Model of Super Spreaders

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Model

Let $x_t = \psi_t + \zeta_t$

where

ψ_t = Number of non-superspreaders

$\zeta_t = x_t - \psi_t$ = Number of superspreaders

Probability

$$P(x_t) = \sum_{\psi_t=0}^{x_t} P(\psi_t) \cdot P(\zeta_t)$$

i. ψ_t - Non super-spreading events

$\psi_t \sim Poisson(a \cdot \lambda_t)$,

$$p(\psi_t) = exp(-a \cdot \lambda_t) \cdot \frac{1}{\psi_t!} \cdot (a \cdot \lambda_t)^{\psi_t},$$

with

$$\lambda_t = \sum_{i=1}^{t-1} x_i \; (\; Gamma((t-i); k, \theta) - Gamma((t-i-1); k, \theta)$$

ii. ζ_t - Super-spreading events

$\zeta_t \sim Poisson(b \cdot \lambda_t)$,

$$p(\zeta_t) = exp(-b \cdot \lambda_t) \cdot \frac{1}{\zeta_t!} \cdot (b \cdot \lambda_t)^{\zeta_t},$$

Likelihood

$$P(x_t) = \sum_{\psi=0}^{x_t} P(\psi) \cdot P(z_t)$$

$$L(a, b, p, \lambda | \, \mathbf{x_t}) = \prod_{t=1}^{Ndays} \sum_{\psi=0}^{x_t} P(\psi) \cdot P(\zeta)$$

$$= \prod_{t=1}^{Ndays} \sum_{\psi=0}^{x_t} \left(p \cdot exp(-a \cdot \lambda_t) \cdot \frac{1}{\psi!} \cdot (a \cdot \lambda_t)^{\psi} \quad + \quad (1-p) \cdot exp(-b \cdot \lambda_t) \cdot \frac{1}{\zeta!} \cdot (b \cdot \lambda_t)^{\zeta} \right)$$

Log Likelihood

$$l(\alpha, \beta, \gamma, \lambda | \, \mathbf{x_t}) = ln \left(\prod_{t=1}^{Ndays} \sum_{\psi=0}^{x_t} P(\psi) \cdot P(\zeta)) \right)$$

$$= \sum_{t=1}^{Ndays} ln \left(\sum_{\psi=0}^{x_t} p \cdot exp(-a \cdot \lambda_t) \cdot \frac{1}{\psi!} \cdot (a \cdot \lambda_t)^{\psi} \quad + \quad (1-p) \cdot exp(-b \cdot \lambda_t) \cdot \frac{1}{\zeta!} \cdot (b \cdot \lambda_t)^{\zeta} \right)$$