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Including Health Insurance offers in the static model

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1 Enviroment

1.1 Households

Consider a measure one of households indexed by their health status $h \in \{g, b\}$ and their risk aversion parameter $\theta \in, [\underline{\theta}, \bar{\theta}]$[1]. The proportion of households from type $(g, \cdot)$ is $\lambda_g \in (0, 1)$.

We will assume that workers' type follows:

$$(\theta, h) \sim F(\theta, h)$$

Households care about consumption c and have a disutility from labor effort l. Preferences are of the GHH form ([Greenwood et al., 1988]):

[1] For a representative household model, the value of θ is usually chosen to be either very close to 1 or equals 2 [Mendoza, 1991]

$$u_{(h,\theta)}(c,l) = \frac{1}{1-\theta}\left[\left(c - \phi\frac{l^{1+\xi}}{1+\xi}\right)^{1-\theta} - 1\right]$$

Where ξ and ϕ are common to every household.

or more generally

$$u_{(h,\theta)}(c,l) = U(c - G(l)), U' > 0, U'' < 0, g' > 0, G'' > 0$$

From the FOCs of GHH preferences with respect to l we get

$$U'(c - G(l))\left(\frac{dc}{dl} - G'(l)\right) = 0$$

which will imply that

$$\frac{dc}{dl} = G'(l)$$

Households decide conditional on their type (h, θ), the level of consumption c, labor l and if they will work for a menu $(w_1, \alpha = 1)$ or for $(w_0, \alpha = 0)$. The optimization problem looks like:

$$\max_{c,l,\alpha\in\{0,1\}} \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(c,l)] \tag{1.1}$$

subject to the following Budget constraint:

$$c \leq \alpha w_1 l + (1-\alpha)[w_0 l - \tilde{m}(h)] \tag{1.2}$$

Where we explicitly write down the realization of the random medical expenditure to depend on current health status h (and perhaps also on health insurance status in the dynamic model).

By Local Non Satiation we can substitute consumption in the utility function and define the indirect utility function $v(w_1, w_0; \theta, h)$ as

$$v(w_1, w_0; \theta, h) = \max_{l,\alpha\in\{0,1\}} \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(\alpha w_1 l + (1-\alpha)[w_0 l - \tilde{m}(h)], l)] \tag{1.3}$$

Now we can define the conditional indirect utility (conditional on choosing or not the menu with health insurance) as follows:

$$v_0 \equiv v(w_0; \theta, h|\alpha = 0) = \max_l \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(w_0 l - \tilde{m}(h), l)] \tag{1.4}$$

$$v_1 \equiv v(w_1; \theta, h|\alpha = 1) = \max_l \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(w_1 l, l)] \tag{1.5}$$

Now, taking FOCs with respect to l

$$\frac{\partial \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(w_0 l - \tilde{m}(h), l)]}{\partial l} = 0 \Rightarrow l_0^*(h, \theta) \tag{1.6}$$

$$\frac{\partial u_{(h,\theta)}(w_1 l, l)}{\partial l} = 0 \Rightarrow l_1^*(h, \theta) \tag{1.7}$$

Now, if we assume that the random variable medical expenditure $\tilde{m}$ has a conditional CDF $H(\tilde{m}|h)$, we can write down equation 1.6 using Leibnitz rule by

$$\int \frac{\partial u_{(h,\theta)}(w_0 l - \tilde{m}, l)}{\partial l} dH(\tilde{m}|h) = 0$$

so if we use the general form of the GHH preferences this will give us

$$\int U'_{(h,\theta)}(w_0 l - \tilde{m} - G(l)) \left(\frac{dc}{dl} - G'(l) \right) dH(\tilde{m}|h) = 0$$

so

$$\int U'_{(h,\theta)}(w_0 l - \tilde{m} - G(l)) \left(w_0 - G'(l) \right) dH(\tilde{m}|h) = 0$$

which implies that

$$l_0^*(h, \theta) = G'^{-1}(w_0) \tag{1.8}$$

and similarly

$$l_1^*(h,\theta) = G'^{-1}(w_1) \tag{1.9}$$

which do not depend on h nor theta in the GHH case, because there is no wealth effect on the labor supply.

Now, if we use the following specification for the GHH preferences:

$$u_{(h,\theta)}(c,l) = U(c - G(l)) = \frac{1}{1-\theta}\left[\left(c - \phi\frac{l^{1+\xi}}{1+\xi}\right)^{1-\theta} - 1\right]$$

we get that $G(l) = \phi\frac{l^{1+\xi}}{1+\xi}$, thus $G'(l) = \phi l^{\xi}$, then $G'^{-1}(w) = \left(\frac{w}{\phi}\right)^{\frac{1}{\xi}}$

So we get

$$l_0^*(h,\theta) = \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}}, \quad l_1^*(h,\theta) = \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} \tag{1.10}$$

Finally the optimal choice will be given by

$$(l^*,\alpha^*) = \begin{cases} (l^* = l_0^*, \alpha^* = 0) & \text{if } \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(w_0 l_0^* - \tilde{m}(h), l_0^*)] > u_{(h,\theta)}(w_1 l_1^*, l_1^*) \\ (l^* = l_1^*, \alpha^* = 1) & \text{otherwise} \end{cases} \tag{1.11}$$

Then after the medical expenditure shock is realized the household consumes his remaining income, i.e,:

$$c^* = \begin{cases} w_0 l_0^* - \tilde{m}(h) & \text{if } \alpha^* = 0 \\ w_1 l_1^* & \text{otherwise} \end{cases} \tag{1.12}$$

From now onward let us assume that we have interior solution for consumption (because for labor effort is already guaranteed by its functional form). This will depend on the distributional assumptions on $H(\tilde{m}|h)$. If we choose an unbounded domain of the distribution for the medical expenditure shock, we can include a minimum consumption threshold $\underline{c}$ such that consumers will consume:

$$c^* = \begin{cases} \max\{w_0 l_0^* - \tilde{m}(h), \underline{c}\} & \text{if } \alpha^* = 0 \\ w_1 l_1^* & \text{otherwise} \end{cases} \tag{1.13}$$

this minimum threshold should ensure that consumption is always nonnegative and for the utility function to be well defined:

$$\underline{c} > \phi \frac{\left(\frac{w_0}{\phi}\right)^{\frac{1+\xi}{\xi}}}{1+\xi}$$

Notice that the inequality must be strict otherwise the household problem is not well defined if $\theta > 1$.

1.2 Labor supply

Given a parameter configuration that satisfies conditions for interiority (see previous notes), we can write the (interior) thresholds where workers are indifferent between the two options as

$$\bar{\theta}_h \in \{\theta \in (0,1) : \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(w_0 l_0^* - \tilde{m}(h), l_0^*)] = u_{(h,\theta)}(w_1 l_1^*, l_1^*)\} \tag{1.14}$$

and if we include the minimum consumption $\underline{c}$ this thresholds should be defined by:

$$\bar{\theta}_h \in \{\theta \in (0,1) : \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(\max\{w_0 l_0^* - \tilde{m}(h), \underline{c}\}, l_0^*)] = u_{(h,\theta)}(w_1 l_1^*, l_1^*)\} \tag{1.15}$$

So the aggregate labor supply of workers for each contract (with and without health insurance provision) for each health status type will be given by:

Aggregate labor supply:

$$L_g^1(w_1, w_0) = \lambda_g l_1^*(h_g)(1-F_g(\bar{\theta}_g)) = \lambda_g G'^{-1}(w_1)(1-F_g(\bar{\theta}_g)) = \lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1-F_g(\bar{\theta}_g)) \tag{1.16}$$

and similarly we get

$$L_g^0(w_1,w_0) = \lambda_g G'^{-1}(w_0) F_g(\bar{\theta}_g) = \lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g) \tag{1.17}$$

$$L_b^1(w_1,w_0) = (1-\lambda_g) G'^{-1}(w_1)(1 - F_b(\bar{\theta}_b)) = (1-\lambda_g) \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1 - F_b(\bar{\theta}_b)) \tag{1.18}$$

$$L_b^0(w_1,w_0) = (1-\lambda_g) G'^{-1}(w_0) F_b(\bar{\theta}_b) = (1-\lambda_g) \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_b(\bar{\theta}_b) \tag{1.19}$$

1.3 Firms

Consider a measure 1 of firms indexed by $i \in [N-1, N]$. Each firm i can produce a task y_i as a monopoly choosing the level of inputs and if they will provide health insurance or not $x_i \in \{0,1\}$, taking as given (due to perfect competition on the labor market) the wages w_0 and w_1 for contracts without health insurance and with health insurance respectively, intermediate good price ψ and rental rate of capital R.

Firms maximize profits choosing the level of labor l_i they want to hire, the amount of intermediates q_i, capital k_i and whether they will hire labor with or without providing health insurance, $x_i \in \{0,1\}$.

As capital and labor are substitutes in production, besides the level of intermediates q_i chosen, firms will either

- Hire labor without health insurance ($x_i = 0$), paying wage w_0 , facing an endogenous proportion of workers of type g, χ^0_{gi}.

- Hire labor with health insurance ($x_i = 1$), paying wage w_1, the expected medical expenditure of the workers it hires, M_i, and a fixed administrative cost of health insurance C^{IN}, facing an endogenous proportion of workers of type g, χ^1_{gi}.

- Use capital in production ($x_i = 0$), paying a marginal cost R and a fixed cost that enables automation for task y_i, C^A_i.

As households are characterized by their health status h, we will assume that the productivity of labor in task y_i for workers of type $h = g$ is given by γ_i and that the productivity of labor for workers of type $h = b$ is given by $\rho\gamma_i$ where $\rho \in (0,1)$. Firms also differ in their productivity of capital z_i and in the elasticity of substitution between factors, ζ_i, capturing that more complex tasks can have a different degree of substitutability between labor and capital.

Production conditional on choosing contract $x_i = I$ is given by the following production function:

$$y^I_i = B[\eta q_i^{\frac{\zeta_i-1}{\zeta_i}} + (1-\eta)\left(z_i k_i + \gamma_i l_i \chi^I_{gi} + \rho\gamma_i l_i(1-\chi^I_{gi})\right)^{\frac{\zeta_i-1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i-1}} \tag{1.20}$$

Each monopoly i faces a demand function given by

$$y_i = Y p_i^{-\sigma} \tag{1.21}$$

Thus the inverse demand function is

$$p_i = \left(\frac{Y}{y_i}\right)^{1/\sigma} \tag{1.22}$$

1.3.1 No Health insurance

First lets take a look to the profit maximization problem of firms, conditional on not providing health insurance:

$$\Pi^0(w_1, w_0, R) = \max_{l_i, q_i, k_i} Y^{1/\sigma}(y_i^0)^{1-1/\sigma} - \psi q_i - w_0 l_i - R k_i - 1_{\{k_i>0\}} C_i^A \tag{1.23}$$

subject to

$$y_i^0 = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)\left(z_i k_i + \gamma_i l_i \chi_{gi}^0 + \rho \gamma_i l_i (1-\chi_{gi}^0)\right)^{\frac{\zeta_i-1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i-1}} \tag{1.24}$$

where

$$B \equiv (1-\eta)^{\zeta/(1-\zeta)}$$

and $\chi_{gi}^0 = \chi_{gi}^0(w_1, w_0)$ is the endogenous proportion of workers of type g that will work under the contract without health insurance when is hired in task y_i and market wages are w_0, w_1.

This measure will be pin down on equilibrium. By market clearing we should have that:

$$\int_{N-1}^{N} (1-x_i) l_i \chi_{gi}^0(w_1, w_0) di = L_g^0(w1, w_0) \tag{1.25}$$

as we want that this endogenous proportion varies over firm's size or productivity, we will assume that:

$$\chi_{gi}^{0}(w_1,w_0)=\delta_i\left(\frac{L_g^0(w1,w_0)}{L_g^0(w1,w_0)+L_b^0(w1,w_0)}\right) \tag{1.26}$$

where δ_i is an increasing and bounded function such that $\chi_{gi}^{I}\in[0,1]$.

Now we can take First Order Conditions to characterize the optimal choices of labor and intermediates.

Now let's define the average labor productivity for contract I by

$$\bar{\gamma}_i^I\equiv\gamma_i\left((1-\rho)\chi_{gi}^I+\rho\right) \tag{1.27}$$

the effective costs of labor

$$\hat{w}_i^0\equiv\frac{w_0}{\bar{\gamma}_i^0} \tag{1.28}$$

$$\hat{w}_i^1\equiv\frac{w_1+M_i}{\bar{\gamma}_i^1} \tag{1.29}$$

and effective cost of capital

$$\hat{R}_i\equiv\frac{R}{z_i} \tag{1.30}$$

So we can rewrite the profit maximization problem in effective units of labor and effective units of capital as:

$$\Pi^0(\hat{w}_i^0,\hat{R}_i)=\max_{\hat{l}_i^0,q_i,\hat{k}_i} Y^{1/\sigma}(y_i^0)^{1-1/\sigma}-\psi q_i-\hat{w}_i^0\hat{l}_i^0-\hat{R}_i\hat{k}_i-1_{\{k_i>0\}}C_i^A \tag{1.31}$$

subject to

$$y_i^0=B[\eta q_i^{\frac{\zeta_i-1}{\zeta_i}}+(1-\eta)\left(\hat{k}_i+\hat{l}_i^0\right)^{\frac{\zeta_i-1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i-1}} \tag{1.32}$$

where

$$\hat{k}_i \equiv z_i k_i \tag{1.33}$$

$$\hat{l}_i^0 \equiv \bar{\gamma}_i^0 l_i \tag{1.34}$$

1.3.2 Conditional Cost minimization problem

Similarly than before, we can write the conditional cost minimization problem with effective units of labor and effective cost of labor as follows:

$$c(y_i^0, \hat{w}_i^0, \hat{R}_i) = \min_{\hat{l}_i^0, q_i, \hat{k}_i} \psi q_i + \hat{w}_i^0 \hat{l}_i^0 + \hat{R}_i \hat{k}_i + 1_{\{\hat{k}_i > 0\}} C_i^A$$

subject to

$$y_i = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)(\hat{k}_i + \hat{l}_i^0)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \tag{1.35}$$

1.3.3 Conditional on hiring labor

Notice that effective units of capital and effective units of labor are perfect substitutes in production so a firm will use capital just if the effective marginal cost of capital is lower enough to offset the automation cost C_i^A. This means that we need to keep track of the level of production, to know when a firm will be indifferent between hiring labor without health insurance and using capital.

To get the conditional input demand we can set one of the inputs to 0 and take FOCs. So, conditional on effective units of labor being the optimal choice of input we can write the lagrangean as

$$\mathcal{L}(\lambda) = \psi q_i + \hat{w}_i^0 \hat{l}_i^0 + \hat{R}_i \hat{k}_i + \lambda \left(y_i - B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)(\hat{l}_i^0)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \right) \tag{1.36}$$

So FOCs are (omitting the subscript i on ζ for simplicity):

$$\hat{w}_i^0 = \lambda B[\eta q_i^{\frac{\zeta - 1}{\zeta}} + (1-\eta)(\hat{l}_i^0)^{\frac{\zeta - 1}{\zeta}}]^{\frac{\zeta}{\zeta - 1} - 1} \frac{\zeta}{\zeta - 1} (1-\eta)(\hat{l}_i^0)^{\frac{\zeta - 1}{\zeta} - 1} \frac{\zeta - 1}{\zeta} \tag{1.37}$$

$$\psi = \lambda B[\eta q_i^{\frac{\zeta-1}{\zeta}} + (1-\eta)(\hat{l}_i^0)^{\frac{\zeta-1}{\zeta}}]^{\frac{\zeta}{\zeta-1}-1} \frac{\zeta}{\zeta-1} \eta q_i^{\frac{\zeta-1}{\zeta}-1} \frac{\zeta-1}{\zeta} \tag{1.38}$$

so we get that the optimal ratio of inputs is given by:

$$\left(\frac{q_i}{\hat{l}_i^0}\right) = \left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi}\right)^{\zeta} \tag{1.39}$$

Finally, plugging in the production function we get

$$y_i = B\left[\eta\left(\hat{l}_i^0\left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi}\right)^{\zeta}\right)^{\frac{\zeta-1}{\zeta}} + (1-\eta)(\hat{l}_i^0)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}$$

$$y_i = B\hat{l}_i^0\left[\eta\left(\left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi}\right)^{\zeta}\right)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\right]^{\frac{\zeta}{\zeta-1}}$$

$$y_i = B\hat{l}_i^0\left[\eta\left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi}\right)^{\zeta-1} + (1-\eta)\right]^{\frac{\zeta}{\zeta-1}}$$

the solutions to this problem are the conditional factor demands:

$$\boxed{\hat{l}_i^0(y_i^0, \hat{w}_i^0) = \frac{y_i^0}{B\left[\eta\left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi}\right)^{\zeta_i-1} + (1-\eta)\right]^{\frac{\zeta_i}{\zeta_i-1}}}} \tag{1.40}$$

$$\boxed{q_i^0(y_i^0, \hat{w}_i^0) = \frac{y_i^0\left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi}\right)^{\zeta_i}}{B\left[\eta\left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi}\right)^{\zeta_i-1} + (1-\eta)\right]^{\frac{\zeta_i}{\zeta_i-1}}}} \tag{1.41}$$

As expected, the conditional factor demands are linear in y_i^I as the production function is CRS.

Now, defining

$$p_i^0 \equiv \left(\frac{\eta \hat{w}_i^0}{(1-\eta)\psi} \right) \tag{1.42}$$

we can write the cost function, conditional on labor without insurance being the optimal input, $c(y_i^0, \hat{w}_i^0)$ as

$$\boxed{c(y_i^0, \hat{w}_i^0) = \frac{y_i^0 \left(\hat{w}_i^0 + \psi (p_i^0)^{\zeta_i} \right)}{B \left[\eta (p_i^0)^{\zeta_i - 1} + (1-\eta) \right]^{\frac{\zeta_i}{\zeta_i - 1}}} \tag{1.43}}$$

Now the profit maximization problem can be written as

$$\Pi^0(\hat{w}_i^0, y_i^0) = \max_{y_i^0} p(i) y_i^0 - c(y_i^0, \hat{w}_i^0) \tag{1.44}$$

where

$$y(i) = Y p(i)^{-\sigma} \tag{1.45}$$

Thus the inverse demand function is

$$p(i) = \left(\frac{Y}{y(i)} \right)^{1/\sigma} \tag{1.46}$$

Then, as the monopolist optimal condition is where Marginal Cost equals Marginal revenue, we get

$$\frac{\hat{w}_i^0 + \psi (p_i^0)^{\zeta_i}}{B \left[\eta (p_i^0)^{\zeta_i - 1} + (1-\eta) \right]^{\frac{\zeta_i}{\zeta_i - 1}}} = Y^{\frac{1}{\sigma}} \left(\frac{\sigma - 1}{\sigma} \right) (y_i^0)^{1 - \frac{1}{\sigma} - 1} \tag{1.47}$$

Then the output function $y_i^0(\hat{w}_i^0)$ is defined by

$$y_i^0(\hat{w}_i^0) = Y\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\frac{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma} \tag{1.48}$$

Finally the conditional profit function gets

$$\Pi^0(\hat{w}_i^0, y_i^0) = Y^{\frac{1}{\sigma}}(y_i^0)^{\frac{\sigma-1}{\sigma}} - \frac{y_i^0[\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}]}{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}} \tag{1.49}$$

It can be shown using Envelope Theorem that the profit function is decreasing in $\hat{w}_i^0$ and thus increasing in i. Now, if we plug in equation 1.48 we get:

$$\Pi^0(\hat{w}_i^0, y_i^0) = Y^{\frac{1}{\sigma}}(y_i^0)^{\frac{\sigma-1}{\sigma}} - \frac{y_i^0[\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}]}{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}$$

$$\Pi^0(\hat{w}_i^0) = Y^{\frac{1}{\sigma}}\left(Y\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\frac{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} - \frac{Y\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\frac{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma}}{\frac{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}}$$

$$\Pi^0(\hat{w}_i^0) = \left[\frac{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} Y\left[1-\left(\frac{\sigma-1}{\sigma}\right)\right]$$

$$\Pi^0(\hat{w}_i^0) = \left[\frac{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}$$

$$\Pi^0(\hat{w}_i^0) = \left[\frac{B[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma} \tag{1.50}$$

1.3.4 Conditional on using capital

Following the same derivations we can solve the conditional cost minimization problem when the optimal choice of input is effective units of capital $\hat{k}_i$. This will give rise to the following conditional input demand functions:

$$\boxed{\hat{k}_i(y_i^k, \hat{R}_i) = \frac{y_i^k}{B\left[\eta\left(\frac{\eta\hat{R}_i}{(1-\eta)\psi}\right)^{\zeta_i-1} + (1-\eta)\right]^{\frac{\zeta_i}{\zeta_i-1}}}} \tag{1.51}$$

$$\boxed{q_i^k(y_i^k, \hat{R}_i) = \frac{y_i^k\left(\frac{\eta\hat{R}_i}{(1-\eta)\psi}\right)^{\zeta_i}}{B\left[\eta\left(\frac{\eta\hat{R}_i}{(1-\eta)\psi}\right)^{\zeta_i-1} + (1-\eta)\right]^{\frac{\zeta_i}{\zeta_i-1}}}} \tag{1.52}$$

so the conditional cost function gets

$$\boxed{c(y_i^k, \hat{R}_i) = \frac{y_i^k\left(\hat{R}_i + \psi(p_i^k)^{\zeta_i}\right)}{B\left[\eta(p_i^k)^{\zeta_i-1} + (1-\eta)\right]^{\frac{\zeta_i}{\zeta_i-1}}}} \tag{1.53}$$

$$p_i^k \equiv \left(\frac{\eta\hat{R}_i}{(1-\eta)\psi}\right) \tag{1.54}$$

and the output function gets like

$$\boxed{y_i^k(\hat{R}_i) = Y\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\frac{B[\eta(p_i^k)^{\zeta_i-1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{R}_i + \psi(p_i^k)^{\zeta_i}}\right]^{\sigma}} \tag{1.55}$$

Finally the conditional indirect profit function will be

$$\boxed{\Pi^k(\hat{R}_i) = \left[\frac{B[\eta(p_i^k)^{\zeta_i-1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{R}_i + \psi(p_i^k)^{\zeta_i}}\right]^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{Y}{\sigma} - C_i^A \qquad (1.56)}$$

1.3.5 With Health insurance

For problem the firm solves conditional on providing health insurance, it will need to pay the expected medical expenditures for the composition of workers she will hire plus an administrative fixed cost. The profit maximizing problem can be written as:

$$\Pi^1(w_1, w_0, R) = \max_{l_i, q_i, k_i} Y^{1/\sigma}(y_i^1)^{1-1/\sigma} - \psi q_i - w_1 l_i - M_i l_i - R k_i - 1_{\{k_i>0\}} C_i^A - 1_{\{l_i>0\}} C^{IN} \tag{1.57}$$

subject to

$$y_i^1 = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)\left(z_i k_i + \gamma_i l_i \chi_{gi}^1 + \rho \gamma_i l_i (1 - \chi_{gi}^1)\right)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \tag{1.58}$$

where C^{IN} is a fixed administrative cost of providing health insurance and $M_i l_i$ is the expected medical expenditure for the composition of workers it hires:

$$M_i = \frac{\mathbb{E}^{\tilde{m}}[\tilde{m}|h=g]\chi_{gi}^1 + \mathbb{E}^{\tilde{m}}[\tilde{m}|h=b](1-\chi_{gi}^1)}{\left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}}} \tag{1.59}$$

where the term in the denominator $\left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}}$ translates labor effort l_i to the measure of workers hired, because every worker supplies the same labor effort.

Now we can translate this problem into effective units of labor and effective units of capital like:

$$\Pi^1(\hat{w}_i^1, \hat{R}_i) = \max_{\hat{l}_i^1, q_i, \hat{k}_i} Y^{1/\sigma}(y_i^1)^{1-1/\sigma} - \psi q_i - \hat{w}_i^1 \hat{l}_i^1 - \hat{R}_i \hat{k}_i - 1_{\{\hat{k}_i>0\}} C_i^A - 1_{\{\hat{l}_i^1>0\}} C^{IN} \tag{1.60}$$

subject to

$$y_i^1 = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)\left(\hat{k}_i + \hat{l}_i^1\right)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \tag{1.61}$$

1.3.6 Cost minimization problem

Doing similar calculations than before, we will get the following conditional profit function, conditional on providing health insurance:

$$\boxed{\Pi^1(\hat{w}_i^1) = \left[\frac{B[\eta(p_i^1)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{w}_i^1 + \psi(p_i^1)^{\zeta_i}}\right]^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{Y}{\sigma} - C^{IN} \qquad (1.62)}$$

1.3.7 Thresholds

1.3.8 Indifference between Health Insurance and Not Health Insurance

A natural question that arises is if there is a threshold $\tilde{X} \in [N-1, N]$, such that, conditional on using labor, firms above that threshold will always choose to provide health insurance and firms below that threshold will always choose to do not provide health insurance at the given market wages.

Suppose that $C > 0$, then the indifference point is conditional on the effective units of labor $\hat{l}_i$ the firm will want to hire and as the conditional effective units of labor could be different under the two contracts, we need to use the indirect profit functions to characterize the indifference point. Recall that the two indirect profit functions we obtained through a two step procedure using the conditional cost minimization problems were:

$$\Pi^1(\hat{w}_i^1) = \left[\frac{B[\eta(p_i^1)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{w}_i^1 + \psi(p_i^1)^{\zeta_i}}\right]^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{Y}{\sigma} - C^{IN} \qquad (1.63)$$

and

$$\Pi^0(\hat{w}_i^0) = \left[\frac{B[\eta(p_i^0)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{w}_i^0 + \psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{Y}{\sigma} \qquad (1.64)$$

where

$$p_i^I \equiv \frac{\eta \hat{w}_i^I}{(1-\eta)\psi} \tag{1.65}$$

Then the indifference condition, where the two indirect profit functions cross, will be given by

$$\boxed{\frac{C^{IN}}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^1)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^1+\psi(p_i^1)^{\zeta_i}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1} \quad \text{for } i=\tilde{X}} \tag{1.66}$$

Proposition 9 (see Appendix) states that if this threshold is interior in the range of tasks, , Advantageous selection holds, $\zeta_i = \zeta$ and $C^{IN} = 0$, then firms after the threshold choose to provide health insurance over no health insurance. This implies that the conditional indirect profit functions, conditional on using labor, should look like:

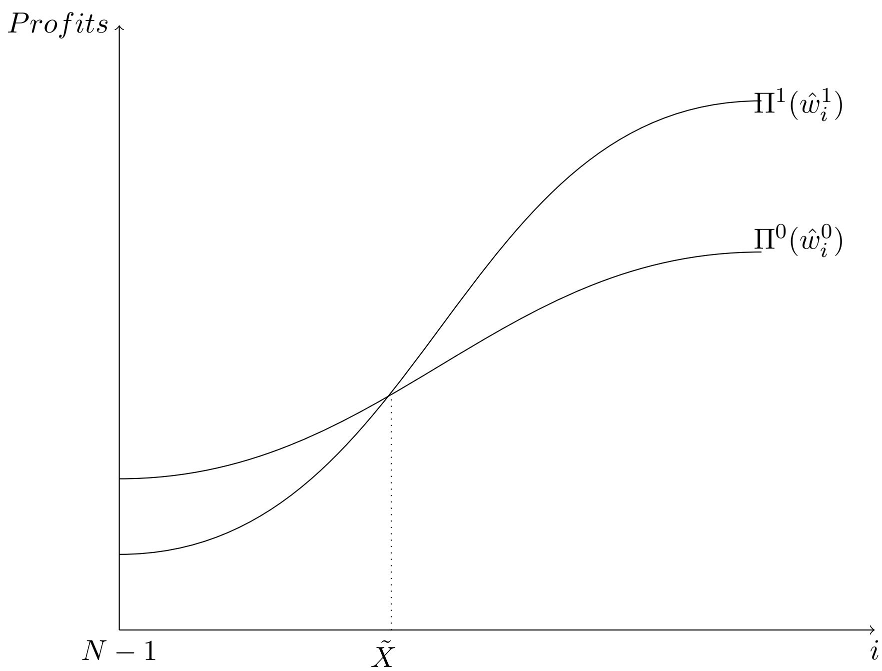

Figure 1.1: Conditional indirect profit functions using labor

> Notice that we haven't been able to prove yet that this threshold will be interior in equilibrium, nor that the previous statement holds for C^{IN}.

1.3.9 Indifference between Capital and No Health Insurance

As firms can produce either with capital, labor without health insurance and labor with health insurance, there will also be a threshold, $\tilde{I}_0$, at which the firm will be indifferent between producing with capital and producing with labor without health insurance. Notice that this threshold depends on the automated cost C_i^A that is task dependent to allow for any possible automation scheme in the range of tasks.

$$\boxed{\frac{C_i^A}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^k)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{R}_i+\psi(p_i^k)^{\zeta_i}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1} \quad (1.67)} \quad \text{for } i=\tilde{I}^0$$

where

$$p_i^k \equiv \frac{\eta\hat{R}_i}{(1-\eta)\psi} \tag{1.68}$$

1.3.10 Indifference between Capital and Health Insurance

The threshold at which the firm is indifferent between using capital and labor with health insurance will depend on both fixed costs. Intuitively there must exist a difference between the two marginal costs such that off sets the difference between the fixed cost administrative C^{IN} and the , task dependent, automation cost C_i^A.

Following similar steps than before, we find that the threshold $\tilde{I}^1$ can be defined by the following equation:

$$\boxed{\frac{C^{IN}-C_i^A}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^1)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{w}_i^1+\psi(p_i^1)^{\zeta}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^k)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{R}_i+\psi(p_i^k)^{\zeta}}\right]^{\sigma-1} \quad (1.69)} \text{for } i=\tilde{I}^1$$

1.3.11 Characterization of the Thresholds

Now, suppose $C^{IN} = 0$ and $C_i^A = 0$ (for a relevant range in of tasks), then the thresholds are defined by comparing the effective marginal costs:

$$\hat{w}^1(\tilde{X}) = \hat{w}^0(\tilde{X}) \tag{1.70}$$

$$\hat{w}^0(\tilde{I}^0) = \hat{R}(\tilde{I}^0) = \frac{R}{z(\tilde{I}^0)} \tag{1.71}$$

$$\hat{w}^1(\tilde{I}^1) = \hat{R}(\tilde{I}^1) = \frac{R}{z(\tilde{I}^1)} \tag{1.72}$$

I think that our extension of ζ_i does not affect this result. But to have that the thresholds are unique we would need some assumptions on $z(i)$. If $z(i)$ is a constant function we have already shown in previous documents that these thresholds if they exist are unique.

Now, for $C^{IN} > 0$ and $C_i^A > 0$ the conditions that define the thresholds are more complicated because the fixed costs force us to compare profits and not only marginal costs:

$$\frac{C^{IN}}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^1)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^1+\psi(p_i^1)^{\zeta_i}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1} \quad \text{for } i=\tilde{X} \tag{1.73}$$

$$\frac{C_i^{A}}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^k)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{R}_i+\psi(p_i^k)^{\zeta}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^0)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta}}\right]^{\sigma-1} \quad \text{for } i=\tilde{I}^0 \tag{1.74}$$

$$\frac{C^{IN}-C_i^{A}}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^1)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{w}_i^1+\psi(p_i^1)^{\zeta}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^k)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{R}_i+\psi(p_i^k)^{\zeta}}\right]^{\sigma-1} \quad \text{for } i=\tilde{I}^1 \tag{1.75}$$

2 Equilibrium

2.1 Possible Equilibrium Configurations

For the general case the thresholds will depend on the assumptions we place on z_i, C_i^A and ζ_i. Take $\zeta_i = \zeta$ for the moment and $z_i = z(i)$ to be a non increasing function of i. Now, depending on how the automation cost function $C^A(i)$ behaves, we will have different equilibrium configurations.

Increasing C_i^A

If tasks with a higher index i, i.e, with higher labor productivity, are more costly to automate, then $C^A(i)$ can be modeled as an increasing function of i (which can be interpreted as a smooth version of the AR framework). Then the indirect profit functions, evaluated at w_0, w_1 and R, could look like:

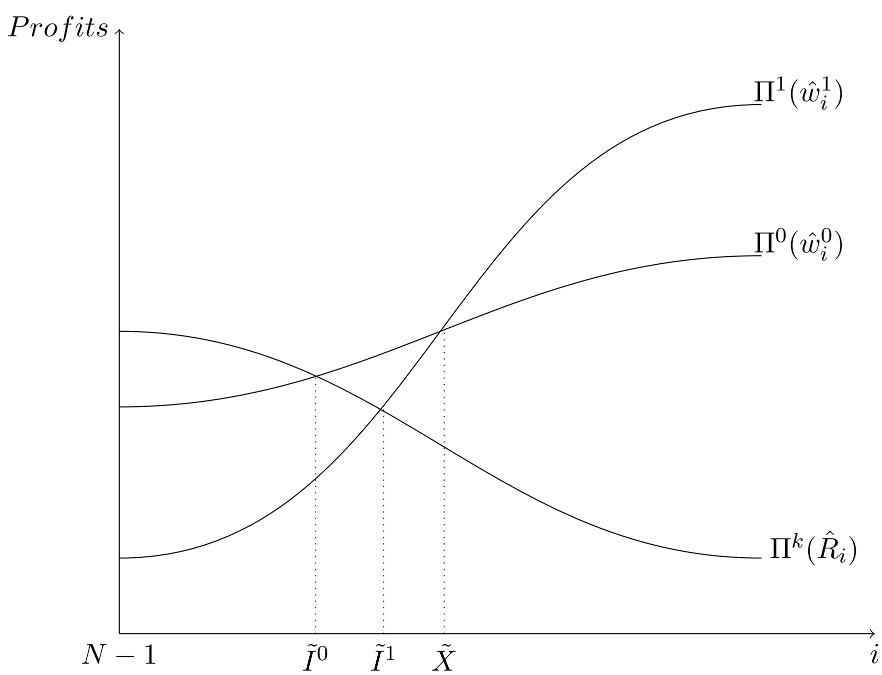

Figure 2.1: Conditional indirect profit functions, increasing automation cost with $\tilde{I}^1 < \tilde{X}$

Firms will produce following the upper envelope of the conditional indirect profit functions, so the corresponding equilibrium configuration would be:

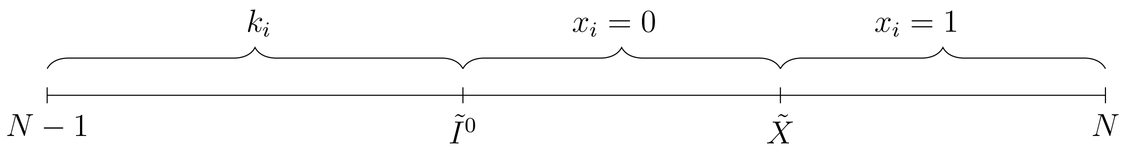

Figure 2.2: Equilibrium configuration for $\tilde{I}^1 < \tilde{X}$

Is easy to see that capital must be used in equilibrium, otherwise we won't have market clearing. However, is not clear if labor with and without health insurance will be provided in equilibrium. If the rental rate of capital R or the automation cost C_i^A is lower enough, we could also have that:

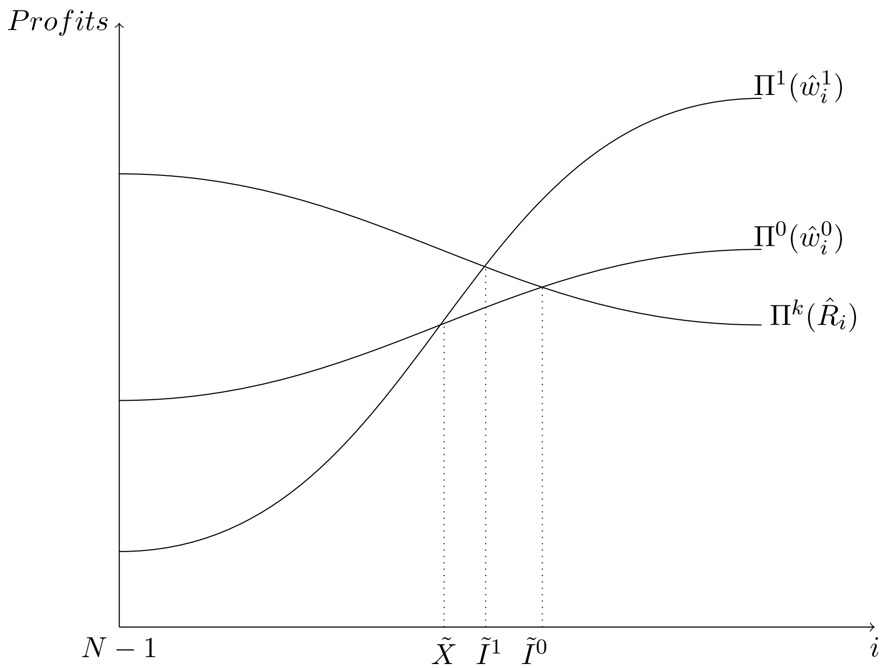

Figure 2.3: Conditional indirect profit functions, increasing automation cost with $\tilde{I}^1 \geq \tilde{X}$

The equilibrium configuration in this case will be to use capital up until the threshold $\tilde{I}^1$ and then to use labor with health insurance:

Figure 2.4: Equilibrium configuration for $\tilde{I}^1 \geq \tilde{X}$

U-shaped C_i^A

The relationship between productivity of labor and its cost of automation could be non monotonic. This is the case Autor and Dorn pointed out in [David and Dorn, 2013], where service occupation is associated with a low productivity of labor but those tasks are hard to codify. In this case, $C^A(i)$ can be modeled as a quadratic function of i. Then the indirect profit functions, evaluated at w_0, w_1 and R, could look like:

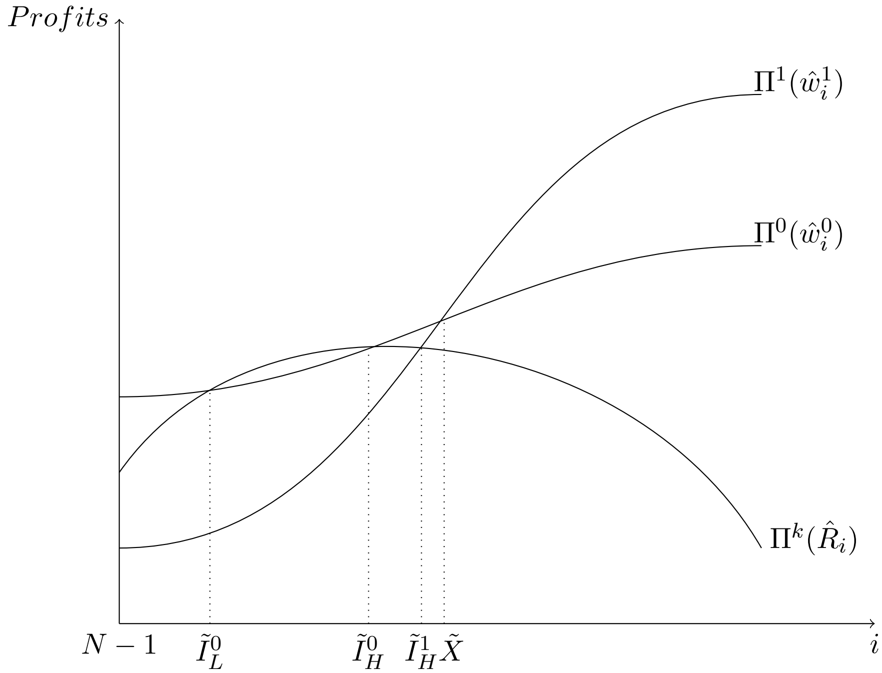

Figure 2.5: Conditional indirect profit functions, U-shaped automation cost

Notice that with a U-shaped automation cost function we will have up to 5 thresholds, because the indirect profit function for capital $\Pi^k(\hat{R}_i)$ can cross the two other indirect profit functions in at most 2 point each. This will give rise to different equilibrium configurations. The one corresponding to the previous figure would be:

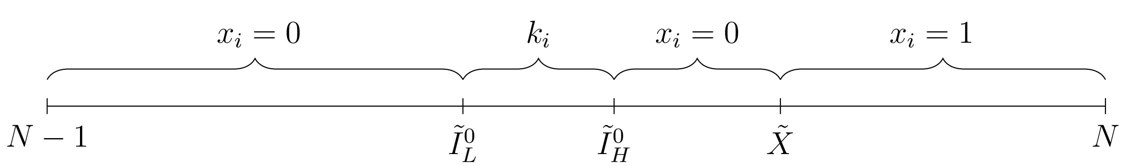

Figure 2.6: Equilibrium configuration for U-shaped C_i^A

This equilibrium configuration implies automation in the middle of the task range, labor with no health insurance at the bottom and the middle of the task range, and labor with health insurance at the top of the task range. This configuration resembles the story behind Autor and Dorn's paper.

2.2 Definition of Equilibrium (Non decreasing C_i^A)

Definition: Equilibrium in the Static Model

An equilibrium for this economy is a set of prices w_0^*, w_1^*, R^*, $[p_i^*]_i$, *allocations* $\{c^*(h,\theta), l^*(h,\theta), \alpha^*(h,\theta)\}_{\theta,h}$, *endogenous thresholds* $\{\bar{\theta}_h^*\}_h$, *production plan* $[y_i^*, l_i^*, q_i^*, k_i^*, x_i^*]_i$, *endogenous production thresholds* $\tilde{I}^0$, $\tilde{I}^1$, $\tilde{X}$, *endogenous proportions* $\{\chi_g^{I*}\}_I$ *and conditional indirect profit functions* Π_i^0,Π_i^1 *and* Π_i^k *s.t*

1. $c^*(h,\theta), l^*(h,\theta), \alpha^*(h,\theta)$ *are given by equation 1.13, 1.10 and 1.11.*

2. $\{\bar{\theta}_h^*\}_h$ *are defined by equation 1.15*

3. $[y_i^*, l_i^*, q_i^*, k_i^*, x_i^*]_i$ *are defined by*

$$(y_i^*, l_i^*, q_i^*, k_i^*, x_i^*) = \begin{cases} (y_i^k, 0, q_i^k, k_i, 0) & \text{if } i \in \left[N-1, \min\{\tilde{I}^1, \tilde{I}^0\}\right] \\ (y_i^0, l_i^0, q_i^0, 0, 0) & \text{if } i \in \left[\tilde{I}^0, \tilde{X}\right] \text{ and } \tilde{I}^0 < \tilde{I}^1 \\ (y_i^1, l_i^1, q_i^1, 0, 1) & \text{if } i \in \left[\max\{\tilde{I}^1, \tilde{X}\}, N\right] \end{cases}$$

where $[y_i^0, y_i^1, y_i^k, l_i^0, l_i^1, q_i^0, q_i^1, q_i^k, k_i]_i$ *are solutions to the conditional problems of firm* i. *So firms between* $N-1$ *and* $\min\{\tilde{I}^1, \tilde{I}^0\}$ *will produce using capital, firms between* $\max\{\tilde{I}^1, \tilde{X}\}$ *and* N *will produce using labor with health insurance and if* $\tilde{I}^0 < \tilde{I}^1$ *firms between* $\tilde{I}^0$ *and* $\tilde{X}$ *will produce using labor without health insurance, where the production thresholds* $\tilde{X}$, $\tilde{I}^0$ *and* $\tilde{I}^1$ *are given by equations 1.66, 1.67 and 1.69.*

4. $\{\chi_{gi}^{I*}\}_I$ *are given by:*

$$\chi_{gi}^{1*} = \delta_i \left(\frac{\lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1 - F_g(\bar{\theta}_g))}{\lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1 - F_g(\bar{\theta}_g)) + (1-\lambda_g) \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1 - F_b(\bar{\theta}_b))} \right)$$

$$\chi_{gi}^{0*} = \delta_i \left(\frac{\lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g)}{\lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g) + (1-\lambda_g) \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_b(\bar{\theta}_b)} \right)$$

5. $[p_i^*]_i$ *are given by*

$$p_i^* = \left(\frac{Y^*}{y_i^*}\right)^{1/\sigma}$$

where

$$Y^* = \int_{N-1}^{N} y_i^* di$$

6. *Market Clearing:*

$$\int_{N-1}^{\min\{\tilde{I}^1,\tilde{I}^0\}} k_i^* di = K$$

If $\tilde{I}^0 < \tilde{I}^1$*, there will be labor without health insurance in equilibrium, then*

$$\int_{\tilde{I}^0}^{\tilde{X}} l_i^* di = \lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g) + (1-\lambda_g)\left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_b(\bar{\theta}_b)$$

$$\int_{\max\{\tilde{I}^1,\tilde{X}\}}^{N} l_i^* di = \lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1-F_g(\bar{\theta}_g)) + (1-\lambda_g)\left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1-F_b(\bar{\theta}_b))$$

7. *Resource Constraint*

$$C = Y - FC - M - \Pi - RK - \Pi_q - \psi \int_{N-1}^{N} q_i^* di$$

8. *Net profits* Π *are given by (here I'm not considering the net profits of intermediate goods)*

$$\Pi = \int_{N-1}^{N} \max\{\Pi_i^0, \Pi_i^1, \Pi_i^k\} di$$

9. *Conditional indirect profit functions* Π_i^0,Π_i^1 *and* Π_i^k *are given by equations 1.50, 1.62 and 1.56.*

2.3 Equations to compute the equilibrium

The following equations give the optimal consumption, labor effort and insurance choice for households. The last two equations don't need to be computed to solve for the equilibrium though, because they will already be implicitly in the definition of the thresholds and the market clearing for consumption.

Households' optimal choices

$$\underline{w} \equiv \frac{w_0}{1+\xi} \tag{2.1}$$

$$l_0 = \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}}, \quad l_1 = \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} \tag{2.2}$$

$$c = \begin{cases} \max\{w_0 l_0 - \tilde{m}, \underline{c}\} & \text{if } \alpha = 0 \\ w_1 l_1 & \text{otherwise} \end{cases} \tag{2.3}$$

Threshold for household insurance decision

$$\bar{\theta}_h \in \{\theta \in (\theta_L, \theta_H) : \mathbb{E}^{\tilde{m}}[u_{(h,\theta)}(\max\{w_0 l_0 - \tilde{m}, \underline{c}\}, l_0)] = u_{(h,\theta)}(w_1 l_1, l_1)\} \tag{2.4}$$

Endogenous proportions

$$\chi^1_{gi} = \delta_i \left(\frac{\lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} \left(1 - F_g(\bar{\theta}_g)\right)}{\lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} \left(1 - F_g(\bar{\theta}_g)\right) + (1-\lambda_g) \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} \left(1 - F_b(\bar{\theta}_b)\right)} \right) \tag{2.5}$$

$$\chi^0_{gi} = \delta_i \left(\frac{\lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g)}{\lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g) + (1-\lambda_g) \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_b(\bar{\theta}_b)} \right) \tag{2.6}$$

Expected firm's medical expenditure

$$M_i = \frac{\mathbb{E}^{\tilde{m}}[\tilde{m}|h=g]\chi^1_{gi} + \mathbb{E}^{\tilde{m}}[\tilde{m}|h=b](1-\chi^1_{gi})}{\left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}}} \tag{2.7}$$

Production plan

$$B \equiv (1-\eta)^{\zeta/(1-\zeta)} \tag{2.8}$$

$$\bar{\gamma}_i^I \equiv \gamma_i \left((1-\rho)\chi_{gi}^I + \rho\right) \tag{2.9}$$

$$\hat{w}_i^0 \equiv \frac{w_0}{\bar{\gamma}_i^0} \tag{2.10}$$

$$\hat{w}_i^1 \equiv \frac{w_1 + M_i}{\bar{\gamma}_i^1} \tag{2.11}$$

$$\hat{R}_i \equiv \frac{R}{z_i} \tag{2.12}$$

$$p_i^I \equiv \frac{\eta \hat{w}_i^I}{(1-\eta)\psi} \tag{2.13}$$

$$p_i^k \equiv \frac{\eta \hat{R}_i}{(1-\eta)\psi} \tag{2.14}$$

$$\hat{k}_i \equiv z_i k_i \tag{2.15}$$

$$\hat{l}_i^I \equiv \bar{\gamma}_i^I l_i \tag{2.16}$$

$$y_i^I(\hat{w}_i^I) = Y \left(\frac{\sigma - 1}{\sigma}\right)^\sigma \left[\frac{B[\eta(p_i^I)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{w}_i^I + \psi(p_i^I)^{\zeta_i}}\right]^\sigma \tag{2.17}$$

$$\hat{l}_i^I(y_i^I, \hat{w}_i^I) = \frac{y_i^I}{B\left[\eta\left(\frac{\eta \hat{w}_i^I}{(1-\eta)\psi}\right)^{\zeta_i - 1} + (1-\eta)\right]^{\frac{\zeta_i}{\zeta_i - 1}}} \tag{2.18}$$

$$q_i^I(y_i^I, \hat{w}_i^I) = \frac{y_i^I \left(\frac{\eta \hat{w}_i^I}{(1-\eta)\psi}\right)^{\zeta_i}}{B\left[\eta\left(\frac{\eta \hat{w}_i^I}{(1-\eta)\psi}\right)^{\zeta_i - 1} + (1-\eta)\right]^{\frac{\zeta_i}{\zeta_i - 1}}} \tag{2.19}$$

$$y_i^k(\hat{R}_i) = Y\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\frac{B[\eta(p_i^k)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{R}_i+\psi(p_i^k)^{\zeta_i}}\right]^{\sigma} \tag{2.20}$$

$$\hat{k}_i(y_i^k,\hat{R}_i) = \frac{y_i^k}{B\left[\eta\left(\frac{\eta\hat{R}_i}{(1-\eta)\psi}\right)^{\zeta_i-1}+(1-\eta)\right]^{\frac{\zeta_i}{\zeta_i-1}}} \tag{2.21}$$

$$q_i^k(y_i^k,\hat{R}_i) = \frac{y_i^k\left(\frac{\eta\hat{R}_i}{(1-\eta)\psi}\right)^{\zeta_i}}{B\left[\eta\left(\frac{\eta\hat{R}_i}{(1-\eta)\psi}\right)^{\zeta_i-1}+(1-\eta)\right]^{\frac{\zeta_i}{\zeta_i-1}}} \tag{2.22}$$

Firms' production thresholds

$$\frac{C^{IN}}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^1)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^1+\psi(p_i^1)^{\zeta_i}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^0)^{\zeta_i-1}+(1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1} \quad \text{for } i=\tilde{X} \tag{2.23}$$

$$\frac{C_i^{A}}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^k)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{R}_i+\psi(p_i^k)^{\zeta}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^0)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{w}_i^0+\psi(p_i^0)^{\zeta}}\right]^{\sigma-1} \quad \text{for } i=\tilde{I}^0 \tag{2.24}$$

$$\frac{C^{IN}-C_i^{A}}{\left(\frac{B(\sigma-1)}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma}} = \left[\frac{[\eta(p_i^1)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{w}_i^1+\psi(p_i^1)^{\zeta}}\right]^{\sigma-1} - \left[\frac{[\eta(p_i^k)^{\zeta-1}+(1-\eta)]^{\frac{\zeta}{\zeta-1}}}{\hat{R}_i+\psi(p_i^k)^{\zeta}}\right]^{\sigma-1} \quad \text{for } i=\tilde{I}^1 \tag{2.25}$$

Total output

$$Y = \left(\int_{N-1}^{N} y_i^{\frac{\sigma-1}{\sigma}}\,di\right)^{\frac{\sigma}{\sigma-1}} \tag{2.26}$$

where firms between $N-1$ and $\min\{\tilde{I}^1,\tilde{I}^0\}$ will produce using capital ($y_i = y_i^k$), firms between $\max\{\tilde{I}^1,\tilde{X}\}$ and N will produce using labor with health insurance ($y_i = y_i^1$)and if $\tilde{I}^0 < \tilde{I}^1$ firms between $\tilde{I}^0$ and $\tilde{X}$ will produce using labor without health insurance ($y_i = y_i^0$). So the previous integral can be decomposed as:

$$Y = \left(\int_{N-1}^{\min\{\tilde{I}^1,\tilde{I}^0\}} y_i^{k\frac{\sigma-1}{\sigma}} di + \mathbb{1}_{\{\tilde{I}^0<\tilde{I}^1\}} \int_{\tilde{I}^0}^{\tilde{X}} y_i^{0\frac{\sigma-1}{\sigma}} di + \int_{\max\{\tilde{I}^1,\tilde{X}\}}^{N} y_i^{1\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{2.27}$$

Notice that the conditional task functions y_i^k, y_i^0, y_i^1 are linear in total amount of consumption good Y. So the previous equation simplifies to:

$$1 = \int_{N-1}^{\min\{\tilde{I}^1,\tilde{I}^0\}} \tilde{y}_i^{k\frac{\sigma-1}{\sigma}} di + \mathbb{1}_{\{\tilde{I}^0<\tilde{I}^1\}} \int_{\tilde{I}^0}^{\tilde{X}} \tilde{y}_i^{0\frac{\sigma-1}{\sigma}} di + \int_{\max\{\tilde{I}^1,\tilde{X}\}}^{N} \tilde{y}_i^{1\frac{\sigma-1}{\sigma}} di \tag{2.28}$$

where $\tilde{y}_i^k, \tilde{y}_i^0, \tilde{y}_i^1$ are the conditional task functions evaluated at $Y = 1$. This happens because this is a CRS production function. even though the integrands do not depend on Y, the integration limits do.

Looking at the definitions of $\tilde{X}$, $\tilde{I}^0$ and $\tilde{I}^1$, we can see that $\tilde{X}$ is decreasing in Y, $\tilde{I}^0$ is increasing in Y and, providing that $C^{IN} > C_i^A \forall i \in [N-1, N]$, $\tilde{I}^1$ is decreasing in Y. Therefore the second integral is decreasing in Y, the third integral is increasing in Y and the first integral is decreasing if $\tilde{I}^1 < \tilde{I}^0$ and increasing if $\tilde{I}^1 > \tilde{I}^0$.

Task prices

As firms are monopolies, task prices were already plugged in to get the production plan of the firms, so this equations do not need to be used directly to compute the equilibrium.

$$p_i = \left(\frac{Y}{y_i}\right)^{1/\sigma} \tag{2.29}$$

so can be decomposed as

$$p_i = \begin{cases} \left(\frac{Y}{y_i^k}\right)^{1/\sigma} & \text{if } i \in \left[N-1, \min\{\tilde{I}^1, \tilde{I}^0\}\right] \\ \left(\frac{Y}{y_i^0}\right)^{1/\sigma} & \text{if } i \in \left[\tilde{I}^0, \tilde{X}\right] \text{ and } \tilde{I}^0 < \tilde{I}^1 \\ \left(\frac{Y}{y_i^1}\right)^{1/\sigma} & \text{if } i \in \left[\max\{\tilde{I}^1, \tilde{X}\}, N\right] \end{cases} \tag{2.30}$$

Market clearing for capital

$$\int_{N-1}^{\min\{\tilde{I}^1,\tilde{I}^0\}} k_i di = K$$

Market clearing for labor without health insurance

$$\int_{\tilde{I}^0}^{\tilde{X}} l_i^0 di = \lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g) + (1-\lambda_g)\left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_b(\bar{\theta}_b)$$

Market clearing for labor with health insurance

$$\int_{\max\{\tilde{I}^1,\tilde{X}\}}^{N} l_i^1 di = \lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1-F_g(\bar{\theta}_g)) + (1-\lambda_g)\left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1-F_b(\bar{\theta}_b))$$

Resource constraint (Review this one)[2]

The Resource constraint can be derived by aggregating the Household's budget constraint and using the market clearing condition for labor and capital, therefore we don't need to use this equation to compute the equilibrium, although it should hold in equilibrium (I will check this in the computer).

$$C \equiv \lambda_g \int_{\underline{\theta}}^{\bar{\theta}} c^*(g,\theta)dF_g(\theta) + (1-\lambda_g)\int_{\underline{\theta}}^{\bar{\theta}} c^*(b,\theta)dF_b(\theta) = Y - FC - M - \Pi - RK - \Pi_q - \psi\int_{N-1}^{N} q_i di \tag{2.31}$$

where the integrals can be written as (notice that we also need to integrate over medical expenditure shocks):

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} c(h,\theta)dF_h(\theta) &= \int_{\underline{\theta}}^{\bar{\theta}_h} c(h,\theta)dF_h(\theta) + \int_{\bar{\theta}_h}^{\bar{\theta}} c(h,\theta)dF_h(\theta)\\ &= F_h(\bar{\theta}_h)\left(w_0 l_0 - \mathbb{E}\left[\tilde{m}\middle| h=h, \frac{\tilde{m}}{l_0} \leq w_0 - \underline{w}\right]\right)\mathbb{P}\left(\frac{\tilde{m}}{l_0} \leq w_0 - \underline{w}\right)\\ &+ F_h(\bar{\theta}_h)\left(\underline{w} l_0\right)\mathbb{P}\left(\frac{\tilde{m}}{l_0} > w_0 - \underline{w}\right)\\ &+ \left(1 - F_h(\bar{\theta}_h)\right) w_1 l_1\end{aligned}$$

The details of the derivations of this equation are explained in the Appendix 3.2.

Conditional indirect profit functions

$$\Pi^0(\hat{w}_i^0) = \left[\frac{B[\eta(p_i^0)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{w}_i^0 + \psi(p_i^0)^{\zeta_i}}\right]^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{Y}{\sigma} \tag{2.32}$$

[2]Who is paying the medical expenditures to whom? and the reservation wage?

$$\Pi^1(\hat{w}_i^1) = \left[\frac{B[\eta(p_i^1)^{\zeta_i-1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{w}_i^1 + \psi(p_i^1)^{\zeta_i}}\right]^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{Y}{\sigma} - C^{IN} \tag{2.33}$$

$$\Pi^k(\hat{R}_i) = \left[\frac{B[\eta(p_i^k)^{\zeta_i-1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i-1}}}{\hat{R}_i + \psi(p_i^k)^{\zeta_i}}\right]^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{Y}{\sigma} - C_i^{A} \tag{2.34}$$

Total profits

$$\Pi = \int_{N-1}^{N} \max\{\Pi_i^0, \Pi_i^1, \Pi_i^k\} di \tag{2.35}$$

Equations that pin down the endogenous objects

Taking the stock of capital K and the price of intermediates ψ as given, we need to solve for 4 unknowns: w_0, w_1, R, Y. The 4 equations that involve this objects are:

$$\int_{N-1}^{\min\{\tilde{I}^1,\tilde{I}^0\}} k_i di = K$$

$$\int_{\tilde{I}^0}^{\tilde{X}} l_i^0 di = \lambda_g \left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_g(\bar{\theta}_g) + (1-\lambda_g)\left(\frac{w_0}{\phi}\right)^{\frac{1}{\xi}} F_b(\bar{\theta}_b)$$

$$\int_{\max\{\tilde{I}^1,\tilde{X}\}}^{N} l_i^1 di = \lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1 - F_g(\bar{\theta}_g)) + (1-\lambda_g)\left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} (1 - F_b(\bar{\theta}_b))$$

$$1 = \int_{N-1}^{\min\{\tilde{I}^1,\tilde{I}^0\}} \tilde{y}_i^{k\,\frac{\sigma-1}{\sigma}} di + \mathbb{1}_{\{\tilde{I}^0<\tilde{I}^1\}} \int_{\tilde{I}^0}^{\tilde{X}} \tilde{y}_i^{0\,\frac{\sigma-1}{\sigma}} di + \int_{\max\{\tilde{I}^1,\tilde{X}\}}^{N} \tilde{y}_i^{1\,\frac{\sigma-1}{\sigma}} di$$

2.4 Primitives and equations to be parametrized

2.4.1 Equations to be parametrized

From the firms problem we should parametrize the following functions:

Table 2.1: Parametrized Functions

Function	Description	Parametrization	Source
γ_i	Labor productivity	$e^{Ai}, A > 0$	[Acemoglu and Restrepo, 2016]
δ_i	Increasing sorting of workers	$\frac{e^{\lambda i-\alpha}}{1+e^{\lambda i-\alpha}}, \lambda > 0$	-
z_i	Capital productivity	-	-
ζ_i	Elasticity of substitution	-	-
C_i^A	Automation cost	-	-

Notice that we have assumed across the document that γ_i is an increasing function and δ_i is increasing an bounded in $(0, 1)$. For the moment I will take $z_i = 1$, $\zeta_i = \zeta$ and C_i^A to be a strictly increasing function like $C_i^A = e^{Di}$. So we need to specify also values for A, λ, α, D and ζ.

2.4.2 Primitives

Household's problem

From the Household's problem the utility function is given by:

$$u_{(h,\theta)}(c, l) = \frac{1}{1-\theta}\left[\left(c - \phi\frac{l^{1+\xi}}{1+\xi}\right)^{1-\theta} - 1\right]$$

Here we need to give a range $[\underline{\theta}, \bar{\theta}]$ for the relative risk aversion parameter θ and values for the cost of labor effort in consumption units ϕ and for the inverse of the intertemporal elasticity of substitution in labor supply ξ.

Following [Mendoza, 1991], for a representative household model these values are taken to be:

Table 2.2: Representative agent parameter values

Parameters	Description	Values	Source
ξ	Intertemporal elasticity in labor supply	0.455	[Mendoza, 1991]
ϕ	Cost of labor effort	1	[Mendoza, 1991]
θ	Relative risk aversion	1.001 or 2	[Mendoza, 1991]

So we need to decide the parameter range $[\underline{\theta}, \bar{\theta}]$ that includes at least one of these two values and we need to specify the distributions for the households' types and for the medical expenditure shocks:

Table 2.3: Household's parameter values

Parameters	Description	Values	Source
λ_g	Measure of healthy households	-	-
$F(g,\theta)$	Joint distribution of household's types	-	-
$H(\tilde{m}\|h)$	Conditional distribution of medical expenditure	-	-

These distributions should be consistent with

Assumption 1:

$$H(\tilde{m}|b)\ FOSD\ H(\tilde{m}|g)$$

and

Assumption 2:

$$F_g(\theta)\ FOSD\ F_b(\theta)$$

Firm's problem

From the Firm's problem the primitives e need to specify values for are:

Table 2.4: Firm's parameter values

Parameters	Description	Values	Source
N	Upper limit for range of tasks	-	-
η	Distribution parameter of the CES	-	-
ρ	Relative labor productivity of unhealthy workers	-	-
ψ	Price of intermediates	-	-
σ	Elasticity of substitution between tasks	-	-
ζ	Elasticity of substitution between factors	-	-
C^{IN}	Health insurance fixed cost	-	-

Notice that by Proposition 9 if $\sigma > 1$ then l_i^{I*} is increasing in i and by Proposition 10, if $\sigma > 1$ and $\sigma > \zeta$ then q_i^{I*} is increasing in i, so if $\sigma = \zeta$, the optimal choice of intermediates do not vary with i. Moreover, to ensure existence of equilibrium in the static model in [Acemoglu and Restrepo, 2016], the authors state Assumption 1 (see page 10 of the paper). A sufficient condition for this assumption to hold is that $\sigma = \zeta$, ensuring homotheticity.

References

[Acemoglu and Restrepo, 2016] Acemoglu, D. and Restrepo, P. (2016). The race between machine and man: Implications of technology for growth, factor shares and employment. Technical report, National Bureau of Economic Research.

[David and Dorn, 2013] David, H. and Dorn, D. (2013). The growth of low-skill service jobs and the polarization of the us labor market. *The American Economic Review*, 103(5):1553–1597.

[Greenwood et al., 1988] Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, pages 402–417.

[Mendoza, 1991] Mendoza, E. G. (1991). Real business cycles in a small open economy. *The American Economic Review*, pages 797–818.

3 Appendix

3.1 Assumptions and Propositions

Define the willingness to pay for health insurance P as the price for which the household is indifferent between both contracts[3]. This price must satisfy then the following equation:

$$\int \frac{1}{1-\theta}\left[\left(w_0 l_0^* - \tilde{m} - \phi\frac{l_0^{*1+\xi}}{1+\xi}\right)^{1-\theta} - 1\right] dH(\tilde{m}|h) = \frac{1}{1-\theta}\left[\left(w_1 l_1^* - P - \phi\frac{l_1^{*1+\xi}}{1+\xi}\right)^{1-\theta} - 1\right] \tag{3.1}$$

In order for this equation to be well defined, we must impose

$$P < w_1 l_1^* - \phi\frac{l_1^{*1+\xi}}{1+\xi}$$

We can allow for negative prices in order to capture when the household needs to be paid to accept the health insurance at the given equilibrium wages.

We expect the solution to this equation $P(h,\theta,w_0,w_1)$ to be strictly increasing on θ. Because as the household becomes more risk averse (higher θ) he should be willing to pay more for health insurance.

Proposition 1. *$P(h,\theta,w_1,w_0)$ as defined by equation 3.1 is increasing on θ.*

Proof. As can be seen in equation 1.10, the optimal choice of labor does not depend on θ, therefore we can abstract the problem which defines P as follows:

$$\mathbb{E}^{\tilde{m}|h} u_\theta(Y_0 - \tilde{m}) = u_\theta(Y_1 - P)$$

Where Y_0 and Y_1 are fix numbers that do not depend on θ nor h. Solving for P we get

$$P = Y_1 - u_\theta^{-1}\left(\mathbb{E}^{\tilde{m}|h} u_\theta(Y_0 - \tilde{m})\right)$$

[3]Notice that for all these proofs I am not including the reservation wage $\underline{w}$ that ensures non negative argument for the utility function.

Now, take $\hat{\theta} > \theta$. As in our specification a higher θ is associated with a higher coefficient of risk aversion, we know by Arrow-Pratt's theorem that the certainty equivalents will satisfy for any lottery L_h:

$$c(u_\theta, L_h) > c(u_{\hat{\theta}}, L_h)$$

Now taking $L_h = Y_0 - \tilde{m}$ where $\tilde{m}$ is drawn from the distribution $H(\tilde{m}|h)$ we have

$$\begin{aligned}
c(u_\theta, L_h) &> c(u_{\hat{\theta}}, L_h) \\
\Leftrightarrow u_\theta^{-1}\left(\mathbb{E}^{\tilde{m}|h} u_\theta(Y_0 - \tilde{m})\right) &> u_{\hat{\theta}}^{-1}\left(\mathbb{E}^{\tilde{m}|h} u_{\hat{\theta}}(Y_0 - \tilde{m})\right) \\
\Leftrightarrow Y_1 - u_\theta^{-1}\left(\mathbb{E}^{\tilde{m}|h} u_\theta(Y_0 - \tilde{m})\right) &< Y_1 - u_{\hat{\theta}}^{-1}\left(\mathbb{E}^{\tilde{m}|h} u_{\hat{\theta}}(Y_0 - \tilde{m})\right) \\
\Leftrightarrow P(h, \theta) &< P(h, \hat{\theta})
\end{aligned}$$

□

Assumption 1.

$$H(\tilde{m}|b) \ \mathit{FOSD} \ H(\tilde{m}|g)$$

Proposition 2. *For any strictly increasing function u,*

$$\textit{if } F \textit{ FOSD } G \textit{ and } F \neq G \textit{ then } \int u(x)dF(x) > \int u(x)dG(x)$$

Proof. Follows from integration by parts (I also assume differentiability of u and the same support for the distributions F and G):

$$\begin{aligned}
\int_a^b u(x)dF(x) &= u(x)F(x)\Big|_b^a - \int_a^b F(x)u'(x)dx \\
&= u(b) - \int_a^b F(x)u'(x)dx
\end{aligned}$$

likewise

$$
\begin{aligned}
\int_a^b u(x)dG(x) &= u(x)G(x)\Big|_b^a - \int_a^b G(x)u'(x)dx \\
&= u(b) - \int_a^b G(x)u'(x)dx
\end{aligned}
$$

Now, by FOSD we know that $F(x) \leq G(x)$ (with this inequality being strict for some x if $F \neq G$) and as u is strictly increasing,

$$
\int_a^b F(x)u'(x)dx < \int_a^b G(x)u'(x)dx \tag{3.2}
$$

finally we get that

$$
\int_a^b u(x)dF(x) - \int_a^b u(x)dF(x) = \int_a^b G(x)u'(x)dx - \int_a^b F(x)u'(x)dx > 0
$$

$\square$

Proposition 3. $P(b, \theta, w_1, w_0) > P(g, \theta, w_1, w_0) \quad \forall \theta \in [0, 1), w_0, w_1 > 0$

Proof. By the definition of P we know that

$$
\begin{aligned}
& P(b, \theta, w_1, w_0) > P(g, \theta, w_1, w_0) \\
\Leftrightarrow\ & Y_1 - u_\theta^{-1}\left(\mathbb{E}^{\tilde{m}|b} u_\theta(Y_0 - \tilde{m})\right) > Y_1 - u_\theta^{-1}\left(\mathbb{E}^{\tilde{m}|g} u_\theta(Y_0 - \tilde{m})\right) \\
\Leftrightarrow\ & u_\theta^{-1}\left(\mathbb{E}^{\tilde{m}|g} u_\theta(Y_0 - \tilde{m})\right) > u_\theta^{-1}\left(\mathbb{E}^{\tilde{m}|b} u_\theta(Y_0 - \tilde{m})\right) \\
\Leftrightarrow\ & \mathbb{E}^{\tilde{m}|g} u_\theta(Y_0 - \tilde{m}) > \mathbb{E}^{\tilde{m}|b} u_\theta(Y_0 - \tilde{m}) \\
\Leftrightarrow\ & \int u_\theta(Y_0 - \tilde{m})dH(\tilde{m}|g) > \int u_\theta(Y_0 - \tilde{m})dH(\tilde{m}|b)
\end{aligned}
$$

now, define $\hat{u}(\tilde{m}) \equiv u(Y_0 - \tilde{m})$. As $u(\cdot)$ is strictly increasing, then $\hat{u}(\cdot)$ is strictly decreasing so we can write

$$\begin{aligned}
&\int u_\theta(Y_0 - \tilde{m})dH(\tilde{m}|g) > \int u_\theta(Y_0 - \tilde{m})dH(\tilde{m}|b) \\
&\Leftrightarrow \int \hat{u}_\theta(\tilde{m})dH(\tilde{m}|g) > \int \hat{u}_\theta(\tilde{m})dH(\tilde{m}|b) \\
&\Leftrightarrow \int -\hat{u}_\theta(\tilde{m})dH(\tilde{m}|b) > \int -\hat{u}_\theta(\tilde{m})dH(\tilde{m}|g)
\end{aligned}$$

Which is true by Assumption 1 and Proposition 2.

□

Proposition 4. $\bar{\theta}_g > \bar{\theta}_b$

Proof. $\bar{\theta}_h$ can be defined as

$$\bar{\theta}_h \in \{\theta \in [0,1) : P(h,\theta) = 0\} \tag{3.3}$$

Now, from proposition 1 we know that this set is a singleton because $P(h,\theta)$ is strictly increasing in θ. Furthermore, from proposition 2 we know that

$$P(b,\theta) > P(g,\theta) \quad \forall \theta \in [0,1)$$

Thus, at $\bar{\theta}_g$ we have that

$$P(b,\bar{\theta}_g) > P(g,\bar{\theta}_g) = 0$$

again, as $P(h,\theta)$ is strictly increasing in θ and

$$P(b,\bar{\theta}_g) > 0 = P(b,\bar{\theta}_b)$$

It follows that $\bar{\theta}_g > \bar{\theta}_b$.

□

Assumption 2.

$$F_g(\theta)\ FOSD\ F_b(\theta)$$

Proposition 5 (Advantageous Selection).

$$F_g(\bar{\theta}_g) < F_b(\bar{\theta}_b) \text{ iff } \chi^0_{gi} < \chi^1_{gi}$$

Proof. Using equations 1.26 and labor supply equations we can write down the endogenous proportion of healthy workers as follows:

$$\begin{aligned}
\chi^I_{gi} &= \delta_i \left(\frac{L^I_g(w1, w_0)}{L^I_g(w1, w_0) + L^I_b(w1, w_0)} \right) \\
&= \delta_i \left(\frac{1}{1 + \left(\frac{L^I_b}{L^I_g} \right)} \right)
\end{aligned}$$

So we know that the following equivalences must hold

$$\begin{aligned}
\chi^0_{gi} &< \chi^1_{gi} \\
\Leftrightarrow \frac{L^0_g}{L^0_b} &< \frac{L^1_g}{L^1_b} \\
\Leftrightarrow \frac{\lambda_g G'^{-1}(w_0) F_g(\bar{\theta}_g)}{(1-\lambda_g) G'^{-1}(w_0) F_b(\bar{\theta}_b)} &< \frac{\lambda_g G'^{-1}(w_1)(1 - F_g(\bar{\theta}_g))}{(1-\lambda_g) G'^{-1}(w_1)(1 - F_b(\bar{\theta}_b))} \\
\Leftrightarrow \frac{1 - F_b(\bar{\theta}_b)}{F_b(\bar{\theta}_b)} &< \frac{(1 - F_g(\bar{\theta}_g))}{F_g(\bar{\theta}_g)} \\
\Leftrightarrow F_g(\bar{\theta}_g) &< F_b(\bar{\theta}_b)
\end{aligned}$$

□

Notice that in this last result there are two effects going in opposite directions. From one hand proposition 4 establishes that $\bar{\theta}_g > \bar{\theta}_b$ but, on the other hand, assumption 2 establishes that for every $\theta \in [0, 1)$ $F_b(\theta) \geq F_g(\theta)$. If the latter effect dominates we will then have $F_g(\bar{\theta}_g) < F_b(\bar{\theta}_b)$.

Proposition 6. *if* $\chi^0_{gi} < \chi^1_{gi}$ *then* $\chi^1_{gi} - \chi^0_{gi}$ *is increasing on* i

Proof. Using equations 1.26 we know that

$$\chi_{gi}^1 - \chi_{gi}^0 = \delta_i \left(\frac{L_g^1(w1, w_0)}{L_g^1(w1, w_0) + L_b^1(w1, w_0)} - \frac{L_g^0(w1, w_0)}{L_g^0(w1, w_0) + L_b^0(w1, w_0)} \right)$$

under Advantageous Selection the term in parenthesis is positive and we have already assumed that δ_i is an increasing function of i.

□

Proposition 7. *Under Advantageous selection, $\frac{\bar{\gamma}_i^1}{\bar{\gamma}_i^0}$ is increasing in i.*

Proof. We know that

$$\begin{aligned} \frac{\bar{\gamma}_i^1}{\bar{\gamma}_i^0} &= \frac{\gamma_i \left((1-\rho)\chi_{gi}^1 + \rho \right)}{\gamma_i \left((1-\rho)\chi_{gi}^0 + \rho \right)} \\ &= \frac{(1-\rho)\delta_i \left(\frac{L_g^1(w1,w_0)}{L_g^1(w1,w_0)+L_b^1(w1,w_0)} \right) + \rho}{(1-\rho)\delta_i \left(\frac{L_g^0(w1,w_0)}{L_g^0(w1,w_0)+L_b^0(w1,w_0)} \right) + \rho} \end{aligned}$$

for simplicity define

$$a \equiv (1-\rho) \left(\frac{L_g^1(w1, w_0)}{L_g^1(w1, w_0) + L_b^1(w1, w_0)} \right)$$

and

$$b \equiv (1-\rho) \left(\frac{L_g^0(w1, w_0)}{L_g^0(w1, w_0) + L_b^0(w1, w_0)} \right)$$

then

$$\frac{\bar{\gamma}_i^1}{\bar{\gamma}_i^0} = \frac{a\delta_i + \rho}{b\delta_i + \rho}$$

Taking the derivative of $\frac{\bar{\gamma}_i^1}{\bar{\gamma}_i^0}$ with respect to i we get

$$\frac{\partial}{\partial i}\frac{\bar{\gamma}_i^1}{\bar{\gamma}_i^0} = \frac{a\delta_i'(b\delta_i + \rho) - (a\delta_i + \rho)(b\delta_i')}{(b\delta_i + \rho)^2}$$

so

$$\begin{aligned} \frac{\partial}{\partial i}\frac{\bar{\gamma}_i^1}{\bar{\gamma}_i^0} &> 0 \\ \Leftrightarrow a\delta_i'(b\delta_i + \rho) - (a\delta_i + \rho)(b\delta_i') &> 0 \\ \Leftrightarrow \delta_i'[a(b\delta_i + \rho) - b(a\delta_i + \rho)] &> 0 \\ \Leftrightarrow \delta_i'[\rho(a - b)] &> 0 \end{aligned}$$

Which is true since $a > b$ under Advantageous selection and δ_i' is strictly positive by assumption. So we can conclude that $\frac{\bar{\gamma}_i^1}{\bar{\gamma}_i^0}$ is increasing on i and must belong to the interval $(1, 1/\rho)$.

□

Proposition 8. *If* $\exists \tilde{i} \in \mathbb{R}^+ : \hat{w}^1(\tilde{i}) = \hat{w}^0(\tilde{i})$ *and Advantageous selection holds then* $\hat{w}^1(\tilde{i})' < \hat{w}^0(\tilde{i})'$.

Proof. We know that

$$\hat{w}_i^1 = \frac{w_1 + M_i}{\bar{\gamma}_i^1} \tag{3.4}$$

and

$$\hat{w}_i^0 = \frac{w_0}{\bar{\gamma}_i^0} \tag{3.5}$$

where

$$M_i = \frac{m_g\chi_{gi}^1 + m_b(1 - \chi_{gi}^1)}{l_1^*} \tag{3.6}$$

By assumption, we can define

$$\bar{w} \equiv \hat{w}^1(\tilde{i}) = \hat{w}^0(\tilde{i}) \tag{3.7}$$

So

$$\begin{aligned}\hat{w}^0(\tilde{i})' &= \frac{-w_0\bar{\gamma}^0(\tilde{i})'}{\left(\bar{\gamma}^0(\tilde{i})\right)^2} \\ &= \frac{-\bar{w}\bar{\gamma}^0(\tilde{i})'}{\bar{\gamma}^0(\tilde{i})}\end{aligned}$$

and

$$\begin{aligned}\hat{w}^1(\tilde{i})' &= \frac{M(\tilde{i})'\bar{\gamma}^1(\tilde{i}) - (w_1 + M_i)\bar{\gamma}^1(\tilde{i})'}{\left(\bar{\gamma}^1(\tilde{i})\right)^2} \\ &= \frac{M(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})} - \frac{\bar{w}\bar{\gamma}^1(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})} \\ &= \frac{\chi_g^{1'}(\tilde{i})\,(m_g - m_b)}{l_1^*\bar{\gamma}^1(\tilde{i})} - \frac{\bar{w}\bar{\gamma}^1(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})}\end{aligned}$$

So,

$$\begin{aligned}&\hat{w}^0(\tilde{i})' > \hat{w}^1(\tilde{i})' \\ \Leftrightarrow\ &\frac{-\bar{w}\bar{\gamma}^0(\tilde{i})'}{\bar{\gamma}^0(\tilde{i})} > \frac{\chi_g^{1'}(\tilde{i})\,(m_g - m_b)}{l_1^*\bar{\gamma}^1(\tilde{i})} - \frac{\bar{w}\bar{\gamma}^1(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})} \\ \Leftrightarrow\ &\frac{\chi_g^{1'}(\tilde{i})\,(m_b - m_g)}{l_1^*\bar{\gamma}^1(\tilde{i})} > \bar{w}\left(\frac{\bar{\gamma}^0(\tilde{i})'}{\bar{\gamma}^0(\tilde{i})} - \frac{\bar{\gamma}^1(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})}\right) \\ \Leftrightarrow\ &\frac{\delta'(\tilde{i})\left(\frac{L_g^1}{L_g^1+L_b^1}\right)(m_b - m_g)}{\bar{w}l_1^*\bar{\gamma}^1(\tilde{i})} > \frac{\bar{\gamma}^0(\tilde{i})'}{\bar{\gamma}^0(\tilde{i})} - \frac{\bar{\gamma}^1(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})}\end{aligned}$$

Notice that the left hand side of the previous inequality is strictly positive given our assumption of $\delta^{'}(i) > 0, \forall i$ and that the expected medical expenditure of bad health workers is higher than the expected medical expenditure of good health workers ($m_b > m_g$) and that $\bar{w} > 0$ because the effective wage for no health insurance is strictly positive, regardless of the medical expenditure. However, as we do not know which is the value of $\bar{w}$, we need to impose more assumptions to say that this inequality will hold where both effective wages cross. We can write the right hand side as follows:

$$
\begin{aligned}
\frac{\bar{\gamma}^0(\tilde{i})'}{\bar{\gamma}^0(\tilde{i})} - \frac{\bar{\gamma}^1(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})} &= \log\left(\bar{\gamma}^0(\tilde{i})\right)' - \log\left(\bar{\gamma}^1(\tilde{i})\right)' \\
&= \log\left(\frac{\bar{\gamma}^0(\tilde{i})}{\bar{\gamma}^1(\tilde{i})}\right)' \\
&= \left(\frac{\bar{\gamma}^1(\tilde{i})}{\bar{\gamma}^0(\tilde{i})}\right)\left(\frac{\bar{\gamma}^0(\tilde{i})}{\bar{\gamma}^1(\tilde{i})}\right)'
\end{aligned}
$$

Notice that the second term in the previous equality is negative under Advantageous selection (see Proposition 7). Thus we can write:

$$
\frac{\delta'(\tilde{i})\left(\frac{L_g^1}{L_g^1+L_b^1}\right)(m_b - m_g)}{\bar{w}l_1^*\bar{\gamma}^1(\tilde{i})} > 0 > \frac{\bar{\gamma}^0(\tilde{i})'}{\bar{\gamma}^0(\tilde{i})} - \frac{\bar{\gamma}^1(\tilde{i})'}{\bar{\gamma}^1(\tilde{i})}
$$

So we can conclude that if $\exists \tilde{i} \in \mathbb{R}^+ : \hat{w}^1(\tilde{i}) = \hat{w}^0(\tilde{i})$ and Advantageous selection holds then $\hat{w}^1(\tilde{i})' < \hat{w}^0(\tilde{i})'$.

Therefore after $\tilde{i}$ both effective wages will never cross again, strengthening the incentives for high productivity firms to provide health insurance if the fixed cost C^{IN} is low enough.

□

Proposition 9. *Under Advantageous selection, If $\zeta_i = \zeta$, $C^{IN} = 0$ and $\tilde{X}$ defined by equation 1.66 is interior in the range of tasks $[N-1, N]$, then for $i > \tilde{X}$ firms will prefer to hire labor with health insurance than labor without health insurance.*

Proof. To see if after the threshold firms choose to provide health insurance, we need to show that the RHS of equation 1.66 is increasing in i at $i = \tilde{X}$. For simplicity, assume that $\zeta_i = \zeta$ and define the following:

$$
f(\hat{w}_i^I) \equiv \frac{\left[\eta(p_i^I(\hat{w}_i^I))^{\zeta-1} + (1-\eta)\right]^{\frac{\zeta}{\zeta-1}}}{\hat{w}_i^I + \psi(p_i^I(\hat{w}_i^I))^{\zeta}} \tag{3.8}
$$

now, we need to compute and sign $\frac{\partial f(\hat{w}_i^I)}{\partial \hat{w}_i^I}$. Omitting the superscript I for simplicity:

$$\frac{\partial f(\hat{w}_i)}{\partial \hat{w}_i} = \frac{\left[\eta p_i^{\zeta-1} + (1-\eta)\right]^{\frac{\zeta}{\zeta-1}-1} \frac{\zeta}{\zeta-1}(\eta(\zeta-1)p_i^{\zeta-2}p_i')(\hat{w}_i + \psi p_i^{\zeta}) - \left[\eta p_i^{\zeta-1} + (1-\eta)\right]^{\frac{\zeta}{\zeta-1}} (1 + \psi\zeta p_i^{\zeta-1}p_i')}{(\hat{w}_i + \psi p_i^{\zeta})^2}$$

$$= \frac{\left[\eta p_i^{\zeta-1} + (1-\eta)\right]^{\frac{1}{\zeta-1}} \zeta(\eta p_i^{\zeta-2}p_i')(\hat{w}_i + \psi p_i^{\zeta}) - \left[\eta p_i^{\zeta-1} + (1-\eta)\right]^{\frac{\zeta}{\zeta-1}} (1 + \psi\zeta p_i^{\zeta-1}p_i')}{(\hat{w}_i + \psi p_i^{\zeta})^2}$$

$$= \frac{\left[\eta p_i^{\zeta-1} + (1-\eta)\right]^{\frac{1}{\zeta-1}} \left(\zeta(\eta p_i^{\zeta-2}p_i')(\hat{w}_i + \psi p_i^{\zeta}) - \left[\eta p_i^{\zeta-1} + (1-\eta)\right) (1 + \psi\zeta p_i^{\zeta-1}p_i')\right)}{(\hat{w}_i + \psi p_i^{\zeta})^2}$$

so

$$\begin{aligned}
\frac{\partial f(\hat{w}_i^I)}{\partial \hat{w}_i^I} &< 0 \\
\Leftrightarrow \zeta(\eta p_i^{\zeta-2}p_i')(\hat{w}_i + \psi p_i^{\zeta}) - \left(\eta p_i^{\zeta-1} + (1-\eta)\right)(1 + \psi\zeta p_i^{\zeta-1}p_i') &< 0 \\
\Leftrightarrow \zeta(\eta p_i^{\zeta-2}p_i')\hat{w}_i + \zeta(\eta p_i^{\zeta-2}p_i')\psi p_i^{\zeta} - \left(\eta p_i^{\zeta-1} + (1-\eta)\right)(1 + \psi\zeta p_i^{\zeta-1}p_i') &< 0 \\
\Leftrightarrow \zeta\eta p_i^{\zeta-1} + \zeta\eta p_i^{2\zeta-2}p_i'\psi - \left(\eta p_i^{\zeta-1} + (1-\eta)\right)(1 + \psi\zeta p_i^{\zeta-1}p_i') &< 0 \\
\Leftrightarrow \zeta\eta p_i^{\zeta-1} + \zeta\eta p_i^{2\zeta-2}p_i'\psi - \eta p_i^{\zeta-1} - \eta p_i^{2\zeta-2}\psi\zeta p_i' - (1-\eta) - (1-\eta)\psi\zeta p_i^{\zeta-1}p_i' &< 0 \\
\Leftrightarrow \zeta\eta p_i^{\zeta-1} - \eta p_i^{\zeta-1} - (1-\eta) - (1-\eta)\psi\zeta p_i^{\zeta-1}p_i' &< 0 \\
\Leftrightarrow \zeta\eta p_i^{\zeta-1} - \eta p_i^{\zeta-1} - (1-\eta) - (1-\eta)\psi\zeta p_i^{\zeta-1}\frac{\eta}{(1-\eta)\psi} &< 0 \\
\Leftrightarrow \zeta\eta p_i^{\zeta-1} - \eta p_i^{\zeta-1} - (1-\eta) - \zeta p_i^{\zeta-1}\eta &< 0 \\
\Leftrightarrow -\eta p_i^{\zeta-1} - (1-\eta) &< 0
\end{aligned}$$

where we have used that $p_i' = \frac{\eta}{(1-\eta)\psi}$ and $p_i = \hat{w}_i p_i'$.

Now, we need to show that the RHS of equation 1.66 is increasing in i at $i = \tilde{X}$:

$$\begin{aligned}
\frac{\partial f(\hat{w}^1(\tilde{X}))}{\partial i} &> \frac{\partial f(\hat{w}^0(\tilde{X}))}{\partial i} \\
\Leftrightarrow \frac{\partial f(\hat{w}^1(\tilde{X}))}{\partial \hat{w}^1}\frac{\partial \hat{w}^1(\tilde{X})}{\partial i} &> \frac{\partial f(\hat{w}^0(\tilde{X}))}{\partial \hat{w}^0}\frac{\partial \hat{w}^0\tilde{X})}{\partial i}
\end{aligned}$$

As we have assumed $C^{IN} = 0$ for this proof, we know that $\hat{w}^1(\tilde{X}) = \hat{w}^0(\tilde{X})$, and as $\frac{\partial f(\hat{w}_i^I)}{\partial \hat{w}_i^I} < 0$, the previous inequality gets as:

$$\frac{\partial \hat{w}^1(\tilde{X})}{\partial i} < \frac{\partial \hat{w}^0(\tilde{X})}{\partial i}$$

Which is true by Proposition 8. Therefore we can conclude that as i increases there are more incentives for high productivity firms to provide health insurance.

□

Proposition 10. *if $\sigma > 1$ then $\hat{l}_i^{I*}$ is increasing in i.*

Proof. Assume $\sigma > 1$, then the optimal choice of effective labor is well defined and takes the form of

$$\hat{l}_i^{I*} = \tilde{B}\frac{\left[\eta^{\zeta}(\hat{w}_i^I)^{\zeta-1} + (1-\eta)^{\zeta}\psi^{\zeta-1}\right]^{\frac{\sigma-\zeta}{\zeta-1}}}{(\hat{w}_i^I)^{\sigma}} = \tilde{B}\frac{\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}}{(\hat{w}_i^I)^{\sigma}}$$

Let's calculate and sign the derivative of the optimal choice of efficient labor with respect to the efficient cost of labor:

$$\begin{aligned}
\frac{\partial \hat{l}_i^{I*}}{\partial \hat{w}_i^I} &= \tilde{B}\frac{\left(\frac{\sigma-\zeta}{\zeta-1}\right)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^{\zeta}(\zeta-1)(\hat{w}_i^I)^{\zeta-2}(\hat{w}_i^I)^{\sigma} - \sigma\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-1}}{(\hat{w}_i^I)^{2\sigma}} \\
&= \tilde{B}\frac{(\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^{\zeta}(\hat{w}_i^I)^{\zeta+\sigma-2} - \sigma\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-1}}{(\hat{w}_i^I)^{2\sigma}}
\end{aligned}$$

In order to sign this expression we must sign the numerator:

$$\begin{aligned}
\frac{\partial \hat{l}_i^{I*}}{\partial \hat{w}_i^I} &< 0 \\
\Leftrightarrow \tilde{B}\left((\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^{\zeta}(\hat{w}_i^I)^{\zeta+\sigma-2} - \sigma\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-1}\right) &< 0
\end{aligned}$$

Now, as $\sigma > 1$ then $\tilde{B} > 0$ the previous expression is negative if

$$\begin{aligned}
&\tilde{B}\left((\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}-\sigma\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-1}\right)<0\\
&\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}-\sigma\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-1}<0\\
&\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}<\sigma\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-1}\\
&\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{-1}\eta^\zeta(\hat{w}_i^I)^{\zeta-1}<\sigma\\
&\Leftrightarrow (\sigma-\zeta)\eta^\zeta(\hat{w}_i^I)^{\zeta-1}<\sigma f(\hat{w}_i^I)\\
&\Leftrightarrow (\sigma-\zeta)\eta^\zeta(\hat{w}_i^I)^{\zeta-1}<\sigma\eta^\zeta(\hat{w}_i^I)^{\zeta-1}+\sigma(1-\eta)^\zeta\psi^{\zeta-1}\\
&\Leftrightarrow (\sigma-\zeta-\sigma)\eta^\zeta(\hat{w}_i^I)^{\zeta-1}<\sigma(1-\eta)^\zeta\psi^{\zeta-1}\\
&\Leftrightarrow -\zeta\eta^\zeta(\hat{w}_i^I)^{\zeta-1}<\sigma(1-\eta)^\zeta\psi^{\zeta-1}
\end{aligned}$$

Which is true because the LHS is negative for any positive wage and the RHS is strictly positive. Moreover as $\hat{w}_i^I$ is a decreasing function of i we get that:

$$\frac{\partial \hat{l}_i^{I*}}{\partial i} > 0$$

□

Proposition 11. *if $\sigma > 1$ then l_i^{I*} is increasing in i*

Proof. Assume $\sigma > 1$, then the optimal choice of labor is well defined and takes the form of

$$l_i^{I*} = \tilde{B}\frac{\left[\eta^\zeta(\hat{w}_i^I)^{\zeta-1}+(1-\eta)^\zeta\psi^{\zeta-1}\right]^{\frac{\sigma-\zeta}{\zeta-1}}}{\bar{\gamma}_i^I(\hat{w}_i^I)^\sigma} = \tilde{B}\frac{\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}}{\bar{\gamma}_i^I(\hat{w}_i^I)^\sigma}$$

Let's calculate and sign the derivative of the optimal choice of efficient labor with respect to the efficient cost of labor:

$$\frac{\partial l_i^{I*}}{\partial \hat{w}_i^I} = \tilde{B}\frac{\left(\frac{\sigma-\zeta}{\zeta-1}\right)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\zeta-1)(\hat{w}_i^I)^{\zeta-2}\bar{\gamma}_i^I(\hat{w}_i^I)^\sigma - \left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\bar{\gamma'}_i^I(\hat{w}_i^I)^\sigma + \sigma\bar{\gamma}_i^I(\hat{w}_i^I)^{\sigma-1})}{(\bar{\gamma}_i^I(\hat{w}_i^I)^\sigma)^2}$$

$$= \tilde{B}\frac{(\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}\bar{\gamma}_i^I - \left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\bar{\gamma'}_i^I(\hat{w}_i^I)^\sigma + \sigma\bar{\gamma}_i^I(\hat{w}_i^I)^{\sigma-1})}{(\bar{\gamma}_i^I(\hat{w}_i^I)^\sigma)^2}$$

In order to sign this expression we must sign the numerator:

$$\frac{\partial l_i^{I*}}{\partial \hat{w}_i^I} < 0$$

$$\Leftrightarrow \tilde{B}\left((\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}\bar{\gamma}_i^I - \left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\bar{\gamma'}_i^I(\hat{w}_i^I)^\sigma + \sigma\bar{\gamma}_i^I(\hat{w}_i^I)^{\sigma-1})\right) < 0$$

Now, as $\sigma > 1$ then $\tilde{B} > 0$ the previous expression is negative if

$$\begin{aligned}
&\tilde{B}\left((\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}\bar{\gamma}_i^I - \left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\bar{\gamma'}_i^I(\hat{w}_i^I)^\sigma + \sigma\bar{\gamma}_i^I(\hat{w}_i^I)^{\sigma-1})\right) < 0\\
&\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}\bar{\gamma}_i^I - \left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\bar{\gamma'}_i^I(\hat{w}_i^I)^\sigma + \sigma\bar{\gamma}_i^I(\hat{w}_i^I)^{\sigma-1}) < 0\\
&\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{-1}\eta^\zeta(\hat{w}_i^I)^{\zeta+\sigma-2}\bar{\gamma}_i^I - (\bar{\gamma'}_i^I(\hat{w}_i^I)^\sigma + \sigma\bar{\gamma}_i^I(\hat{w}_i^I)^{\sigma-1}) < 0\\
&\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{-1}\eta^\zeta(\hat{w}_i^I)^{\zeta-1}\bar{\gamma}_i^I - (\bar{\gamma'}_i^I\hat{w}_i^I + \sigma\bar{\gamma}_i^I) < 0\\
&\Leftrightarrow (\sigma-\zeta)\eta^\zeta(\hat{w}_i^I)^{\zeta-1}\bar{\gamma}_i^I - \left[f(\hat{w}_i^I)\right](\bar{\gamma'}_i^I\hat{w}_i^I + \sigma\bar{\gamma}_i^I) < 0\\
&\Leftrightarrow (\sigma-\zeta)\eta^\zeta(\hat{w}_i^I)^{\zeta-1}\bar{\gamma}_i^I - \left[\eta^\zeta(\hat{w}_i^I)^{\zeta-1} + (1-\eta)^\zeta\psi^{\zeta-1}\right](\bar{\gamma'}_i^I\hat{w}_i^I + \sigma\bar{\gamma}_i^I) < 0\\
&\Leftrightarrow (\sigma-\zeta-\frac{\bar{\gamma'}_i^I\hat{w}_i^I}{\bar{\gamma}_i^I}-\sigma)\eta^\zeta(\hat{w}_i^I)^{\zeta-1}\bar{\gamma}_i^I - (1-\eta)^\zeta\psi^{\zeta-1}(\bar{\gamma'}_i^I\hat{w}_i^I + \sigma\bar{\gamma}_i^I) < 0\\
&\Leftrightarrow -(\zeta+\frac{\bar{\gamma'}_i^I\hat{w}_i^I}{\bar{\gamma}_i^I})\eta^\zeta(\hat{w}_i^I)^{\zeta-1}\bar{\gamma}_i^I - (1-\eta)^\zeta\psi^{\zeta-1}(\bar{\gamma'}_i^I\hat{w}_i^I + \sigma\bar{\gamma}_i^I) < 0
\end{aligned}$$

Which is true because the LHS is strictly negative for any wage. Moreover as $\hat{w}_i^I$ is a decreasing function of i we get that:

$$\frac{\partial l_i^{I*}}{\partial i} > 0$$

So more productive firms will be bigger in equilibrium.

$\square$

Proposition 12. *if $\sigma > 1$ and $\sigma > \zeta$ then q_i^{I*} is increasing in i*

Proof. Assume $\sigma > 1$, then the optimal choice of intermediates is well defined and takes the form of

$$q_i^{I*} = \frac{\tilde{\eta}\tilde{B}}{\psi^{\zeta}} \frac{\left[\eta^{\zeta}(\hat{w}_i^I)^{\zeta-1} + (1-\eta)^{\zeta}\psi^{\zeta-1}\right]^{\frac{\sigma-\zeta}{\zeta-1}}}{(\hat{w}_i^I)^{\sigma-\zeta}} \tag{3.9}$$

Let's calculate and sign the derivative of the optimal choice of intermediates with respect to the efficient cost of labor:

$$\begin{aligned}
\frac{\partial q_i^{I*}}{\partial \hat{w}_i^I} &= \frac{\tilde{\eta}\tilde{B}}{\psi^{\zeta}} \frac{\left(\frac{\sigma-\zeta}{\zeta-1}\right)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1} \eta^{\zeta}(\zeta-1)(\hat{w}_i^I)^{\zeta-2}(\hat{w}_i^I)^{\sigma-\zeta} - (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-\zeta-1}}{(\hat{w}_i^I)^{2(\sigma-\zeta)}} \\
&= \frac{\tilde{\eta}\tilde{B}}{\psi^{\zeta}} \frac{(\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1} \eta^{\zeta}(\hat{w}_i^I)^{\zeta+\sigma-\zeta-2} - (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-\zeta-1}}{(\hat{w}_i^I)^{2(\sigma-\zeta)}}
\end{aligned}$$

In order to sign this expression we must sign the numerator:

$$\begin{aligned}
&\frac{\partial q_i^{I*}}{\partial \hat{w}_i^I} < 0 \\
\Leftrightarrow \frac{\tilde{\eta}\tilde{B}}{\psi^{\zeta}} &\left((\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1} \eta^{\zeta}(\hat{w}_i^I)^{\zeta+\sigma-\zeta-2} - (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-\zeta-1}\right) < 0
\end{aligned}$$

Now, as $\sigma > 1$ then $\frac{\tilde{\eta}\tilde{B}}{\psi^{\zeta}} > 0$ and the previous expression is negative if

$$\frac{\tilde{\eta}\tilde{B}}{\psi^{\zeta}}\left((\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^{\zeta}(\hat{w}_i^I)^{\zeta+\sigma-\zeta-2}-(\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-\zeta-1}\right)<0$$

$$\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}\eta^{\zeta}(\hat{w}_i^I)^{\sigma-2}-(\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{\frac{\sigma-\zeta}{\zeta-1}}(\hat{w}_i^I)^{\sigma-\zeta-1}<0$$

$$\Leftrightarrow (\sigma-\zeta)\left[f(\hat{w}_i^I)\right]^{-1}\eta^{\zeta}(\hat{w}_i^I)^{\zeta-1}<(\sigma-\zeta)$$

$$\Leftrightarrow (\sigma-\zeta)\eta^{\zeta}(\hat{w}_i^I)^{\zeta-1}<(\sigma-\zeta)f(\hat{w}_i^I)$$

$$\Leftrightarrow (\sigma-\zeta)[\eta^{\zeta}(\hat{w}_i^I)^{\zeta-1}-\eta^{\zeta}(\hat{w}_i^I)^{\zeta-1}-(1-\eta)^{\zeta}\psi^{\zeta-1}]<0$$

$$\Leftrightarrow -(\sigma-\zeta)[(1-\eta)^{\zeta}\psi^{\zeta-1}]<0$$

$$\Leftrightarrow \zeta<\sigma$$

Moreover as $\hat{w}_i^I$ is a decreasing function of i we get that if $\zeta < \sigma$ and $\sigma > 1$:

$$\frac{\partial q_i^{I*}}{\partial i} > 0$$

□

Proposition 13. *Under Assumptions 3 and 4 (stated below), there does not exist an equilibrium where all firms choose to provide health insurance (without taking care of Capital decisions).*

Proof. By contradiction. Suppose $\exists(w_0, w_1)$, equilibrium wages, such that all firms choose to provide health insurance, i.e, $x_i = 1 \forall i \in [N-1, N]$. Then, for any non negative fixed cost of insurance C^{IN}, it must be the case that the effective marginal cost of hiring labor without insurance is strictly higher than the one of hiring lebor with health insurance:

$$\frac{w_0}{\bar{\gamma}_i^0} > \frac{w_1 + M_i}{\bar{\gamma}_i^1} \tag{3.10}$$

As no worker is willing to provide labor for the contract with health insurance, the aggregate labor supply is given by:

$$L_g^1 = \lambda_g \left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} \tag{3.11}$$

$$L_b^1 = (1-\lambda_g)\left(\frac{w_1}{\phi}\right)^{\frac{1}{\xi}} \tag{3.12}$$

$$L_g^0 = 0 \tag{3.13}$$

$$L_b^0 = 0 \tag{3.14}$$

Notice that this imply that the proportion of healthy workers, under the contract with health insurance, that each firm faces in equilibrium is given by:

$$\chi_{gi}^1 = \delta_i \lambda_g \tag{3.15}$$

and for the the contract that is not offered in equilibrium, beliefs could be anything, so, if there is sorting of workers:

$$\chi_{gi}^0 \in [0, \delta_i] \tag{3.16}$$

And if there is no sorting, we can extend this range to be $\chi_{gi}^0 \in [0, 1]$. Without restricting beliefs, we can write equation 3.10 as:

$$\boxed{\frac{w_0}{\gamma_i\left((1-\rho)\chi_{gi}^0 + \rho\right)} > \frac{w_1 + M_i}{\gamma_i\left((1-\rho)\chi_{gi}^1 + \rho\right)}} \tag{3.17}$$

this inequality can be further reduced by assuming that, for at least one firm $i \in [N-1, N]$, beliefs for the contract without health insurance (that no one is offering in equilibrium) are as good as the beliefs that firm has over the proportion of healthy workers she will attract under the contract with health insurance.

Assumption 3. $\exists i \in [N-1, N] \quad st \quad \chi_{gi}^0 \geq \chi_{gi}^1$

This assumption can be justified in some way if we look at the least productive firm $i = N - 1$. If we further assume that $\underline{\delta} \equiv \delta(N-1) = 0$, that is that the lowest productivity firm will always get unhealthy workers regardless of the contract she chooses, then the previous assumption will be satisfied. Now, a more compelling and economically founded assumption could be that the lowest productivity firm if she deviates and is the only firm offering the contract without insurance, will always attract healthy workers. The reasoning behind this is because those are the workers with the lowest willingness to pay for insurance and healthy and unhealthy workers come from a distribution with the same support on the risk aversion parameter, so we can always get a measure of just healthy workers.

Let's suppose that Assumption 3 holds for $i = N - 1$, then the previous inequality will imply that:

$$w_0 > w_1 + M(N-1) \tag{3.18}$$

Now, turning to the other side of the market, as (w_0, w_1) are equilibrium wages, by market clearing we must have that no worker is willing to provide labor for the contract without health insurance. This has to be true for every worker, so in particular has to be true for the worker who has the lowest willingness to pay for health insurance: a risk neutral healthy worker. For this worker, wages must be such that:

$$\begin{aligned}
w_0 l_0^* - \phi \frac{l_0^{*1+\xi}}{1+\xi} - m_g &< w_1 l_1^* - \phi \frac{l_1^{*1+\xi}}{1+\xi} \\
\Leftrightarrow l_0^* \left(w_0 - \frac{w_0}{1+\xi} \right) &< l_1^* \left(w_1 - \frac{w_1}{1+\xi} \right) + m_g \\
\Leftrightarrow \frac{w_0^{\frac{1}{\xi}}}{\phi^{\frac{1}{\xi}}} \left(w_0 - \frac{w_0}{1+\xi} \right) &< \frac{w_1^{\frac{1}{\xi}}}{\phi^{\frac{1}{\xi}}} \left(w_1 - \frac{w_1}{1+\xi} \right) + m_g \\
\Leftrightarrow \frac{w_0^{\frac{1}{\xi}}}{\phi^{\frac{1}{\xi}}} \left(\frac{(1+\xi)w_0 - w_0}{1+\xi} \right) &< \frac{w_1^{\frac{1}{\xi}}}{\phi^{\frac{1}{\xi}}} \left(\frac{(1+\xi)w_1 - w_1}{1+\xi} \right) + m_g \\
\Leftrightarrow w_0^{\frac{1}{\xi}+1} &< w_1^{\frac{1}{\xi}+1} + C m_g
\end{aligned}$$

where $C \equiv \frac{(1+\xi)\phi^{\frac{1}{\xi}}}{\xi}$

The last inequality can be further reduced by using the fact that under Assumption 3, $w_0 > w_1$ as follows:

$$\begin{aligned}
w_0^{\frac{1}{\xi}+1} &< w_1^{\frac{1}{\xi}+1} + C m_g \\
\Leftrightarrow w_0^{\frac{1}{\xi}+1} &< w_1^{\frac{1}{\xi}} \left(w_1 + \frac{C m_g}{w_1^{\frac{1}{\xi}}} \right) \\
\Rightarrow w_0^{\frac{1}{\xi}+1} &< w_0^{\frac{1}{\xi}} \left(w_1 + \frac{C m_g}{w_1^{\frac{1}{\xi}}} \right) \\
\Leftrightarrow w_0 &< w_1 + \frac{C m_g}{w_1^{\frac{1}{\xi}}}
\end{aligned}$$

Assumption 4.

$$\frac{1+\xi}{\xi}m_g < m_g\underline{\delta}\lambda_g + m_b(1-\underline{\delta}\lambda_g)$$

Now, using Assumption 4 we can rewrite the previous inequality:

$$\begin{aligned}
& w_0 < w_1 + \frac{Cm_g}{w_1^{\frac{1}{\xi}}} \\
\Leftrightarrow\ & w_0 < w_1 + \frac{(1+\xi)\phi^{\frac{1}{\xi}}m_g}{\xi w_1^{\frac{1}{\xi}}} \\
\Leftrightarrow\ & w_0 < w_1 + \frac{(1+\xi)\phi^{\frac{1}{\xi}}m_g}{\xi w_1^{\frac{1}{\xi}}} \\
\Leftrightarrow\ & w_0 < w_1 + \frac{(1+\xi)m_g}{\xi l_1^*} \\
\Rightarrow\ & w_0 < w_1 + \frac{m_g\underline{\delta}\lambda_g + m_b(1-\underline{\delta}\lambda_g)}{l_1^*} \\
\Leftrightarrow\ & w_0 < w_1 + M(N-1)
\end{aligned}$$

Which reaches a contradiction with inequality 3.18. Thus, under Assumptions 1 and 2 it can not exist an equilibrium where all firms choose to provide health insurance.

□

3.2 Resource Constraint

The market clearing condition for consumption can be derived from the Household's budget constraint as follows (without minimum consumption threshold):

$$C \equiv \lambda_g \int_{\underline{\theta}}^{\bar{\theta}} c(g,\theta)dF_g(\theta) + (1-\lambda_g)\int_{\underline{\theta}}^{\bar{\theta}} c(b,\theta)dF_b(\theta)$$

where

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} c(h,\theta)dF_h(\theta) &= \int_{\underline{\theta}}^{\bar{\theta}_h} c(h,\theta)dF_h(\theta) + \int_{\bar{\theta}_h}^{\bar{\theta}} c(h,\theta)dF_h(\theta) \\
&= F_h(\bar{\theta}_h)\left(w_0 l_0 - \mathbb{E}^h\left[\tilde{m}\right]\right) + \left(1-F_h(\bar{\theta}_h)\right)w_1 l_1
\end{aligned}$$

so using the equations of aggregate labor supply:

$$C = w_0(L_g^0 + L_b^0) + w_1(L_g^1 + L_b^1) - \lambda_g F(\bar{\theta}_g)\mathbb{E}^g[\tilde{m}] - (1-\lambda_g)F(\bar{\theta}_b)\mathbb{E}^b[\tilde{m}]$$

where the last two terms account for the total (expected) medical expenditure of workers without health insurance, which can be defined as

$$M^0 \equiv \lambda_g F(\bar{\theta}_g)\mathbb{E}^g[\tilde{m}] + (1-\lambda_g)F(\bar{\theta}_b)\mathbb{E}^b[\tilde{m}] \tag{3.19}$$

In a similar way we can define the expected medical expenditure of the workers with health insurance (paid by the firms) by:

$$M^1 \equiv \lambda_g \left(1 - F(\bar{\theta}_g)\right)\mathbb{E}^g[\tilde{m}] + (1-\lambda_g)\left(1 - F(\bar{\theta}_b)\right)\mathbb{E}^b[\tilde{m}] \tag{3.20}$$

Now, if we add to the RHS the profits of the final good producers Π_F, the task producers Π and the intermediate goods producers Π_q we get:

$$C = w_0(L_g^0 + L_b^0) + w_1(L_g^1 + L_b^1) - M^0 + \Pi_F + \Pi + \Pi_q$$

where this terms can be split into

$$\Pi_F = Y - \int p_i y_i di \tag{3.21}$$

$$\Pi = \int p_i y_i - \psi q_i - w_0 l_i^0 - w_1 l_i^1 - M_i l_i^1 - R k_i - \mathbb{1}_{\{l_i^1 > 0\}} C^{IN} - \mathbb{1}_{\{k_i > 0\}} C_i^A di \tag{3.22}$$

$$\Pi_q = \int \psi q_i - \mu\psi q_i di \tag{3.23}$$

Now, noticing that

$$M^1 = \int M_i l_i^1 di \tag{3.24}$$

And defining the total medical expenditure by

$$M \equiv M^1 + M^0 \tag{3.25}$$

And the aggregate Fixed costs of the economy by

$$FC \equiv \int \mathbb{1}_{\{l_i^1>0\}} C^{IN} + \mathbb{1}_{\{k_i>0\}} C_i^A di \tag{3.26}$$

We get,by subtracting profits from the RHS, that the Resource constraint of the economy can be written as

$$C = Y - FC - M - \Pi - RK - \Pi_q - \psi \int_{N-1}^{N} q_i di \tag{3.27}$$

If we add the minimum threshold $\underline{c}$, this shouldn't change the Resource constraint, because this can be seen as a transfer from the Government to the households and those resources can be paid through a lump sum tax on the firms which does not alter the Resource constraint of the economy.