Health insurance market with automation decisions

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Outline

Introduction

- We are trying to build a General Equilibrium model of the Health insurance market where firms can automate their process, replacing labor by "robots".
- We will follow closely the framework given in [Acemoglu and Restrepo, 2016], adapting their static model to include the health insurance decisions on the firm side and labor decisions with heterogeneous agents on the households' side.
- Furthermore, we will extend their framework, allowing for more flexible automation structures incorporating aspects of [David and Dorn, 2013] framework.

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1. Introduction
2. Model
   - Households
   - Firms
   - Equilibrium
3. Simulation

Households

- Consider a measure one of households indexed by their health status $h \in \{g, b\}$ and their risk aversion parameter $\theta \in, (\underline{\theta}, \bar{\theta})$.
- The proportion of households from type $(g, \cdot)$ is $\lambda_g \in (0, 1)$. We will assume that workers' type has a joint CDF as follows:

$$(\theta, h) \sim F(\theta, h)$$

- Households care about consumption c and are endowed with one unit of labor. Households preferences are of CARA form:

$$u_\theta(c) = -e^{-\theta c}$$

- Depending on their health status h, each household is subject to a medical expenditure shock:

$$\tilde{m} \sim H(\tilde{m}|h)$$

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Households

- Households decide conditional on their type (h, θ), if they will work for a contract with health insurance that pays wage w_1 and fully insures them against the medical expenditure shock $\tilde{m}$ $(\alpha = 1)$ or for a contract without health insurance that pays a wage w_0 $(\alpha = 0)$.
- After the medical expenditure shock is realized, the household chooses the level of consumption c to maximize their utility:

$$\max_{c,l \geq 0, \alpha \in \{0,1\}} \mathbb{E}^{\tilde{m}}[u_{(\theta)}(c)|h] \tag{1}$$

s.t

$$c \leq \alpha w_1 l + (1-\alpha)[w_0 l - \tilde{m}] \tag{2}$$

$$l \leq 1 \tag{3}$$

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Households

- Finally the optimal choice will be given by

$$\alpha^* = \begin{cases} 0 & \text{if } \mathbb{E}^{\tilde{m}|h}[u_{(\theta)}(w_0 l_0^* - \tilde{m})] > u_{(\theta)}(w_1 l_1^*) \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

Then after the medical expenditure shock is realized the household consumes his remaining income, i.e,:

$$c^* = \begin{cases} w_0 l_0^* - \tilde{m} & \text{if } \alpha^* = 0 \\ w_1 l_1^* & \text{otherwise} \end{cases} \tag{5}$$

- Given households do not value leisure (in this version of the model), labor is perfectly inelastic within the choice of contract, i.e,

$$l_0^* = l_1^* = 1$$

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Willingness to pay

- Define the willingness to pay for health insurance P as the price for which, after payment, the household is indifferent between both contracts. This price, if exists, must satisfy then the following equation:

$$\mathbb{E}^{\tilde{m}|h}[u_{(\theta)}(w_0 l_0^* - \tilde{m})] = u_{(\theta)}(w_1 l_1^* - P) \tag{6}$$

- Define also the interior thresholds on the risk aversion parameter θ for which households are indifferent between the two contracts as follows:

$$\bar{\theta}_h \in \{\theta \in (\underline{\theta}, \bar{\theta}) : \mathbb{E}^{\tilde{m}|h}[u_{(\theta)}(w_0 l_0^* - \tilde{m})] = u_{(\theta)}(w_1 l_1^*)\} \tag{7}$$

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Assumptions and Propositions

Proposition 1

$P(h, \theta, w_1, w_0)$ as defined by equation 6 is strictly increasing on θ.

Assumption 1

$$H(\tilde{m}|b)\ FOSD\ H(\tilde{m}|g)$$

Proposition 2

Under Assumption 1

$$P(b, \theta, w_1, w_0) > P(g, \theta, w_1, w_0) \quad \forall \theta \in (\underline{\theta}, \bar{\theta}), w_0, w_1 > 0$$

Assumptions and Propositions

Proposition 3

Under Assumption 1, if $\bar{\theta}_h$ is interior $\forall h$, then

$$\bar{\theta}_g > \bar{\theta}_b$$

Assumption 2

$$F_g(\theta)\ FOSD\ F_b(\theta)$$

Labor Supply

- Given Proposition 1 we know that $\bar{\theta}_h$ is a singleton, so we can define the aggregate labor supply as follows:

$$L_g^1(w_1, w_0) = \lambda_g l_1^*(h_g)(1 - F_g(\bar{\theta}_g)) = \lambda_g(1 - F_g(\bar{\theta}_g))$$

$$L_g^0(w_1, w_0) = \lambda_g F_g(\bar{\theta}_g)$$

$$L_b^1(w_1, w_0) = (1 - \lambda_g)(1 - F_b(\bar{\theta}_b))$$

$$L_b^0(w_1, w_0) = (1 - \lambda_g) F_b(\bar{\theta}_b)$$

Firms

- As in [Acemoglu and Restrepo, 2016], the economy has a unique final good Y which is produced competitively by combining a continuum of tasks y_i with an elasticity of substitution between tasks $\sigma \in (0, \infty)$:

$$Y = \left(\int_{N-1}^{N} y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{8}$$

- Each firm indexed by $i \in [N-1, N]$ produce a task y_i as a monopoly choosing the level of inputs and whether they will provide health insurance or not $x_i \in \{0, 1\}$.
- Inputs for production of each task are a task specific intermediate good q_i, labor l_i and capital k_i.

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Firms

- The task specific intermediate goods q_i are being produced by monopolies, but there is competitive fringe that can sell these intermediate goods at a fixed price ψ.
- There is a fixed supply K of capital that is rent at a price R and capital owners are not part of the economy (absent owners).
- Each monopoly i, who produce task y_i, take wages for contracts without health insurance w_0, with health insurance w_1 and prices ψ and R as given, and face a demand for task y_i given by:

$$y_i = Yp_i^{-\sigma} \tag{9}$$

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Firms

- As households are characterized by their health status h, we will assume that the productivity of labor in task y_i for workers of type $h = g$ is given by γ_i and that the productivity of labor for workers of type $h = b$ is given by $\rho\gamma_i$ where $\rho \in (0, 1)$.
- Firms also differ in their productivity of capital z_i and in the elasticity of substitution between factors, ζ_i.
- Production of task y_i, conditional on choosing contract $x_i = l$ is then given by the following CES production function:

$$y_i^l = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)\left(z_i k_i + \gamma_i l_i \chi_{gi}^l + \rho\gamma_i l_i (1 - \chi_{gi}^l)\right)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \quad (10)$$

- Where χ_{gi}^l is the endogenous proportion of workers of type g who will work for contract l in task y_i, η is a CES aggregator and B is a normalizing constant.

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Firms

- To increase incentives of high productivity firms to provide health insurance in equilibrium we will assume that there exist an increasing function δ_i that sorts healthy workers on each firm i.
- Thus, in equilibrium, the endogenous proportion of healthy workers for task y_i and contract I will be given by:

$$\chi^I_{gi} = \frac{\delta_i L^I_g(w_1, w_0)}{\delta_i L^I_g(w_1, w_0) + L^I_b(w_1, w_0)}$$

s.t

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Firms

Proposition 4 (Advantageous Selection)

$$F_g(\bar{\theta}_g) < F_b(\bar{\theta}_b) \text{ iff } \chi^0_{gi} < \chi^1_{gi}$$

Firms

As capital and labor are substitutes in production, besides the level of intermediates q_i chosen, firms will either

- Hire labor without health insurance ($x_i = 0$), paying wage w_0 , facing an endogenous proportion of workers of type g, χ^0_{gi}.
- Hire labor with health insurance ($x_i = 1$), paying wage w_1, the expected medical expenditure of the workers it hires, M_i, and a fixed administrative cost of health insurance C^{IN}, facing an endogenous proportion of workers of type g, χ^1_{gi}.
- Use capital in production ($x_i = 0$), paying a marginal cost R and a fixed cost that enables automation for task y_i, C^A_i.

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Firms

So the problem of the firm can be split in three conditional problems:

- Conditional on hiring labor without health insurance:

$$\Pi_i^0(w_0) = \max_{l_i, q_i} Y^{1/\sigma}(y_i^0)^{1-1/\sigma} - \psi q_i - w_0 l_i \tag{11}$$

s.t

$$y_i^0 = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)\left(\gamma_i l_i \chi_{gi}^0 + \rho \gamma_i l_i (1 - \chi_{gi}^0)\right)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \tag{12}$$

Firms

- Conditional on hiring labor with health insurance:

$$\Pi_i^1(w_1) = \max_{l_i, q_i} Y^{1/\sigma}(y_i^1)^{1-1/\sigma} - \psi q_i - w_1 l_i - M_i l_i - C^{IN} \qquad (13)$$

s.t

$$y_i^1 = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)\left(\gamma_i l_i \chi_{gi}^1 + \rho\gamma_i l_i(1-\chi_{gi}^1)\right)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \qquad (14)$$

where

$$M_i = \mathbb{E}^{\tilde{m}}[\tilde{m}|h=g]\chi_{gi}^1 + \mathbb{E}^{\tilde{m}}[\tilde{m}|h=b](1-\chi_{gi}^1) \qquad (15)$$

Firms

- Conditional on using capital:

$$\Pi_i^k(R) = \max_{k_i, q_i} Y^{1/\sigma}(y_i^k)^{1-1/\sigma} - \psi q_i - Rk_i - C_i^A \tag{16}$$

s.t

$$y_i^k = B[\eta q_i^{\frac{\zeta_i - 1}{\zeta_i}} + (1-\eta)(z_i k_i)^{\frac{\zeta_i - 1}{\zeta_i}}]^{\frac{\zeta_i}{\zeta_i - 1}} \tag{17}$$

Firms

- We can rewrite the previous problems in terms of effective units of labor and capital and effective cost of labor and capital. Lets define the average labor productivity and effective costs of labor and capital as:

$$\bar{\gamma}_i^l \equiv \gamma_i \left((1-\rho)\chi_{gi}^l + \rho\right) \tag{18}$$

$$\hat{w}_i^0 \equiv \frac{w_0}{\bar{\gamma}_i^0} \tag{19}$$

$$\hat{w}_i^1 \equiv \frac{w_1 + M_i}{\bar{\gamma}_i^1} \tag{20}$$

$$\hat{R}_i \equiv \frac{R}{z_i} \tag{21}$$

Firms

- In the same fashion we can define the effective units of labor and capital as:

$$\hat{l}_i^0 \equiv \bar{\gamma}_i^0 l_i \tag{22}$$

$$\hat{l}_i^1 \equiv \bar{\gamma}_i^1 l_i \tag{23}$$

$$\hat{k}_i \equiv z_i k_i \tag{24}$$

Firms

- The solutions to the previous problems are the conditional factor demands $\hat{l}_i^0$, q_i^0, $\hat{l}_i^1$, q_i^1, $\hat{k}_i$, q_i^k and the conditional profit functions Π_i^0, Π_i^1, Π_i^k.

$$\Pi_i^0(\hat{w}_i^0) = \left[\frac{B[\eta(p_i^0)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{w}_i^0 + \psi(p_i^0)^{\zeta_i}} \right]^{\sigma - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \frac{Y}{\sigma} \quad (25)$$

$$\Pi^1(\hat{w}_i^1) = \left[\frac{B[\eta(p_i^1)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{w}_i^1 + \psi(p_i^1)^{\zeta_i}} \right]^{\sigma - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \frac{Y}{\sigma} - C^{IN} \quad (26)$$

$$\Pi^k(\hat{R}_i) = \left[\frac{B[\eta(p_i^k)^{\zeta_i - 1} + (1-\eta)]^{\frac{\zeta_i}{\zeta_i - 1}}}{\hat{R}_i + \psi(p_i^k)^{\zeta_i}} \right]^{\sigma - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \frac{Y}{\sigma} - C_i^A \quad (27)$$

Firms

- Where

$$p_i^0 \equiv \frac{\eta \hat{w}_i^0}{(1-\eta)\psi} \tag{28}$$

$$p_i^1 \equiv \frac{\eta \hat{w}_i^1}{(1-\eta)\psi} \tag{29}$$

$$p_i^k \equiv \frac{\eta \hat{R}_i}{(1-\eta)\psi} \tag{30}$$

Thresholds

- A natural question that arises is whether there exist thresholds in the range of tasks where firms are indifferent between some pair of the three previous conditional problems. For an interior equilibrium to exist, this must be the case.
- Define $\tilde{X} \in [N-1, N]$ as the indifference point such that firm $i = \tilde{X}$ is indifferent between the contract without health insurance and the contract with health insurance, i.e,

$$\Pi^0_{\tilde{X}}(\hat{w}^0_{\tilde{X}}) = \Pi^1_{\tilde{X}}(\hat{w}^1_{\tilde{X}}) \tag{31}$$

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Possible Equilibrium Configurations

- For the general case the thresholds will depend on the assumptions we place on z_i, C_i^A and ζ_i. Take $\zeta_i = \zeta$ for the moment and z_i to be a non increasing function of i.
- Now, depending on how the automation cost function $C^A(i)$ behaves, we could have different equilibrium configurations.
- If tasks with a higher index i, i.e, with higher labor productivity, are more costly to automate, then $C^A(i)$ can be modeled as an increasing function of i.

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- Now, depending on how the automation cost function $C^A(i)$ behaves, we could have different equilibrium configurations.
- If tasks with a higher index i, i.e, with higher labor productivity, are more costly to automate, then $C^A(i)$ can be modeled as an increasing function of i.

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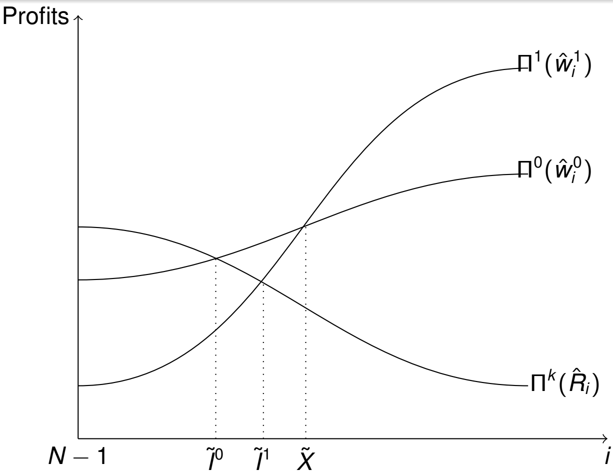

Figure 1: Conditional indirect profit functions, increasing automation cost with $\tilde{I}^1 < \tilde{X}$

Possible Equilibrium Configurations

- Firms will produce following the upper envelope of the conditional indirect profit functions, so the corresponding equilibrium configuration would be:

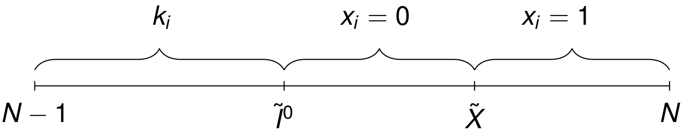

Figure 2: Equilibrium configuration for $\tilde{l}^1 < \tilde{X}$

- Is easy to see that capital must be used in equilibrium, otherwise we won't have market clearing. However, is not clear if labor with and without health insurance will be provided in equilibrium. If the the automation cost C_i^A is low enough, we could also have that:

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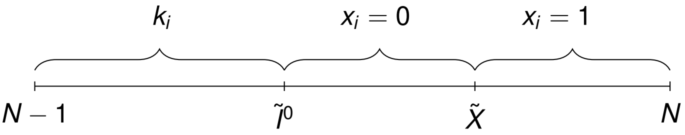

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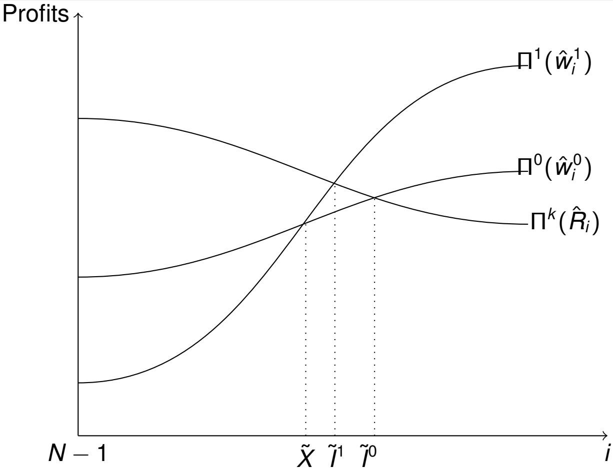

Figure 3: Conditional indirect profit functions, increasing automation cost with $\tilde{I}^1 \geq \tilde{X}$

Possible Equilibrium Configurations

- The relationship between productivity of labor and its cost of automation could be non monotonic. This is the case Autor and Dorn pointed out in [David and Dorn, 2013], where service occupation is associated with a low productivity of labor but those tasks are hard to codify.
- In this case, $C^A(i)$ can be modeled as a quadratic function of i. Then the indirect profit functions, evaluated at w_0, w_1 and R, could look like:

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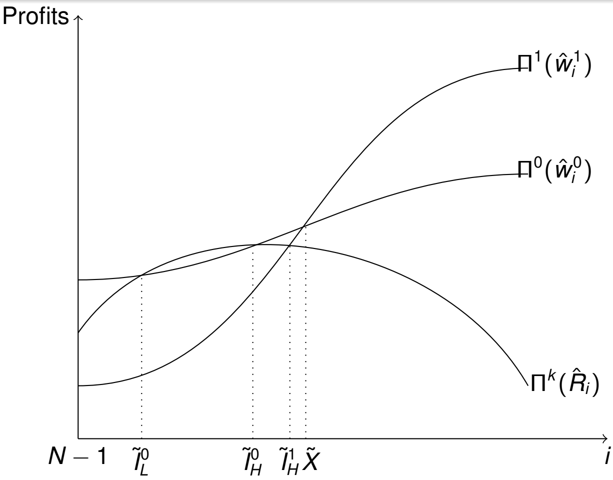

Figure 4: Conditional indirect profit functions, U-shaped automation cost

Possible Equilibrium Configurations

- Notice that with a U-shaped automation cost function we will have up to 5 thresholds, because the indirect profit function for capital $\Pi^k(\hat{R}_i)$ can cross the two other indirect profit functions in at most 2 point each.
- This will give rise to different equilibrium configurations. The one corresponding to the previous figure would be:

Figure 5: Equilibrium configuration for U-shaped C_i^A

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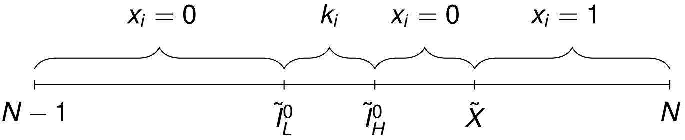

Figure 5: Equilibrium configuration for U-shaped C_i^A

Equilibrium in the Static Model I

An equilibrium for this economy is a set of prices $w_0^*, w_1^*, R^*, [p_i^*]_i$, *allocations* $\{c^*(h,\theta), l^*(h,\theta), \alpha^*(h,\theta)\}_{\theta,h}$, *endogenous thresholds* $\{\bar{\theta}_h^*\}_h$, *production plan* $[y_i^*, l_i^*, q_i^*, k_i^*, x_i^*]_i$, *endogenous production thresholds* $\tilde{I}^0$, $\tilde{I}^1$, $\tilde{X}$, *endogenous proportions* $\{\chi_g^{l*}\}_l$ *and conditional indirect profit functions* Π_i^0,Π_i^1 *and* Π_i^k *s.t*

1. $c^*(h,\theta), l^*(h,\theta), \alpha^*(h,\theta)$ solve the households problem.
2. $\{\bar{\theta}_h^*\}_h$ are defined by equation 7
3. $[y_i^*, l_i^*, q_i^*, k_i^*, x_i^*]_i$ are defined by

$$(y_i^*, l_i^*, q_i^*, k_i^*, x_i^*) = \begin{cases} (y_i^k, 0, q_i^k, k_i, 0) & \text{if } i \in \left[N-1, \min\{\tilde{I}^1, \tilde{I}^0\}\right] \\ (y_i^0, l_i^0, q_i^0, 0, 0) & \text{if } i \in \left[\tilde{I}^0, \tilde{X}\right] \text{ and } \tilde{I}^0 < \tilde{I}^1 \\ (y_i^1, l_i^1, q_i^1, 0, 1) & \text{if } i \in \left[\max\{\tilde{I}^1, \tilde{X}\}, N\right] \end{cases}$$

Equilibrium in the Static Model II

where $[y_i^0, y_i^1, y_i^k, l_i^0, l_i^1, q_i^0, q_i^1, q_i^k, k_i]_i$ are solutions to the conditional problems of firm i and production thresholds $\tilde{X}$, $\tilde{l}^0$ and $\tilde{l}^1$ are given by equations 31, 32 and 26.

4. $\{\chi_{gi}^{l*}\}_l$ are given by:

$$\chi_{gi}^{1*} = \frac{\delta_i \lambda_g (1 - F_g(\bar{\theta}_g^*))}{\delta_i \lambda_g (1 - F_g(\bar{\theta}_g^*)) + (1 - \lambda_g)(1 - F_b(\bar{\theta}_b^*))}$$

$$\chi_{gi}^{0*} = \frac{\delta_i \lambda_g F_g(\bar{\theta}_g^*)}{\delta_i \lambda_g F_g(\bar{\theta}_g^*) + (1 - \lambda_g) F_b(\bar{\theta}_b^*)}$$

Equilibrium in the Static Model III

5. $[p_i^*]_i$ are given by

$$p_i^* = \left(\frac{Y^*}{y_i^*}\right)^{1/\sigma}$$

where

$$Y^* = \left(\int_{N-1}^{N} y_i^{*\,\frac{\sigma-1}{\sigma}}\, di\right)^{\frac{\sigma}{\sigma-1}}$$

6. Market Clearing:

$$\int_{N-1}^{\min\{\tilde{I}^1, \tilde{I}^0\}} k_i^* di = K$$

If $\tilde{I}^0 < \tilde{I}^1$, there will be labor without health insurance in equilibrium, then

$$\int_{\tilde{I}^0}^{\tilde{X}} l_i^* di = \lambda_g F_g(\bar{\theta}_g^*) + (1 - \lambda_g) F_b(\bar{\theta}_b^*)$$

Equilibrium in the Static Model IV

$$\int_{\max\{\tilde{I}^1, \tilde{X}\}}^{N} I_i^* \, di = \lambda_g (1 - F_g(\bar{\theta}_g^*)) + (1 - \lambda_g)(1 - F_b(\bar{\theta}_b^*))$$

7. Resource Constraint (redundant to compute the equilibrium, not sure if we need to include it)

$$C = Y - FC - M - \Pi - RK - \Pi_q - \psi \int_{N-1}^{N} q_i^* \, di$$

8. Net profits Π are given by (here I'm not considering the net profits of intermediate goods)

$$\Pi = \int_{N-1}^{N} \max\{\Pi_i^0, \Pi_i^1, \Pi_i^k\} di$$

9. Conditional indirect profit functions Π_i^0,Π_i^1 and Π_i^k are given by equations 25, 26 and 27.

Outline

1. Introduction
2. Model
3. Simulation

Functions and Parameters

Table 1: Parametrized Functions

Function	Description	Parametrization
γ_i	Labor productivity	$A_0 e^{Ai}$
δ_i	Increasing sorting of workers	$\left(\frac{e^{\lambda i-\alpha}}{1+e^{\lambda i-\alpha}}\right)/M$
z_i	Capital productivity	1
ζ_i	Elasticity of substitution	2
C_i^A	Automation cost	$D_0 e^{Di}$
A		1
A_0		1
λ_d		10
α_d		5
M		$\int_{N-1}^{N}\left(\frac{e^{\lambda i-\alpha}}{1+e^{\lambda i-\alpha}}\right)di$
D_0		0.1
D		1

Functions and Parameters

Table 2: Household's parameter values

Parameters	Description	Values
λ_g	Measure of healthy households	0.5
$F(g, \theta)$	Conditional dist of risk aversion	$Gamma(2, 1)$
$F(b, \theta)$	Conditional dist of risk aversion	$Gamma(0.2, 1)$
$P_0(g)$	Prob of 0 medical exp	0.5
$P_0(b)$	Prob of 0 medical exp	0.3
$H(\tilde{m}\|g)$	Conditional dist of positive medical exp	$Exp(1)$
$H(\tilde{m}\|b)$	Conditional dist of positive medical exp	$Exp(0.25)$

Functions and Parameters

Table 3: Firm's parameter values

Parameters	Description	Values
N	Upper limit for range of tasks	1
η	Distribution parameter of the CES	0.5
ρ	Relative labor productivity of unhealthy workers	0.8
ψ	Price of intermediates	1
σ	Elasticity of substitution between tasks	2
$\zeta_i = \zeta$	Elasticity of substitution between factors	2
C^{IN}	Health insurance fixed cost	0.5
K	Capital stock	1

Functions and Parameters

Table 4: Firm's parameter values

Parameters	Description	Values
N	Upper limit for range of tasks	1
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$\zeta_i = \zeta$	Elasticity of substitution between factors	2
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Figure 6: Conditional profits

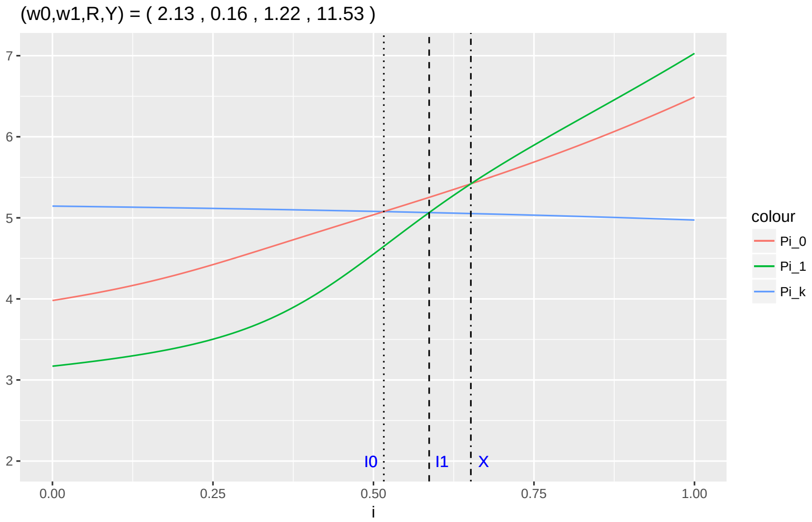


Figure 7: Effective prices

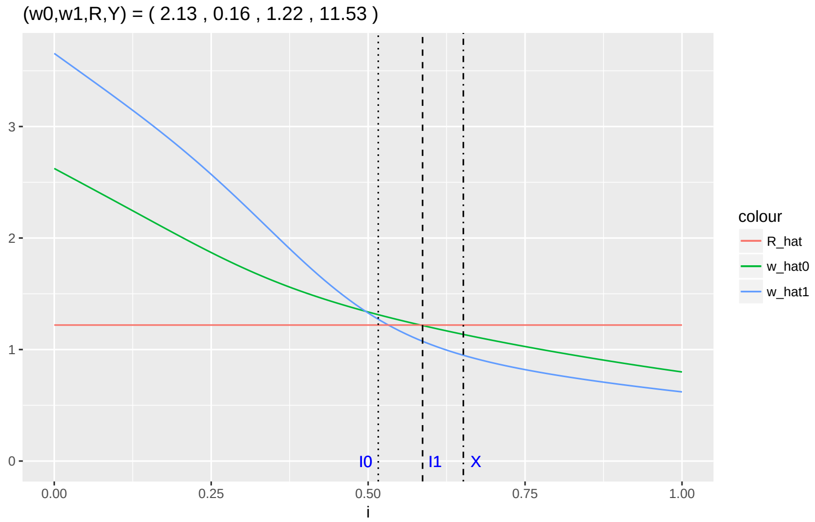


Figure 8: Endogenous proportion

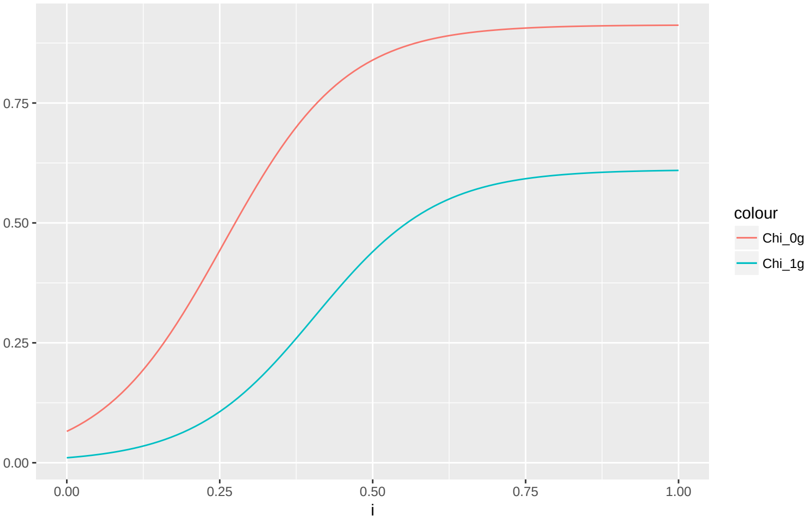


Figure 9: Average labor productivity

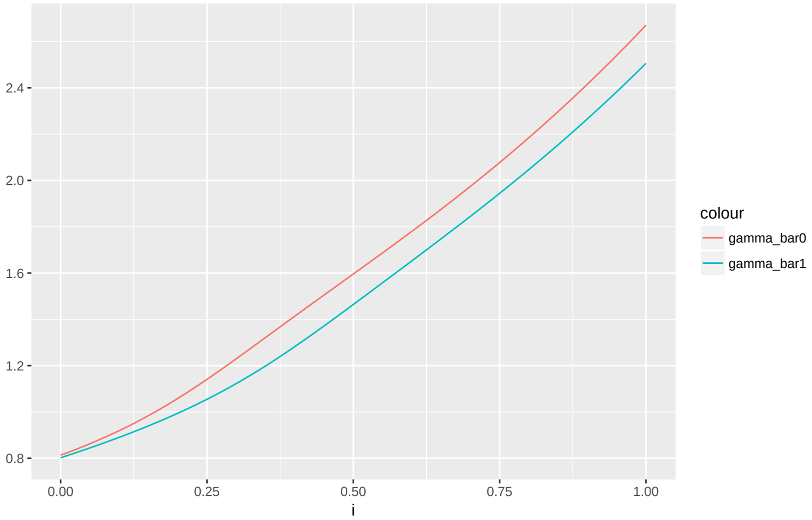
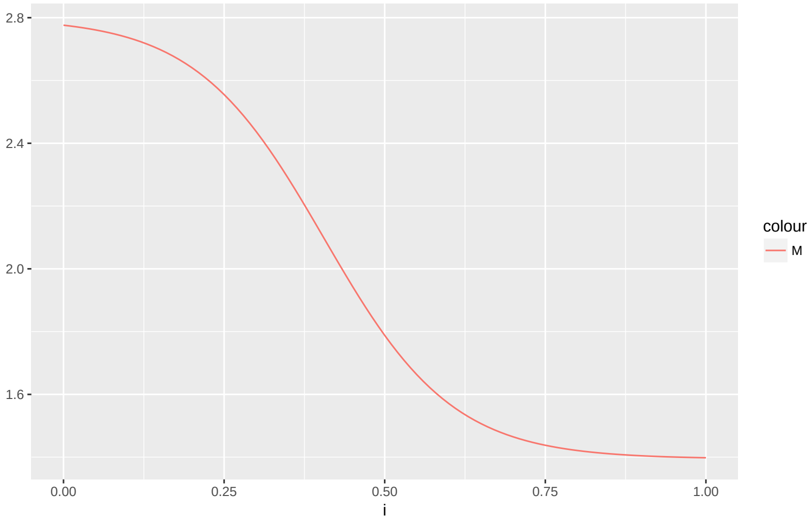


Figure 10: Expected Medical Expenditure



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