent exponential random variables with unit intensities, Z is Gamma$(\gamma, 1)$ random variable independent of X_1 and X_2, and $\lambda_1, \lambda_2, \theta_1, \theta_2 \in \mathbb{R}$. Finally, if the bivariate distribution of (m, Y) satisfies a linear projection as in (2) (with Y taking the place of S_T) and $S_T = f(Y)$ for some strictly increasing function f, then $\mathrm{Cov}(m, Y) < 0$ is equivalent to strict monotonicity of m. Furthermore, in the bivariate elliptical case, Stein's lemma implies that $\mathrm{Cov}(m, Y) < 0$ is equivalent to positivity of the risk premium.

The final example shows that Bick (1990)'s notion of viability is more restrictive than

[12] When CAPM holds, it is well known there is an SDF m which is affine in aggregate wealth, W, and therefore the same joint distributional assumptions can be applied to (W, S_T).

[13] The same result holds for the generalized elliptical distribution (see Xu and Hou (2008)).

[14] See example Example 2.3 in Furman and Zitikis (2008). See their paper also for the case of bivariate gamma distribution, which also implies (2).

strict monotonicity because it requires the SDF (not just the projection) to be a decreasing function of the terminal stock price.

Example 15 (viable diffusions) *Suppose the stock price process S satisfies*

$$\frac{dS_t}{S_t} = \mu(t, S_t)\, dt + \sigma(t, S_t)\, dB_t, \tag{3}$$

where B is standard Brownian motion, and the coefficients μ and σ are sufficiently integrable. Bick (1990) terms (3) *viable if the state-price density process π satisfies $\pi_T = u'(S_T)$ for some differentiable and strictly concave function u. Viability is sufficient but not necessary for strict monotonicity, because the latter requires only that the projection $m(s) = E(\pi_T/\pi_0 \; |S_T = s)$ be strictly decreasing in s. For example, if μ and σ are deterministic but time varying, then viability is essentially equivalent to $(\mu_t - r_t)/\sigma_t^2 = \kappa$ for all $t \in [0, T]$ for some strictly positive constant κ;*[15] *but strict monotonicity is equivalent to the weaker condition $\int_0^T (\mu_t - r_t)\, dt > 0$.*

II. Data

The options data are from the OptionMetrics Ivy DB database, which contains information on the entire exchange-traded US equity option market from 1996 to 2008 and includes daily volume, open interest, best daily closings bid and ask quotes, option Greeks and implied volatilities. The implied volatilities and Greeks are calculated using the binomial model of Cox, Ross, and Rubinstein (1979). The data set also includes information on daily prices, returns and dividend distributions of all exchange-traded stocks.

Because our strike-price monotonicity results apply only to European-style options, we restrict our analysis to option contracts for which the underlying stock has no ex-dividend dates during the remaining life of the contract. The early exercise premium for such call

[15] If $(\mu_t - r_t)/\sigma_t^2$ is not constant, then Bick's necessary condition (i)(d) is violated because $\frac{d}{dx}k(t, S, T, x)$ can be shown to depend on t as well as x.

options is therefore zero. There is some bias in our put return results, however, because of the possibility of early exercise. In principal, the holding period return of an American put could be adjusted as follows. Assuming a one-factor diffusion model, the idea behind the representation in Theorem 1 in Carr, Jarrow, and Myneni (1992) can be used to obtain the following expression for gross return on an American put option over the interval (t, T), including any interest earned on the strike over any subinterval in which early exercise is optimal:

$$\frac{P(S_T, T) + rK \int_t^T e^{r(T-u)} 1_{\{S_u \leq B_u\}} du}{P(S_t, t)},$$

where r is the short rate, P is the American put price, and B_t is the critical time-t stock price below which early exercise is optimal at t. We ignore the adjustment term in the numerator. In our analysis of average put option returns, we believe that the adjustment is small because of our short holding periods (one week) and the low interest rates within our sample period.

Following the standard practice in empirical option studies, we choose only those call and put option contracts for which the bid price is greater than or equal to \$0.125. We also eliminate contracts for which the recorded ask price is lower than the bid price.[16] We impose a filter that requires option prices, estimated as the bid-ask midpoint, exceed the no-arbitrage lower bounds $S - Ke^{-r\tau}$ for calls and $Ke^{-r\tau} - S$ for puts, where S is the price of the underlying asset, K is the strike price, r is the risk free rate and τ is the time to expiration. In an unreported robustness check, our results hold when we also require that option prices satisfy put-call parity bounds. That is, the bid and ask prices are required to satisfy $C_{bid} - P_{ask} \leq S - Ke^{-r\tau} \leq C_{ask} - P_{bid}$, where C and P are the call and put prices respectively. On each expiration Friday from January, 1996 to June, 2005, we first identify option contracts that expire on the next expiration Friday. Following Coval and Shumway (2001), we then use prices observed on Tuesdays to calculate weekly returns for only those option contracts identified in the previous step. Our initial sample consists of 2,123,004 call

[16]Significant violations were observed in the data.

returns, and the sample reduces to 1,643,925 after the restrictions have been applied. The mean (median) number of unique stocks on each buying date is 1,380 (1,421) and the mean (median) number of strikes for each stock on each buying date is 3.43 (3.00). The thin trading in deep in-the-money (ITM) and deep out-of-the-money (OTM) options increases the possibility of stale quotes. Therefore as a robustness test we repeat our analysis with only those option contracts which have a positive volume on the buying date. The number of calls in our sample that have a positive volume on the buying date falls to 598,826. The results however remain qualitatively similar.

The initial sample for put options consists of 2,224,293 return observations, with 1,982,501 observations satisfying our sample restrictions, and 450,931 returns having a positive volume on the buying date. The mean (median) number of unique stocks on each buying date is 1,423 (1,461) and the mean (median) number of strikes for each stock on each buying date is 3.72 (3.00).

III. Empirical Results

We first test weak monotonicity by testing whether call and put expected returns are increasing in the strike price. We then test strict monotonicity by testing the monotonicity of expected returns in strike for additional option strategies with log-concave payout functions. Each pairwise test, presented in SubsectionA, assigns strategies into strike groups, computes return differences between consecutive strike groups (higher strike return minus lower strike return), and then tests whether the average return differences between groups are positive. Subsection B compliments the pairwise tests by conducting a joint test of monotonicity of expected returns in strike using the Page test for ordered alternatives, which generates a single test statistic.

A. Pairwise Tests

We first assign returns into strike groups. For any underlying stock on any buying date, we assign to strike group 3 the option-strategy return with strike closest to at the money. The next two higher strikes are assigned to strike groups 4 and 5, respectively, and the next two lower strikes are assigned to strike groups 2 and 1, respectively. Note that the returns are first divided into strike groups before any filters are applied to the data. Average return differences are reported in Table II with, for example, the column " $3-2$ " presenting average strike-group-3 minus strike-group-2 returns. For a given strategy (e.g., long calls), each difference used to compute the average is obtained by subtracting the strike-group-3 and strike-group-2 returns for the same stock on the same week. Under the hypothesis of weak monotonicity, each call and put return difference has a positive expected value. Under the hypothesis of strict monotonicity, all return differences have positive expected values. The return differences are averaged across different stocks to form a weekly time series of average return differences. The average of the weekly time series and the corresponding t-statistics are reported in Table II.

[*Insert Table II here*]

For vanilla calls and puts and binary calls we compute an additional difference. For vanilla call options, the column " $1 - R_S$ " presents the average difference between each strike-group-1 return and the return of the underlying stock (which represents a zero-strike call). For put options, the column " $R_f - 5$ " presents the average difference between the risk-free rate (the limiting return of a put as the strike price increases towards infinity) and the strike-group-5 return. And for binary calls, the column "$1 - R_f$ " presents the average difference between the strike-group-1 return and the risk-free rate (the return on a binary call with zero strike).

Table II also provides a skewness-adjusted t-statistic for the average return difference. Central-limit approximations are problematic because of the small sample sizes and the heavy skewness of option returns (most OTM option returns are -100%). We use skewness-adjusted standard errors developed by Chen (1995) as an extension of Johnson (1978) and Sutton (1993). It is derived using Hall's t-type inversion of the Edgeworth expansion and has been shown to be accurate for sample sizes as small as 13 and for distributions as asymmetric as the exponential distribution. Further details on this test statistic are provided in the appendix.

A.1. Weak Monotonicity Tests

Results for call option returns, presented in Table II Panel A, indicate that the average return differences are all positive and statistically significant at the 5% level of significance (return differences " $4-3$ " and " $5-4$ " are significant at the 1% level) suggesting that expected call returns are monotonically increasing in the strike price. For example, call options in strike group 1 earn on average 1% higher weekly return than that of underlying stocks, and call options in strike group 4 earn about 10% higher average weekly return than call options in strike group 3. The skewness-adjusted t-statistic for OTM contracts are significantly higher than the usual t-statistics reflecting the strong positive skew in OTM call return distributions. This is not surprising because of the following reason: It is well known that the distribution of the t-statistic inherits a skewness that is opposite of the parent distribution. If the original distribution is positively skewed, then the t-distribution is negativiely skewed, and confidence intervals are to the left of where they should be to ensure symmetric coverage. In the context of our hypothesis testing this translates into true upper tail probabilities that are higher than normal resulting in fewer rejections. Thus to find a more accurate and powerful test with a common critical point for a given significance level α, we use a modified t-statistic that appropriately accounts for the skewness. See appendix for more details.

The put-option results in Panel B are somewhat weak. None of the return differences except for " R_f-5 " are significant at the 5% level. The average " $5-4$ " return difference is significant at the 10% level and the average " $3-2$ " return difference is negative, although not statistically significant. However in an unreported test, we confirm that the average return difference " $5-1$ " is positive and significant at the 5% level of significance. Moreover, the Page-test result in Section B is consistent with strict monotonicity.

A.2. Strict Monotonicity Tests

Butterfly spreads A butterfly spread with call options is composed of two short calls with strike price K, and long one call each with strikes $K - \Delta K$ and $K + \Delta K$.[17] To calculate the return differences for the butterfly spreads, we impose the following restrictions: 1) There should exist at least four valid strikes for any underlying stock on any buying date to estimate at least one return difference; 2) The estimated price of the spread must be greater than \$0.125 to eliminate no-arbitrage convexity violations and reduce the possibility of unrealistic returns; and 3) The value of ΔK should be the same across all butterfly spreads on the same underlying stock and the same buying date. We allow the spreads to have overlapping strikes: that is, the middle and right strike of a spread could be the left and middle strike respectively of the next higher spread. The total number of valid butterfly returns after the restrictions on the calls and the spreads are imposed is 181,976.

Results from Table II Panel C indicate that two out of the four differences are positive and significant at the 1% level . The difference " $2-1$ " is negative but statistically insignificant. The difference " $5-4$" is positive and statistically significant at the 5% level.

Binary call options Estimating returns for non-exchange traded securities is more involved because binary options were not exchange traded in our sample period. We estimate their prices by using a smooth implied volatility curve estimated using calls on all available strikes on the same underlying stock and same trading day. Let $B(K) = E\left\{m 1_{\{S_T > K\}}\right\}$ denote the price of the binary option with strike K that pays a dollar if it expires in the

[17] Butterfly spread strike groups are assigned based on the middle strike.

money. Assuming only differentiability, it is well known that

$$\frac{d}{dK}C(K) = \frac{d}{dK}E\left\{m \cdot (S_T - K) 1_{\{S_T > K\}}\right\} = -B(K).$$

Because strikes are insufficiently dense, a better way to compute $\frac{d}{dK}C(K)$ (rather than differencing market premiums at different strikes) is to estimate a smooth Black-Scholes (B-S) implied volatility curve $\sigma(K)$ and then estimate $B(K)$ from the B-S model in the following way. Letting r denote the annualized continuously compounded zero-coupon-bond yield from 0 to T, then (assuming a smooth B-S implied volatility curve $\sigma()$):

$$B(K) = -\frac{d}{dK}C^{BS}(S, K, \sigma(K)) = e^{-rT}\left\{N(d_2) - KN'(d_2)\sqrt{T}\sigma'(K)\right\}.$$

We apply two obvious no-arbitrage restrictions:

$$B(K) \in \left(0, e^{-rT}\right), \quad \frac{d}{dK}B(K) \leq 0.$$

The second can be checked using

$$\begin{aligned}\frac{d^2C}{dK^2} &= e^{-rT}\frac{N'(d_2)}{\sigma(K)}\left\{1 + d_2K\sqrt{T}\sigma'(K)\right\}\left\{\frac{1}{K\sqrt{T}} + d_1\sigma'(K)\right\} \\ &\quad + e^{-rT}KN'(d_2)\sqrt{T}\sigma''(K).\end{aligned}$$

To estimate a smooth B-S implied volatility curve, implied volatilities for all calls on the same stock and same buying date are used to fit a smooth curve using cubic spline interpolation. To ensure better estimation of the implied volatility curve, we restrict our sample to only those calls whose underlying stocks have at least four strikes on any trading day.[18] Finally, to reduce the possibility of unrealistic returns, we omit binary calls with estimated prices below \$0.01, giving us a total of 94,009 binary call returns that satisfies all

[18]We get similar results when we require at least three strikes instead of four.

the restrictions. Results from Table II Panel D indicate that all average return differences are positive, with three out of five significant at the 1% level. Binary calls for strike group 1 earn on average 4.9% higher weekly return than the risk free whereas binary calls for strike group 4 are 5.9% higher than those for strike group 3.

Modified bullish call spreads Once we have the prices of the cash-or-nothing binary call option, computing the prices of the modified bullish call spread is straightforward. Recall that we defined a modified bullish call spread as a portfolio that is long a call with strike K, short a call with strike $K + \Delta K$, where $\Delta K > 0$, and short a cash-or-nothing binary call with payoff $\Delta K \cdot 1_{\{S_T > K + \Delta K\}}$ (all on the same stock and same expiration T). We impose no other restriction on the bullish call spreads other than requiring a minimum price of 0.125, which results in a total of 63,291 bullish call return observations. Table II Panel E shows that the return differences are all positive and significant at the 1% level of significance. For example, bullish call spread returns for strike group 2 earn on average 9.7% higher weekly return than strike group 2, and returns for strike group 3 are 4.7% higher than those for strike group 2.

B. Page test for ordered alternatives

The results in Table II are overall consistent with strict SDF monotonicity. A problem with the pairwise tests, however, is that four or five separate tests statistics are provided for each strategy. If all are positive and significant, this clearly supports the hypothesis of monotonicity, but mixed results can be difficult to interpret statistically. Letting $\bar{r}_i$ denote the mean strike-group-i return, the Page test (or Page's L test) for ordered alternatives provides a single statistic to test the null hypothesis of equal means returns,

$$H_0 : \bar{r}_1 = \bar{r}_2 = \cdots = \bar{r}_k,$$

against the alternative hypothesis that the means are monotonically increasing across groups:

$$H_A : \bar{r}_1 \leq \bar{r}_2 \leq \cdots \leq \bar{r}_k.$$

However, the distribution of the Page test statistic relies on the (typically unstated) assumption that each $k!$ ranks are equally likely under the null.[19] In our application this assumption is questionable because, for example, in the Black-Scholes model under the null (which is equivalent to zero risk premia) only two ranks of realized call-option returns would be observed for a particular stock over some small period: returns monotonically increasing in strike if the market return exceeds the riskless return, and returns monotonically decreasing in strike otherwise. This tends to increase the variance of the test statistic, though the effect is mitigated by both averaging of returns across stocks each period and by other factors (such as volatility) affecting call-option returns. Therefore we also compute a modified test statistic. Letting $\left(X_t^1, \ldots, X_t^k\right)$ denote the time-t ranks (which is some permutation of $\{1, \ldots, k\}$) of the k average returns for each strike group, the null hypothesis of the modified test statistic is that the true covariance between the rank and the index position is zero every period:

$$E \sum_{j=1}^{k} \left(j - \frac{k+1}{2} \right) X_t^j = 0, \quad t = 1, \ldots, T.$$

That is, for each period the true regression slope of the points $\left\{ \left(j, X_t^j\right) ; j = 0, \ldots k \right\}$ is zero . A special case is if each permutation is equally likely under the null (as in the classical Page test). Another special case[20] is if the distribution of the rank is symmetric under H_0 in the sense that the distribution of the random vector $V = \left\{ X_t^1 - \frac{k+1}{2}, \ldots X_t^k - \frac{k+1}{2} \right\}$ is the same as the distribution of $-V$. This includes the case in which there are only two equally

[19] This is satisfied, for example, if the k random variables are independent and identically distributed under H_0.

[20] Note that

$$\sum_{j=1}^{K_i} \left(j - \frac{K_i + 1}{2} \right) X_t^{i,j} = \sum_{j=1}^{K_i} \left(X_t^{i,j} - \frac{K_i + 1}{2} \right) j.$$

likely rankings: strictly ascending or strictly descending.

To apply the test, we take the average return differences from our pairwise tests (Section A) and, for each period, cumulate them across strike groups to get an average return for each group (strike group zero is assigned a zero average return; average returns for all other groups are relative to strike group zero). The average returns each period are ranked to get a vector of time-t ranks $\left(X_t^1, \ldots, X_t^k\right)$. The test statistic is

$$L = \frac{1}{\sigma\sqrt{T}} \sum_{t=1}^{T} L_t \quad \text{where} \quad L_t = \sum_{j=1}^{k} \left(j - \frac{k+1}{2}\right) X_t^j.$$

Under the assumption of equally likely ranks (the classical Page test)

$$\sigma^2 = \frac{1}{144} k^2 \left(k+1\right)^2 \left(k-1\right).$$

Our modified test is based on estimating $\hat{\sigma}$ in the usual manner:

$$\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^{T} \left(L_t - \bar{L}\right)^2 \quad \text{where} \quad \bar{L} = \frac{1}{T} \sum_{t=1}^{T} L_t.$$

Under either H_0 (and using the appropriate variance term) the test statistic is asymtotically standard normal.

Table III reports the Page test statistics for all the option strategies. To be consistent with the analysis in Table II, the maximum number of strike groups considered is five. Panel A reports the results for the classical Page test and panel B reports the results for the modified Page test which does not rely on the assumption that ranks are equally likely under the null.

[*Insert Table III here*]

From Table II, we see that the Page statistics for all the option strategies are positive and highly significant suggesting that the average returns of the option strategies are ordered in an increasing across strike groups. For example, the statistic for call options is 9.73 for the classical Page test and 6.11 for the restricted sample. These high statistic values result because, in the call-option samples, for example if we look at a stock-by-stock case, 72% of the stocks have average-return ranks that are monotonically increasing in strike (that is, rank vectors $(1, 2, \ldots, k_i)$). The results are even stronger if we further increase the required minimum number of weekly observations . For example, restricting the number of minimum weekly observations to 150 (250) increases the number of stocks that have average call returns strictly increasing in strike to 81%(90%). Note also that although the pairwise test results for puts from Table II Panel B were somewhat weak, the joint test provides convincing evidence of strict monotonicity.

C. Heterogeneity and Test Design

The previous two subsections provide strong evidence in support of strict monotonicity of the SDF. These results contradict the results of Ni (2007), who finds violations of weak monotonicity. Ni (2007) sorts call option contracts on each buying date into five portfolios based on moneyness and shows that the portfolio with OTM option contracts earns significantly lower returns than the portfolio with ITM option contracts. She suggests that investors are sometimes risk-seeking and a preference for idiosyncratic skewness leads to a premium for deep OTM options and could be a possible explanation for the puzzling call returns. We will show that such a testing procedure can bias her results because it fails to control for the stocks underlying the options in each portfolio, unlike our pairwise tests, in which matched pairs of returns (same underlying stock and same time period) for different strikes are subtracted before averaging.

[*Insert Table IV here*]

To illustrate the problem, consider the Black-Scholes setting, in which the excess (over the riskless rate) option return equals the product of the option elasticity and the excess stock return. Elasticities depend on the option moneyness and stock volatility (as well as the short rate and time to expiration). Table IV presents elasticities for AIG and USRX call options for a particular date. The Black-Scholes elasticity of the lowest strike call on AIG is almost equal to the elasticity of the highest strike call on USRX. The heterogeneity between these two stocks would not bias the results if AIG and USRX each had one option in each of the five strike groups each day. However, if both stocks have calls in strike group 4, but USRX calls are overrepresented in strike group 5, this may induce a downward bias in strike-group 5 returns relative to strike-group 4 returns. Many stocks offer only a limited number of strikes, especially after applying reasonable filtering rules (e.g., eliminating no-arbitrage violations). The problem is likely to be most severe in the deep ITM and deep OTM groups, where strike listings are thinnest, and where thin trading is most likely to result in more frequent filter violations. The combination of the underlying stock heterogeneity and the heterogeneity of strike-group representation would have the most severe impact in the deep OTM group because of the much higher elasticities in this group compared to the deep ITM group.

[*Insert Table V here*]

Table V reports the average Black-Scholes elasticities of calls sorted into strike groups based on the option's moneyness. The sample and moneyness cutoffs [21] used are from Ni (2007). The mean and median elasticities of deep OTM calls are in fact smaller than that of ATMs, despite the fact that, ceteris paribus, elasticities are increasing in strike. Assuming, for illustration, identical stock betas, positive risk premia, and the validity of CAPM, the expected return of the ATM group would therefore exceed that of the OTM group, despite the fact that expected returns of calls would be increasing in strike in the B-S setting. Also note that the standard deviation of the elasticities are enormous suggesting huge variations within strike groups.

[21] The moneyness cutoffs used are as follows: $K/S \leq 0.85, 0.85 < K/S \leq 0.95, 0.95 < K/S \leq 1.05, 1.05 < K/S \leq 1.15, K/S > 1.15$.

Further compounding the heterogeneity issue is the moneyness sorting rule of Ni (2007). Consider a call option with a strike price of 35 when the underlying stock is worth $30. The moneyness of this option is 1.17 and would be assigned to strike group five using the cutoffs used in Ni (2007). In fact, all OTM calls in this case will be assigned to the same strike group.[22]. Alternatively, when the stock price is high ($175 for example), most of the out of the money calls will be assigned to strike group three. In other words, the strike-increment to price ratio varies significantly with the price level, which has a disparate impact on the strike-group assignments for different stocks.

IV. Conclusion

This paper develops a simple nonparametric method for testing the strict monotonicity of the projected SDF using the expected returns of option strategies. Our application is to stocks, but the method is applicable to any process (whether or not a traded asset) on which options are traded, such as energy or commodity products. We show that strict monotonicity with respect to stocks is implied by a variety of different models of the joint distribution of returns and the SDF. Using data on option contracts on individual stocks, we find that the average weekly returns of the tested option trading strategies is consistent with strict monotonicity. Our method can be applied in future work to stock-index options, for example, to test classes of asset-pricing models such as representative agent models with expected utility.

[22]For a underlying stock price of 30, strikes would generally increase in intervals of 5

Appendix A SDF monotonicity

Proof of Proposition 1

(i) $\iff$ (ii) follows because

$$R(S_T) = \frac{E(S_T)}{E(mS_T)} > \frac{1}{E(m)} = R(1) \tag{A1}$$

is equivalent to $0 > E(mS_T) - E(S_T)E(m)$, which is equivalent to (i). Rearranging $(A1)$ gives $E(S_T) > E(mS_T)/E(m)$, which is equivalent to (iii). Finally,(i) $\iff$ (iv) because

$$\frac{d}{dK}R(S_T - K) = \frac{E(S_T - K)E(m) - E(m(S_T - K))}{\{E(m[S_T - K])\}^2} = \frac{E(S_T)E(m) - E(mS_T)}{\{E(m[S_T - K])\}^2},$$

and because $R(X) = R(-X)$ for any payout $X \in F_T$.

Proof of Proposition 3

(i) $\iff$ (iv) follows, as shown in Coval and Shumway (2001), from

$$\begin{aligned}
\frac{d}{dK}R\left((S_T - K)^+\right) &= \frac{d}{dK}\frac{E\left\{(S_T - K)^+\right\}}{E\left\{m(S_T - K)^+\right\}} \\
&= \frac{E\left\{(S_T - K)^+\right\}E\left(m1_{\{S_T > K\}}\right) - E(m(S_T - K))P(S_T > K)}{\left(E\left\{m(S_T - K)^+\right\}\right)^2} \\
&= -\left(\frac{P(S_T > K)}{E\left\{m(S_T - K)^+\right\}}\right)^2 Cov(S_T, m \mid S_T > K).
\end{aligned}$$

and, similarly,

$$\frac{d}{dK}R\left((K - S_T)^+\right) = -\left(\frac{P(S_T < K)}{E\left\{m(K - S_T)^+\right\}}\right)^2 Cov(S_T, m \mid S_T < K).$$

(i) $\iff$ (ii) follows because

$$R\left(S_T 1_{\{S_T \in [\alpha,\beta]\}}\right) = \frac{E\left(S_T 1_{\{S_T \in [\alpha,\beta]\}}\right)}{E\left(mS_T 1_{\{S_T \in [\alpha,\beta]\}}\right)} > \frac{P(S_T \in [\alpha,\beta])}{E\left(m1_{\{S_T \in [\alpha,\beta]\}}\right)} = R\left(1_{\{S_T \in [\alpha,\beta]\}}\right)$$

is equivalent to

$$0 > E\left(mS_T 1_{\{S_T\in[\alpha,\beta]\}}\right) P\left(S_T \in [\alpha,\beta]\right) - E\left(S_T 1_{\{S_T\in[\alpha,\beta]\}}\right) E\left(m1_{\{S_T\in[\alpha,\beta]\}}\right). \tag{A2}$$

The right-hand side of $(A2)$ is equal to $Cov\left(S_T, m \mid S_T \in [\alpha,\beta]\right)\left\{P\left(S_T \in [\alpha,\beta]\right)\right\}^2$. The inequality in (iii) is

$$E\left(S_T \mid\ S_T \in [\alpha,\beta]\right) = \frac{E\left(mS_T 1_{\{S_T\in[\alpha,\beta]\}}\right)}{P\left(S_T \in [\alpha,\beta]\right)} > \frac{E\left(mS_T 1_{\{S_T\in[\alpha,\beta]\}}\right)}{E\left(m1_{\{S_T\in[\alpha,\beta]\}}\right)} = E^Q\left(S_T \mid\ S_T \in [\alpha,\beta]\right).$$

Together with $(A2)$ we get (i) $\Longleftrightarrow$ (iii).

Derivation of Example 5

Let $m_i = m(s_i)$ for $i \in \{1,\dots,4\}$. We derive necessary and sufficient conditions on the m_is for weak monotonicity. For the truncated regressions it is sufficient to consider conditioning on $\{S_T \le s_i\}$ or $\{S_T \ge s_i\}$, for $i \in \{2,3\}$. Obviously (as in a two-point regression)

$$Cov\left(S_T, m \mid S_T \le s_2\right) < 0 \quad \Longleftrightarrow \quad m_1 > m_2$$

and

$$Cov\left(S_T, m \mid S_T \ge s_3\right) < 0 \quad \Longleftrightarrow \quad m_3 > m_4.$$

Noting that $E\left(S_T \mid S_T \le s_3\right) = s_2$ (recall the s_i's are equally spaced):

$$Cov\left(S_T, m \mid S_T \le s_3\right) = E\left(\left(S_T - s_2\right)m \mid S_T \le s_3\right) = \frac{1}{3}\left\{\left(s_1 - s_2\right)m_1 + \left(s_2 - s_2\right)m_2 + \left(s_3 - s_2\right)m_3\right\}$$

and therefore

$$Cov\left(S_T, m \mid S_T \le s_3\right) < 0 \quad \Longleftrightarrow \quad m_1 > m_3.$$

The analogous argument gives

$$Cov\left(S_T, m \mid S_T \geq s_2\right) < 0 \Longleftrightarrow m_2 > m_4.$$

Finally, the unconditional covariance satisfies (using $E\left(S_T\right) = \left(s_2 + s_3\right)/2 = \bar{s}$)

$$Cov\left(S_T, m\right) = E\left(\left(S_T - \bar{s}\right) m\right) = \frac{1}{4}\left\{\left(s_1 - \bar{s}\right) m_1 + \left(s_2 - \bar{s}\right) m_2 + \left(s_3 - \bar{s}\right) m_3 + \left(s_4 - \bar{s}\right) m_4\right\}$$

and therefore (multiplying both sides by 8 and defining $\Delta s = s_{i+1} - s_i$ as the common stock-price increment)

$$Cov\left(S_T, m\right) < 0 \quad \Longleftrightarrow \quad \Delta s \cdot m_3 + 3\Delta s \cdot m_4 < 3\Delta s \cdot m_1 + \Delta s \cdot m_2.$$

That is, a negative unconditional covariance is equivalent to

$$m_3 - m_2 < 3\left(m_1 - m_4\right); \tag{A3}$$

but $(A3)$ follows trivially from the earlier conditions $m_1 > m_3$ and $m_2 > m_4$ (which together imply $m_3 - m_2 < m_1 - m_4$).

Proof of Proposition 7

We assume, for simplicity, that G is right continuous at x_1 and left continuous at x_2 (that is, $G\left(x_1\right) = \lim_{s\downarrow x_1} G\left(s\right)$ and $G\left(x_2\right) = \lim_{s\uparrow x_2} G\left(s\right)$).

a) Sufficiency of (1): Let F be some absolutely continuous distribution function F and let $\alpha = \int_0^\infty G\left(u - K\right) dF\left(u\right)$ (which represents the expected payout under F). Define the distribution function W:

$$W\left(s; K\right) = \frac{1}{\alpha}\int_0^s G\left(u - K\right) dF\left(u\right) \quad s \in [0, \infty). \tag{A4}$$

The inverse of the expected return is

$$\frac{1}{R(K)} = \frac{E(mG(S_T - K))}{E(G(S_T - K))} = \int_0^\infty m(s)\, dW(s;K). \tag{A5}$$

We show that $R(K)$ is strictly increasing in K when $m(s)$ is strictly decreasing in s by showing that $W(s;K)$ is decreasing in K for all $s \in (0,\infty)$ and $K \geq 0$. That is, $W(\cdot;K_2)$ stochastically dominates $W(\cdot;K_1)$ in the first order sense for any $0 \leq K_1 < K_2$.[23] If $s - K < x_1$, then $W(s;k) = 0$ for k in some neighborhood of K and therefore $\frac{d}{dK} W(s;K) = 0$. We therefore consider only $s \geq K + x_1$. Differentiating $(A4)$, we have $\frac{d}{dK} W(s;K) \leq 0$ is equivalent to $H(s) \leq 0$ where

$$H(s) = -\alpha \int_0^s F'(u)\, dG(u-K) + \int_0^\infty F'(u)\, dG(u-K) \int_0^s G(u-K)\, dF(u).$$

Using the notation $W'(s;K) = \frac{d}{ds} W(s;K)$ and substituting $W'(s;K) = \alpha^{-1} G(s-K) F'(s)$, then $\frac{d}{dK} W(s;K) \leq 0$ if and only if

$$\int_0^\infty 1_{\{u-K \in [x_1,x_2]\}} W'(u;K) \frac{dG(u-K)}{G(u-K)} \leq \int_0^s 1_{\{u-K \in [x_1,x_2]\}} \frac{W'(u;K)}{W(s;K)} \frac{dG(u-K)}{G(u-K)}.$$

We are computing a weighted average of $dG(u-K)/G(u-K)$ on both sides of the above[24], but the weight function on the right side is more concentrated to the left (that is, on smaller values of u). The inequality therefore holds because of first-order stochastic dominance[25] and the monotonicity condition in (1) (which implies that $dG(u-K)/G(u-K)$ is nonincreasing in u).

b) Necessity of (1): Consider first the case of a jump in $G'()/G()$, and suppose contrary to

[23] See Huang and Litzenberger (1988, Ch 2).

[24] Note that $\int_0^\infty W'(u;K)\, du = \int_0^s \frac{W'(u;K)}{W(s;K)} du = 1$.

[25] The distribution function defined by the density function $W'(u;K)$ first-order dominates that defined by the density $\frac{W'(u;K)}{W(s;K)} 1_{\{u \leq s\}}$.

the hypothesis that

$$\lim_{s\uparrow\hat{s}}\frac{G'(s)}{G(s)}<\lim_{s\downarrow\hat{s}}\frac{G'(s)}{G(s)}\text{ some }\hat{s}\in(x_1,x_2),$$

and define the midpoint

$$p=\frac{1}{2}\left(\lim_{s\uparrow\hat{s}}\frac{G'(s)}{G(s)}+\lim_{s\downarrow\hat{s}}\frac{G'(s)}{G(s)}\right).$$

Using piecewise continuity, choose $\varepsilon>0$ sufficiently small that $G'(x)/G(x)<p$ for $x\in[\hat{s}-\varepsilon,\hat{s})$ and $G'(x)/G(x)>p$ for $x\in(\hat{s},\hat{s}+\varepsilon)$, and concentrate F on $[\hat{s}-\varepsilon,\hat{s}+\varepsilon]$ (that is, let $F(\hat{s}-\varepsilon)=0$, and $F(\hat{s}+\varepsilon)=1$).[26] Define

$$\Phi(s)=\int_{\hat{s}-\varepsilon}^{s}\frac{G'(u)}{G(u)}dW(u;0).$$

Then $H(\hat{s})\leq 0$ is equivalent to $\Phi(\hat{s}+\varepsilon)\leq\Phi(\hat{s})/W(\hat{s};0)$ Using the identity

$$\Phi(\hat{s}+\varepsilon)=W(\hat{s};0)\left(\frac{\Phi(\hat{s})}{W(\hat{s};0)}\right)+\{1-W(\hat{s};0)\}\left(\frac{\Phi(\hat{s}+\varepsilon)-\Phi(\hat{s})}{1-W(\hat{s};0)}\right)$$

together with

$$\frac{\Phi(\hat{s}+\varepsilon)-\Phi(\hat{s})}{1-W(\hat{s};0)}>p>\frac{\Phi(\hat{s})}{W(\hat{s};0)}$$

gives

$$\Phi(\hat{s}+\varepsilon)>W(\hat{s};0)\left(\frac{\Phi(\hat{s})}{W(\hat{s};0)}\right)+\{1-W(\hat{s};0)\}\frac{\Phi(\hat{s})}{W(\hat{s};0)}=\frac{\Phi(\hat{s})}{W(\hat{s};0)}.$$

But $\Phi(\hat{s}+\varepsilon)>\Phi(\hat{s})/W(\hat{s};0)$ implies $H(\hat{s})>0$, contradicts monotonicity in K. Therefore $G'()/G()$ cannot increase at a jump.

Now consider the possibility of an increase in G'/G over any interval in which G'/G is continuous. Suppose

$$\frac{G'(s_2)}{G(s_2)}>\frac{G'(s_1)}{G(s_1)}\tag{A6}$$

for some $s_1<s_2$, where $s_1,s_2\in(x_1,x_2)$. Let $s^*\in\arg\min\{G'(s)/G(s);s\in[s_1,s_2]\}$.

[26] It is straightforward (but tedious) to modify the proof to use only distribution functions F with support equal to the positive real line (which is our standing assumption).

Concentrating F on $[s^*, s_2]$ (that is, let $F(s^*) = 0$ and $F(s_2) = 1$), then, for any $s \in (s^*, s_2)$, $H(s) \leq 0$ is equivalent to

$$\int_{s^*}^{s_2} \frac{G'(u)}{G(u)} dW(u;0) \leq \int_{s^*}^{s} \frac{G'(u)}{G(u)} \frac{dW(u;0)}{W(s;0)}.$$

Letting $s \downarrow s^*$,

$$\int_{s_1}^{s_2} \frac{G'(u)}{G(u)} dW(u;0) \leq \frac{G'(s^*)}{G(s^*)},$$

which contradicts the fact that s^* minimizes G'/G in this interval.

The following lemma relaxes the standing assumption that F is absolutely continuous.

Lemma A1 *Suppose Y is a random variable with distribution function F. Let the support of Y be $[a, b]$, where $-\infty \leq a < b \leq \infty$, and let $\sigma(Y)$ denote the information set generated by Y. a) If g is a strictly decreasing function of finite variation then*

$$Cov(Y, g(Y)|\, A) < 0 \quad \text{for all } A \in \sigma(Y) \text{ and } P(A) > 0.$$

b) If g is continuously differentiable, and F is strictly increasing on $[a, b]$ (no zero probability intervals), then g is strictly decreasing if and only if

$$Cov(Y, g(Y)|\, Y \in [\alpha, \beta]) < 0 \quad \text{for all } a \leq \alpha < \beta \leq b. \tag{A7}$$

Proof. Part a) Define

$$h(x; A) = \int_{\alpha}^{x} \{E(Y|\, A) - y\}\, 1_{\{y \in A\}} \frac{dF(y)}{P(A)},$$

which satisfies $h(a-) = h(b) = 0$ and $h(x) > 0$ for all x such that $P(Y \in A \text{ and } Y \leq x) > 0$.

Using integration by parts:

$$\begin{aligned}\mathrm{Cov}(Y, g(Y)|\ A) &= \int_{a-}^{b} (y - E(Y|\ A))\, 1_{\{y\in A\}} g(y) \frac{dF(y)}{P(A)} \\ &= -\int_{a-}^{b} g(y)\, dh(y; A) \\ &= \int_{a-}^{b} h(y-; A)\, dg(y).\end{aligned}$$

If g is strictly decreasing then the left hand side must be strictly negative.

Part b) The sufficiency of g strictly decreasing follows from Part a. To prove necessity, suppose $(A7)$ holds, but g is not strictly decreasing. Then there must exist points $c < e$ such that $g(c) \leq g(e)$, and therefore a point $d \in (c, e)$ such that $g'(d) > 0$ (we can rule out $g'(x) = 0$ for all $x \in (c, e)$, because this contradicts $\mathrm{Cov}\big(Y, g(Y)\, 1_{\{Y\in[c,e]\}}\big) < 0$). Continuity of g' implies $g'(x) > 0$ for x in a neighborhood of d, implying $\mathrm{Cov}\big(Y, g(Y)\, 1_{\{Y\in[d-\varepsilon,d+\varepsilon]\}}\big) > 0$ for some $\varepsilon > 0$, contradicting $(A7)$. ■

Proof of Proposition 8

The proof of (i) $\Longleftrightarrow$ (ii) follows from Lemma A1 above. The proof of the equivalence of (ii), (iii) and (iv) is the same as in Proposition 3 (the weak monotonicity case). The sufficiency part of (ii) follows from Proposition 7 above. The necessity proof of (ii) follows.

Suppose $m()$ is not strictly decreasing. Then there must exist points $c < e$ such that $m(c) \leq m(e)$. If $m()$ is constant in the interval $[c, e]$, then fix some $\bar{K}$ and define some G satisfying Condition 6 so that $G\left(s - \bar{K}\right)$ is positive only for s within some interval within the interior of $[c, e]$. Then it is easy to show that $R(G(S_T - K))$ will be constant for K within some neighborhood of $\bar{K}$. If $m()$ is not constant in the interval $[c, e]$, there must exist a point $d \in (c, e)$ such that $g'(d) > 0$. Continuity of g' implies $g'(x) > 0$ for x in an interval $[d_1, d_2]$ containing d. Fix some $\bar{K}$ and construct a G that satisfies Condition 6 so that $G\left(s - \bar{K}\right)$ is positive only for $s \in [d_1, d_2]$. It is easy to show from the sufficiency part of the proof of Proposition 7 that $R(G(S_T - K))$ is decreasing in K within some neighborhood of $\bar{K}$.

Appendix B Skewness-adjusted t-statistics

The proposed statistic for testing the mean of positively skewed distributions is taken from Chen (1995) and is derived using the Edgeworth expansion as follows:

$$P\left\{\frac{\sqrt{n}(\bar{X}-\mu)}{S} \leq x - \frac{1}{6\sqrt{n}}\hat{\beta}_1(1+2x^2)\right\} = \Phi(x) + o(n^{-1/2}), \tag{B1}$$

where $\Phi(x)$ is the standard normal distribution, $S^2 = \frac{\sum(X_i-\bar{X})^2}{(n-1)}$, and $\hat{\beta}_1 = \frac{n^{-1}\sum(X_i-\bar{X})^3}{S^3}$ (For more details on the Edgeworth expansion, see Hall (1983)). To test the hypothesis $H_0 : \mu_x = \mu_0$ against $H_0 : \mu_x > \mu_0$ is to reject H_0 when

$$\frac{\sqrt{n}(\bar{X}-\mu_0)}{S} > z_\alpha - \frac{1}{6\sqrt{n}}\hat{\beta}_1(1+2z^2) \tag{B2}$$

where z_α satisfies the equation $1-\Phi(z_\alpha) = \alpha$. Chen (1995) argues that the critical point of the above hypothesis test depends on the skewness of each sample. Thus to find a more accurate and powerful test with a common critical point for a given significance level α, we use the following approach. We first solve $\frac{\sqrt{n}(\bar{X}-\mu_0)}{S} > x - \frac{1}{6\sqrt{n}}\hat{\beta}_1(1+2x^2)$ for x. Let $\alpha = \frac{\hat{\beta}_1}{6\sqrt{n}}$ and $t = \frac{\sqrt{n}(\bar{X}-\mu)}{S}$. Thus, when n is large such that $1-8a(t+a) \geq 0$, we have from equation B1,

$$P\left\{\frac{1-\sqrt{1-8a(t+a)}}{4a} > z_\alpha \text{ or } \frac{1+\sqrt{1-8a(t+a)}}{4a} < z_\alpha\right\} = \alpha + o(n^{-1/2})$$

Thus we have

$$P\left\{\frac{1-\sqrt{1-8a(t+a)}}{4a} > z_\alpha\right\} \leq \alpha$$

Using Taylor series expansion we then have

$$\frac{1-\sqrt{1-8a(t+a)}}{4a} = t + a + 2at^2 + 4a^2(t+2t^3) + o(n^{-1/2}) \tag{B3}$$

Equation B3 is the skewness adjusted t statistic that we have used in this paper. An advantage of this test statistic is that it can be used for sample sizes as small as 13 and for distributions as asymmetric as the exponential distribution. Expressing equation B3 in terms of the original variables, we get the following expression:

$$t_{skew} = \frac{\sqrt{n}(\bar{X}-\mu_0)}{S} + \frac{1}{6\sqrt{n}}\hat{\beta}_1\left[1+2\left(\frac{\sqrt{n}(\bar{X}-\mu_0)}{S}\right)^2\right] + \frac{1}{9n}\hat{\beta}_1^2\left[\frac{\sqrt{n}(\bar{X}-\mu_0)}{S}+2\left(\frac{\sqrt{n}(\bar{X}-\mu_0)}{S}\right)^2\right] \quad \text{(B4)}$$

Appendix C Page Test Appendix

Let

$$L_t = \sum_{j=1}^{k}\left(j-\frac{k+1}{2}\right)X_t^j$$

where $\left\{X_t^1,\ldots,X_t^k\right\}$ denotes ranks of the k time-t treatments (time-t average returns over the k strike groups in our application). The mean and variance of L_t are computed under the equally-likely rank assumption of the classical Page test.

Lemma C1 *Assuming each of the* $k!$ *ranks are equally likely then*

$$\begin{aligned} E\left(L_t\right) &= 0, \\ Var\left(L_t\right) &= \frac{1}{144}k^2\left(k+1\right)^2\left(k-1\right). \end{aligned}$$

Proof. Let p denote a random permutation of $\{1,\ldots,k\}$ with each $k!$ permutation equally likely, and let $p(j)$ denoting the jth element of p. Then

$$E\left(L_t\right) = \sum_{j=1}^{k}\left(j-\frac{k+1}{2}\right)E\left(p\left(j\right)\right) = E\left(p\left(1\right)\right)\sum_{j=1}^{k}\left(j-\frac{k+1}{2}\right) = 0.$$

To compute the variance, we first show that

$$Cov\left(p\left(j\right), p\left(i\right)\right) = -\frac{1}{k-1} Var\left(p\left(j\right)\right) \quad i \neq j,\ i, j \in \{1, \ldots, k\}.$$

This is done by defining the random variable

$$x = \begin{cases} p\left(i\right) & \text{with probability} \quad (k-1)/k \\ p\left(j\right) & \text{with probability} \quad 1/k \end{cases}$$

Note that x is uniformly distributed on $\{1, \ldots, k\}$ and is independent of $p\left(j\right)$. Therefore

$$0 = Cov\left(p\left(j\right), x\right) = \frac{k-1}{k} Cov\left(p\left(j\right), p\left(i\right)\right) + \frac{1}{k} Var\left(p\left(j\right)\right).$$

Using $Var\left(L_t\right) = Var\left(\sum_{j=1}^{k} jX_j\right)$, we compute the variance:

$$\begin{aligned} Var\left(\sum_{j=1}^{k} jX_j\right) &= \sum_{j=1}^{k}\sum_{i=1}^{k} ijCov\left(p\left(j\right), p\left(i\right)\right) \\ &= Var\left(p\left(1\right)\right)\left\{\sum_{j=1}^{k} j^2 - \frac{1}{k-1}\sum_{j=1}^{k}\sum_{i\neq j} ij\right\} \\ &= Var\left(p\left(1\right)\right)\left\{\sum_{j=1}^{k} j^2 - \frac{1}{k-1}\left(\left[\sum_{j=1}^{k} j\right]^2 - \sum_{j} j^2\right)\right\} \\ &= Var\left(p\left(1\right)\right)\frac{k^2\left(k+1\right)}{12} \end{aligned}$$

Finally substitute

$$Var\left(p\left(1\right)\right) = \frac{1}{k}\sum_{j=1}^{k}\left(j - \frac{k+1}{2}\right)^2 = \frac{1}{12}\left(k+1\right)\left(k-1\right),$$

where we have used, for any $k \in \{1, 2, \ldots\}$,

$$\sum_{j=1}^{k} j^2 = \frac{k\,(k+1)\,(2k+1)}{6}.$$

■

References

Ait-Sahalia, Yacine, and A. W. Lo, 2000, Nonparametric risk management and implied risk aversion, *Journal of Econometrics* 94, 9–51.

Ait-Sahalia, Yacine, Yubo Wang, and Francis Yared, 2001, Do option markets correctly price the probabilities of movement of the underlying assets?, *Journal of Econometrics* 102, 67–110.

Bakshi, Gurdip, and Zhiwu Chen, 2001, Stock valuation in dynamic economies, Working Paper.

Bakshi, Gurdip, Nikunj Kapadia, and Dilip Madan, 2003, Stock return characteristics, skew laws, and the differential pricing of individual equity options, *The Review of Financial Studies* 16, 101–143.

Bakshi, G., and D. Madan, 2009, Investor heterogeneity and the non-monotonicity of the aggregate marginal rate of substitution in market index, Working paper.

Bergman, Yaacov Z., Bruce D. Grundy, and Zvi Weiner, 1996, General properties of option prices, *Journal of Finance* 51, 1573–1610.

Bick, Avi, 1990, On viable diffusion price processes of the market portfolio, *The Journal of Finance* 45, 673–689.

Black, Fisher, and Myron S. Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–659.

Blackburn, Douglas W., and Andrey D. Ukhov, 2006, Estimating preferences toward risk: evidence from dow jones, working paper, Indiana University at Bloomington.

Broadie, Mark, Mikhail Chernov, and Michael Johannes, 2007, Understanding index option returns, Working Paper.

Brown, David, and Jens Carsten Jackwerth, 2004, The pricing kernel puzzle: Reconciling index option data and economic theory, Working Paper.

Cambell, John Y., and John H. Cochrane, 2000, Explaining the poor performance of consumption-based asset pricing models, *Journal of Finance* 55, 2863–2878.

Carr, P., H. Geman, D. Madan, and M. Yor, 2002, The fine structure of asset returns: An empirical investigation, *Journal of Business* 75, 305–332.

Carr, Peter, Robert Jarrow, and Ravi Myneni, 1992, Alternative characterizations of american put options, *Mathematical Finance* 2, 87–106.

Chabi-Yo, Fousseni, Ren Garcia, and Eric Renault, 2008, State dependence can explain the risk aversion puzzle, *Review of Financial Studies* 21, 973–1011.

Chen, Ling, 1995, Testing the mean of skewed distributions, *Journal of the American Statistical Association* 90, 767–772.

Chen, Ren-Raw, and Oded Palmon, 2002, An empirical-distribution-based option pricing model: A solution to the volatility smile puzzle, Working Paper.

Chernov, Mikhail, and Eric Ghysels, 2000, Towards a unified approach to the joint estimation of objective and risk neutral measures for the purpose of options valuation, *Journal of Financial Economics* 56, 407–458.

Constantinides, George, 1992, A theory of the nominal term structure of interest rates, *Review of Financial Studies* 5, 531–552.

Coval, Joshua, and Tyler Shumway, 2001, Expected option returns, *Journal of Finance* 56, 983–1009.

Cox, John C., Stephen A. Ross, and Mark Rubinstein, 1979, Option pricing: A simplified approach, *Journal of Financial Economics* 7, 229–263.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Ferreira, Eva, Monica Gago, and Rubio Gonzalo, 2003, A semiparametric estimation of liquidity effects on option pricing, *Spanish Economic Review* 5, 1–24.

Finucane, Thomas J, 1991, Put-call parity and expected returns, *The Journal of Financial and Quantitative Analysis* 26, 445–457.

Friedman, Milton, and Leonard J. Savage, 1948, The utility analysis of choices involving risk, *Journal of Political Economy* 56, 279–304.

Furman, E., and R. Zitikis, 2008, General stein-type decompositions of covariances and the capital asset pricing model, Working Paper.

Giovannini, Alberto, and Philippe Weil, 1989, Risk aversion and intertemporal substitution in the capital asset pricing model, NBER Working Paper.

Hall, Peter, 1983, Inverting an edgeworth expansion, *The Annals of Statistics* 11, 569–576.

Han, B, 2004, Limits of arbitrage, sentiment, and pricing kernel: Evidences from index options, Working Paper, Department of Finance, Ohio State University.

Hansen, Lars P., and Ravi Jagannathan, 1997, Assessing specification errors in stochastic discount factor models, *Journal of Finance* 52, 557–590.

Hansen, L. P., and K. J. Singleton, 1982, General instrumental variables estimation of non-linear rational expectations model, *Econometrica* 50, 1269–1286.

——— , 1983, Stochastic consumption, risk aversion,and the temporal behavior of asset returns, *Journal of Political Economy* 91, 249–265.

Harvey, C. R., and R. E. Whaley, 1992, Market volatiltiy prediction and the efficiency of the s&p 100 index option market, *Journal of Financial Economics* 30, 43–73.

Huang, C., and R. Litzenberger, 1988, *Foundations for Financial Economics* (North-Hollland).

Jackwerth, Jens Carsten, 2000, Recovering risk aversion from option prices and realized returns, *Review of Financial Studies* 13, 433–451 Working Paper.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–92.

Johnson, N. J., 1978, Modified t tests and confidence intervals for asymmetrical populations, *Journal of the American Statistical Association* 73, 536–544.

Kumar, Alok, 2005, Institutional skewness preferences and the idiosyncratic skewness premium, Working paper, University of Texas at Austin, working paper.

Mayhew, Stewart, and Vassil Mihov, 2004, How do exchanges select stocks for option listing,, *Journal of Finance* 59, 447–471.

McDonald, Robert, and Daniel Seigel, 1984, Option pricing when the underlying asset earns a below-equilibrium rate of return, *The Journal of Finance* 39, 261–265.

McEnally, Richard W., 1974, A note on the return behavior of high risk common stocks, *Journal of Finance* 29, 199–202.

Merton, Robert C., 1973(a), An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

——— , 1973(b), Theory of rational option pricing, *The Bell Journal of Economics and Management Science* 4, 141–183.

Mitton, Todd, and Keith Vorkink, 2005, Equilibrium underdiversification and the preference for skewness, Working paper, Bringhan Young University.

Newey, Whitney K., and Kenneth D. West, 1987, A simple positive-definite heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Ni, Sophie Xiaoyan, 2007, Stock option returns: A puzzle, Working paper, University of Illinois at Urbana-Champaign.

Ofek, Eli, and Matthew Richardson, 2003, Dotcom mania: the rise and fall of internet stock prices, *The Journal of Finance* 58, 1113–1137.

Page, E B., 1963, Ordered hypotheses for multiple treatments: A significance for linear ranks, *Journal of the American Statistical Association* 58, 216–230.

Pan, Jun, 2002, The jump-risk premia implicit in options: Evidence from an integrated time-series study, *Journal of Financial Economics* 63, 3–50.

Pirie, W R., 1985, Page test for ordered alternatives, *Encyclopedia of Statistical Sciences* 6, 553–555.

Post, Thierry, and Haim Levy, 2005, Does risk seeking drive stock prices? a stochastic dominance analysis of aggregate investor preferences and beliefs, *Review of Financial Studies* 18, 925–953.

Quiggin, John, 1991, On the optimal design of lotteries, *Econometrica* 58, 1–16.

Rosenberg, Joshua, and Robert Engle, 2002, Empirical pricing kernels, *Journal of Financial Economics* 64, 341–372.

Rubinstein, Mark, 1984, A simple formula for the expected rate of return of an option over a finite holding period, *The Journal of Finance* 39, 1503–1509.

Santa-Clara, Pedro, and Alessio Saretto, 2005, Option strategies, good deals and margin calls, Working Pape.

Shive, S., and T. Shumway, 2006, Is the pricing kernel monotonic?, Working paper. University of Notre Dame.

Sutton, C. D., 1993, Computer-intensive methods for tests about the mean of an asymmetrical distribution, *Journal of the American Statistical Association* 88, 802–810.

Vanden, Joel M, 2006, Option coskewness and capital asset pricing, Working Paper.

Vershik, A.M., 1964, Some characteristic properties of gaussian stochastic processes, *Theory of Probability and Its Applications* 9, 353–356.

Xu, X., and C. Hou, 2008, Capital asset pricing model with generalized elliptical distribution, *Systems Engineering - Theory & Practice* 26, 17–23.

Yaari, Menahem E., 1987, The dual theory of choice under risk, *Econometrica* 55, 95–115.

Ziegler, A, 2007, Why does implied risk aversion smile?, *Review of Financial Studies* 20, 859–904.

Table I

Average Returns for option strategies on the S&P 500 index

This table reports weekly average buy and hold returns for some option trading strategies on the S&P 500 index that are special cases of the log-concave class of payout functions. Strike groups are defined using the following procedure: For each buying date we find the option contract which has a strike price that is closest to the price of the underlying security and assign that to strike group 3. The next two higher strikes are assigned to groups 4 and 5 respectively. Similarly, the previous two lower strikes are assigned to groups 1 and 2 respectively. Returns are calculated based on prices observed on Tuesdays. " $*$ " indicates the strike group for which the average return difference between the strike group and its immediate lower strike group is negative and statistically significant at the 5% level thus violating strict monotoncity.

Table I: Average returns by strike group

Panel A: call returns					
Strike Group	1	2	3	4	5
Average returns	0.024	0.028	0.036	0.042	0.046
t statistics	0.774	0.822	0.927	0.899	0.830

Panel B: put returns					
Strike Groups	1	2	3	4	5
Average returns	-0.140	-0.120	-0.105	-0.090	-0.078
t statistics	-3.172	-2.834	-2.618	-2.419	-2.258

Panel C: binary call returns					
Strike Groups	1	2	3	4	5
Average returns	-0.029	0.004	-0.008	0.041	0.042
t statistics	-1.550	0.208	-0.344	1.488	1.257

Panel D: bullish call returns					
Strike Groups	1	2	3	4	5
Average returns	0.004	-0.008	0.041	0.042	0.068
t statistics	0.208	-0.344	1.488	1.257	1.706

Panel E: (modified) butterfly spread returns					
Strike Groups	1	2	3	4	5
Average returns	0.470	0.507	0.276 *	0.392	0.525
t statistics	2.840	4.008	2.584	2.803	4.389

Table II
Average Return Differences for option strategies on individual stocks

This table reports differences of weekly average buy and hold returns for some option trading strategies that are special cases of the log-concave class of payout functions. Strike groups are defined using the following procedure: For each underlying stock and buying date we find the option contract which has a strike price that is closest to the price of the underlying stock and assign that to strike group 3. The next two higher strikes are assigned to groups 4 and 5 respectively. Similarly, the previous two lower strikes are assigned to groups 1 and 2 respectively. The options must satisfy the following conditions to be included in the sample: (1) The bid price is strictly larger than \$0.125, (2) the ask price is greater than the bid price, (3) the underlying stock does not have an ex-dividend date prior to maturity and (4) the option prices satisfies a no-arbitrage restriction. R_s denotes the return on the underlying stock and R_f denotes the risk-free rate.

Table II: Average return differences by strike group

Panel A: call returns

Strike Groups	1-R_s	2 -1	3 -2	4 -3	5 -4
Average returns	0.010	0.006	0.014	0.102	0.070
t statistics	2.33	2.25	2.23	8.81	3.82
Skewness adjusted t statistics	2.30	2.25	2.28	15.07	6.27

Panel B: put returns

Strike Groups	2 -1	3 -2	4 -3	5 -4	R_f - 5
Average returns	0.002	-0.004	0.013	0.012	0.016
t statistics	0.07	-0.36	0.97	1.73	2.27
Skewness adjusted t statistics	0.17	-0.39	0.85	1.63	2.19

Panel C: butterfly spread returns

Strike Groups		2 -1	3 -2	4 -3	5 -4
Average returns	-	-0.009	0.079	0.187	0.133
t statistics	-	-0.99	4.36	4.14	1.98
Skewness adjusted t statistics	-	-1.01	4.62	7.14	2.26

Panel D: binary call returns

Strike Groups	1-R_f	2 -1	3 -2	4 -3	5 -4
Average returns	0.049	0.000	0.006	0.059	0.225
t statistics	8.19	0.04	0.54	3.30	3.04
Skewness adjusted t statistics	7.68	0.04	0.63	3.70	5.63

Panel E: (modified) bullish call returns

Strike Groups		2 -1	3 -2	4 -3	5 -4
Average returns	-	0.097	0.047	0.185	0.099
t statistics	-	8.47	3.19	9.49	2.94
Skewness adjusted t statistics	-	5.31	4.07	9.82	6.00

Table III
Page Test for Ordered Alternatives

Table III, reports the results of the Page Test for Ordered Alternatives for option strategies on individual stocks. Average return differences from our pairwise tests for each period are cumulated across strike groups to get an average return for each group The average returns each period are ranked to get a vector of time-t ranks. Panel A reports the results for the classical Page test and Panel B reports the results for the modified Page test that does not depend on the equal rank assumption under the null.

Table III: Page Test

Panel A

	No. of Periods	Page statistic
call options	377	9.47
put options	377	11.67
butterfly spreads	377	12.48
binary call options	377	5.41
modified bullish call spreads	377	15.04

Panel B

	No. of Periods	Page statistic
call options	377	6.11
put options	377	7.96
butterfly spreads	377	4.24
binary call options	377	2.41
modified bullish call spreads	377	7.85

Table IV
Option Elasticities for AIG and USRX

This table illustrates the heterogeneity in option elasticities across stocks.

Table IV						
date	ticker	option id	stock price	strike	moneyness	elasticity
19-Jan-96	AIG	10415728	94.25	80.00	0.85	6.24
19-Jan-96	AIG	10394678	94.25	85.00	0.90	9.19
19-Jan-96	AIG	11008635	94.25	90.00	0.95	14.86
19-Jan-96	AIG	11132774	94.25	95.00	1.01	24.34
19-Jan-96	AIG	10105028	94.25	100.00	1.06	33.24
date	ticker	option id	stock price	strike	moneyness	elasticity
19-Jan-96	USRX	11566685	89.50	75.00	0.84	4.27
19-Jan-96	USRX	10755696	89.50	80.00	0.89	4.54
19-Jan-96	USRX	11569192	89.50	85.00	0.95	5.29
19-Jan-96	USRX	10055356	89.50	90.00	1.01	5.95
19-Jan-96	USRX	11499707	89.50	95.00	1.06	6.90

Table V
Average Deltas and Elasticities by Strike Group

This table reports the mean, median and standard deviation of the elasticities and deltas of call options on each buying date sorted by moneyness. Strike Groups are estimating using the Ni (2007) moneyness cutoffs. They are as follows: $[K/S \leq 0.085, 0.85 < K/S \leq 0.95, 0.95 < K/S \leq 1.05, 1.05 < K/S \leq 1.15, K/S > 1.15]$. Call options are selected on each option expiration date that mature on the next expiration date if the following conditions are satisfied: (1) The bid price is strictly larger than $0.125; (2) The ask price is greater than the bid price; (3) The underlying stock does not have an ex-dividend date prior to maturity; and (4) The call and put prices satisfies a no-arbitrage restriction.

Table V					
Panel A: Call deltas sorted by moneyness					
Strike Group	1	2	3	4	5
mean	0.92	0.81	0.54	0.30	0.20
median	0.94	0.81	0.55	0.30	0.19
std. deviation	0.05	0.09	0.12	0.09	0.09
Panel B: Call elasticities sorted by moneyness					
mean	3.82	7.20	12.87	13.60	9.51
median	3.79	6.83	11.13	12.37	8.80
std. deviation	1.05	2.18	7.52	6.49	3.59