

Lecture 8: Integrating Learning and Planning

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Outline

- 1 Introduction
- 2 Model-Based Reinforcement Learning
- 3 Integrated Architectures
- 4 Simulation-Based Search

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Model-Based Reinforcement Learning

- *Last lecture*: learn **policy** directly from experience
- *Previous lectures*: learn **value function** directly from experience
- *This lecture*: learn **model** directly from experience
- and use **planning** to construct a value function or policy
- Integrate learning and planning into a single architecture

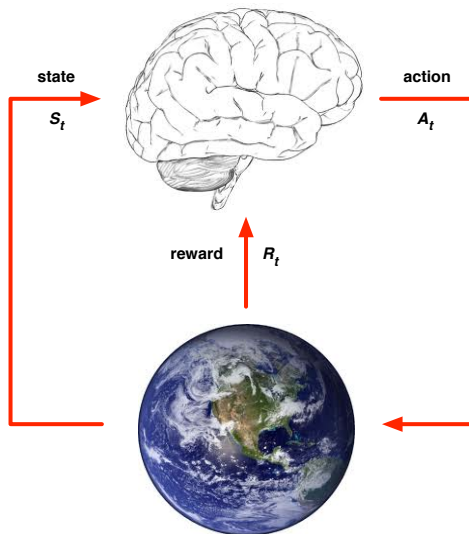
Model-Based and Model-Free RL

- Model-Free RL
 - No model
 - **Learn** value function (and/or policy) from experience

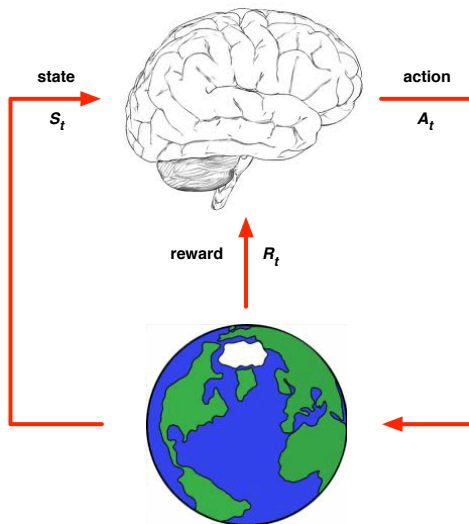
Model-Based and Model-Free RL

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- Model-Based RL
 - Learn a model from experience
 - **Plan** value function (and/or policy) from model

Model-Free RL



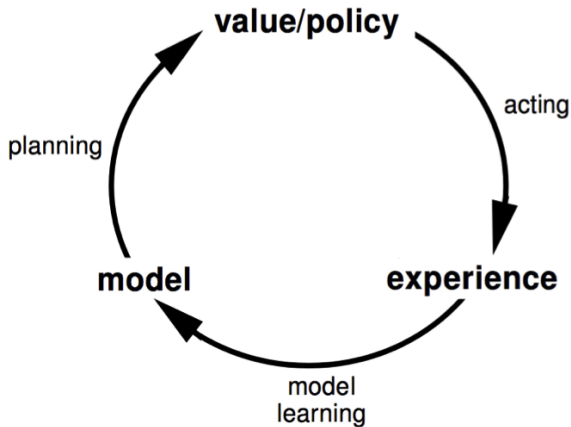
Model-Based RL



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Model-Based RL



Advantages of Model-Based RL

Advantages:

- Can efficiently learn model by supervised learning methods
- Can reason about model uncertainty

Disadvantages:

- First learn a model, then construct a value function
⇒ two sources of approximation error

What is a Model?

- A *model* \mathcal{M} is a representation of an MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$, parametrized by η
- We will assume state space \mathcal{S} and action space \mathcal{A} are known
- So a model $\mathcal{M} = \langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$ represents state transitions $\mathcal{P}_\eta \approx \mathcal{P}$ and rewards $\mathcal{R}_\eta \approx \mathcal{R}$

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$

$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

- Typically assume conditional independence between state transitions and rewards

$$\mathbb{P}[S_{t+1}, R_{t+1} \mid S_t, A_t] = \mathbb{P}[S_{t+1} \mid S_t, A_t] \mathbb{P}[R_{t+1} \mid S_t, A_t]$$

Model Learning

- Goal: estimate model \mathcal{M}_η from experience $\{S_1, A_1, R_2, \dots, S_T\}$
- This is a supervised learning problem

$$S_1, A_1 \rightarrow R_2, S_2$$

$$S_2, A_2 \rightarrow R_3, S_3$$

$$\vdots$$

$$S_{T-1}, A_{T-1} \rightarrow R_T, S_T$$

- Learning $s, a \rightarrow r$ is a *regression* problem
- Learning $s, a \rightarrow s'$ is a *density estimation* problem
- Pick loss function, e.g. mean-squared error, KL divergence, ...
- Find parameters η that minimise empirical loss

Examples of Models

- Table Lookup Model
- Linear Expectation Model
- Linear Gaussian Model
- Gaussian Process Model
- Deep Belief Network Model
- ...

Table Lookup Model

- Model is an explicit MDP, $\hat{\mathcal{P}}, \hat{\mathcal{R}}$
- Count visits $N(s, a)$ to each state action pair

$$\hat{\mathcal{P}}_{s,s'}^a = \frac{1}{N(s, a)} \sum_{t=1}^T \mathbf{1}(S_t, A_t, S_{t+1} = s, a, s')$$

$$\hat{\mathcal{R}}_s^a = \frac{1}{N(s, a)} \sum_{t=1}^T \mathbf{1}(S_t, A_t = s, a) R_t$$

- Alternatively
 - At each time-step t , record experience tuple $\langle S_t, A_t, R_{t+1}, S_{t+1} \rangle$
 - To sample model, randomly pick tuple matching $\langle s, a, \cdot, \cdot \rangle$

AB Example

Two states A , B ; no discounting; 8 episodes of experience

$A, 0, B, 0$

$B, 1$

$B, 1$

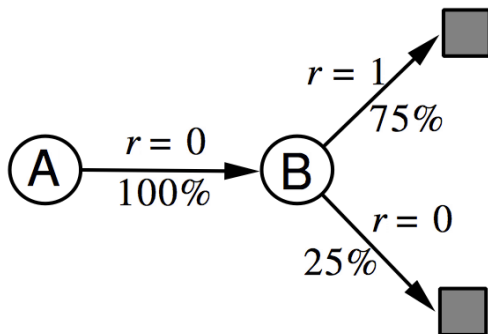
$B, 1$

$B, 1$

$B, 1$

$B, 1$

$B, 0$



We have constructed a **table lookup model** from the experience

Planning with a Model

- Given a model $\mathcal{M}_\eta = \langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$
- Solve the MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$
- Using favourite planning algorithm
 - Value iteration
 - Policy iteration
 - Tree search
 - ...

Sample-Based Planning

- A simple but powerful approach to planning
- Use the model **only** to generate samples
- **Sample** experience from model

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$

$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

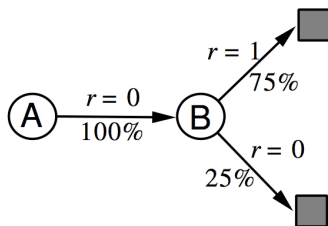
- Apply **model-free** RL to samples, e.g.:
 - Monte-Carlo control
 - Sarsa
 - Q-learning
- Sample-based planning methods are often more efficient

Back to the AB Example

- Construct a table-lookup model from real experience
- Apply model-free RL to sampled experience

Real experience

A, 0, B, 0
 B, 1
 B, 1
 B, 1
 B, 1
 B, 1
 B, 1
 B, 1
 B, 0



Sampled experience

B, 1
 B, 0
 B, 1
 A, 0, B, 1
 B, 1
 A, 0, B, 1
 B, 1
 B, 0

e.g. Monte-Carlo learning: $V(A) = 1, V(B) = 0.75$

Planning with an Inaccurate Model

- Given an imperfect model $\langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle \neq \langle \mathcal{P}, \mathcal{R} \rangle$
- Performance of model-based RL is limited to optimal policy for approximate MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$
- i.e. Model-based RL is only as good as the estimated model
- When the model is inaccurate, planning process will compute a suboptimal policy
- Solution 1: when model is wrong, use model-free RL
- Solution 2: reason explicitly about model uncertainty

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Real and Simulated Experience

We consider two sources of experience

Real experience Sampled from environment (true MDP)

$$S' \sim \mathcal{P}_{s,s'}^a$$

$$R = \mathcal{R}_s^a$$

Simulated experience Sampled from model (approximate MDP)

$$S' \sim \mathcal{P}_\eta(S' \mid S, A)$$

$$R = \mathcal{R}_\eta(R \mid S, A)$$

Integrating Learning and Planning

- Model-Free RL
 - No model
 - **Learn** value function (and/or policy) from real experience

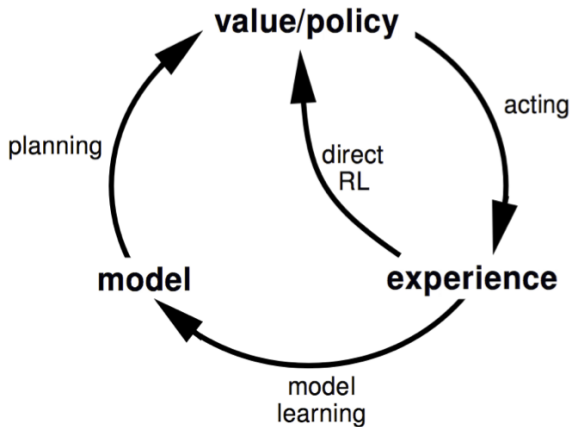
Integrating Learning and Planning

- Model-Free RL
 - No model
 - **Learn** value function (and/or policy) from real experience
- Model-Based RL (using Sample-Based Planning)
 - Learn a model from real experience
 - **Plan** value function (and/or policy) from simulated experience

Integrating Learning and Planning

- Model-Free RL
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 - **Learn** value function (and/or policy) from real experience
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- Dyna
 - Learn a model from real experience
 - **Learn and plan** value function (and/or policy) from real and simulated experience

Dyna Architecture



Dyna-Q Algorithm

Initialize $Q(s, a)$ and $Model(s, a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$

Do forever:

(a) $S \leftarrow$ current (nonterminal) state

(b) $A \leftarrow \varepsilon$ -greedy(S, Q)

(c) Execute action A ; observe resultant reward, R , and state, S'

(d) $Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$

(e) $Model(S, A) \leftarrow R, S'$ (assuming deterministic environment)

(f) Repeat n times:

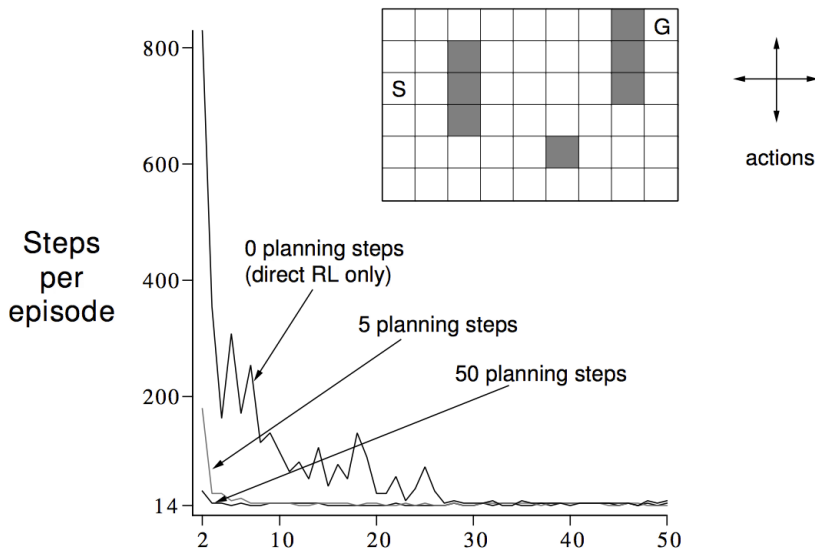
$S \leftarrow$ random previously observed state

$A \leftarrow$ random action previously taken in S

$R, S' \leftarrow Model(S, A)$

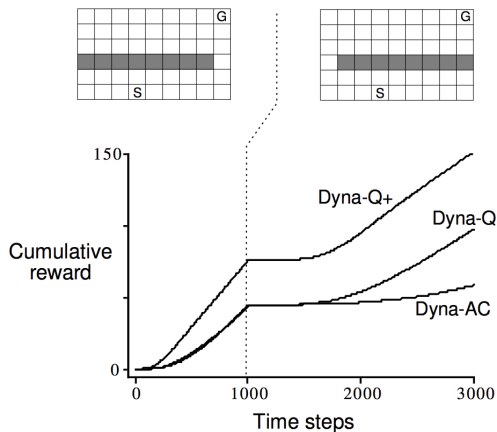
$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$

Dyna-Q on a Simple Maze



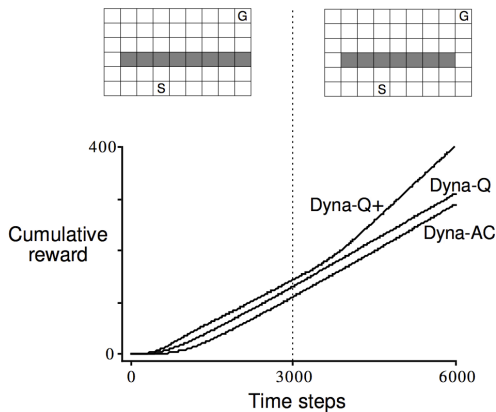
Dyna-Q with an Inaccurate Model

- The changed environment is **harder**



Dyna-Q with an Inaccurate Model (2)

- The changed environment is **easier**

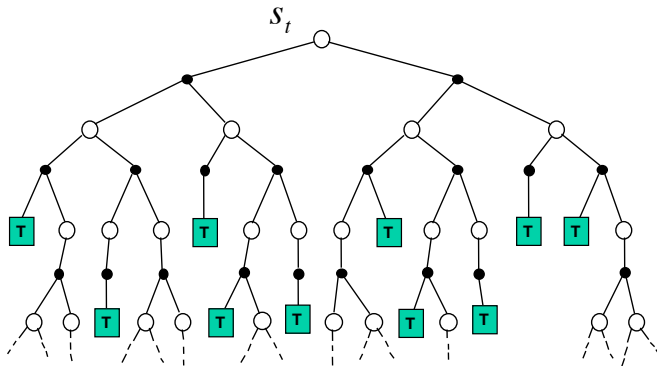


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Forward Search

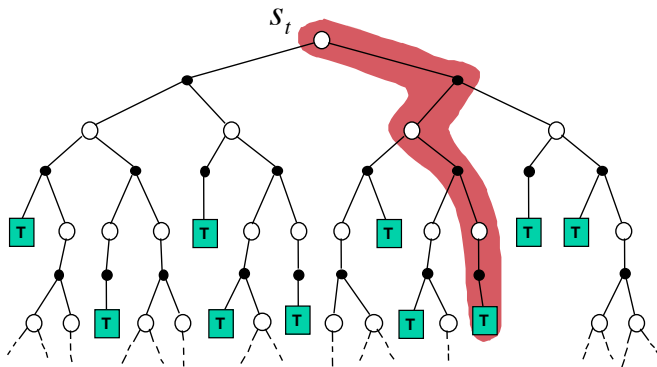
- **Forward search** algorithms select the best action by **lookahead**
- They build a **search tree** with the current state s_t at the root
- Using a **model** of the MDP to look ahead



- No need to solve whole MDP, just sub-MDP starting from **now**

Simulation-Based Search

- **Forward search** paradigm using sample-based planning
- **Simulate** episodes of experience from **now** with the model
- Apply **model-free** RL to simulated episodes



Simulation-Based Search (2)

- **Simulate** episodes of experience from **now** with the model

$$\{s_t^k, A_t^k, R_{t+1}^k, \dots, S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu$$

- Apply **model-free** RL to simulated episodes
 - Monte-Carlo control \rightarrow Monte-Carlo search
 - Sarsa \rightarrow TD search

Simple Monte-Carlo Search

- Given a model \mathcal{M}_ν and a **simulation policy** π
- For each action $a \in \mathcal{A}$
 - Simulate K episodes from current (real) state s_t

$$\{\mathbf{s}_t, \mathbf{a}, R_{t+1}^k, S_{t+1}^k, A_{t+1}^k, \dots, S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu, \pi$$

- Evaluate actions by mean return (**Monte-Carlo evaluation**)

$$Q(\mathbf{s}_t, \mathbf{a}) = \frac{1}{K} \sum_{k=1}^K G_t \xrightarrow{P} q_\pi(s_t, a)$$

- Select current (real) action with maximum value

$$a_t = \operatorname{argmax}_{a \in \mathcal{A}} Q(s_t, a)$$

Monte-Carlo Tree Search (Evaluation)

- Given a model \mathcal{M}_ν
- Simulate K episodes from current state s_t using current simulation policy π

$$\{\textcolor{red}{s}_t, A_t^k, R_{t+1}^k, S_{t+1}^k, \dots, S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu, \pi$$

- Build a search tree containing visited states and actions
- **Evaluate** states $Q(s, a)$ by mean return of episodes from s, a

$$Q(\textcolor{red}{s}, \textcolor{red}{a}) = \frac{1}{N(s, a)} \sum_{k=1}^K \sum_{u=t}^T \mathbf{1}(S_u, A_u = s, a) G_u \xrightarrow{P} q_\pi(s, a)$$

- After search is finished, select current (real) action with maximum value in search tree

$$a_t = \operatorname{argmax}_{a \in \mathcal{A}} Q(s_t, a)$$

Monte-Carlo Tree Search (Simulation)

- In MCTS, the simulation policy π **improves**
- Each simulation consists of two phases (in-tree, out-of-tree)
 - **Tree policy** (improves): pick actions to maximise $Q(S, A)$
 - **Default policy** (fixed): pick actions randomly
- Repeat (each simulation)
 - **Evaluate** states $Q(S, A)$ by Monte-Carlo evaluation
 - **Improve** tree policy, e.g. by ϵ – greedy(Q)
- **Monte-Carlo control** applied to **simulated experience**
- Converges on the optimal search tree, $Q(S, A) \rightarrow q_*(S, A)$

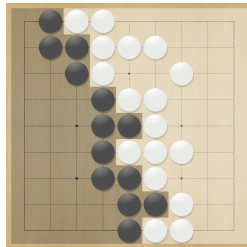
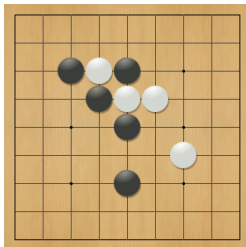
Case Study: the Game of Go

- The ancient oriental game of Go is 2500 years old
- Considered to be the hardest classic board game
- Considered a grand challenge task for AI (*John McCarthy*)
- Traditional game-tree search has failed in Go



Rules of Go

- Usually played on 19x19, also 13x13 or 9x9 board
- Simple rules, complex strategy
- Black and white place down stones alternately
- Surrounded stones are captured and removed
- The player with more territory wins the game



Position Evaluation in Go

- How good is a position s ?
- Reward function (undiscounted):

$$R_t = 0 \text{ for all non-terminal steps } t < T$$

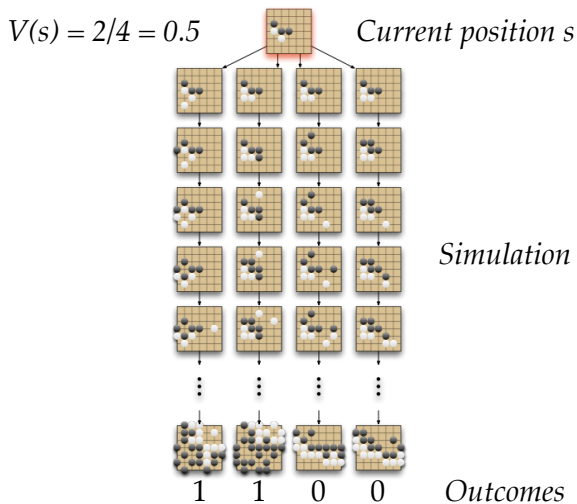
$$R_T = \begin{cases} 1 & \text{if Black wins} \\ 0 & \text{if White wins} \end{cases}$$

- Policy $\pi = \langle \pi_B, \pi_W \rangle$ selects moves for both players
- Value function (how good is position s):

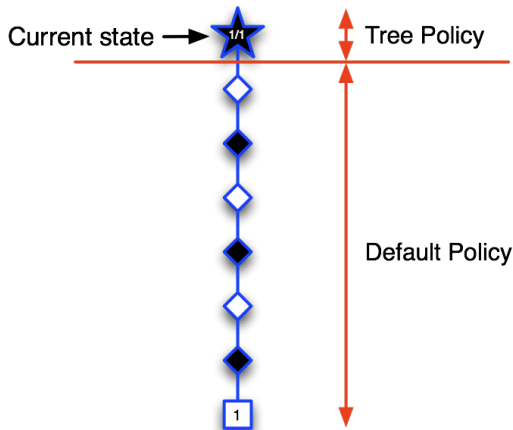
$$v_\pi(s) = \mathbb{E}_\pi [R_T \mid S = s] = \mathbb{P} [\text{Black wins} \mid S = s]$$

$$v_*(s) = \max_{\pi_B} \min_{\pi_W} v_\pi(s)$$

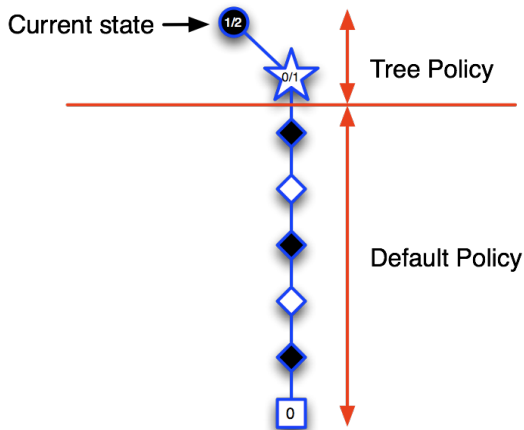
Monte-Carlo Evaluation in Go



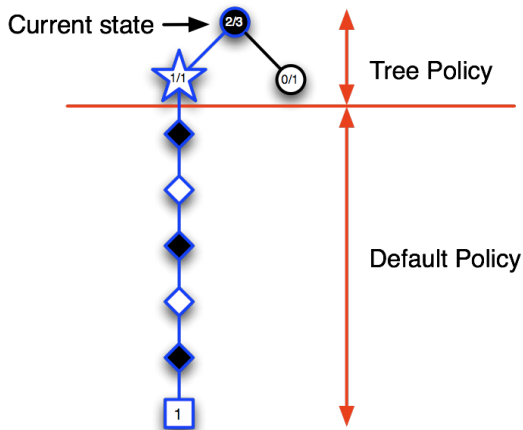
Applying Monte-Carlo Tree Search (1)



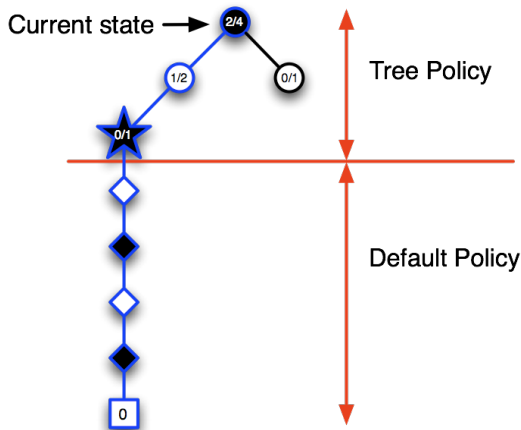
Applying Monte-Carlo Tree Search (2)



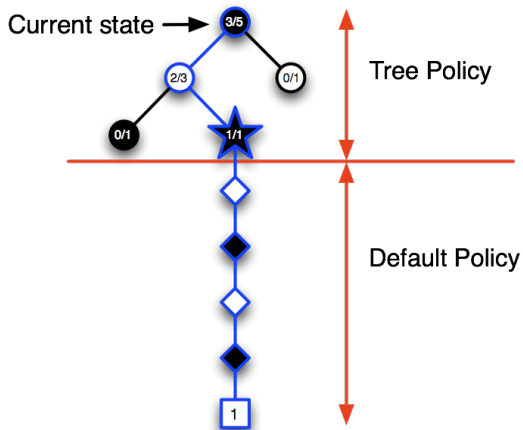
Applying Monte-Carlo Tree Search (3)



Applying Monte-Carlo Tree Search (4)



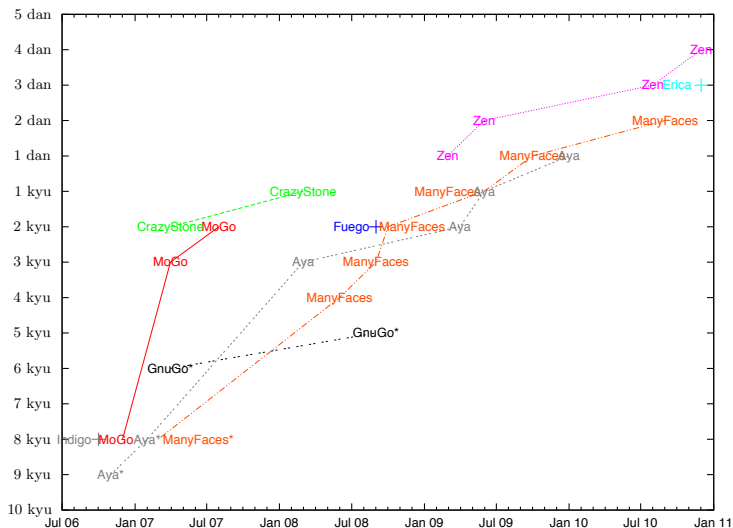
Applying Monte-Carlo Tree Search (5)



Advantages of MC Tree Search

- Highly selective best-first search
- Evaluates states *dynamically* (unlike e.g. DP)
- Uses sampling to break curse of dimensionality
- Works for “black-box” models (only requires samples)
- Computationally efficient, anytime, parallelisable

Example: MC Tree Search in Computer Go



Temporal-Difference Search

- Simulation-based search
- Using TD instead of MC (bootstrapping)
- MC tree search applies MC control to sub-MDP from now
- TD search applies Sarsa to sub-MDP from now

MC vs. TD search

- For model-free reinforcement learning, bootstrapping is helpful
 - TD learning reduces variance but increases bias
 - TD learning is usually more efficient than MC
 - $TD(\lambda)$ can be much more efficient than MC
- For simulation-based search, bootstrapping is also helpful
 - TD search reduces variance but increases bias
 - TD search is usually more efficient than MC search
 - $TD(\lambda)$ search can be much more efficient than MC search

TD Search

- Simulate episodes from the current (real) state s_t
- Estimate action-value function $Q(s, a)$
- For each step of simulation, update action-values by Sarsa

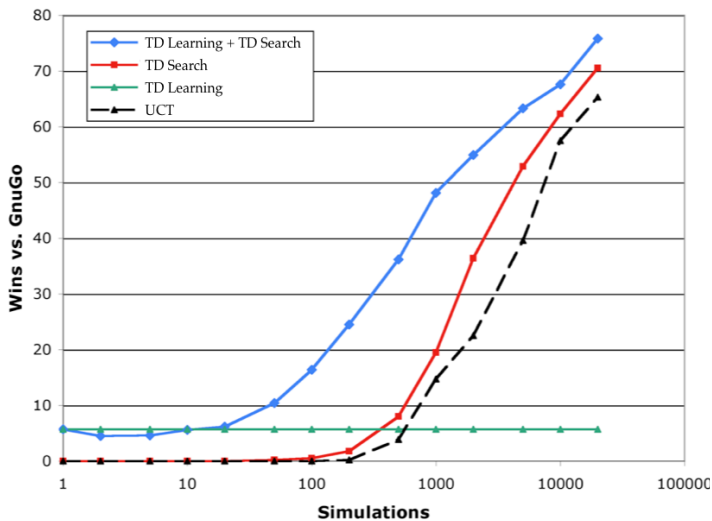
$$\Delta Q(S, A) = \alpha(R + \gamma Q(S', A') - Q(S, A))$$

- Select actions based on action-values $Q(s, a)$
 - e.g. ϵ -greedy
- May also use function approximation for Q

Dyna-2

- In Dyna-2, the agent stores two sets of feature weights
 - Long-term memory
 - Short-term (working) memory
- Long-term memory is updated from real experience using TD learning
 - General domain knowledge that applies to any episode
- Short-term memory is updated from simulated experience using TD search
 - Specific local knowledge about the current situation
- Over value function is sum of long and short-term memories

Results of TD search in Go



Questions?