SF2955

Computer Intensive Methods In Mathematical Statistics

Project 1

Monte Carlo-based mobility tracking in cellular networks

Baixi Jin
20000607-T005
baixi@kth.se

Hanmo Chen
19990904-T072
hanmoc@kth.se

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1 A hidden Markov model for mobility tracking

1.1 Motion model

Consider a target moving in R^2, the motion model can be formulated as follows:

$$\boldsymbol{X}_{n+1} = \boldsymbol{\Phi}\boldsymbol{X}_{n+1} + \boldsymbol{\Psi}_z\boldsymbol{Z}_n + \boldsymbol{\Psi}_\omega\boldsymbol{W}_{n+1} \tag{1}$$

1.1.1 $\{\boldsymbol{X}_n\}_{n\in N^*}$ is not a Markov chain

According to fundamental Markov property, if the statement holds, $\boldsymbol{X}_{n+1}$ should be conditionally independent of $(\boldsymbol{X}_0, \boldsymbol{X}_1, ...)$ given $\boldsymbol{X}_n$. However, in this case, $\boldsymbol{Z}_n$ is a Markov chain and $\boldsymbol{Z}_{n+1}$ is related with $\boldsymbol{Z}_n$. Thus, $\{\boldsymbol{X}_n\}$ is not conditionally independent, so it's not a Markov chain.

1.1.2 $\left\{\tilde{\boldsymbol{X}}_n\right\}_{n\in N^*}$ is a Markov chain

$$\tilde{\boldsymbol{X}}_n = (\boldsymbol{X}_n^T, \boldsymbol{Z}_n^T)^T \tag{2}$$

First of all, $\{\boldsymbol{W}_{n+1}\}$ are independent noise variables of $(\boldsymbol{X}_0, \boldsymbol{X}_1, ...)$. Secondly, if $\tilde{\boldsymbol{X}}_n$ is given, it means both $\boldsymbol{X}_n$ and $\boldsymbol{Z}_n$ are known. Since $\boldsymbol{Z}_{n+1}$ only depends on $\boldsymbol{Z}_n$ and is therefore conditionally independent of $(\tilde{\boldsymbol{X}}_0, \tilde{\boldsymbol{X}}_1, ...)$, $\left\{\tilde{\boldsymbol{X}}_n\right\}_{n\in N^*}$ is a Markov chain.

1.1.3 simulation of the trajectory

We set m=1000. Based on equation (1), the trajectory plot can be obtained.[1]

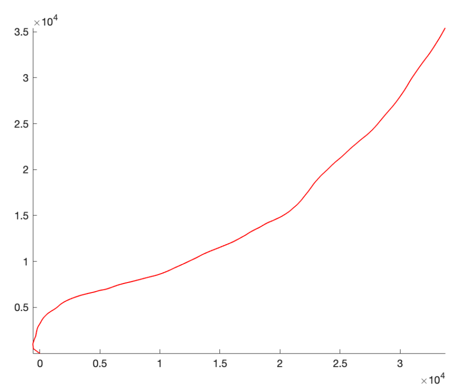

Figure 1: trajectory of the target under m=1000

1.2 Observation model

As the target is moving, it can be detected by received signal strength indication(RSSI) from $s = 6$ basic stations(BS). Locations of BS are denoted by $\{\boldsymbol{\pi}_l\}_{l=1}^{s}$ and are already known. The RSSI that the mobile unit received from the lth BS can be modeled as

$$Y_n^l = v - 10\eta \log_{10} \left\|(X_n^1, X_n^2)^T - \pi_l\right\| + V_n^l \tag{3}$$

where V_n^l are independent Gaussian noises and v,η are constants. $\|\bullet\|$ denotes the Euclidean distance. The RSSIs received at time n from all BSs are denoted by $\boldsymbol{Y}_n = (Y_n^1, ..., Y_n^s)^T$.

1.2.1 $\left\{(\tilde{\boldsymbol{X}}_n, \boldsymbol{Y}_n)\right\}_{n\in N^*}$ is a hidden Markov model

As known before, $\left\{\tilde{\boldsymbol{X}}_n\right\}$ is a Markov chain. But in the observation model, the state $\left\{\tilde{\boldsymbol{X}}_n\right\}$ is not directly visible. However, the output $\{\boldsymbol{Y}_n\}_{n\in N^*}$ which is dependent on $\left\{\tilde{\boldsymbol{X}}_n\right\}_{n\in N^*}$ is visible. Since V_n^l is independent of time n, $\boldsymbol{Y}_n$ is only dependent on (X_n^1, X_n^2). Thus $\{\boldsymbol{Y}_n\}_{n\in N^*}$ are conditionally independent given $\left\{\tilde{\boldsymbol{X}}_n\right\}_{n\in N^*}$. i.e.

$$P(\boldsymbol{Y}_0, \boldsymbol{Y}_1, ...|\tilde{\boldsymbol{X}}_0, \tilde{\boldsymbol{X}}_1, ...) = P(\boldsymbol{Y}_0|\tilde{\boldsymbol{X}}_0)P(\boldsymbol{Y}_1|\tilde{\boldsymbol{X}}_1)... \tag{4}$$

$\left\{(\tilde{\boldsymbol{X}}_n, \boldsymbol{Y}_n)\right\}_{n\in N^*}$ is a hidden Markov model.

1.2.2 The transition density

$$p(y_n|\tilde{x_n}) = p(y_n|x_n, z_n) = p(y_n|x_n) = p(y_n|x_n^1, x_n^2) \tag{5}$$

With (X_m^1, X_m^2) given, the only variable left in y_n is v_n^l. According to equation (5),

$$\boldsymbol{Y}_n^l|\tilde{\boldsymbol{X}}_n \sim\ N(v - 10\eta \log_{10} \left\|(X_n^1, X_n^2)^T - \pi_l\right\|, \varsigma^2) \tag{6}$$

Hence,

$$\boldsymbol{Y}_n|\tilde{\boldsymbol{X}}_n \sim\ N(v - 10\eta \log_{10} \left\|(X_n^1, X_n^2)^T - \pi_1\right\|, ..., v - 10\eta \log_{10} \left\|(X_n^1, X_n^2)^T - \pi_6\right\|, \varsigma^2 \boldsymbol{I}_{6\times 6}) \tag{7}$$

$p(y_n|\tilde{x_n})$ is the density function of the multivariate Gaussian distribution indicated in equation (7).

2 Mobility Tracking Using SMC Methods

Sequential Monte Carlo(SMC) methods can be applied into estimation of positions of the target. Expected positions are denoted by

$$\tau_n^1 = E[X_n^1|\boldsymbol{Y}_{0:n} = \boldsymbol{y}_{0:n}] \tag{8}$$

and

$$\tau_n^2 = E[X_n^2|\boldsymbol{Y}_{0:n} = \boldsymbol{y}_{0:n}] \tag{9}$$

More specifically, the goal is to produce a sequence $(\tilde{X}_{0:n}^i, \omega_n(\tilde{X}_{0:n}^i))_{i=1}^N$, yet only last particle generation of $(\tilde{X}_{0:n}^i)_{i=1}^N$ will be used. Since

$$\tau_n^i = \int x_n^i f(x_n^i|\boldsymbol{y_{0:n}}) dx_n^i \tag{10}$$

This way, τ_n^1 and τ_n^2 can be approximated. Sampling from the densities $f(\tilde{\boldsymbol{x}}_n|\boldsymbol{y}_{0:n})$ can be achieved by implementing sequential importance sampling(SIS) and sequential importance sampling with resampling (SISR) algorithms, respectively.

2.1 Sequential Importance Sampling (SIS)

First, we use SIS to sample. In our experiment, the particle sample size N = 10000. In order to obtain better visualization, we have plotted the estimated trajectory of the plane together with locations of basic stations in figure 2. Obviously, the result is not so satisfying.

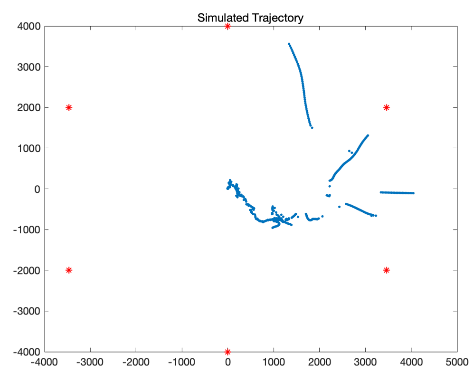

Figure 2: estimates of trajectory of the plane

We have also plotted histograms of the importance weights under n=50, 100 and 200 in figure 3. It is clearly seen that weight degeneration is a universal problem with the SIS method since the weight distribution is skew.

The efficient sample size can also be given by $N/(1 + CV_n^2)$, where

$$CV_n = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(N\frac{\omega_n^i}{\Omega_n} - 1)^2} \tag{11}$$

The lower CV_n is, the greater efficient sample size is. The efficient sample sizes at n=50, 100 and 200 are 6227, 2741 and 121, respectively. Figure 4 shows the efficient sample sizes at time points $0 : 500$.

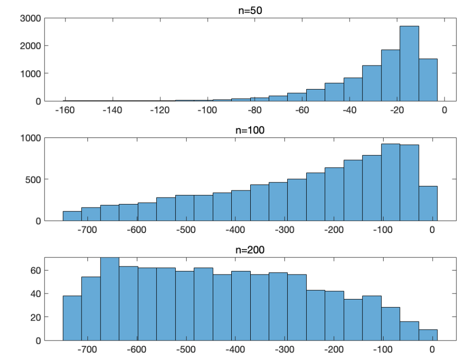

Figure 3: importance weights at n=50, 100, 200

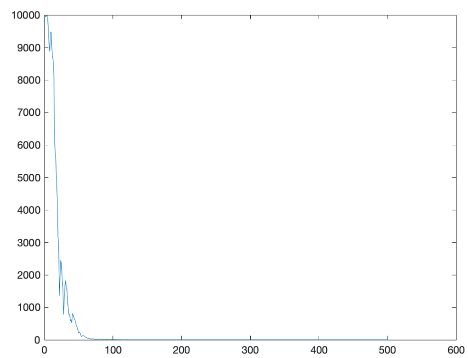

Figure 4: efficient sample sizes at n=0,...,500

2.2 Sequential Importance Sampling with Resampling(SISR)

Compared to SIS, we are now implementing SISR algorithm by simply adding a resampling procedure. In our experiment, the particle sample size N = 10000. In order to obtain better visualization, we have plotted the estimated trajectory of the plane together with locations of basic stations in Figure 5.

We have also plotted histograms of the importance weights under n=50, 100 and 200 in Figure 6. The problem of weight degeneration becomes less severe compared to SIS methods. Among these three time points, n=200 is the best with respect to skewness of weight distribution.

The efficient sample sizes at n=50, 100, 200 are 8813, 8670 and 9187, respectively. Figure 7 shows the efficient sample sizes at time points 0 : 500.

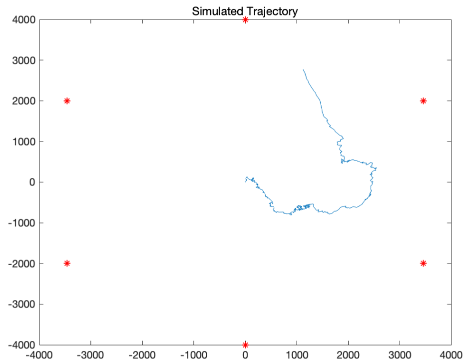

Figure 5: estimates of trajectory of the plane

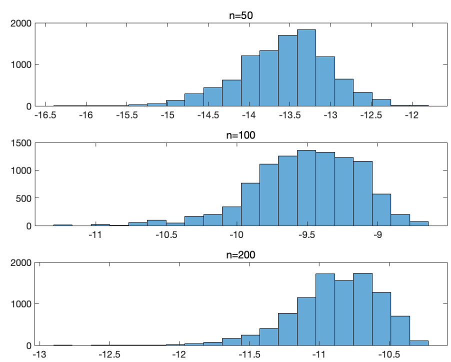

Figure 6: importance weights at n=50, 100, 200

3 SMC-based model calibration

Assume all parameters except ς are calibrated. We calibrate ς by maximizing the normalized log-likelihood function

$$\varsigma \mapsto l_m(\varsigma, \boldsymbol{y}_{0:m}) = m^{-1} \ln L_m(\varsigma, \boldsymbol{y}_{0:m}) \tag{12}$$

where

$$L_m(\varsigma, \boldsymbol{y}_{0:m}) = f_\varsigma(\boldsymbol{y}_{0:m}) \tag{13}$$

In the experiment, we first design the grid ς_j from 0.5 to 3 with increment of 0.05. For each ς_j, the SISR algorithm in Problem 4 is implemented and log-likelihood is also estimated.

$$\hat{\zeta}_m = 2.20 \tag{14}$$

The estimated trajectory under $\hat{\varsigma}_m$ is displayed in Figure 9.

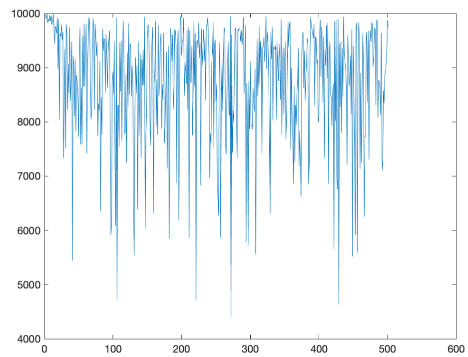

Figure 7: efficient sample sizes at n=0,...,500

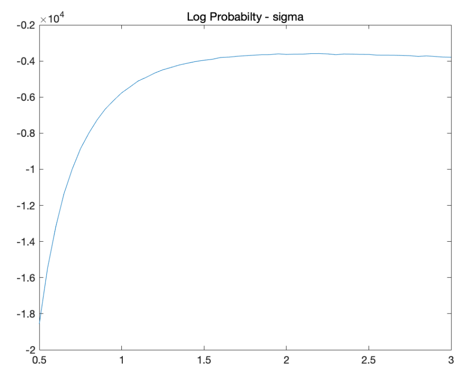

Figure 8: Log Likelihood Estimated

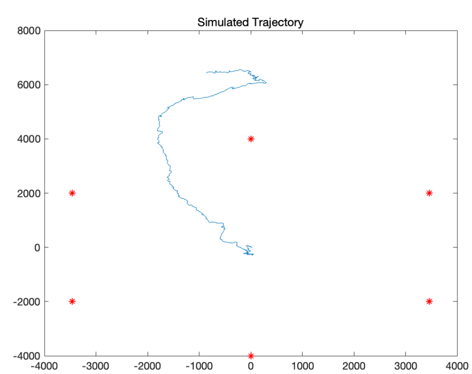

Figure 9: estimates of trajectory of the plane