COMP0124 Multi-agent Artificial Intelligence

Bayesian Games

- Games with Incomplete Information

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Content

- Lecture 1: Multiagent AI and basic game theory
- Lecture 2: Potential games, and extensive form and repeated games
- Lecture 3: Solving ("Learning") Nash Equilibria
- Lecture 4: Bayesian Games, auction theory and mechanism design
- Lecture 5: Learning and deep neural networks
- Lecture 6: Single-agent Learning (1)
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Subject to change

Incomplete Information

- In many game theoretic situations, one agent is unsure about the payoffs or preferences of others.
- Incomplete information introduces additional strategic interactions and also raises questions related to “learning”.
- Examples:
  - Bargaining (how much the other party is willing to pay is generally unknown to you)
  - Auctions (how much should you bid for an object that you want, knowing that others will also compete against you?)
  - Market competition (firms generally do not know the exact cost of their competitors)
  - Signaling games (how should you infer the information of others from the signals they send)
  - Social learning (how can you leverage the decisions of others in order to make better decisions)

Bayesian Game Example

- The battle of the sexes game, which was a complete information “coordination” game.
  Both parties want to meet, but they have different preferences on “Ballet” and “Football”.

	B	F
B	$(2, 1)$	$(0, 0)$
F	$(0, 0)$	$(1, 2)$

- In this game there are two pure strategy equilibria (one of them better for player 1 and the other one better for player 2), and a mixed strategy equilibrium.
- Now imagine that player 1 does not know whether player 2 wishes to meet or wishes to avoid player 1. Therefore, this is a situation of incomplete information—also sometimes called asymmetric information.

Bayesian Game Example

- Suppose player 2 having two different types, one type that wishes to meet player 1 and the other wishes to avoid him.
  - e.g, suppose that these two types have probability 1/2 each. Then the game takes the form one of the following two with probability 1/2.

	B	F
B	$(2, 1)$	$(0, 0)$
F	$(0, 0)$	$(1, 2)$

	B	F
B	$(2, 0)$	$(0, 2)$
F	$(0, 1)$	$(1, 0)$

- Note that player 2 knows which game it is (she knows the state of the world), but player 1 does not.
- What are strategies in this game?

Bayesian Game Example

- From player 1's point of view, player 2 has two possible types
  - or equivalently, the world has two possible states each with 1/2 probability and only player 2 knows the actual state.
- How do we reason about equilibria here?
  - Idea: Use Nash Equilibrium concept in an expanded game, where each different type of player 2 has a different strategy
  - or equivalently, form beliefs about other player's actions in each state and act optimally given these beliefs

Bayesian Game Example

- Consider the strategy profile (B,(B,F)), which means that player 1 will play B, and while in state 1, player 2 will also play B (when she wants to meet player 1) and in state 2, player 2 will play F (when she wants to avoid player 1).
  - Clearly, given the play of B by player 1, the strategy of player 2 is a best response.
- Let us now check that player 1 is also playing a best response.
  - Since both states are equally likely, the expected payoff of player 1 is

$$\mathbb{E}[B,(B,F)] = \frac{1}{2} \times 2 + \frac{1}{2} \times 0 = 1.$$

  - If, instead, he deviates and plays F, his expected payoff is

$$\mathbb{E}[F,(B,F)] = \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2}.$$

- Therefore, the strategy profile (B,(B,F)) is a (Bayesian) Nash equilibrium

	B	F
B	$(2,1)$	$(0,0)$
F	$(0,0)$	$(1,2)$

	B	F
B	$(2,0)$	$(0,2)$
F	$(0,1)$	$(1,0)$

Bayesian Game Example

- Note that meeting at Football, which is the preferable outcome for player 2 is no longer a Nash equilibrium. Why not?
  - Suppose that the two players will meet at Football when they want to meet. Then the relevant strategy profile is (F,(F,B)) and

$$\mathbb{E}[F,(F,B)] = \frac{1}{2} \times 1 + \frac{1}{2} \times 0 = \frac{1}{2}.$$

  - If, instead, player 1 deviates and plays B, his expected payoff is

$$\mathbb{E}[B,(F,B)] = \frac{1}{2} \times 0 + \frac{1}{2} \times 2 = 1.$$

- Therefore, the strategy profile (F,(F,B)) is not a (Bayesian) Nash equilibrium.

	B	F
B	$(2,1)$	$(0,0)$
F	$(0,0)$	$(1,2)$

	B	F
B	$(2,0)$	$(0,2)$
F	$(0,1)$	$(1,0)$

Bayesian Game Definition

- A Bayesian Game (aka. Incomplete information game) consists of
  - A set of players I;
  - A set of actions (pure strategies) for each player i: S_i;
  - A set of types for each player i: $\vartheta_i \in \Theta_i$;
  - A payoff function for each player i: $u_i(s_1,\dots,s_I,\vartheta_1,\dots,\vartheta_I)$;
  - A (joint) probability distribution $p(\vartheta_1, \dots, \vartheta_I)$ over types.
- More generally, one could also allow for a signal for each player,
  - so that the signal is correlated with the underlying type vector.

Bayesian Game Definition

- Importantly, throughout in Bayesian games,
  - the strategy spaces,
  - the payoff functions,
  - possible types, and
  - the prior probability distribution

  are assumed to be common knowledge.
- Strong assumption, but very mathematically convenient,
  - as any private information is encoded in the types and
  - others can form beliefs about this type and each player understands others' beliefs about his or her own type, and so on, and so on.
- A (pure) strategy for player i is a map $s_i : \Theta_i \rightarrow S_i$ prescribing an action for each possible type of player i.
  - e.g., a joint pure strategy: (F,(F,B))

Bayesian Games

- Recall that player types are drawn from some prior probability distribution $p(\vartheta_1, \ldots, \vartheta_I)$.
- Given $p(\vartheta_1, \ldots, \vartheta_I)$ we can compute the conditional distribution $p(\vartheta_{-i} \mid \vartheta_i)$ using Bayes rule.
  - Hence the label "Bayesian games".
- Player i knows her own type and evaluates her expected payoffs according to the conditional distribution $p(\vartheta_{-i} \mid \vartheta_i)$, where

$$\vartheta_{-i} = (\vartheta_1, \ldots, \vartheta_{i-1}, \vartheta_{i+1}, \ldots, \vartheta_I).$$

Bayesian Games

- Since the payoff functions, possible types, and the prior probability distribution are common knowledge, we can compute expected payoffs of player i of type ϑ_i as
  - When types are finite:

$$U\left(s_i', s_{-i}, \theta_i\right) = \sum_{\theta_{-i}} p(\theta_{-i} \mid \theta_i) u_i(s_i', s_{-i}(\theta_{-i}), \theta_i, \theta_{-i})$$

  - When types not finite:

$$= \int u_i(s_i', s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) P(d\theta_{-i} \mid \theta_i)$$

Bayesian Nash Equilibria

- (Bayesian Nash Equilibrium) The strategy profile s(·) is a (pure strategy) Bayesian Nash equilibrium if for all i ∈ I and for all $\vartheta_i \in \Theta_i$, we have that
  - When types are finite:

$$s_i(\theta_i) \in \arg\max_{s_i' \in S_i} \sum_{\theta_{-i}} p(\theta_{-i} \mid \theta_i) u_i(s_i', s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}),$$

  - When types not finite:

$$s_i(\theta_i) \in \arg\max_{s_i' \in S_i} \int u_i(s_i', s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) P(d\theta_{-i} \mid \theta_i) \ .$$

- Hence a Bayesian Nash equilibrium is a Nash equilibrium of the "expanded game" in which each player i's space of pure strategies is the set of maps from Θ_i to S_i.

Existence of Bayesian Nash Equilibria

- Theorem
  - Consider a finite incomplete information (Bayesian) game. Then a mixed strategy Bayesian Nash equilibrium exists.
- Theorem
  - Consider a Bayesian game with continuous strategy spaces and continuous types. If strategy sets and type sets are compact, payoff functions are continuous and concave in own strategies, then a pure strategy Bayesian Nash equilibrium exists.
- The ideas underlying these theorems and proofs are identical to those for the existence of equilibria in (complete information) strategic form games.

Example: Incomplete Information Cournot

- Suppose that two firms both produce at constant marginal cost.
  - Demand is given by P (Q) as in the usual Cournot game.
  - Firm 1 has marginal cost equal to C (and this is common knowledge).
  - Firm 2's marginal cost is private information. It is equal to C_L with probability ϑ and to C_H with probability $(1 - \vartheta)$, where $C_L < C_H$.
  - This game has 2 players, 2 states (L and H) and the possible actions of each player are $q_i \in [0, \infty)$, but firm 2 has two possible types.
  - The payoff functions of the players, after quantity choices are made, are given by

$$u_1((q_1,q_2),t) = q_1(P(q_1 + q_2) - C)$$
$$u_2((q_1,q_2),t) = q_2(P(q_1 + q_2) - C_t),$$

  where $t \in \{L,H\}$ is the type of player 2.

Example: Incomplete Information Cournot

- A strategy profile can be represented as
  - (q_1^*, q_L^*, q_H^*) [or equivalently as $(q_1^*, q_2^*(\vartheta_2))$], where q_L^* and q_H^* denote the actions of player 2 as a function of its possible types.
- We now characterize the Bayesian Nash equilibria of this game by computing the best response functions (correspondences) and finding their intersection.
- There are now three best response functions and they are are given by

$$
\begin{aligned}
B_1(q_L, q_H) = \arg\max_{q_1 \geq 0} \{&\theta(P(q_1 + q_L) - C)q_1 \\
&+ (1-\theta)(P(q_1 + q_H) - C)q_1\} \\
B_L(q_1) = \arg\max_{q_L \geq 0} &\{(P(q_1 + q_L) - C_L)q_L\} \\
B_H(q_1) = \arg\max_{q_H \geq 0} &\{(P(q_1 + q_H) - C_H)q_H\}.
\end{aligned}
$$

Example: Incomplete Information Cournot

- The Bayesian Nash equilibria of this game are vectors (q_1^*, q_L^*, q_H^*) such that

$$B_1(q_L^*, q_H^*) = q_1^*, \quad B_L(q_1^*) = q_L^*, \quad B_H(q_1^*) = q_H^*.$$

- To simplify the algebra, let us assume that P(Q) = α – Q, Q ≤ α. Then we can compute:

$$\begin{aligned} q_1^* &= \frac{1}{3}(\alpha - 2C + \theta C_L + (1-\theta) C_H) \\ q_L^* &= \frac{1}{3}(\alpha - 2C_L + C) - \frac{1}{6}(1-\theta)(C_H - C_L) \\ q_H^* &= \frac{1}{3}(\alpha - 2C_H + C) + \frac{1}{6}\theta(C_H - C_L). \end{aligned}$$

Example: Incomplete Information Cournot

- Note that $q_L^* > q_H^*$. This reflects the fact that with lower marginal cost, the firm will produce more.
- However, incomplete information also affects firm 2's output choice.
  - given this demand function, if both firms knew each other's marginal cost, then the unique Nash equilibrium involves output of firm i given by

$$\frac{1}{3}(\alpha - 2C_i + C_j).$$

- With incomplete information, firm 2's output is more if its cost is C_H and less if its cost is C_L.
  - If firm 1 knew firm 2's cost is high, then it would produce more.
  - However, its lack of information about the cost of firm 2 leads firm 1 to produce a relatively moderate level of output, which then allows firm 2 to be more "aggressive".
- Hence, in this case, firm 2 benefits from the lack of information of firm 1 and it produces more than if 1 knew his actual cost.

Auctions

- A major application of Bayesian games is to auctions, which are historically and currently common method of allocating scarce goods across individuals with different valuations for these goods.
- This corresponds to a situation of incomplete information because the valuations of different potential buyers are unknown.
- For example, if we were to announce a mechanism, which involves giving a particular good (for example a seat in a sports game) for free to the individual with the highest valuation, this would create an incentive for all individuals to overstate their valuations.
- In general, auctions are designed by profit-maximizing entities, which would like to sell the goods to raise the highest possible revenue.

References

- Fudenberg and Tirole, Game Theory, Sections 6.1-6.5.
- Slides based on Asu Ozdaglar, Game Theory with Engineering Applications, 2010

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Online Auctions

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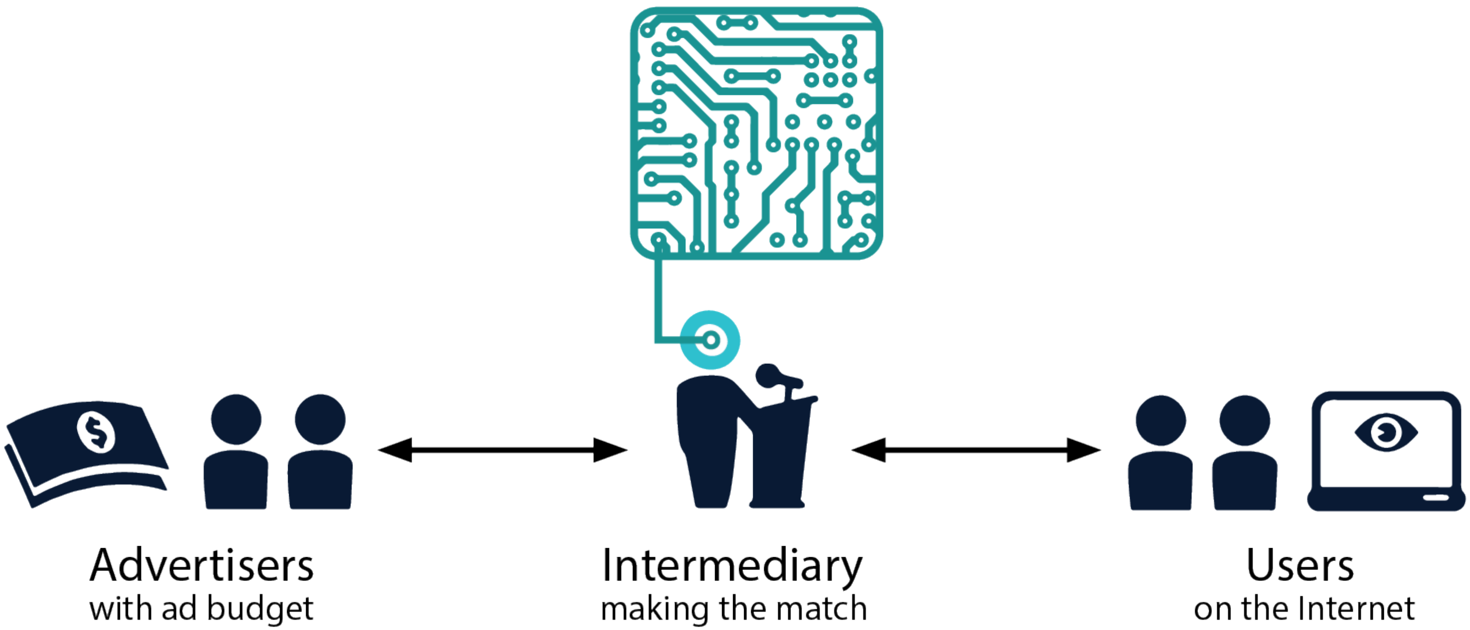
Subject to change

eBay

- eBay was founded by Pierre Omidyar in 1995
  - Started *AuctionWeb* (the eBay name came later)
  - Wanted to create a "perfect market" where buyers and sellers could interact freely
  - The Internet would enable just such a market
- One Internet business model that works
  - *Gross margin = Revenue – Cost of goods sold*
  - A revolutionary business model that Cost of goods is almost zero

Online Advertising



- Design **algorithms** to make the best match between the advertisers and Internet users with economic constraints

•Transformed from a low-tech process to highly optimized, mathematical, computer-centric (Wall Street-like) process

• Key directions: operations research, estimating CTR/AR; auction systems; learning algorithms; behavioral targeting; fighting spam (Click fraud)

Online Auctions

- An auction is a process of buying and selling goods or services by offering them up for bid, taking bids, and then selling the item to the highest bidder
- Auctions are popular
  - Historical sale tool
    - Bonds, treasury bills, land leases, privatization, art, etc.
  - Internet marketplace
    - eBay changed the landscape as a gigantic auctioneer
    - Sponsored search (Google, Facebook, etc.)

Settings

- Imagine we want to sell a ***single*** item
  - Later we'll extend this to multiple items
- We don't know what it's generally worth
  - Just what it's worth to us
  - We may set a *reserve price* (later)
- Each *bidder* (player) has her own *intrinsic value* of the item.
  - Willing to purchase it up to this price
  - Values are independent
- But we don't know these values
  - **Differs from our previous game theory assumptions about knowledge of payoffs**
- *How should we proceed?*

First steps

- We could just ask how much people are willing to pay
  - But would they lie?
  - Or manipulate the outcome?

- Problem:
  - How do we motivate buyers to reveal their true values?

- Auction theory: a sub-field of *Mechanism Design*
  - We design the market,"Economists as engineers"
  - Design an auction so that *in equilibrium* we get the results that we want

Goal of auctions

A seller ("auctioneer") may have several goals.

<u>Most common goals:</u>

1. Maximize revenue (profit)

2. Maximize social welfare (efficiency)
   - Give the item to the buyer that wants it the most. (regardless of payments.)

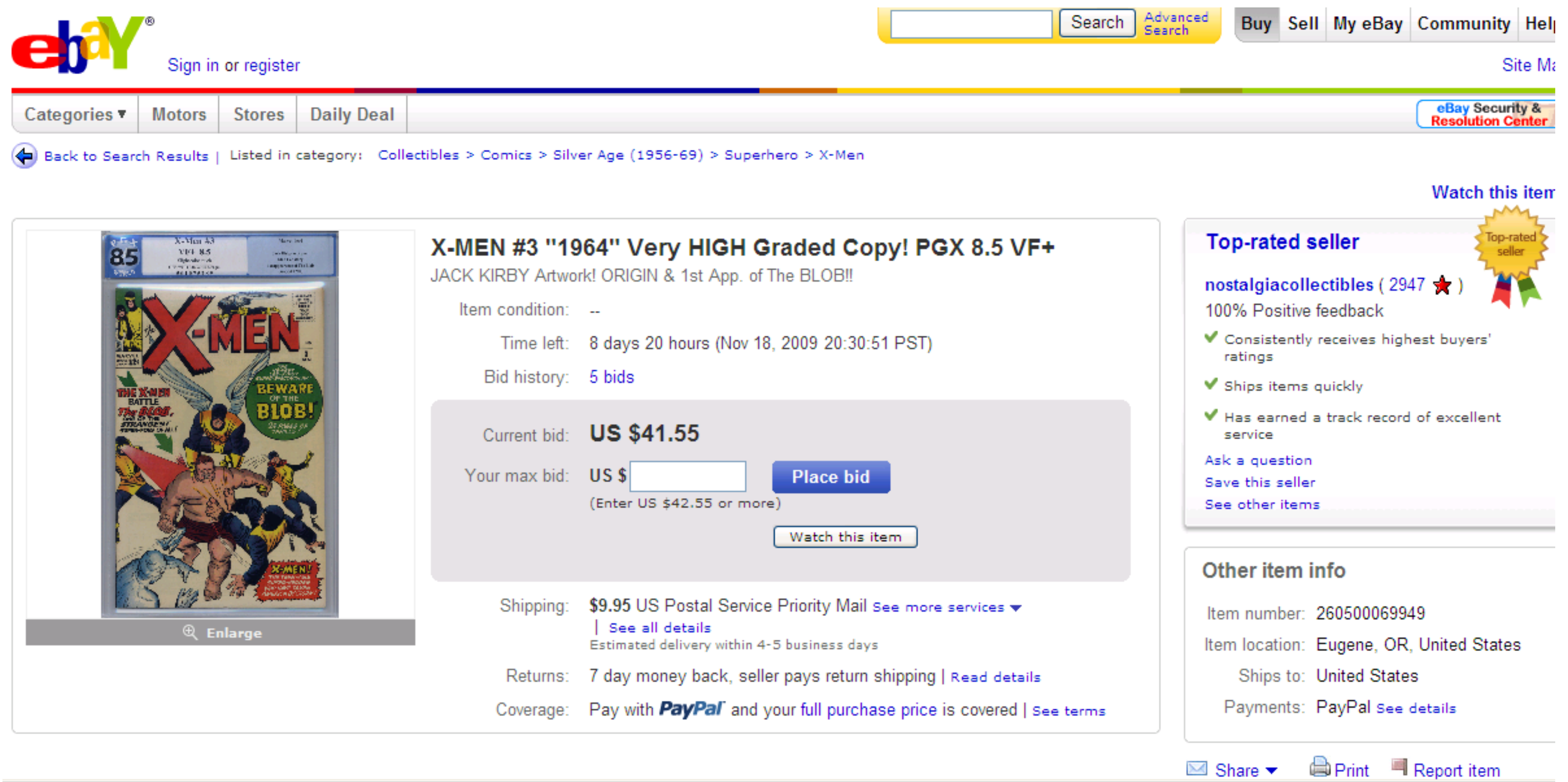
This is our focus today.

3. Fairness:
   for example, give items to the poor.

Types of auctions

- Ascending-bid (English auctions)

English Auctions at ebay



English auction - rules

- Price *p* is announced each time.
  - At the beginning, *p=0*.

- <u>Raising hand by a buyer:</u>
  Agreeing to buy the item for *p + $1.*

- If no bidder raised his hand for 1 minute, the item is sold.
  - To the bidder who made the last offer.
  - She pays her last offer.

at $3

p=0

bid=3

bid=2

Dutch Auctions

Dutch Flower Market

Dutch auction - rules

- Price *p* is announced each time.
  - At the beginning,
    p = maximum price.
- Seller lowers the price by $1 at each period.
- First buyer to raise his hand, wins the item.
  - He pays the current price.

at $97

p=100

Me!

ebay

Sign in or register

Search | Advanced Search

Buy | Sell | My eBay | Community | Help

Site Map

Categories ▾ | Motors | Stores | Daily Deal

eBay Security & Resolution Center

Back to Search Results | Listed in category: Coins & Paper Money > Coins: US > Halves > Liberty Walking (1916-47) > 1916-40

Watch this item

Enlarge

1939-D PCGS MS65 WALKER 50c -DUTCH AUCTION- ID#W90

Item condition: --

Time left: 25 days 14 hours (Dec 05, 2009 14:37:47 PST)

Quantity: 1 2 available

Price: US $155.00 Buy It Now

or

Best Offer: Make Offer

Watch this item

Shipping: $8.50 US Postal Service Priority Mail | See all details

Estimated delivery within 5-6 business days

Returns: 7 day money back, buyer pays return shipping | Read details

Coverage: Pay with PayPal and your full purchase price is covered | See terms

Seller info

fairtraderz (23876) me

99.8% Positive feedback

Ask a question

Save this seller

See other items

Other item info

Item number: 370286184080

Item location: Sacramento, United States

Ships to: United States

Payments: PayPal, Visa/MasterCard

See details

Share ▾ | Print | Report item

Description | Shipping and payments

1939-D PCGS MS65 WALKER 50c -DUTCH AUCTION- ID#W90

Description

1939-D PCGS MS65 WALKER 50c -DUTCH AUCTION- ID#W90

PLEASE READ, THIS IS A DUTCH AUCTION. WITH TWO COINS FOR SALE STARTING AT $155.00 EACH

This auction is a dutch auction with several coins available. The winner with the highest bid will be able to choose from the group by its serial number. If the high bid is tied, the earlier bidder will choose.

CCT

PLEASE READ.... SHIPPING

Dutch auctions - trivia

1. One advantage: quick.
   - Only requires one bid!

2. US department of treasury sells bonds using Dutch auctions.

3. The IPO for Google's stock was done using a variant of a Dutch auction.

Four auctions

We will now present the following auctions.

1. English Auctions
2. Dutch Auctions

"Open Cry" auctions

3. 1^{st}-price/"pay-your-bid" auctions
4. 2^{nd}-price/Vickrey auctions

"Sealed bid" auctions

1^{st} -price auctions

- Each bidder writes his bid in a sealed envelope.

- The seller:
  - Collects bids
  - Open envelopes.

- <u>Winner:</u>
  bidder with the highest bid.
- <u>Payment:</u>
  winner pays his bid.

Note: bidders do not see the bids of the other bidders.

2^{nd} -price auctions

- Each bidder writes his bid in a sealed envelope.

- The seller:
  - Collects bids
  - Open envelopes.

- Winner:
  bidder with the highest bid.
  Payment:
  winner pays the **2^{nd} highest bid**.

Note: bidders do not see the bids of the other bidders.

at $5

2^{nd}-price auction = Vickrey actions

- Second-price auctions are also known as Vickrey auctions.
- Auction defined by ***William Vickrey*** in 1961.
- Won the Nobel prize in economics in 1996
  - we will see his name again later in the course

Relations between auctions

English Auction

Dutch auction

1^{st}-price auction

2^{nd}-price auction

How do they relate to each other?

Equivalent auctions 1

- 1^{st}-price auctions are *strategically equivalent* to Dutch auctions (expected profits are identical)

Strategies:

1st-price: given that no one has a higher bid, what is the maximum I am willing to pay?

Dutch: Given that nobody has raised their hand, when should I raise mine?

- no new information is revealed during the auction!

Equivalent auctions 2

2st-price auctions are equivalent* to English auctions.

- In English auctions, bidders gradually drop out as the seller steadily raises the price
- The winner is the last bidder remaining, and pays the price at which the second-to-last bidder drops out

Equivalent auctions 2

2st-price auctions are equivalent* to English auctions.

- The outcomes in the two auctions are the same.

* Actually, in English auctions bidders observe additional information: bids of other players. (possible effect: herd phenomena- people tend to go along with what others do or think without considering their actions)

But do bidders bid truthfully?

Modeling

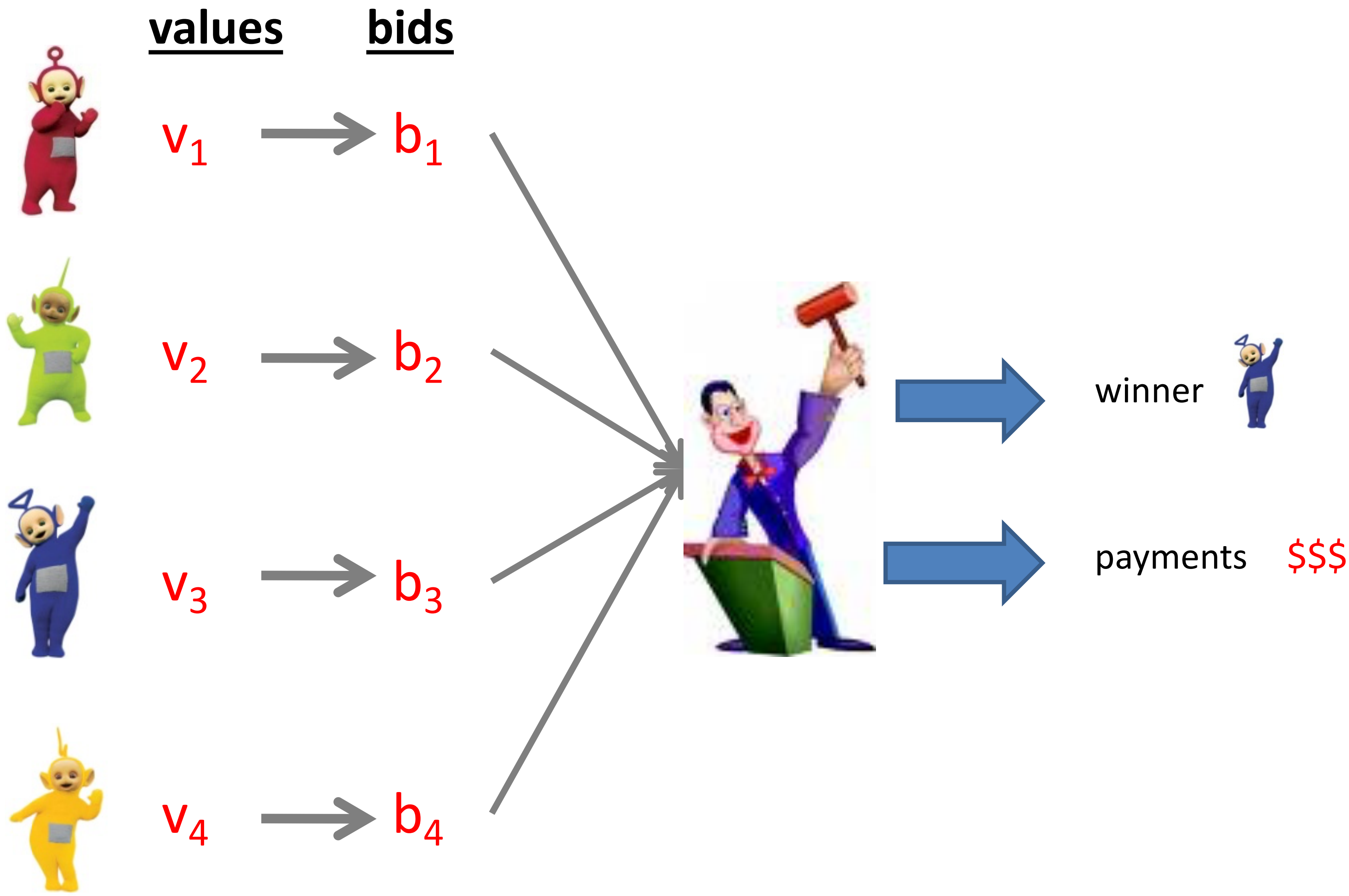
- n bidders

- Each bidder has value v_i for the item
  - "willingness to pay"
  - Known only to him – "private value"

- If Bidder i wins and pays p_i, his utility is $v_i - p_i$
  - In addition, her utility is 0 when she loses.

- <u>Note:</u> bidders prefer losing than paying more than their value.

Auctions scheme



Strategy

- A strategy for each bidder:

  how to bid given your intrinsic value?

- Examples for strategies:
  - $b_i(v_i) = v_i$ (truthful)
  - $b_i(v_i) = v_i/2$
  - $b_i(v_i) = v_i/n$
  - *If* $v<50$, $b_i(v_i) = v_i$
    otherwise, $b_i(v_i) = v_i+17$

	B(v)=v	**B(v)=v/2**	**B(v)=v/n**	**....**
B(v)=v				
...				

- Can be modeled as normal form game, where these strategies are the pure strategies.
- Example for a *game with incomplete information*.

Strategies and equilibrium

- An equilibrium in the auction is a profile of strategies $B_1,B_2,\ldots,B_n$ such that:
  - Dominant strategy equilibrium: each strategy is optimal whatever the other strategies are.
  - Nash equilibrium: each strategy is a best response to the other strategies.
- Again: a strategy here is a *function*, a plan for the game. Not just a bid.

	B(v)=v	**B(v)=v/2**	**B(v)=v/n**	**....**
B(v)=v				
...				

Equilibrium behavior in 2nd-price auctions

Theorem: In 2nd-price auctions truth-telling $B(v)=v$ is a dominant strategy.

- in English auctions too (with private values)

That is, no matter what the others are doing, I will never gain anything from lying.

- Bidding is easy, independent from our beliefs on the value of the others.

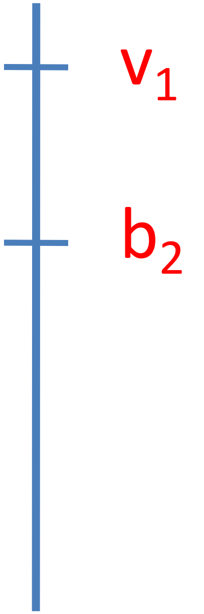
Conclusion: 2nd price auctions are efficient (maximize social welfare).

- Selling to bidder with highest bid is actually selling to the bidder with the highest value.

Truthfulness: proof

- Let's prove now that truthfulness is a dominant strategy.
- We will show that Bidder 1 will never benefit from bidding a bid that is not v_1. (v_1 is Bidder 1's true value)

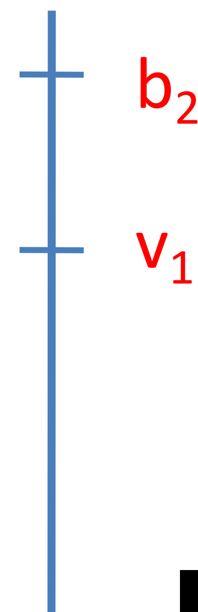
- <u>Case 1:</u> Bidder 1 wins when bidding v_1.
  - v_1 is the highest bid, b_2 is the 2nd highest.
  - His utility is $v_1 - b_2 > 0$.
  - Bidding above b_2 will not change anything (no gain from lying).
  - Bidding less than b_2 will turn him into a loser - from positive utility to zero (no gain from lying).



Truthfulness: proof

- Let's prove now that truthfulness is a dominant strategy.
- We will show that Bidder 1 will never benefit from bidding a bid that is not v_1. (v_1 is Bidder 1's true value)

- Case 2: Bidder 1 loses when bidding v_1.
  - Let b_2 be the highest bid now.
  - Bidder 1's utility 0 (losing).
  - Any bid below b_2 will gain him zero utility (no gain from lying).
  - Any bid above b_2 will gain him a utility of $v_1 - b_2 < 0$ - losing is better (no gain from lying).



∎

Efficiency in 2nd-price auctions

- Since 2nd-price is truthful, we can conclude it is *efficient*:

- That is, in equilibrium, the auction allocates the item to the bidder with the highest value.
  - With the actual highest value, not just the highest bid.
  - Without assuming anything on the values
    - (For every profile of values).

What we saw so far…

- 2^{nd} price and English auctions are:
  - Equivalent*
  - Have a truthful dominant-strategy equilibrium.
  - Efficient in equilibrium.

- 1^{st} -price and Dutch auctions are:
  - Equivalent.
  - Truthful?
  - Efficient?

1st price auctions

- Truthful? NO!

$31

$30 ~~$100~~

Bayesian analysis

- There is no dominant strategy in 1st price auctions.

- How do people behave?
  - They have beliefs on the preferences of the other players!

- Beliefs are modeled with probability distributions.

Bayes-Nash equilibrium

- Definition: A set of bidding strategies is a **Nash equilibrium** if each bidder's strategy maximizes his payoff given the optimal strategies of the others.
  - In auctions: bidders do not know their opponent's values, i.e., there is *incomplete information.*

  → Each bidder's strategy must maximize her *expected* payoff accounting for the uncertainty about opponent values.

Continuous distributions

A brief reminder of basic notions in statistics/probability.

Continuous distributions

Reminder:

- Let *V* be a random variable that takes values from *[0,t]*.

- Cumulative distribution function *F:[0,t]→[0,1]*
  F(x) = {Probability that V<x} = Pr{V<x}

- The *density* of *F* is the density distribution *f(x)=F'(x).*

- The expectation of *V*: $\mathrm{E[V]} = \int_0^t x \cdot f(x)dx$

Example: The Uniform Distribution

What is the probability that $V<x$?

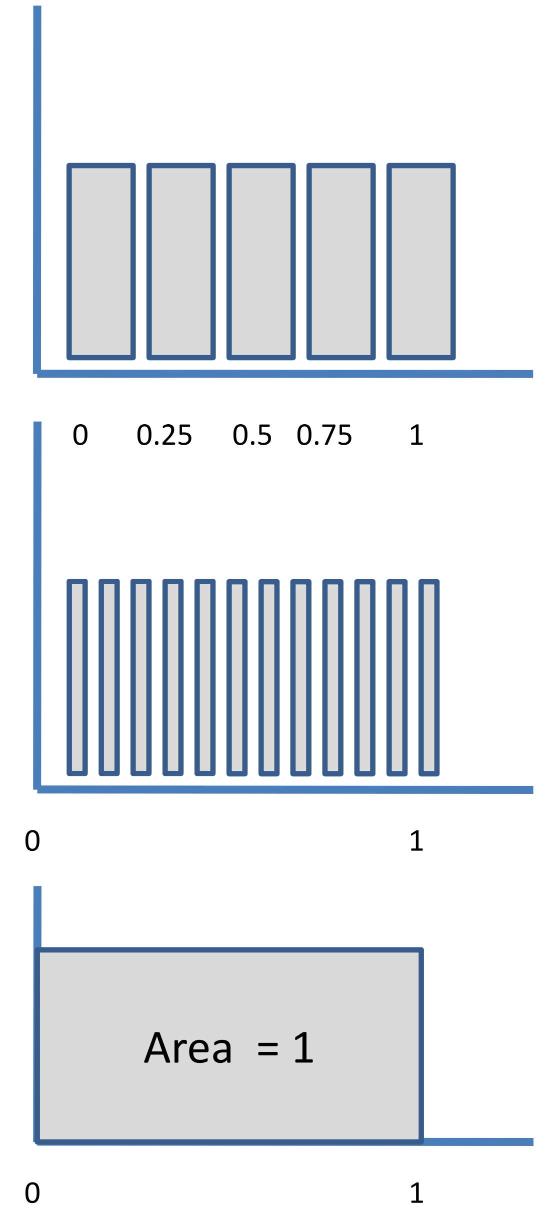
$$F(x)=x.$$

Density: $f(x)=1$

Expectation:

$$\mathrm{E[V]}=\int_0^1 x\cdot f(x)dx$$

$$=\int_0^1 x\cdot 1dx \quad =\left.\frac{x^2}{2}\right|_0^1=\frac{1}{2}$$



Auctions with uniform distributions

A simple Bayesian auction model:

- 2 buyers
- Values are between 0 and 1.
- Values are distributed uniformly on *[0,1]*

What is the equilibrium in this game of incomplete information?

Are 1^{st}-price auctions efficient?

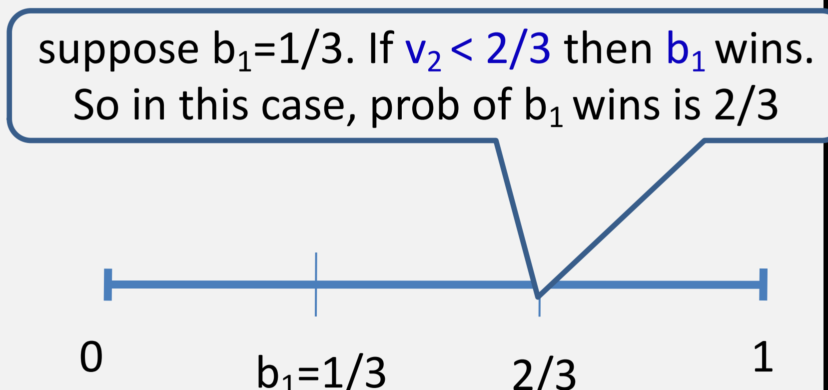
Equilibrium in 1st-price auctions

<u>Claim:</u> bidding *$b(v)=v/2$* is an equilibrium

– 2 bidders, uniform distribution.

<u>Proof:</u>

- Assume that Bidder 2's strategy is $b_2(v)=v_2/2$.
- We show: $b_1(v)=v_1/2$ is a best response for Bidder 2.
  – (clearly, no need to bid above 1).



- Bidder 1's expected utility is:

$Prob[\, b_1 > b_2 \,] \times (v_1-b_1) =$

$Prob[\, b_1 > v_2/2 \,] \times (v_1-b_1) =$

$2b_1 * (v_1-b_1)$

- $[\, 2b_1 * (v_1-b_1) \,]' = 2v_1-4b_1 = 0$ (maximize for b_1)

→ $b_1 = v_1/2$ (→ it is a best response for $b_2=v_2/2$)

Equilibrium in 1st-price auctions

<u>We proved:</u> bidding b(v)=v/2 is an equilibrium

– 2 bidders, uniform distribution.

<u>For *n* players:</u>

bidding $b_i(v_i) = \frac{(n-1)}{n} v_i$ by all players is a Nash equilibrium.

(with more competition, you will bid closer to your true value)

<u>Conclusion:</u>

1st-price auctions are *efficient*

(not truthful, but in equilibrium the bidder with the highest intrinsic value wins).

Equilibrium in 1st-price auctions

- We proved: 1st-price auction is efficient for the uniform distribution.

- What about general distributions?
  - Turns out: **Yes!**
  - Let us go into the math next

Equilibrium in 1st-price auctions

- We proved: 1st-price auction is efficient for the uniform distribution.

- What about general distributions?

- Notation:
  let $v_1,\ldots,v_{n-1}$ be n-1 draws from a distribution F.
  Let $\boldsymbol{max}_{[n-1]} = max\{v_1,\ldots,v_{n-1}\}$ (highest-order statistic)

Equilibrium in 1st-price auctions

When $v_1,\ldots,v_n$ are distributed i.i.d. from F

- F is strictly increasing

Claim: the following is a symmetric Nash equilibrium

$$b_i(v_i) = E\left[\max_{[n-1]} \middle| \max_{[n-1]} < v_i\right]$$

- That is, each bidder will bid the expectation of the second-highest bidder's value, given that v_i wins.
- The above $b_i(v_i)$ is strictly monotone in v_i → 1st-price auction is efficient.
- Example: uniform distribution $b_i(v_i) = \frac{(n-1)}{n} v_i$

Equilibrium in 1st-price auctions

- Suppose bidder *i*'s value is v_i in [0,1], which is only known by bidder *i*.
- Given this value, bidder *i* must submit a sealed bid b_i (v_i)
- We view bidder *i*'s strategy as a bidding function b_i : [0,1] -> $R_{+.}$ Some properties:
  - Bidders with higher values will place higher bids. So b_i is a strictly increasing function
  - Bidders are also *symmetric*. So bidders with the same value will submit the same bid: b_i = b (*symmetric Nash equilibrium*)

Equilibrium in 1st-price auctions

- Bidder *1*'s payoff

$$\begin{cases} v_1 - b_1 & if \ \ b_1 > \max\{b(v_2),\ldots,b(v_n)\} \\ 0 & if \ \ b_1 \le \max\{b(v_2),\ldots,b(v_n)\} \end{cases}$$

- The expected payoff of bidding b_1 is given by

$$\begin{aligned} \pi(b_1) &= (v_1 - b_1)P(b_1 > \max\{b(v_2),\ldots,b(v_n)\}) \\ &= (v_1 - b_1)P(b_1 > b(v_2),\ldots,b_1 > (v_n)) \end{aligned}$$

- An optimal strategy b_i should maximize $\pi(b_1)$
- It is not immediately clear how to optimize a function b_i

Equilibrium in 1st-price auctions

- An elegant device: bidder can implement their deviation by keeping the strategy b() but supplying a different "true value" to it
  - Suppose that bidder *i* cannot attend the auction and that he sends a friend to bid for him
    - The friend knows the equilibrium bidding function b^* but doe not know v_i
    - Bidder tells his friend the value as x and wants him to submit the bid $b^*(x)$
  - The expected pay off in this case is

$$\pi(b^*, x) = (v_1 - b^*(x))P(b^*(x) > b^*(v_2), \ldots, b^*(x) > b^*(v_n))$$

$$= (v_1 - b^*(x))P(x > v_2, \ldots, x > v_n) = (v_1 - b^*(x))F^{N-1}(x)$$

Equilibrium in 1st-price auctions

- The expected payoff must be maximized when reporting his true value v_i to his friend ($x = v_i$)

$$F(v_i)^{n-1}(v_i - b^*(v_i)) \geq F(v_i)^{n-1}(v_i - b^*(x))$$

- In other words, so if we differentiate the expected payoff with respect to x, the resulting derivative must be zero when $x = v_i$:

$$\frac{d\pi(b^*, x)}{dx} = \frac{d(v_1 - b^*(x))F^{N-1}(x)}{dx}$$

$$= (N-1)F^{N-2}(x)f(x)(v_1 - b^*(x)) - F^{N-1}(x)b^{*\,\prime}(x)$$

Equilibrium in 1st-price auctions

- Setting $x = v_1$ and rearranging yields:

$$
\begin{aligned}
&(N-1)F^{N-2}(v_1)f(v_1)v_1 \\
&= F^{N-1}(v_1)b^{*\,\prime}(v_1) + (N-1)F^{N-2}(v_1)f(v_1)b^{*}(v_1) \\
&= \frac{dF^{N-1}(v_1)b^{*}(v_1)}{dv}
\end{aligned}
$$

Chapter 9, Advanced Microeconomic Theory, Geoffrey A. Jehle, Philip J. Reny

Equilibrium in 1st-price auctions

- We thus have

$$F^{N-1}(v_1)b^*(v_1) = (N-1)\int_0^{v_i} xf(x)F^{N-2}(x)\,dx + \text{constant}$$

- If we assume a bidder with value zero must bid zero, the above constant is zero. Therefore, we have (replace v_i with v)

$$b^*(v) = \frac{(N-1)\int_0^{v} xf(x)F^{N-2}(x)\,dx}{F^{N-1}(v)} = \frac{\int_0^{v} x\,dF^{N-1}(x)}{F^{N-1}(v)}$$

- It shows that in the equilibrium, each bidder bids the expectation of the second-highest bidder's value conditional on winning the auction.

Equilibrium in 1st-price auctions

- We thus have

$$F^{N-1}(v_1)b^{*}(v_1) = (N-1)\int_{0}^{v_i} xf(x)F^{N-2}(x)\,dx + \text{constant}$$

- If we assume a bidder with value zero must bid zero, the above constant is zero. Therefore, we have (replace

> The insight: if my value v is the highest among all bidders, then in a symmetric equilibrium where strategies are increasing, it suffices for me to bid just to outbid the opponent with the second highest valuation.

$$b^{*}(v\ ) = \frac{\quad}{F^{N-1}(v)} - \frac{\quad}{F^{N-1}(v\ )}$$

- It shows that in the equilibrium, each bidder bids the expectation of the second-highest bidder's value conditional on winning the auction.

Exercise

1. Show that the equilibrium bidding function is strictly increasing in *v*. (hint: differentiate $b^*(v)$)
2. Suppose we have n bidders and each bidder's value is uniformly distributed on [0,1].
   - What the equilibrium bidding function would look like?
   - What is the seller's expected revenue?

Answers to Q2

- Replacing F(v)=v and f(v)=1 gives

$$b^*(v) = \frac{\int_0^v x\,dF^{N-1}(x)}{F^{N-1}(v_1)} = \frac{\int_0^v x\,dx^{N-1}}{v^{N-1}}$$

$$= \frac{\int_0^v x(N-1)x^{N-2}\,dx}{v^{N-1}} = \frac{(N-1)\int_0^v x^{N-1}\,dx}{v^{N-1}}$$

$$= \frac{(N-1)}{v^{N-1}}\frac{1}{N}v^N = v - \frac{v}{N}$$

Equilibrium in 1st-price auctions

- The expected payoff must be maximized when reporting his true value v_i to his friend ($x = v_i$)

$$F(v_i)^{n-1}(v_i\text{-b}^*(v_i)) \geq F(v_i)^{n-1}(v_i - \text{b}*(x))$$

- In other words, so if we differentiate the expected payoff with respect to x, the resulting derivative must be zero when $x = v_i$:

$$\frac{d\pi(b^*,x)}{dx} = \frac{d(v_1 - b^*(x))F^{N-1}(x)}{dx}$$

$$= (N-1)F^{N-2}(x)f(x)(v_1 - b^*(x)) - F^{N-1}(x)b^{*\prime}(x)$$

Alternative Answers to Q2

- Recall that the expected payoff must be maximized when reporting his true value v_i to his friend ($x = v_i$)

$$F(v_i)^{n-1}(v_i\text{-}b^*(v_i)) \geq F(v_i)^{n-1}(v_i - b*(x))$$

- Replacing F(v)=v and f(v)=1 gives

$$v_i^{\,n-1}(v_i\text{-}b^*(v_i)) \geq v_i^{n-1}(v_i - b*(x))$$

- Taking derivative of the expected payoff gives

$$(n-1)v_i^{\,n-2}v_i\text{-}(n-1)\,v_i^{n-2}\,b^*(v_i)\text{-}\,v_i^{n-1}\,b'*(x)=0$$

-> $$b'*(x)=(n-1)(1-b^*(v_i)/v_i\,)$$

-> $$b*(x)=((n-1)/n)v_i$$

Answers to Q2

- We have the expected revenue as:

$$R = \int_0^1 b^*(v)\, dF^N(v) = N\int_0^1 b^*(v) f(v) F^{N-1}(v)\, dv$$

- Replacing F(v)=v and f(v)=1 and b*(v) = v-v/N) gives

$$R = N\int_0^1 b^*(v) f(v) F^{N-1}(v)\, dv$$

$$= N\int_0^1 [v - v/N] v^{N-1}\, dv$$

$$= (N-1)\int_0^1 v^N\, dv = \frac{N-1}{N+1}\int_0^1 dv^{N+1} = \frac{N-1}{N+1}$$

Equilibrium in 2rd-price auctions

- bidder *1*'s payoff

$$\begin{cases} v_1 - b_i & if \;\; b_1 > b_i > \max\{b(v_2),\ldots,b(v_{i-1}),b(v_{i+1}),\ldots,b(v_n)\} \\ 0 & if \;\; b_1 \le \max\{b(v_2),\ldots,b(v_n)\} \end{cases}$$

- The expected payoff of bidding b_1 is given by

$$\pi(v_1,b_1) = \int_0^{b_1} (v_1 - x)\, dF^{N-1}(x) = \int_0^{b_1} (N-1)(v_1 - x) f(x) F^{N-2}(x) dx$$

- Suppose $b_1 < v_1$, if b_1 is increased to v_1 the integral increases by the amount

$$\int_{b_1}^{v_1} (N-1)(v_1 - x) f(x) F^{N-2}(x) dx$$

- The reverse happens if $b_1 > v_1$

Equilibrium in 2rd-price auctions

- bidder *1*'s payoff

$$\begin{cases} v_1 - b_i & \text{if } b_1 > b_i > \max\{b(v_2),\ldots,b(v_{i-1}),b(v_{i+1}),\ldots,b(v_n)\} \\ 0 & \text{if } b_1 \le \max\{b(v_2),\ldots,b(v_n)\} \end{cases}$$

- The expected payoff of bidding b_1 is given by

$$\pi(v_1,b_1 \ldots \int^{b_1} \ldots \int^{b_1} \ldots F^{N-2}(x)dx$$

So telling the truth $b_1 = v_1$ is a Bayesian Nash equilibrium bidding strategy!

- Su … egral

in…

$$\int_{b_1}^{v_1} (N-1)(v_1 - x) f(x) F^{N-2}(x)dx$$

- The reverse happens if $b_1 > v_1$

Exercise

1. Suppose that each bidder's value is uniformly distributed on [0,1].
   - What is the seller's expected revenue for a 2rd price auction?
   - Show that first and second price auctions generate the same expected revenue!

What we saw so far…

- 2nd price and English auctions are:
  - Equivalent*
  - Have a truthful dominant-strategy equilibrium.
  - Efficient <u>in equilibrium</u>. (“efficient”)

- 1st -price and Dutch auctions are:
  - Equivalent.
  - Truthful??? **No!**
  - Efficient??? **Yes!**
    - Actually true for all distributions, not just the uniform distribution.

Revenue equivalence

Suppose n numbers are drawn independently from the uniform distribution on the interval [0, 1] and then sorted from smallest to largest. The expected value of the number in the kth position on this list is k/n + 1.

- How much does the seller make?
- 1^{st} -price and Dutch auctions:
  - Bidders reduce their bids by a factor of **(n-1) / n**
  - Expect largest bid to be **n** / **(n+1)**
    - Expect revenue: **(n-1) / (n+1)**
- 2^{nd} price and English auctions:
  - Seller commits to collecting less than max. bid
  - Look at highest and second-highest bids
    - Expect revenue: **(n-1) / (n+1)**
- *This holds for very many auction formats!*

Reserve Prices and Entry Fees

- *Reserve Prices*: the seller is assumed to have committed to not selling below the reserve
  - Reserve prices are assumed to be known to all bidders
  - The reserve prices = the minimum bids
- *Entry Fees*: those bidders who enter have to pay the entry fee to the seller
- They reduce bidders' incentives to participate, but they might increase revenue as 1) the seller collects extra revenues 2) bidders might bid more aggressively

Reserve Prices and Entry Fees in 1st Price Auctions

- If we define reserve price as r and entry fee as delta δ, the expected payoff for a bidder is:

$$\pi_1 = E[(v-b)\, I_{b>\max\{b(Z),r\}}\,] - \delta \qquad I = 1 \; when \; b > \max\{b(Z),r\}, \; otherwise \; 0$$

$$where \text{ Z=}\max\{v_j : v_j \geq \rho, j = 2,\dots,n\}, \text{ and } r = b(\rho)$$

- Maximizing it leads to the equilibrium bidding function (HOW?)

Chapter 3 An Introduction to Auction Theory, Flavio M. Menezes, Paulo K. Monteiro

$$b^* = \begin{cases} v - \dfrac{\delta + \int_\rho^v F^{N-1}(x)\,dx}{F^{N-1}(v)} & if \;\; v > \rho \\ \text{not bid} & othewise \end{cases}$$

Reserve Prices and Entry Fees in 1st Price Auctions

- It shows that if we set $\delta = 0$, the following

$$\rho = \frac{1 - F(\rho)}{f(\rho)}$$

maximizing the expected revenue of a seller using a first-price auction

- How can we obtain the result when *F* is uniform distribution?

Model and real life

We discussed several simplified models. Real auctions are more complicated

- Do bidders know their values?
  - If so many people are willing to pay more than $100, it possibly worth it. (English auctions may help discover the value.)
- Auctions are (usually) repeated, and not stand-alone.
- Budgets and wealth effects.
  - I think that this TV is worth $1000, but my wife will divorce me if I pay more than $100.
- Manipulation is not only with bids
- Do bidders have accurate probabilities?
- Do bidders behave rationally?

Common values and the Winner's Curse

- So far, we have assumed that
  - Bidders values for the item being auctioned are independent
  - Each bidder knows her own value for the item
  - And it is not concerned with how much it is worth to anyone else
- However, there are other cases where there is a *common eventual value* for the item
  - E.g., the amount it will generate on resale
- Each bidder i may have some private information about the common value v, leading to an estimate v_i of this value

Common values and the Winner's Curse

- Individual bidder estimates will typically be slightly wrong and they also will typically not be independent of each other
  - A simple model would be: $v_i = v + noise_i$
- In a sealed bid second-price auction, is it still a dominate strategy for the bidder to bid v_i ?
- The answer is no
  - Suppose there are many bidders and that each bids her estimate
  - If a bidder wins, it is very likely her estimate is an *overestimate*
  - The second-place bid – which is what she paid – is also likely to be an overestimate. As a result she will likely lose money on the resale relative to what she paid

Common values and the Winner's Curse

- Thus, rational bidders should take the winner's curse into account in deciding on their bids
- In a common-value auction,
  - Bidders will shade their bids downward even when the second-price format is used
  - With the first-price format, bids will be reduced even further

References

- Slides are partly based on http://www.ymsir.com/networks/
- Chapter 3 An Introduction to Auction Theory, Flavio M. Menezes, Paulo K. Monteiro
- Chapter 9, Advanced Microeconomic Theory, Geoffrey A. Jehle, Philip J. Reny
- Weinan Zhang, Shuai Yuan, and Jun Wang. 2014. Optimal real-time bidding for display advertising. ACM KDD

Computational Advertising

with applications to Auctions and Coalition games

Dr. Jun Wang

Computer Science, UCL

Outline

- Introduction and history
- Sponsored Search
  - First-price auction
  - Second-price auction
  - VCG auction
- Conversion Attribution

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- Introduction and history
- Sponsored Search
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What is Advertising?

- **Advertising:** tell the public about a product or service in order to persuade them to buy it
- A form of *communication*:
    - •To deliver marketing messages

        To inform potential customers about products and services and how to obtain and use them

    - •Branding - through the creation and reinforcement of brand image and brand loyalty

•Major medium: television, radio, movies, magazines, newspapers, video games, the Internet, and billboards

"Half the money I spend on advertising is wasted; the trouble is I don't know which half."

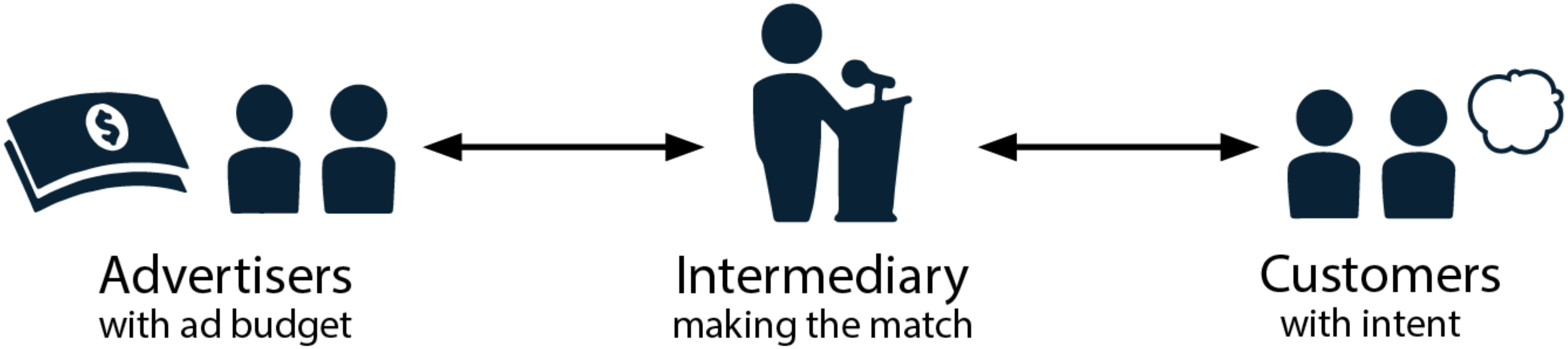
- *John Wanamaker*

(1838-1922)

Father of modern advertising and a pioneer in marketing

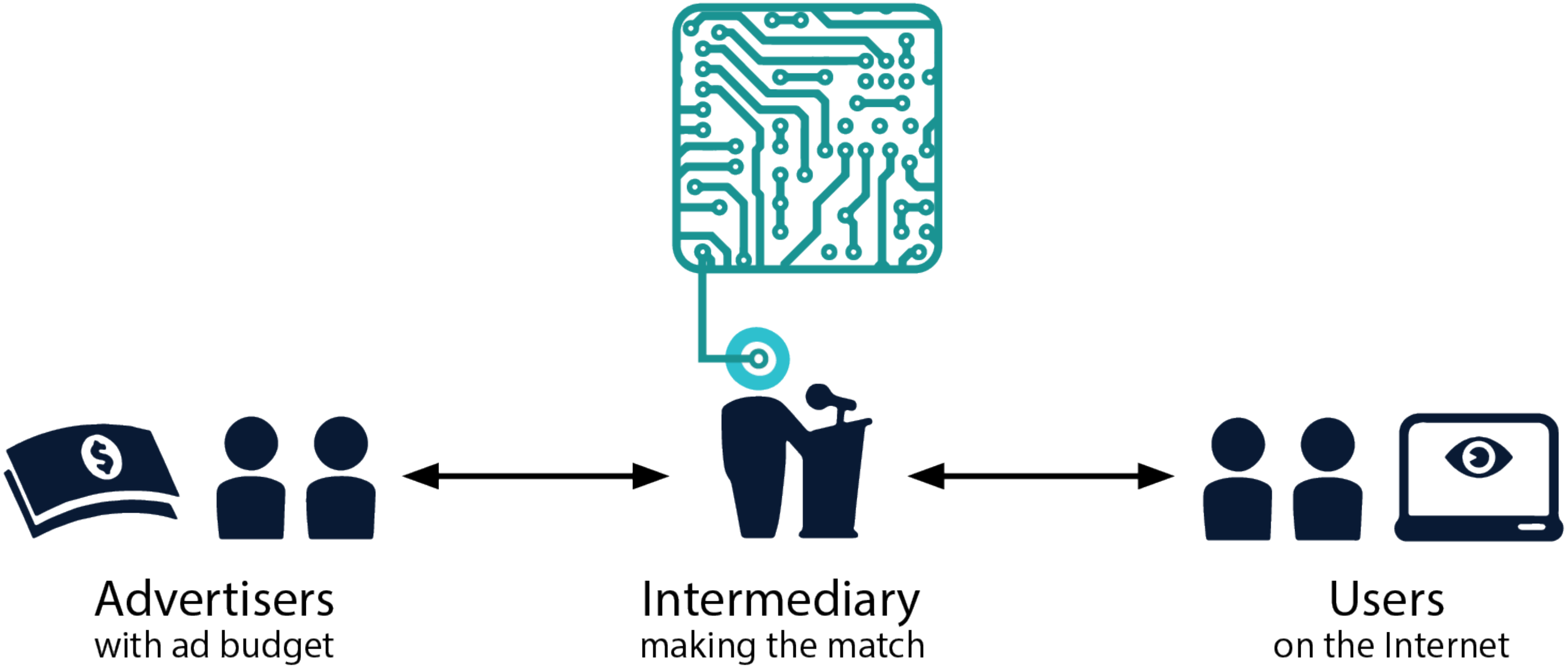
Online Advertising

- **Online Advertising:** use the Internet and Web to deliver marketing messages



- Make the best match between advertisers and customers with economic constraints

Computational Advertising



- Design **algorithms** to make the best match between the advertisers and Internet users with economic constraints

•Transformed from a low-tech process to highly optimized, mathematical, computer-centric (Wall Street-like) process

• Key directions: operations research, estimating CTR/AR; auction systems; learning algorithms; behavioral targeting; fighting spam (Click fraud)

The Scale of Online Advertising

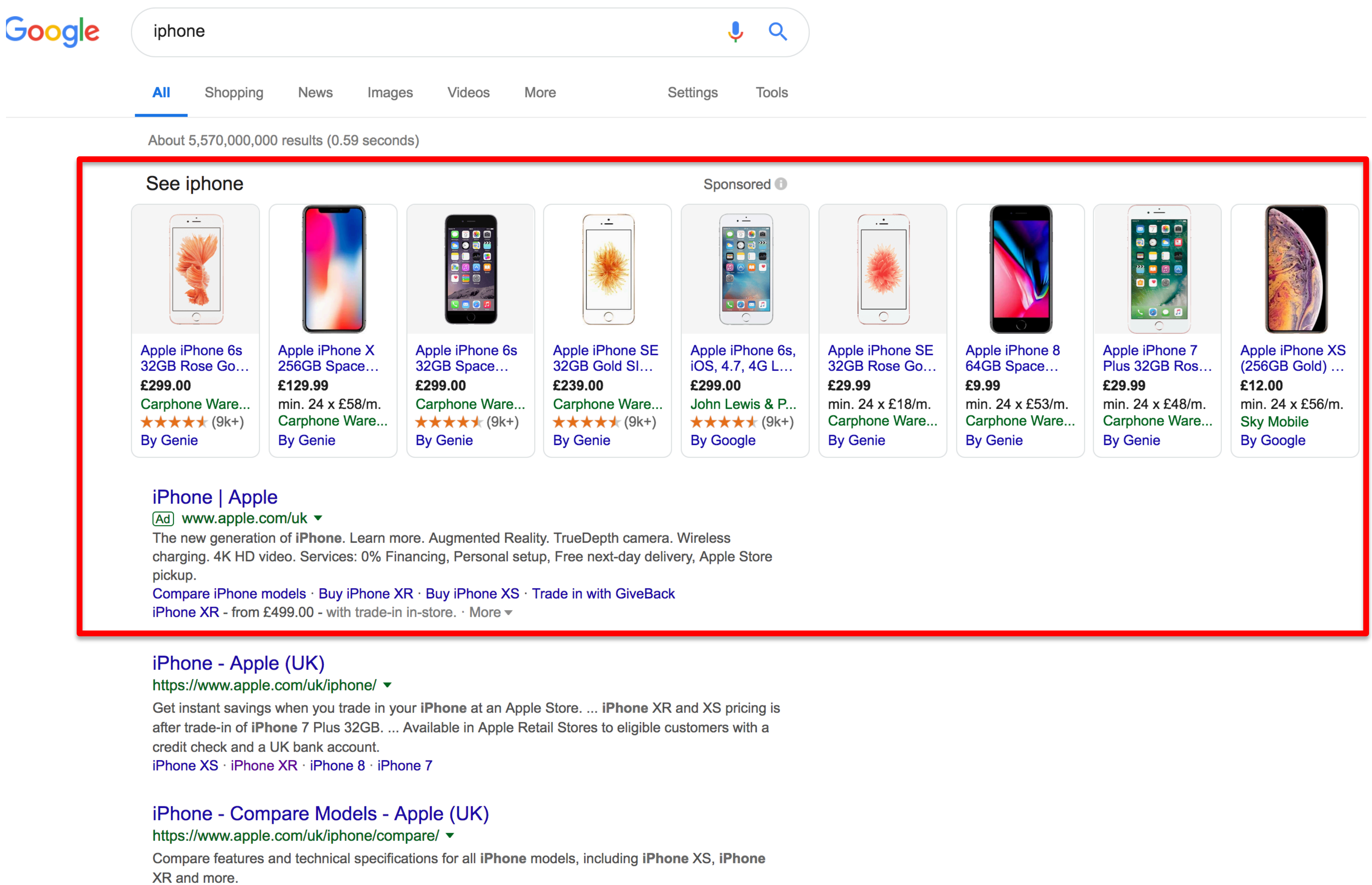
- Every online **ad view** can be evaluated, bought, and sold, all **individually**, and all **instantaneously**.
- Instead of buying keywords or a bundle of ad views, advertisers are now **buying users** directly.

	DSP/Exchange	Daily traffic
RTB advertising	iPinYou, China	18 billion impressions
	YOYI, China	5 billion impressions
	Fikisu, US	32 billion impressions
	Appnexus, US	**100+billion impressions**
Web search	Google search	**~3.5 billion searches/impressions**
Financial markets	New York stock exchange	12 billion shares daily
	Shanghai stock exchange	14 billion shares daily

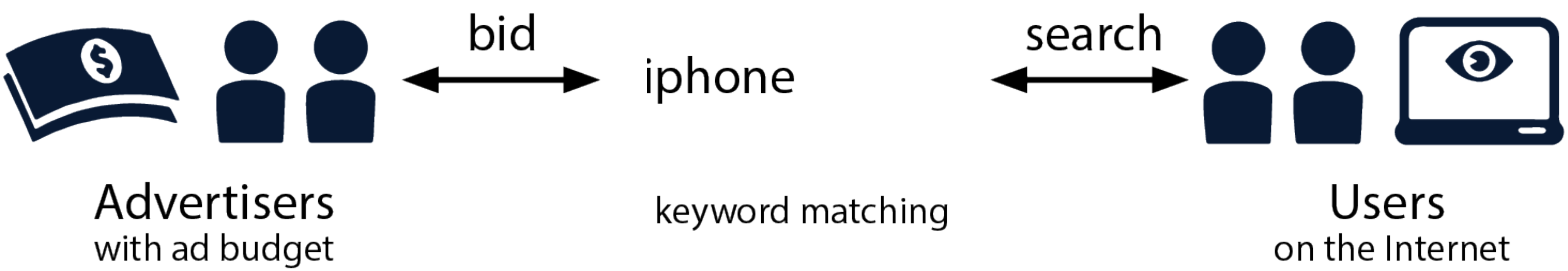
http://www.internetlivestats.com/google-search-statistics/

Shen, Jianqiang, et al. "From 0.5 Million to 2.5 Million: Efficiently Scaling up Real-Time Bidding." Data Mining (ICDM), 2015 IEEE International Conference on. IEEE, 2015.

Example 1 Sponsored Search



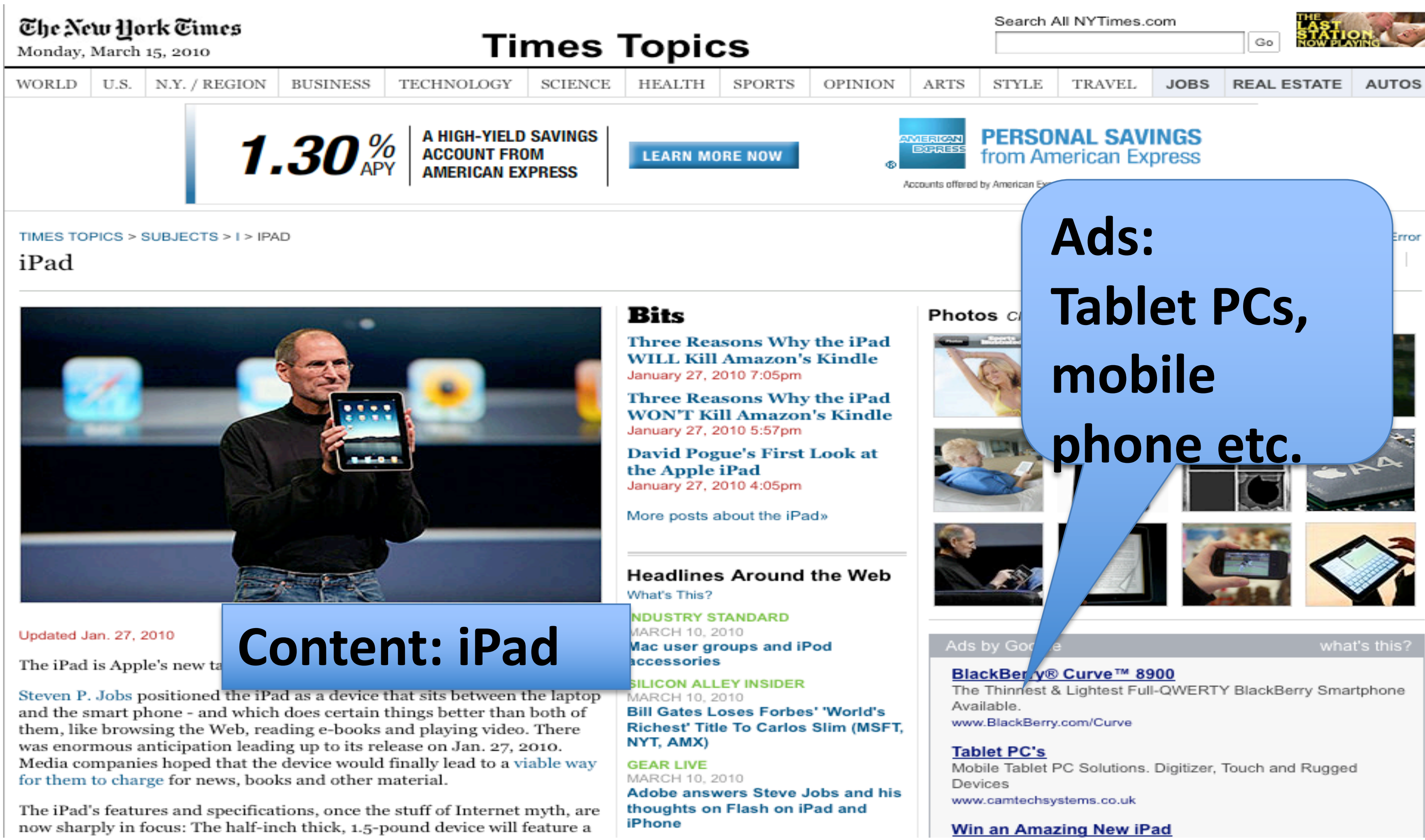
Sponsored Search



- Advertiser sets a bid price for the keyword
- User searches the keyword
- Search engine hosts the auction to ranking the ads

Example 2: Contextual ads

- Contextual Advertising: relevant to the content on the page



Example 2: Contextual ads

- "Content ads"
  - Use keyword matching engine to pick ads related to content on page
  - Matching algorithm tweaking to deal with multiple content on page
  - Auction needs tweaking to deal with position
  - Generally lower CTRs, lower conversion performance, adjustments made in payment
- Very simple to set up

Example 3: Display ads

- Display Ads (many types): Standardized ad shapes with images
- Normally not automatically related to content but may be related to audience

Example 3: Display ads
Display formats

- Floating ad: An ad which moves across the user's screen or floats above the content.
- Expanding ad: An ad which changes size and which may alter the contents of the webpage.
- Wallpaper ad: An ad which changes the background of the page being viewed.
- Trick banner: A banner ad that looks like a dialog box with buttons. It simulates an error message or an alert.
- Pop-up: A new window which opens in front of the current one, displaying an advertisement, or entire webpage.
- Pop-under: Similar to a Pop-Up except that the window is loaded or sent behind the current window so that the user does not see it until they close one or more active windows.
- Video ad: similar to a banner ad, except that instead of a static or animated image, actual moving video clips are displayed.
- Map ad: text or graphics linked from, and appearing in or over, a location on an electronic map such as on Google Maps.
- Mobile ad: an SMS text or multi-media message sent to a cell phone.

Why Online?

- An example: $200 iPhone price cut 2007. Search keywords “iphone

Why Online?

- Why is web search potentially more attractive for advertisers than TV spots, newspaper ads or radio spots?
  - Someone who just searched for “*BMW X5*” is infinitely more likely to buy one than a random person watching TV
  - Relevant, yet no obtrusive
  - Most importantly, the advertiser only pays if the customer took an action indicating interest i.e., clicking on the ad (CPC, cost per click).

A little History:

- First radio commercial
  - Apartment rentals in NYC
- Challenge to advertising
  - Could only survive in areas with dense population
  - Need enough potential buyers to pay for content that people would want to listen to
  - So only viable in big cities

Hal Varian, 2009

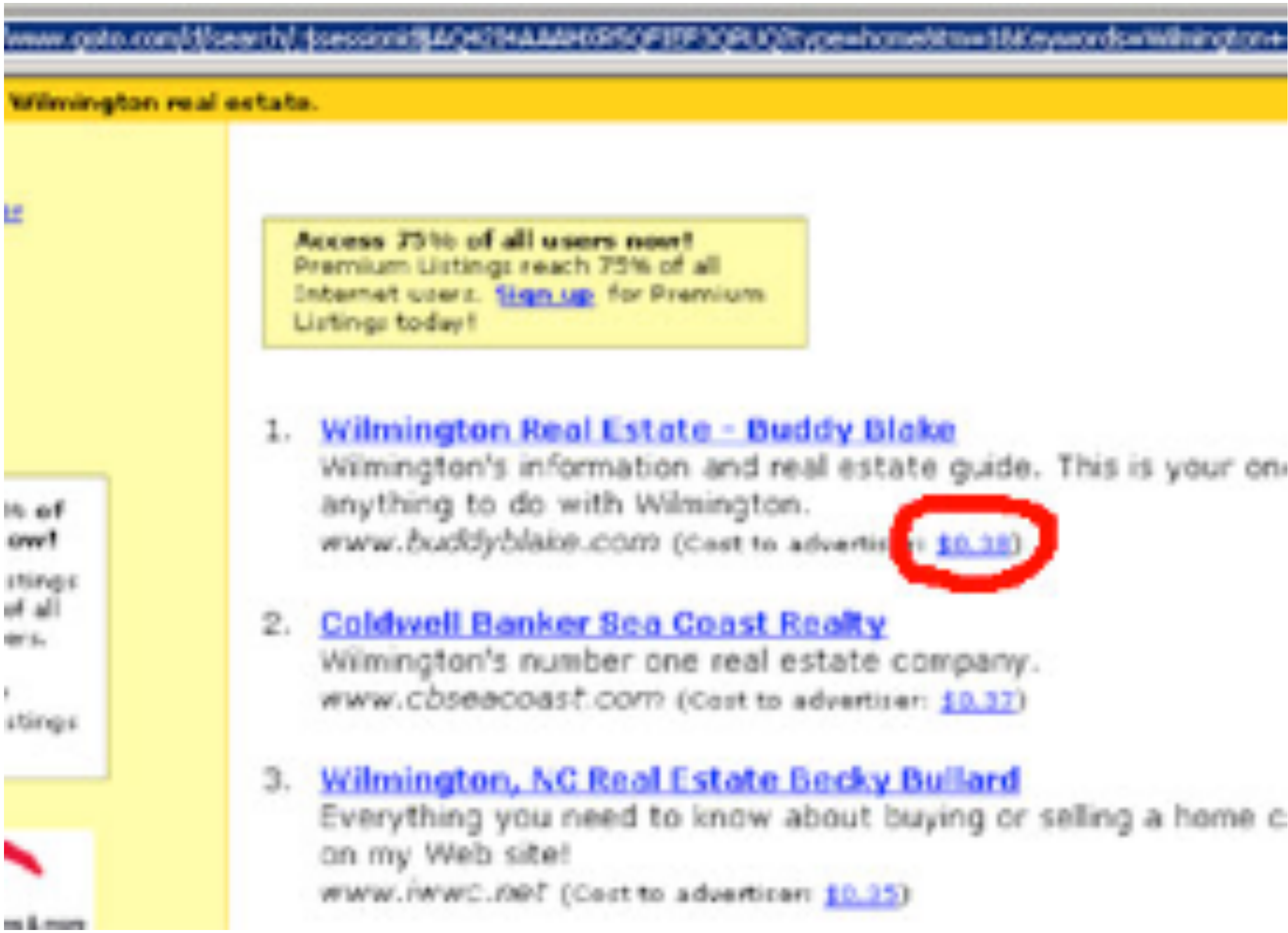
A little History

- rebroadcast nationwide
  - Built national networks
- Didn’t have to make very much per person to cover cost of producing content
  - National brand marketing
- TV took this model over when it arrived

- Hal Varian, 2009

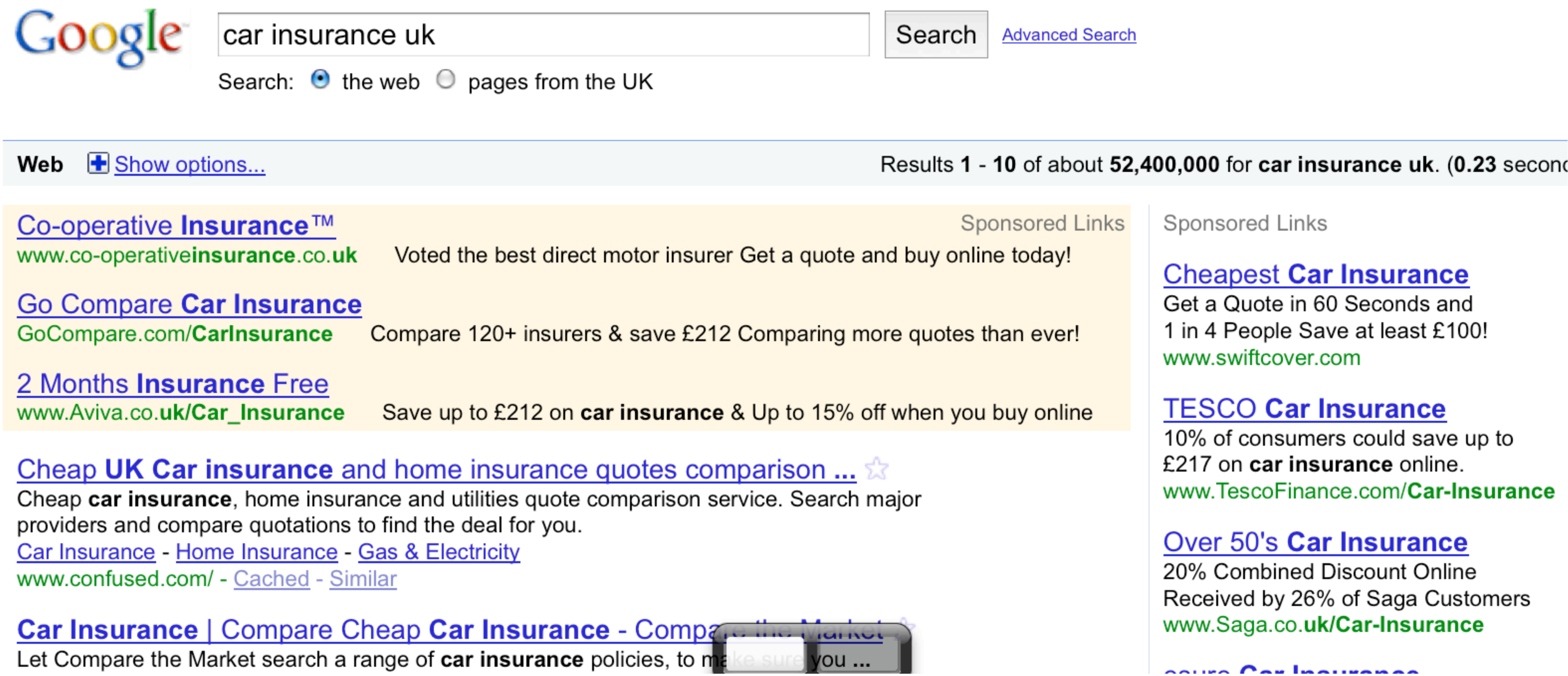
A little History: first gen. search ads

- First generation of search ads Goto (1996): No separation of ads/docs. Just one result!
- *Buddy Blake* bid the maximum ($0.38) for this search.
- He paid $0.38 to Goto every time somebody clicked on the link.



- *Upfront and honest*. No relevance ranking, but Goto did not pretend there was any.

A little History: Second generation of search ads



- Google (2000/2001): strict separation of search results and search ads

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Sponsored Search: auctions

- Ad revenue from auctions
  - Google (2017): $73.8 billion dollars
  - 33 percent of the world's $223.7 billion in digital ad revenue in 2017
- Early (pre-1997!) models
  - Sold per-impression
  - Flat fees to show ads a fixed number of times

Sponsored search

- ***How should we set prices?***
  - *Lots of different keywords*
- Should we post prices (like a store)?
  - Too many keywords and advertisers
- What if there were just one ad slot?
  - Just a single-item auction!
  - Sealed-bid second-price auction appealing
- But we have multiple ad slots
  - How do we deal with that?

Overture (then GoTo and then Yahoo!)

- **Generalized First-Price Auctions (GFP)** (1997)
  - •Advertiser submitted a per click bid
  - •They paid this every time the ad was clicked. Sold one click at a time (rather than 1000 blocks)
  - •Ease of use, more entry
- Example:
  - •2 slots on a page and 3 bidders
  - •1st slot gets 200 clicks (per hour), 2nd 100 clicks
  - •Bidders value per click of \$10, \$4 and \$2

•Suppose if B2 bids \$4.01 she gets a slot and so B1 need not bid more than \$4.02 to get 1st slot. But in this case, B2 will find it worthwhile to increase their bid to \$4.03, etc.

winning bids were posted

- •The price would not be stable?

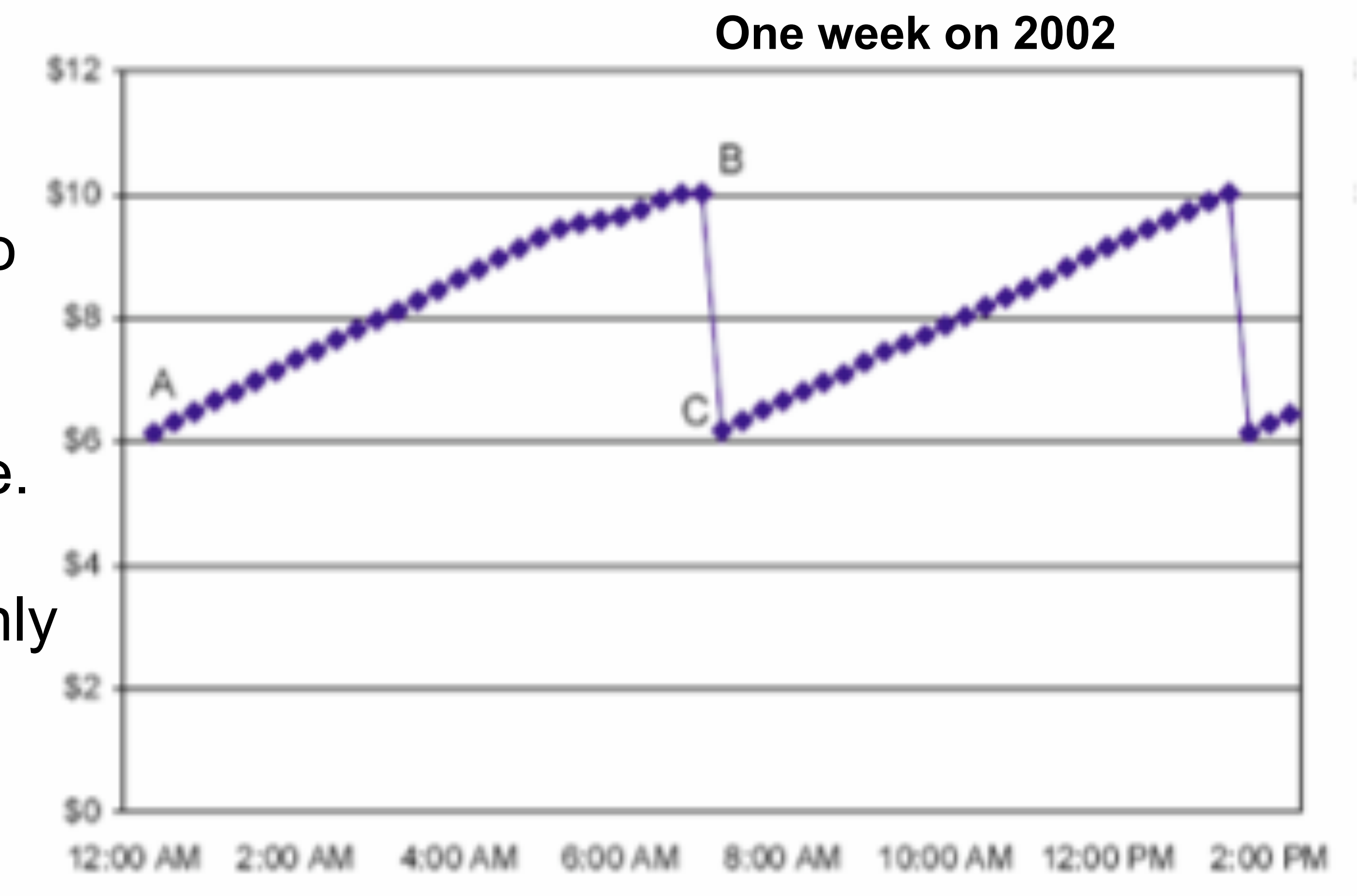
Generalized First Price Auction

- As we learned from Auction Theory that
  - when bidders are charged the full value of their bids, they will generally *underreport*
  - This is what happened here
- Moreover, since the auctions are running *continuously over time*,
  - advertisers constantly adjust their bids by small increments to experiment with the outcome, and
  - to try slightly outbidding competitors

GFP is Unstable

The constant price experimentation led to prices for all queries being updated essentially all the time.

This resulted in a highly turbulent market



- Led to buyer’s remorse and gaming
- A -> B: two bidders raise bid prices to get the first position
- B-> C: One of them reaches its maximum and then goes for the second position

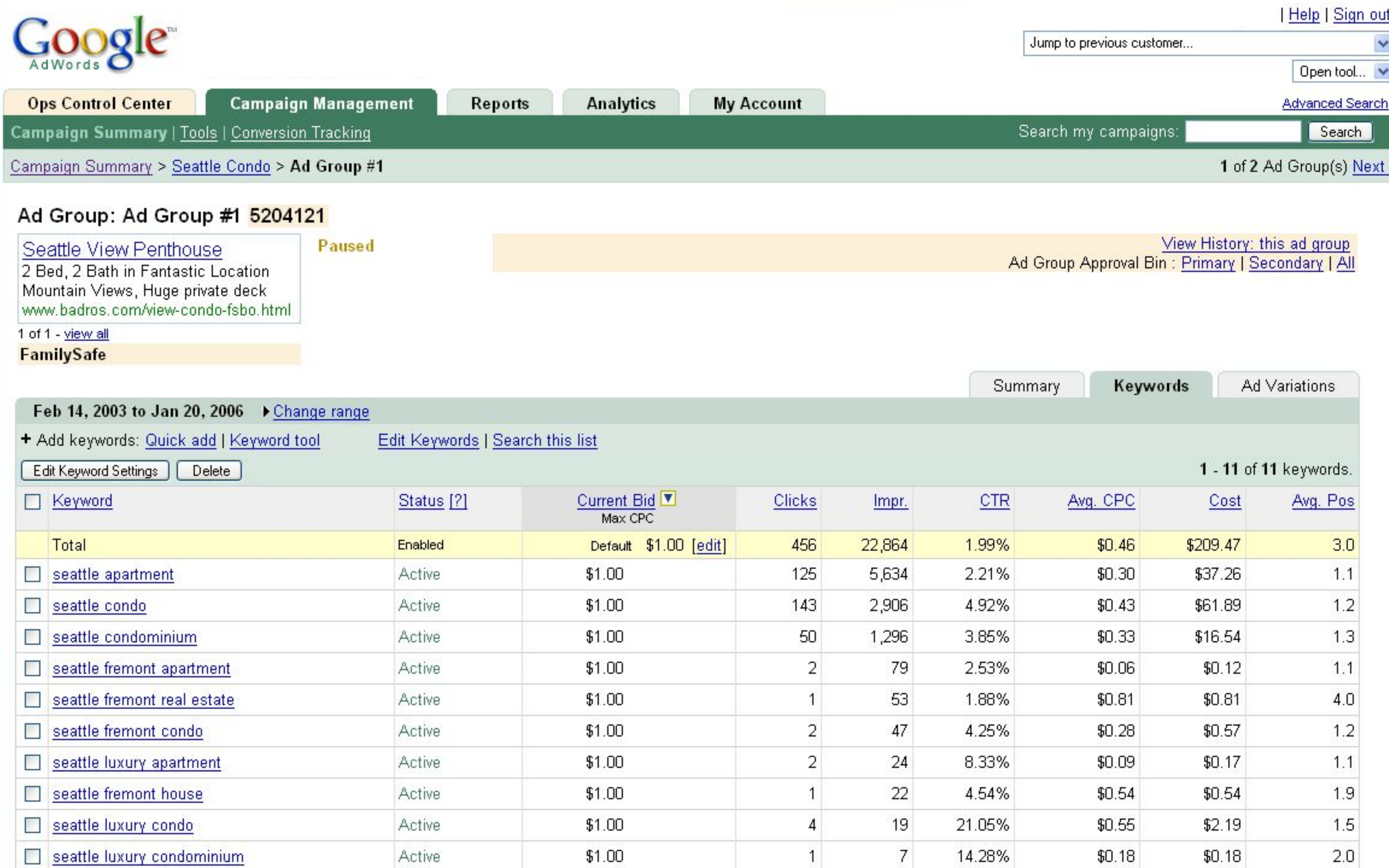
Google's Rules

- Each bidder can specify a rich rule for determining how to bid as a function of the search terms and the site from which the search originates.
  - Google sets a reserve or minimum price for each search term.
- Google estimates the "click-through rate" that each bidder would have if it were listed in the first spot.
- Google ranks the bids according to the product of the clickthrough-rate and the bid; it assigns ad slots in that order.

  Rank Score = Bid Price X Quantity Measure (clickthrough etc)

- Google is paid only if a bidder's link is clicked.
- Also, it receives *the smallest price the bidder could have bid to get its ranking:*
  – **Generalized second price auction**
- Yahoo! switched to this.

Google's Rules: Creating an AdWords Ad



| Help | Sign out

Jump to previous customer...

Open tool...

Ops Control Center | Campaign Management | Reports | Analytics | My Account

Advanced Search

Campaign Summary | Tools | Conversion Tracking

Search my campaigns: Search

Campaign Summary > Seattle Condo > Ad Group #1

1 of 2 Ad Group(s) Next ›

Ad Group: Ad Group #1 5204121

Seattle View Penthouse
2 Bed, 2 Bath in Fantastic Location
Mountain Views, Huge private deck
www.badros.com/view-condo-fsbo.html

Paused

View History: this ad group
Ad Group Approval Bin : Primary | Secondary | All

1 of 1 - view all

FamilySafe

Summary | Keywords | Ad Variations

Feb 14, 2003 to Jan 20, 2006 Change range

+ Add keywords: Quick add | Keyword tool Edit Keywords | Search this list

Edit Keyword Settings Delete

1 - 11 of 11 keywords.

Keyword	Status [?]	Current Bid Max CPC	Clicks	Impr.	CTR	Avg. CPC	Cost	Avg. Pos
Total	Enabled	Default $1.00 [edit]	456	22,864	1.99%	$0.46	$209.47	3.0
seattle apartment	Active	$1.00	125	5,634	2.21%	$0.30	$37.26	1.1
seattle condo	Active	$1.00	143	2,906	4.92%	$0.43	$61.89	1.2
seattle condominium	Active	$1.00	50	1,296	3.85%	$0.33	$16.54	1.3
seattle fremont apartment	Active	$1.00	2	79	2.53%	$0.06	$0.12	1.1
seattle fremont real estate	Active	$1.00	1	53	1.88%	$0.81	$0.81	4.0
seattle fremont condo	Active	$1.00	2	47	4.25%	$0.28	$0.57	1.2
seattle luxury apartment	Active	$1.00	2	24	8.33%	$0.09	$0.17	1.1
seattle fremont house	Active	$1.00	1	22	4.54%	$0.54	$0.54	1.9
seattle luxury condo	Active	$1.00	4	19	21.05%	$0.55	$2.19	1.5
seattle luxury condominium	Active	$1.00	1	7	14.28%	$0.18	$0.18	2.0

Google's Rules: Setting up ad campaign

- Choose your maxCPC (click per cost)
  - Value of a click = prob of sale * profit per sale
  - Can estimate how many clicks you will get
- Set daily budget

Example (no CTR adjustment)

- If 3 bidders, "*act truthfully*", bid $10, $4 and $2.
- Payments will be $4 and $2 so their total payments are $800 and $200 respectively.
- No incentive to change their bids so it is rather stable

Bidder B1 pays: $ 4 x 200 = $800

Bidder B2 pays: $ 2 x 100 = $200

Revenues under GSP is $1,000

Example (with CTR)

- I bid $1 and have an estimated click-through rate of .50.
- You have bid $2 and have an estimated click through rate of .2.
- The reserve price is 0.1.
- My score is .5; yours is .4, so my ad ranks first.
  - I could have won with a bid as low as .81, so that is what I pay if my link is clicked.
  - You could have had your slot for as low as .1, so that is what you pay if your link is clicked.

First price (GFP) vs. Second Price GSP

- Generalized First Price Auction is unstable
- Second Price Auction (Single Item)
  - Truth-telling is the *dominant strategy* – (i.e., no buyer's regret when bidding true value)
- Generalized 2ndPrice (GSP) Auction (Multiple items)
  - Tailored to the unique environment of online ads (ranked items)
  - BUT truth-telling is NOT a dominant strategy
- Recall that, *dominant strategy* is the one that is better than another strategy for one player, no matter how that player's opponents may play

Truth-telling is not a dominant strategy under GSP strategy

- Intuition: sometimes, bid below your true valuation.
  You may get less traffic, but you'll earn greater profits
- Suppose there are 3 bidders but 2 positions.
  - Positions have click-through rates 100 and 80.
  - Bidder 3's valuation is $10

Bidder	Bid
1	$8
2	$5

Option 1: Bidder 3 bids $10 pays $8

Option 1 payoff: ($10-$8)*100 = $200

Option 2: Bidder 3 bids $6 pays $5

Option 2 payoff: ($10-$5)*80 = $400

$400>$200. So Bidder 3 should place a bid below its valuation.

VCG Auctions

- Vickery-Clarke-Groves Auction (not used yet)
  - With one slot, identical to GSP auction
  - VCG charges bidder *i* the *harm* they impose on others (their decrease in value of clicks because of *i*'s presence).
  - Truth-telling is a dominant strategy under the VCG Auction

VCG Auctions – An example

$$\text{Payment\_VCG}_i = (\text{Click}_i - \text{Click}_{i+1}) * \text{Bid}_{i+1} + \text{Payment\_VCG}_{i+1}$$

- Three bidders bid $10, $4 and $2 respectively
- B2 still pays $200. B1 now pays $600 (rather than $800) = $400 (harm on B2, pushed to second)+$200 (harm on B3, pushed out)
- In this case, VCG has lower revenues when bidders tell truth
- But under GSP, bidders may not tell truth.

Sponsored Links

Bid = $ 10
CPC = $ 4

Postgraduate Degrees (UK)
Full & part-time/distance & online
Business, Management & Law Courses
www.rgu.ac.uk/abs/postgraduate

200 clicks per hour

Bid = $ 4
CPC = $ 2

Msc
Masters Degree by Distance Learning
Browse and Apply online now!
www.rdi.co.uk/Masters

100 clicks per hour

Bidder B1 pays: (200-100)x $ 4 + 200 = $600

Bidder B2 pays: (100-0) x $ 2 + 0 = $200

Revenues under VCG is $800 < GSP

Factors affecting revenue for search engine

$$\text{Monetization (RPM)} = \frac{\text{Revenue}}{\text{Queries}} \times (1K)$$

$$= \frac{\text{Revenue}}{\text{Clicks}} \times \frac{\text{Clicks}}{\text{Queries}}$$

$$= \frac{\text{Revenue}}{\text{Clicks}} \times \frac{\text{Queries w/ Ads}}{\text{Queries}} \times \frac{\text{Ads}}{\text{Queries w/ Ads}} \times \frac{\text{Clicks}}{\text{Ads}}$$

$$= \underbrace{\textit{CPC}}_{\text{Price}} \times \underbrace{\textit{Coverage} \times \textit{Depth}}_{\text{Quantity}} \times \underbrace{\textit{CTR per Ad}}_{\text{Quality}}$$

Hal Varian, 2009

Increasing revenue of search engines

- Increase CPC
  - Create higher conversion prob for advertiser
  - Capture more value by increasing competition
- Increasing coverage
  - Get more keywords
  - Match more broadly (affects CTR)
- Increase depth
  - Get more advertisers, more ads
- Increase CTR
  - Show more relevant ads
  - Show high quality ads in prominent positions

Engineering challenge: Predicting CTR

- Dizzying set of factors could affect clickthrough
  - Country, time of day, targeted text vs query, …
- How does one automatically figure out which factor is more relevant?
  - How to update model quickly in face of change
  - How do you estimate CTR for not-yet-shown ads?
- Huge machine learning model for predicting clicks and other aspects of ad quality

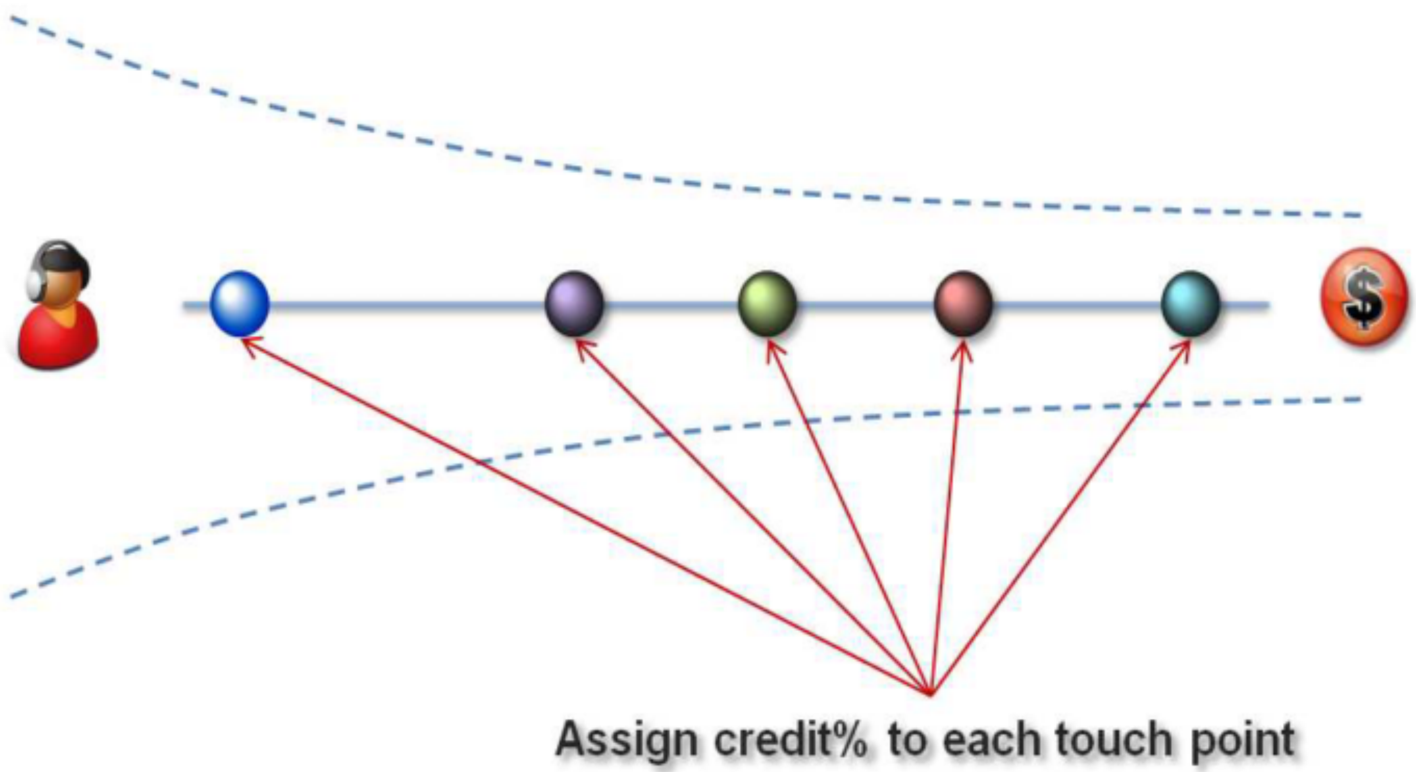
Future Direction: Behavioral targeting

- **Audience Extension**: Extending valuable inventory by labeling an audience in one environment and recognizing that audience elsewhere. Example: readers of financial site see ads for Mercedes on financial site and extended elsewhere
- **Retargeting**: Recognizing consumers who have already seen or expressed interest in a product or brand and showing targeted advertisements in different media or locations
  - Search Retargeting: Showing search-relevant ads on a page visited immediately after a search execution
- **Audience Amplification**: Finding similar users to those you're already targeting
  - E.g., Amazon collaborative filtering
- **Surround Session**: Enable a single advertiser to own ad space for a user's entire session

Outline

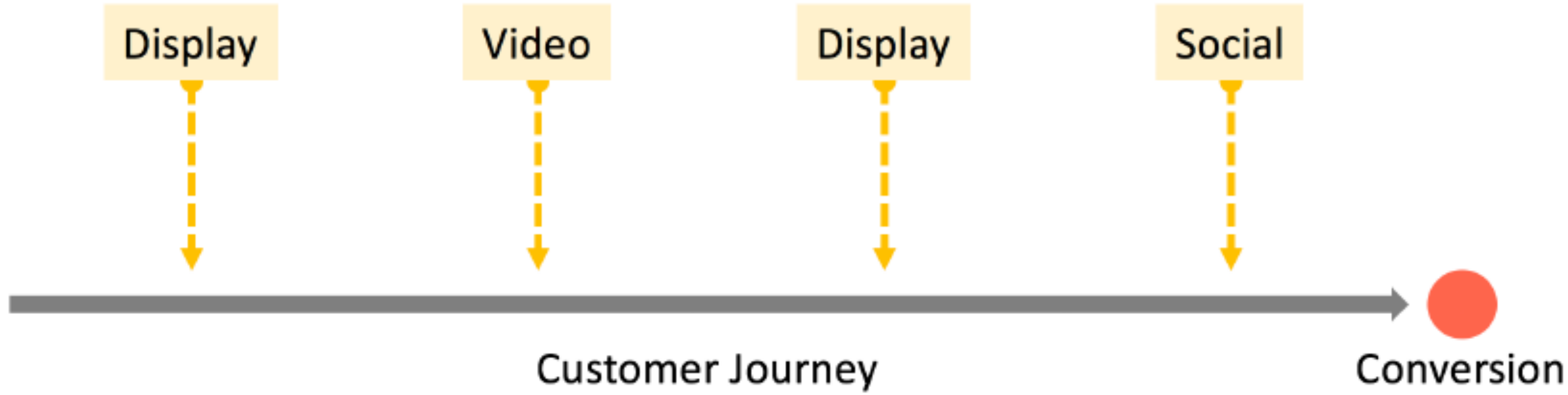
- Introduction and history
- Sponsored Search
  - First-price auction
  - Second-price auction
  - VCG auction
- Conversion Attribution

Conversion Attribution



- Assign credit% to each channel according to contribution
- Current industrial solution: last-touch attribution

Heuristics-based Attribution



Model	Attribution			
Last Touch	0%	0%	0%	100%
First Touch	100%	0%	0%	0%
Linear	25%	25%	25%	25%
Time Decay	10%	20%	30%	40%
Position Based	40%	10%	10%	40%

A Good Attribution Model

- Fairness
  – Reward an individual channel in accordance with its ability to affect the likelihood of conversion
- Data driven
  – Using ad touch and conversion data for each campaign to build its model
- Interpretability
  – Generally accepted by all parties

[Dalessandro et al. Casually Motivated Attribution for Online Advertising. ADKDD 11]

Bagged Logistic Regression

Display	Search	Mobile	Email	Social	Convert?
1	1	0	0	1	1
1	0	1	1	1	0
0	1	0	1	0	1
0	0	1	1	1	0

- For M iterations
  - Sample 50% data instances and 50% features
  - Train a logistic regression and record the weights
- Average the feature weights

[Shao et al. Data-driven multi-touch attribution models. KDD 11]

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Shapley Value based Attribution

- Coalition game:
  - the attribution that player *i* gets

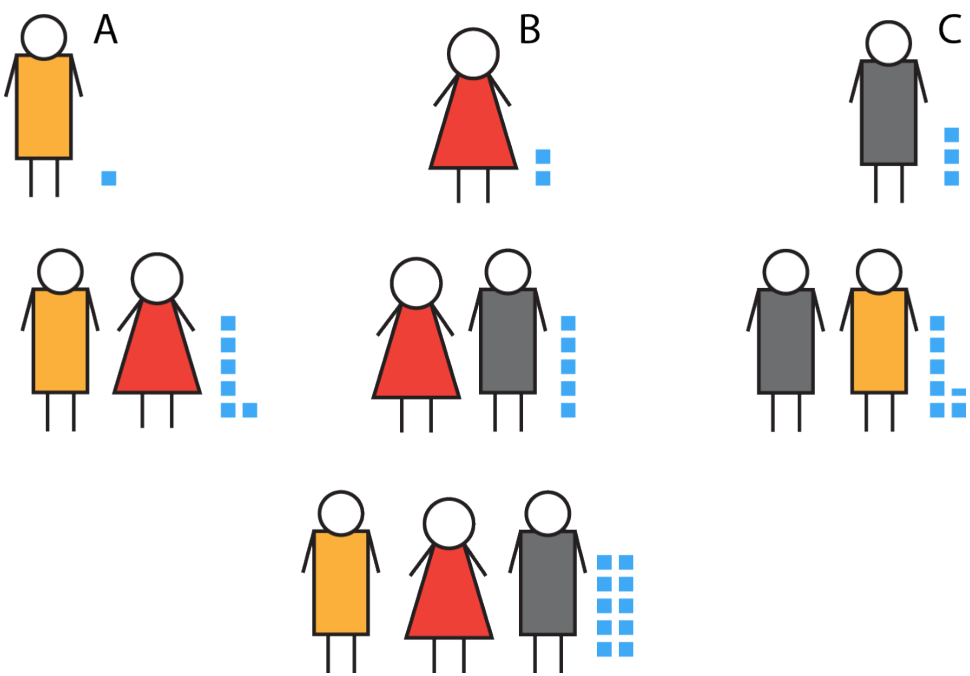
$$\phi_i(v) = \frac{1}{\text{number of players}} \sum_{\text{coalitions excluding } i} \frac{\text{marginal contribution of } i \text{ to coalition}}{\text{number of coalitions excluding } i \text{ of this size}}$$

Imagine the coalition being formed one player at a time, with each player demanding their marginal contribution $v(S\cup\{i\}) - v(S)$ as a fair compensation, and then for each player take the average of this contribution over the possible different permutations in which the coalition can be formed. *V(S):* the value from a subset coalitions *S*

https://en.wikipedia.org/wiki/Shapley_value

Shapley Value based Attribution

- Coalition game
  - How much does a player contribute in the game



[Fig source: https://pjdelta.wordpress.com/2014/08/10/group-project-how-much-did-i-contribute/]

The idea: form the whole group by adding people one-by-one, and see how much more output is generated by each additional person
Consider the sequence A-B-C. the marginal contribution of (A,B,C) is (10,50,40).
Alternative sequences:

B-C-A -> (A,B,C) = (50,20,30);
C-A-B -> (A,B,C) = (35,35,30);
A-C-B-> (A,B,C) =(10,35,55);
C-B-A -> (A,B,C) =(50,20,30);
B-A-C -> (A,B,C) =(40,20,40).

Now, take average of the contributions in these six sequences:
A=(10+50+35+10+50+40)/6=195/6=**32.5**
B=(50+20+35+35+20+20)/6=180/6=**30**
C=(40+30+30+55+30+40)/6=225/6=**37.5**
These are called the **Shapley Values** of the three people.

Shapley Value based Attribution

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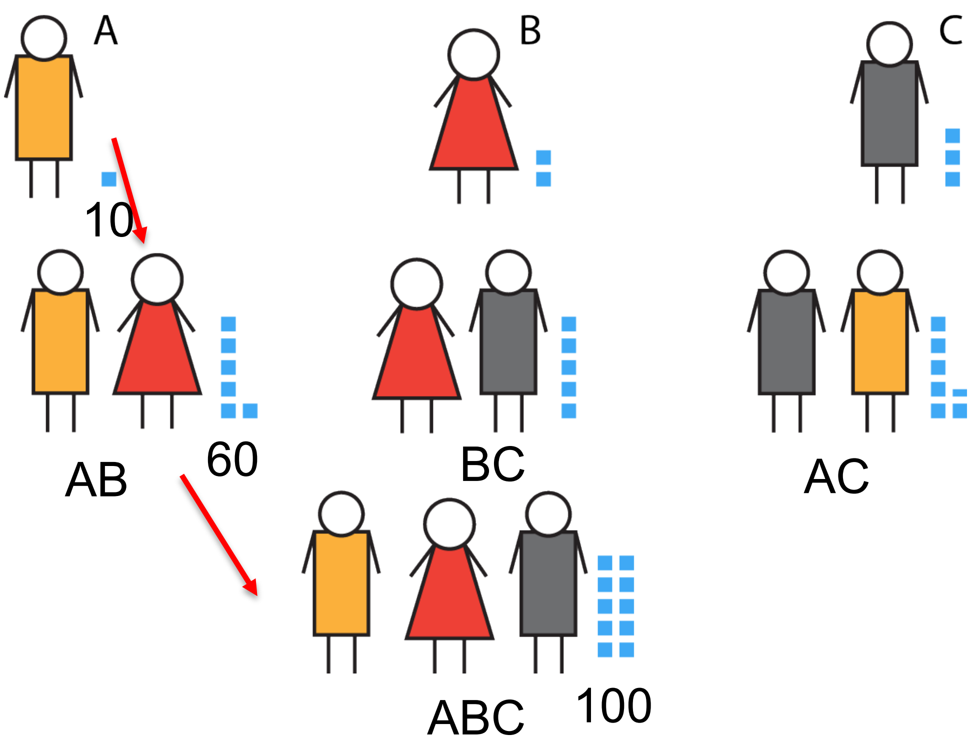

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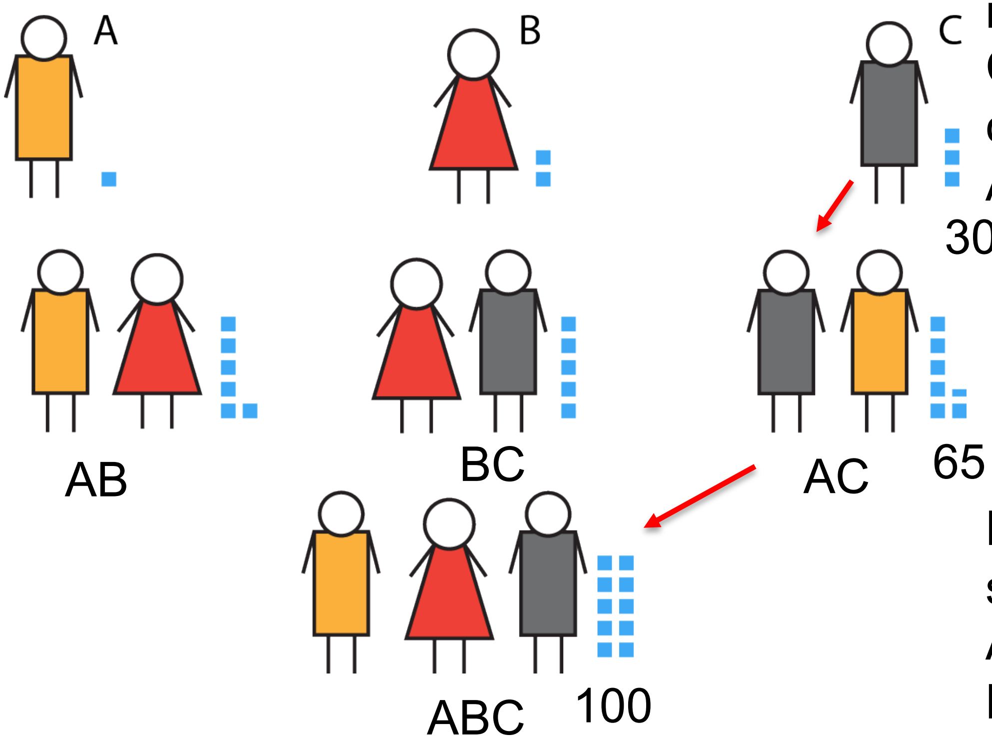

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