Summary

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Nov 2023

1 Introduction

Innovations in aerial robotics control systems are critical for advancing the capabilities of unmanned aerial vehicles (UAVs), particularly quadrotors. This research focuses on the application of the Alternating Direction Method of Multipliers (ADMM) to address the optimal control problem (OCP) with aerodynamic drag, which traditional approaches often neglect.

2 Quadrotor Dynamics

The dynamics of a quadrotor, including the influence of aerodynamic drag, are modeled by quadratic cost functions and deterministic linear systems. The state vector $\mathbf{x}$ and control input $\mathbf{u}$ encapsulate the system's behavior over time.

2.1 State and Control Input

The state vector of the quadrotor is represented as:

$$\mathbf{x} = \begin{bmatrix} p \\ v \\ q \\ \omega \end{bmatrix}$$

where p is the position, v is the velocity, q represents the Euler angles, and ω is the angular rate.

The control input vector is:

$$\mathbf{u} = \begin{bmatrix} F_{thrust} \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}$$

2.2 Dynamics Equations

The dynamics are described by the following differential equations:

$$
\begin{aligned}
\dot{p} &= v \\
\dot{v} &= g + \frac{1}{m} R F_{thrust} - k_d v \\
\dot{q} &= f(p, q, r) \\
\dot{\omega} &= J^{-1}(\tau - \omega \times J\omega)
\end{aligned}
$$

3 ADMM for Linear OCPs

The ADMM approach for linear OCPs is realized by the following updates, which yield a two-degree-of-freedom (2DOF) optimal controller with both feed-forward and feedback control terms.

3.1 ADMM Iterates

The ADMM iterations, when instantiated on the OCP, become:

$$
\begin{aligned}
r^{k+1} &=_r C_r(r) + \frac{\rho}{2} \|x^k - r + v^k\|_2^2 && (1) \\
x^{k+1}, u^{k+1} &=_{x,u} \frac{\rho}{2} \|x - r^{k+1} + v^k\|_2^2 + C_u(u) && (2) \\
v^{k+1} &= v^k + x^{k+1} - r^{k+1} && (3)
\end{aligned}
$$

where $C_r(r)$ and $C_u(u)$ represent the cost functions for the state and control inputs, respectively.

4 Results and Discussion

The ADMM algorithm's application to the quadrotor's OCP demonstrates that a control structure capable of zero tracking error can be achieved. This marks a substantial improvement over traditional control strategies.

5 Future Work

The ongoing work will focus on refining the control strategy by optimizing the r-sub-problem with JAXopt and implementing closed-loop feedback control. We plan to run extensive tests on the RotorPy simulator to verify the convergence and performance of the ADMM approach in realistic flight scenarios.