Likelihood

$$L(\lambda, k, \theta |\ \mathbf{x}) = \prod_{t=1}^{Ndays} exp(-\lambda)\frac{1}{x_t!}\lambda^{x_t}$$

where

$$\lambda = R0\sum_{i=1}^{t-1} x_i\ (\ Gamma((t-i); k, \theta) - Gamma((t-i-1); k, \theta)$$

Log likelihood

$$l = \sum_{i=1}^{Ndays} y_i ln(\lambda) - n\lambda$$

where

$$\lambda = R0\sum_{i=1}^{t-1} x_i\ (\ Gamma((t-i); k, \theta) - Gamma((t-i-1); k, \theta)$$

Bayesian Inference

Likelihood

$$L(\lambda, k, \theta |\ \mathbf{x}) = \prod_{t=1}^{Ndays} exp(-\lambda)\frac{1}{x_t!}\lambda^{x_t}$$

where

$$\lambda = R0\sum_{i=1}^{t-1} x_i\ (\ Gamma((t-i); k, \theta) - Gamma((t-i-1); k, \theta)$$

Prior

$$p(r0) = Gamma(\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} r0^{\alpha-1} e^{-\beta r0}$$

Bayesian Inference II

Posterior $p(r0|x)$

Given Gamma(1, 1) prior on r0;

$$p(r0|x) \propto \prod_{t=1}^{Ndays} exp(-r0\lambda_t)\frac{1}{x_t!}(r0\lambda_t)^{x_t} \times r0^{\alpha-1} e^{-\beta r0}$$

$$\propto exp(-r0(\sum_{t=1}^{Ndays} \lambda_t + 1)) \times (r0\lambda_t)^{\sum_{t=1}^{Ndays} x_t}$$

$$\therefore p(r0|x) \propto Gamma(\sum_{t=1}^{Ndays} x_t + 1, \sum_{t=1}^{Ndays} \lambda_t + 1)$$

$$\propto Gamma(\sum_{t=1}^{Ndays} x_t + \alpha, \sum_{t=1}^{Ndays} \lambda_t + \beta)$$

where

$$\lambda_t = \sum_{i=1}^{t-1} x_i\ (\ Gamma((t-i); k, \theta) - Gamma((t-i-1); k, \theta)$$

Prior

$$p(r0) = Gamma(\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} r0^{\alpha-1} e^{-\beta r0}$$