Mathematical Foundations in Computer Science

1 Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 2 P.M. each day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix:

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}.$$

Prove that the chain is irreducible, and determine the steady-state probabilities.

2 Jobs submitted to a university departmental file server can be divided into three classes:

Type	*Relative frequency*	*Mean execution time (in seconds)*
Student jobs	0.8	1
Faculty jobs	0.1	20
Administrative jobs	0.1	5

Assuming that, within a class, execution times are one-stage, two-stage, and three-stage Erlang, respectively, compute the average number of jobs in the server assuming a Poisson overall arrival stream of jobs with average rate of 0.1 jobs per second. Assume that all classes are treated equally by the scheduler.

3 A group of telephone subscribers is observed continuously during a 80-min busy-hour period. During this time, they make 30 calls, and the total conversation time is 4200 s. Estimate the call arrival rate and the traffic intensity.

4 The arrival of large jobs at a server forms a Poisson process with rate two per hour. The service times of such jobs are exponentially distributed with mean 20 min. Only four large jobs can be accommodated in the system at a time. Assuming that the fraction of computing power utilized by smaller jobs is negligible, determine the probability that a large job will be turned away because of lack of storage space.

5 The grades of a class of 9 students on a midterm report (x) and on the final examination (y) are as follows

x	77	50	71	72	81	94	96	99	67
y	82	66	78	34	47	85	99	99	68

(a) Estimate the linear regression line.
(b) Estimate the final examination grade of a student who received a grade of 85 on the midterm report.

6 Use the data set

y	x
7	2
50	15
100	30
40	10
70	20

a) Plot the data.
(b) Fit a regression line through the origin.
(c) Plot the regression line on the graph with the data.
(d) Give a general formula (in terms of the y_i and the slope b_1) for the estimator of σ^2.
(e) Give a formula for $Var(\hat{y}_i)$, $i = 1, 2, \ldots, n$, for this case.
(f) Plot 95% confidence limits for the mean response on the graph around the regression line.

7 The amounts of solids removed from a particular material when exposed to drying periods of different lengths are as shown:

x (hours)	y (grams)	
4.4	13.1	14.2
4.5	9.0	11.5
4.8	10.4	11.5
5.5	13.8	14.8
5.7	12.7	15.1
5.9	9.9	12.7
6.3	13.8	16.5
6.9	16.4	15.7
7.5	17.6	16.9
7.8	18.3	17.2

(a) Estimate the linear regression line.
(b) Test at the 0.05 level of significance whether the linear model is adequate.