Tsinghua-Berkeley Shenzhen Institute
Information Theory and Statistical Learning
Fall 2020

Homework 1

YOUR NAME September 14, 2020

- **Acknowledgments:** This template takes some materials from course CSE 547/Stat 548 of Washington University: `https://courses.cs.washington.edu/courses/cse547/17sp/index.html`.

  If you refer to other materials in your homework, please list here.

- **Collaborators:** I finish this template by myself. If you finish your homework all by yourself, make a similar statement. If you get help from others in finishing your homework, state like this:
  - 1.2 (b) was solved with the help from ______.
  - Discussion with ______ helped me finishing 1.3.

- *I certify that all solutions are entirely in my words and that I have not looked at another student's solutions. I have credited all external sources in this write up.*

Your signature

You may use `enumerate` to generate answers for each question:

1.1. You may find `https://en.wikibooks.org/wiki/LaTeX` useful.

   (a) Writing LaTeX online may be easier for beginners. You may find Overleaf (`https://www.overleaf.com/`) useful.

1.2. You may need aligned equations for your homework, here are several examples:

   Total probability rule:

$$\begin{aligned}\mathbb{P}(X=x) &= \sum_{y\in\mathcal{Y}} \mathbb{P}(X=x, Y=y)\\ &= \sum_{y\in\mathcal{Y}} \mathbb{P}(X=x|Y=y)\,\mathbb{P}(Y=y),\end{aligned}$$

   or

$$\begin{aligned}P_X(x) &= \sum_{y\in\mathcal{Y}} P_{XY}(x,y)\\ &= \sum_{y\in\mathcal{Y}} P_{X|Y}(x|y)P_Y(y).\end{aligned}$$

   Indicator function:

$$\mathbb{1}_A(\omega) = \begin{cases}1, & \text{if } \omega \in A,\\ 0, & \text{if } \omega \notin A.\end{cases}$$

1.3. You may need to add figures and source codes in your homework. Figure 1 is an example that compares the empirical distribution (histogram) and probability density function of a Gaussian random variable.

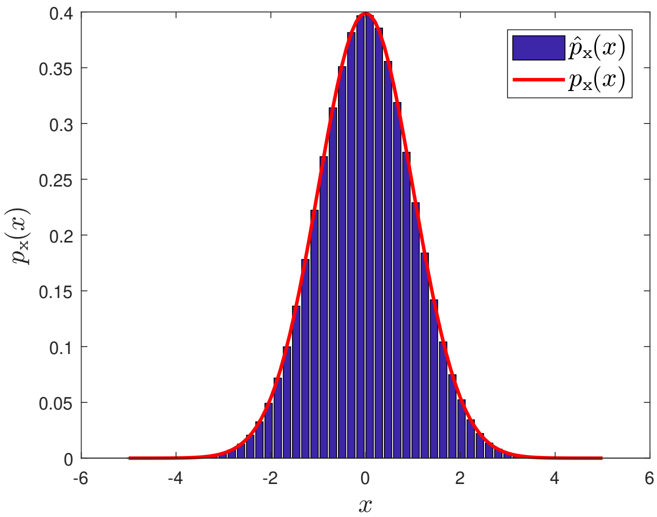

Figure 1: Gaussian PDF and histogram of samples

The source code to plot Figure 1 could be found in Appendix A. Here are the core codes:

```
4 [cnt, x_hist] = hist(data, nbins); % not to plot, only to get
      emperical distribution.
```

```
6 cnt = cnt / n / (x_hist(2) - x_hist(1)); % normalization, be
      careful :)
7 bar(x_hist, cnt); % plot the hist using bar()
```

To understand line 6, note that if we have n samples of X denoted by $x^{(i)}, i = 1, 2, \cdots, n$, then the probability density function p_X can be estimated as

$$\begin{aligned} p_X(x_0) &= \frac{\mathrm{d}}{\mathrm{d}x} \mathbb{P}(X \leq x) \bigg|_{x=x_0} \\ &\approx \frac{\mathbb{P}(x_0 - \Delta x < X \leq x_0)}{\Delta x} \\ &\approx \frac{1}{n\Delta x} \sum_{i=1}^{n} \mathbb{1}_{x^{(i)} \in (x_0 - \Delta x, x_0]} . \end{aligned}$$

A Source code

The source code for plotting Figure 1 is shown as follows.

```
n = 1e6;  % n samples
data = randn(1e6, 1); % Generate n Random Gaussian samples.
nbins = 50; % bins in your histogram
[cnt, x_hist] = hist(data, nbins); % not to plot, only to get emperical distribution.
figure;
cnt = cnt / n / (x_hist(2) - x_hist(1)); % normalization, be careful :)
bar(x_hist, cnt); % plot the hist using bar()
hold on;
x = -5 : 0.1 : 5;
plot(x, normpdf(x), 'r', 'linewidth', 2);
legend({'$\hat{p}_{\sf{x}}(x)$', '$p_{\sf{x}}(x)$'}, 'Interpreter', 'LaTeX', 'fontsize', 15);
xlabel('$x$', 'Interpreter', 'LaTeX', 'fontsize', 15); % You may change the size accordingly
ylabel('$p_{\sf{x}}(x)$', 'Interpreter', 'LaTeX', 'fontsize', 15);
```