**Assignment 1**

**Robotics 811, Fall 2017**

DUE: Thursday, September 14, 2017

1. Implement the $PA = LDU$ decomposition algorithm directly yourself (in other words, do not just call a built-in Gaussian elimination algorithm in MatLab, for instance). You may assume that the matrix $A$ is square and invertible. Also: Do not worry about column interchanges, just row interchanges. Show that your implementation works properly.

2. Compute the $PA = LDU$ decomposition and the SVD decomposition for each of the following matrices:

   (It is perhaps easiest if you compute the $PA = LDU$ decompositions by hand and use a pre-defined routine to compute the SVD decompositions.)

   (a)
$$A_1 = \begin{pmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{pmatrix}$$

   (b)
$$A_2 = \begin{pmatrix} 4 & 8 & 0 & 0 \\ 2 & 0 & -2 & 0 \\ 0 & 4 & -1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$$

   (c)
$$A_3 = \begin{pmatrix} 2 & 2 & 5 \\ 3 & 2 & 5 \\ 1 & 1 & 5 \end{pmatrix}$$

3. Solve the system of equations $Ax = b$ for the given values of $A$ and $b$. Specify whether each system has zero, one, or more solutions. If the system has zero solutions give the SVD solution. If the system has a unique solution compute that solution. If the system has more than one solution specify both the SVD solution and all solutions. Relate your answers to the SVD decomposition.

   (a)
$$A = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ 5 & 5 & 5 \end{pmatrix} \qquad b = \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix}$$

   (b)
$$A_1 = \begin{pmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$$

   (c)
$$A_1 = \begin{pmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 18 \\ -12 \\ 8 \end{pmatrix}$$

4. Suppose that $u$ is an $n$-dimensional column vector of unit length in $\Re^n$, and let $u^T$ be its transpose. Then $uu^T$ is a matrix. Consider the $n \times n$ matrix $A = I - uu^T$.

   (a) Describe the action of the matrix $A$ geometrically.

   (b) Give the eigenvalues of $A$.

   (c) Describe the null space of $A$.

   (d) What is $A^2$?

5. The following problem arises in a large number of robotics and vision problems: Suppose $\mathbf{p}_1, \ldots, \mathbf{p}_n$ are the 3D coordinates of $n$ points located on a rigid body in three-space. Suppose further that $\mathbf{q}_1, \ldots, \mathbf{q}_n$ are the 3D coordinates of these same points after the body has been translated and rotated by some unknown amount. Devise and demonstrate an algorithm in which SVD plays a central role for inferring the body's translation and rotation.

   **Comment:** Your algorithm should make use of all the information available. True, in principle you only need three pairs of points – but if you use more points your solution will be more robust, something that might come in handy some day when you need to do this for real with noisy data.

   **Caution:** A common mistake is to find the best affine transformation, rather than the best rigid body transformation.

   **Hint:** It may help to think in terms of inertia and/or correlation matrices.