SF2955 - HA1

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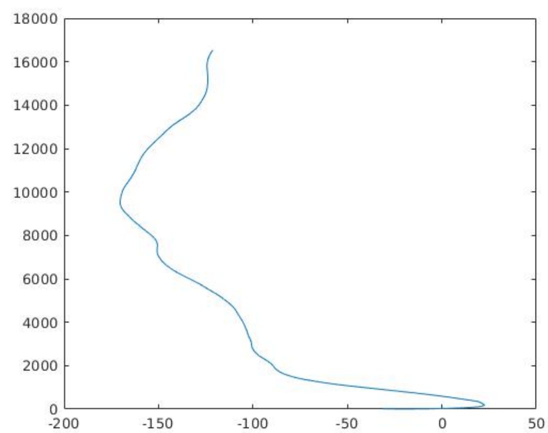
1 Motion model - Problem 1

Target is moving according to:

$$X_{n+1} = \phi X_n + \psi_z Z_n + \psi_w W_{n+1} \tag{1}$$

A Markov chain is a sequence of possible events where the next event depends only on the state for the previous event not any events before that. Since $\{Z\}_n$ is a bivariate Markov chain and depends on the previous step $\{Z\}_{n-1}$ we get that X_{n+1} does not only depend on X_n, Z_n and W_{n+1} but also on the previous step Z_{n-1}. This gives that $\{X\}_n$ is not a Markov chain.
Introducing $\tilde{X}_n = (X_n^T, Z_n^T)^T$ solves this problem since $\tilde{X}_n$ depends on the previous events X_{n-1} and Z_{n-1} where Z_{n-1} is given by Z_n and can be used to calculate X_n . So $\tilde{X}_n$ full fills the criteria for a Markov chain.



Figur 1: Trajectory of moving target for m = 200. The Result looks like a reasonable trajectory.

2 Observation model - Problem 2

Task: Calculate the transition density $p(y_n|\tilde{x}_n)$ of $Y_n|\tilde{X}_n$.

By definition of density functions:

$$p(y_n|\tilde{x}_n) = \frac{\partial}{\partial t} F_{Y_n|\tilde{X}_n}(y_n|\tilde{x}_n) \tag{2}$$

The distribution function can be written as the following probability:

$$F_{Y_n|\tilde{X}_n}(y_n|\tilde{x}_n) = P(Y_n < y_n|\tilde{X}_n = \tilde{x}_n) = P(v - 10\eta log_{10}||(X_n^1, X_n^2) - \pi_l|| + V_n^l < y_n^l) \tag{3}$$

Know that $\{V_n^l\}_{l=1}^s$ are independent Gaussian noise variables with mean zero and standard deviation ς. then:

$$\begin{aligned} & P(v - 10\eta log_{10}||(X_n^1, X_n^2) - \pi_l|| + V_n^l < y_n^l) & (4)\\ &= P(V_n^l < y_n^l - (v - 10\eta log_{10}||(X_n^1, X_n^2) - \pi_l||)) & (5)\\ &= P(\gamma_n^l < \frac{y_n^l - (v - 10\eta log_{10}||(X_n^1, X_n^2) - \pi_l||)}{\sqrt{\varsigma}}) & (6)\end{aligned}$$

Set $\mu_n^l = v - 10\eta log_{10}||(X_n^1, X_n^2) - \pi_l||$ Where the distribution of γ is the standard normal distribution: $N(0,1)$
This gives:

$$F_{Y_n|\tilde{X}_n}(y_n|\tilde{x}_n) = P(\gamma_n^l < \frac{y_n^l - \mu_n^l}{\sqrt{\varsigma}}) = \Phi(\frac{y_n^l - \mu_n^l}{\sqrt{\varsigma}}) \tag{7}$$

The derivation of this is the the transition density function. Since it is normal distributed will the derivation also be normal distributed and we get the transition density function to be:

$$\Phi(\frac{y_n^l - \mu_n^l}{\sqrt{\varsigma}}) \tag{8}$$

3 Mobility tracking using SMC methods

Problem 3

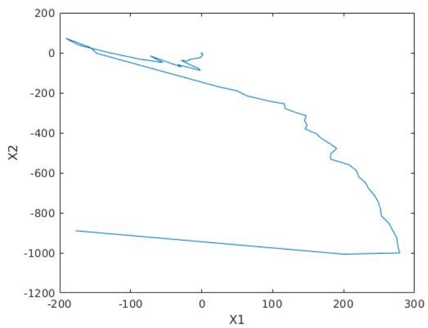
Implementation of the SIS algorithm is made to provide estimates of $\{(\tau_n^1, \tau_n^2)\}_{n=0}^m$. The prior transition density is used as proposal kernel and N = 10000 particles is made. To calculate the efficient sample size following formulas where used:

$$\textit{Efficient sample size} = \frac{N}{1 + CV_N^2} \tag{9}$$

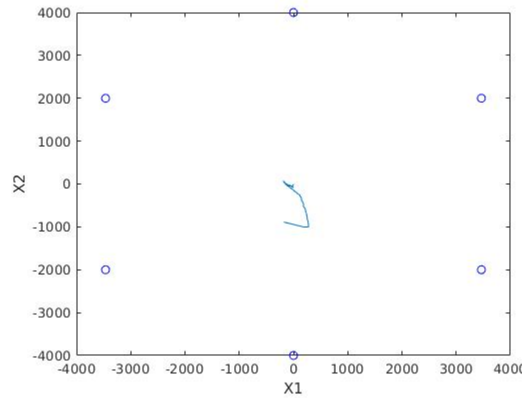
Where CV is the coefficient of variation.

$$CV_N = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(N\frac{\omega_n^i}{\Omega_n} - 1\right)^2} \tag{10}$$

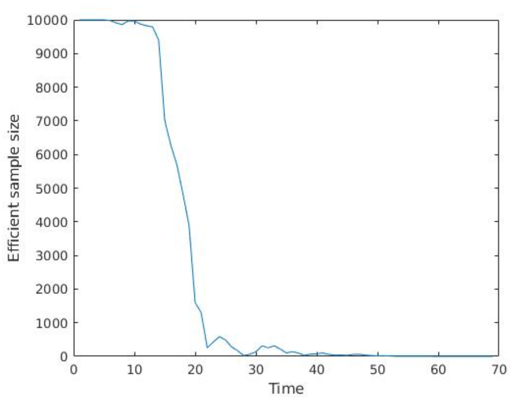
The results are shown in figures (a)-(d).



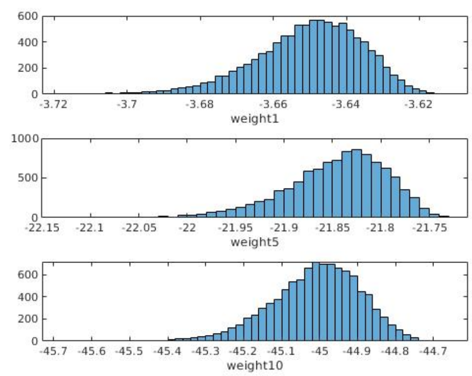
(a) Estimated trajectory of moving target



(b) Trajectory of moving target with Basis Stations.



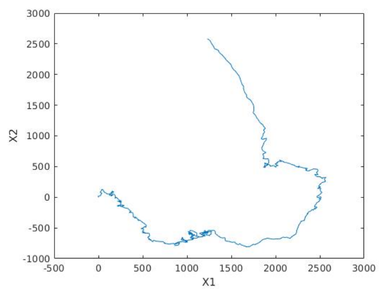
(c) Efficient sample size.



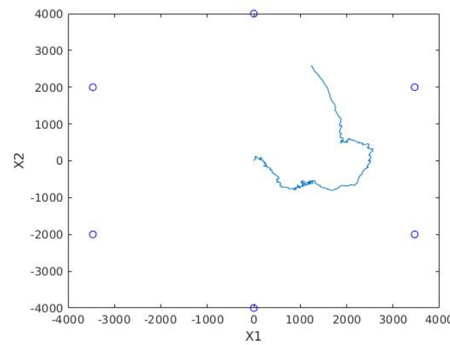
(d) Histogram of importance weights.

Problem 4

The SISR algorithm is implemented for the same densities as before and with N = 10000 particles. The reason to use SISR instead of SIS is to get weights that is not degenerated. The most probable driving command is also calculated and investigated.



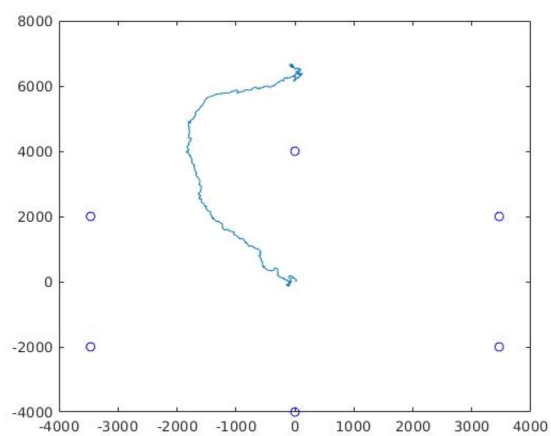
(a) Estimated trajectory of moving target



(b) Trajectory of moving target with Basis Stations.

SMC-based model calibration - Problem 5

In this task is an maximum likelihood estimation made to find best value for ς. The previus SISR algorithm is used on the space (0,3) of ς with step size 0.2. The resulting best value for ς is 3. Trajectory can be seen in figure 5.



Figur 4: Trajectory for estimated τ for $\varsigma = 3$