$FV = C_0(1+\frac{r}{m})^{m \times T}$ $PV = \frac{C}{r}$ $PV = \frac{C}{r-g}$

ordinary Annuity

$PV = \frac{C}{r}[1 - \frac{1}{(1+r)^T}]$

$PV = \frac{C}{r-g}[1 - (\frac{1+g}{1+r})^T]$

$FV = \frac{C}{r}[(1+r)^T - 1]$

Annuity due:

$PV = \frac{C}{r}[1 - \frac{1}{(1+r)^n}](1+r)$

$FV = \frac{C}{r}[(1+r)^n - 1](1+r)$

Characteristics	1 AAR	2 Payback Period	3 Discounted Payback Period	4 NPV	5 PI	6 IRR	7 MIRR
(+) Takes into account the time value of money	No	No	Yes	Yes	Yes	Yes	Yes
(+) Uses "real discount rate" to discount cash flows	NA	NA	Yes	Yes	Yes	No	Yes
(+) Towards firm value maximization	No	No	No	Yes	Yes(*)	Yes(*)	Yes(*)
(+) Easy to calculate	Yes	Yes	Yes(*)	Yes(*)	Yes(*)	Yes	Yes(*)
(Info needed to calculate)	BV Inv, NI	CF	CF, discount rate	CF, discount rate	CF, discount rate	CF	CF, discount rate
(-) Ignores later cash flows	No	Yes	Yes	No	No	No	No
(-) Arbitrary cutoff	Yes	Yes	Yes	No	No	No	No

AAR = Average Net Income / Average Book Value of Investment

over Invesment's lifetime

↗ Cost − Acc. Depreciation ★ 3年: $\frac{90+60+30+0}{4}$

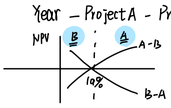
Sales
− Expense
E.B.D
− Depreciation
E.B.T
− Taxes ≥ × t
NI

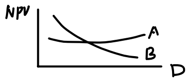
MIRR: $FV_{Initial} = FV_{CFs}$

eg. $CF_0(1+MIRR)^3 = CF_1(1+r)^2 + CF_2(1+r)^1 + CF_3$

8. Crossover Rate $0 = \sum_{i=0}^{n} \frac{\Delta CF_i}{(1+IRR)^i}$ $\Delta CF_i = CF_{Ai} - CF_{Bi}$

Year − Project A − Project B − (A−B) − (B−A) ⇒ NPV = 0

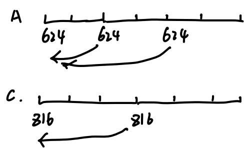




PI = PV(Cash Inflow) / PV(Cash Outflow)

Estimating Cash flow

Chain Replication Approach



Equivalent Annual NPV (EANPV) approach

$EANPV = \frac{\text{Project NPV} \cdot k}{[1 - \frac{1}{(1+k)^n}]}$

(EAC)

★ $OCF = (Revenue - Cash\ cost)(1-t) + CCA \cdot t$

$Q(P-VC) - FC$; + ax s−le

−Accelerated depreciation

(Declining balance)

CCA expense = CCA rate × UCC

C = cost of asset

d = CCA tax rate

T_c = corporate tax rate

k = discount rate

S = minimum of salvage value / purchase price

n = number of periods asset is in use

$$PV_{CCA\ Tax\ Shield} = \frac{C \cdot d \cdot T_c}{k+d} \times \frac{[1+1.5k]}{1+k} - \frac{S \cdot d \cdot T_c}{k+d} \times \frac{1}{(1+k)^n}$$

Replacement Decisions:

[initial invest, WC] [salvage value, WC] land

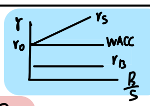
NPV = PV(Δ Initial Outlay) + PV(Δ Ending CFs) + PV(Δ Operating CFs, *after tax*) + PV(ΔCCA Tax shield) 单独算出来!

$$PV(\Delta CCATS) = \frac{\Delta C_0 \cdot d \cdot T_c}{k+d} \times \frac{[1+1.5k]}{1+k} - \frac{\Delta S V_n \cdot d \cdot T_c}{k+d} \times \frac{1}{(1+k)^n}$$

$\Delta C_0 = C_0^{New} - C_0^{old}$ $\Delta SV_n = SV_n^{new} - SV_n^{old}$

Inflation 1+ Nominal rate = (1+ Real rate) × (1+ Inflation Rate)

Risk & Return



(1) Accounting Break-even

$\pi = (TR - TC)(1 - T_c)$

TR = P × Q

TC = FC + VC + D ↘ Depreciation.

VC = C × Q

⇒ P − C = contribution margin per unit

$Q = \frac{FC + D}{P - C}$

(2) NPV Break Even

$CF = [(P-VC)Q - FC](1-T) + T \cdot D$

NPV = 0 = − Initial Investment
+ PV of salvage
+ PV of Tax shield
+ PV of operating CF, after tax
↳ PV of $[(P-C) \cdot Q - FC] \times (1-t)$

EAC Approach:

$B/E = \frac{EAC_c + FC(1-T) - EAC_{TS}}{(P-VC) \times (1-T)}$

(3) Sensitivity (what-if)

$$\frac{\%\Delta NPV}{\%\Delta Variable} = \frac{(NPV_t - NPV_0)/NPV_0}{(Variable_t - Variable_0)/Variable_0}$$

Revenue ⇒ P
− VC ⇒ VC/unit
− FC
− D
Pre tax profit
− Tax
Net Profit
+ D
CF

(4) Senario Analysis

NPV worst, best, avg — CF (Expected Variables) × probability "weighted average"

cost of capital

cost of debt

$$WACC = \frac{S}{S+B} \times r_S + \frac{B}{S+B} \times r_B \times (1 - T_c)$$

cost of Equity: CAMP, DDM; B − bond valuation

DDM $k_e = \frac{Div_1}{P_e}$ $k_e = \frac{D_1}{P_0} + g$ $k_p = \frac{Div_p}{P_p}$

price of shares: $P_0 = \frac{Div_1}{r}$ (ppm) $P_0 = \frac{Div_1}{r-g}$

CAPM (SML) $k_e = R_f + \beta_e \times (R_m - R_f)$

$$\beta = \frac{COV(R_i, R_m)}{Var(R_m)} = \frac{\sigma_{i,m}}{\sigma_m^2}$$

$$\beta_{Equity} = \beta_{Asset} \times [1 + (1-T_c)\frac{Debt}{Equity}]$$

$$\beta_{Asset} = \frac{\beta_{equity}}{1 + (1-T_c)\frac{Debt}{Equity}} \quad \text{(firm Beta)}$$

Flotation cost $f = \frac{B}{B+S} f_B + \frac{S}{B+S} \cdot f_S$

$$\text{True cost of Investment} = \frac{\text{amount of Investment}}{1 - f_0}$$

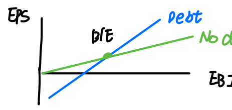
$$f_W = (\frac{E}{V}) f_E \times (1 - \%RE) + (\frac{D}{V}) f_D + (\frac{P}{V}) f_P$$

Capital Structure

Equity at end of current year = Eat end of last year + Income available to SH − Dividends

$$EPS = \frac{(EBIT - Interest)(1-t)}{\# shares} \approx NI$$

debt↑, interest↑, Esp volatile↑



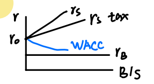
RoA = NI / Asset **RoE** = NI / Equity

Break even EBIT

EPS (all equity) = EPS (levered)

MM perfect world: (no taxes, transaction costs, risk of 破产; perfect information, debt risk free, equal access)

	Without taxes	With tax	Trade off Theory
Firm Value	$V_U = V_L$; $V_U = \frac{EBIT}{r_0} = S_L + B = V_L$	↑ $V_L = V_U + B \times T_C$; $V_U = \frac{EBIT(1-T_c)}{r_0}$	$V_L = V_U + PV(\text{Tax shield}) - PV(\text{cost of financial distress})$
WACC	constant; $WACC = r_S \cdot \frac{S}{S+B} + r_B \frac{B}{B+S}$	D↑, WACC↓; $WACC = r_S \frac{S}{B+S} + r_B(1-T_c)\frac{B}{B+S}$	D↑, WACC 先↑ 后↓
Cost of equity	D↑, r_S↑; SH risk↑ return↑ ↑ $r_S = r_0 + \frac{B}{S}(r_0 - r_B)$; $r_S = \frac{NI}{S} = \frac{EBIT - I}{S}$	D↑, r_S↑; ↑ $r_S = r_0 + \frac{B}{S}(r_0 - r_B)(1-T_c)$; PV of CFs ($r = k_e$) EV = D+S	D↑, r_S↑
Optimal capital structure	Any	100% Debt	0% ~ 100% debt



- r_B is the interest rate (cost of debt)
- r_S is the return on (levered) equity (cost of equity)
- r_0 is the return on unlevered equity (cost of capital for an all-equity firm)
- B is the market value of debt and S is the market value of equity

FCF = EBIT(1−t) + Depre − CAPX − ΔNWC

PV of TS = $T_c \cdot B$

Net profit = Earning ± interest (personal / firm)

$V_L = V_U + PV(\text{tax shield}) - PV(\text{costs of financial distress})$ (t·B)

expected costs = (distress costs)(probability of distress)

D↑ (r_B↑, r_S↑) vs PVTS ⇒ WACC ↓↑ 先降后升

Optimal capital structure:

Homemade leverage

U→L borrow → same $\frac{B}{S}$

L→U lend D×x, Invest E×(1−x) → same $\frac{B}{S} = \frac{x}{1-x}$

Your personal investment

Firm issue debt:

CF to Equity = (EBIT − Interest)(1−t)

debt ↑ → Interest tax shield; expected cost of bankruptcy (r_B↑, r_S↑)

$E = \frac{CF_e}{\text{cost of equity}}$

max Firm value; min WACC

V = D + E ↑; WACC ↓

debt: bankruptcy, financial distress cost

Promised return = $\frac{FV - MV}{MV}$

expected return = $\frac{EV - MV}{MV}$

equity: shirking & dilution; perquisite / perks given out to management

M & A

Horizontal: same product
vertical: different stage of product
conglomerate: different product.

$synergy = V_{AB} - (V_A + V_B) = \sum_{t=1}^{T} \frac{\Delta CF_t}{(1+r)^t}$

$\Delta CF_t = \Delta Rev_t - \Delta Cost_t - \Delta Taxes_t - \Delta Capital\ Requirements_t$ (cost of capital)

Dubious motives for merge:
① Increase firm size or EPS (low P/E firm)
② diversification

Sources:
Revenue: Market power, Strategic benefit, Management
Cost: Economies of scale, vertical Integration, inefficiency
Tax: net operating loss, debt, surplus funds
Lower cost of capital: economies of scale in raising fund

Takeover defence:
① manager: buy firm → going private
- leveraged buyout: highly debt financing
- press releases, mailing to SH
① Sale of assets
② spin off: SH own shares in 2 firm
③ carve out: division → subsidiary

Share Exchange ratio = # of shares the acquiring company issue for each individual share of target company
= # new shares / # of shares$_T$

我拿它的share ↘ Bidder → Target

Value of Acq or Value of T to AC

$NPV_{merge} = MV_T + Synergies - cost$

cost of Acqui → { cash; stock { new share × merged price/share; % of AC give up × ($MV_{AC} + MV_T$ + synergies) }

Cash: MV_{AC} + NPV ― Value of T to AC
stock: $MV_{AC} + MV_T$ + Synergy ― combined

Boundary Condition

(Syn全给A)　　　　　　　-(Syn全给B)

least target will accept ~ most bidder can get

combined firm's price : $\frac{\text{Pre A value + All synergies}}{\text{\# of shares of A}} \sim \frac{\text{Pre merge A value}}{\text{\# of shares of A}}$

# of total shares: $\frac{\text{value of firms with synergies}}{\text{share price}}$

Cash offer: current B's share price ~ $\frac{\text{synergy (premium)}}{\text{B's \# share}}$ + current B's price

Merged firm's share price = $\frac{\text{Combine Value}}{\text{post merge \# shares}}$

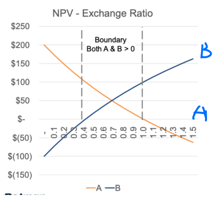
→ Cash: = Pre AC # shares (不变)

merge premium$_T$ = premium/share × # share$_T$

For target: { cash receive / share, or. ; exchange rate × combined share price.

可以用来算 indifferent.

NPV to A & B Shareholders



Valuation

DCF:

from CF investing

FCF = EBIT × (1−T) + D&A − ΔNWC − Capex

EV { Explicit horizon; Terminal Value: $TV_n = \frac{FCF_{n+1}}{r_{WACC} - g_{FCF}} = \frac{FCF_n (1+g)}{r_{WACC} - g_{FCF}}$ }

normative FCF　　小于 gdp growth

D/E 不变 → WACC 不变; discount by WACC

MV of Equity / market capitalization = Enterprise Value − Debt + Cash (PV of FCF)

APV

discount by Ke_u

Adjusted PV = V_u + PV(Interest Tax Shield)　debt↓ WACC↑

PV of FCF: discount rate: cost of equity when no debt (Ke_u)

$ke_L = ke_u + (ke_u - k_d)(D/E)(1-T)$　WACC ⇒ $r_s(ke_L)$ ⇒ $r_0(ke_u)$

Tax saving in Yr t: $t \times D_t \times k_d$ (Interest payment; D_t → Amount of debt at time t)

Marked value of equity = V_u − Debt + Cash + PV(Tax Saving)

Pros:
- ΔCF can be modeled
- Estimate of Intrinsic Value based on Fundamentals
- Flexibility for Assumption, test sensitivity

Cons:
- high grow firm with negative CF
- uncertainty in future CF
- TV: subject estimation error; Small Δ in Wacc, growth → large Δ

Comparable company

- $\frac{EV}{FCF}$, $\frac{EV}{EBITDA}$, $\frac{EV}{EBT}$, $\frac{EV}{sales}$
- similar capital structure

$\frac{\text{Price}}{\text{(CF or BV or Sales or earnings) per share}}$

Sales
− COGS
GP
− SG&A
EBITDA
− D/A
EBIT
− I
EBT
− T
NI

(Sales … EBITDA → EV; normalize I, D, T; EBT … NI → E; NI → P/E)

Pros:
- reasonable approximation (similar company)
- easy available data
- directly from market

Cons:
- Sensitive to market price (over value → over value)
- additional fair takeover premium estimation
- inaccurate bc it does not incorporate value of synergy

Comparable transaction

(use acquisition price)

Pros:
- not require Takeover premium estimation
- Takeover premium directly from recent market
- market price-reduce litigation risk

Cons:
- sensitive to M&A market condition
- inadequate # of comparable transactions
- transaction in diff industry ⇒ inaccurate
- not incorporate synergy ⇒ inaccurate.

MM perfect world: (compromising bond holder)
not account for taxes, brokerage fee, agency cost
uncertainty with investor preference, fututure CF
signaling effects (pay div → company health; no → growing)

Dividend Policy

dividend payout ratio = dividends / net income
Retention ratio = Addition to RE / net income
= 1 − dividend payout ratio

Hold of record: trade date + 2 business day
ex-dividend date: last day seller get dividend

Jan15　Jan28　Jan29　Jan30　Feb16
declaration　Ex-dividend　Record　Payment
(2d, 1d) Price drop by $ of dividend

Homemade dividend
M&M perfect world: dividend policy ⇒ irrelevant (not affect Firm value, stock price)
(no tax or debt)

Residual dividend: dividend add to FV, E

MMPW: take all positive NPV project ⇒ residual earnings payout as dividend

dividend yield = $\frac{\text{Annual dividend Paid}}{\text{share price}}$

Firm value V_0 = PV of V_1 + dividends

$P_0 = \frac{V_0}{\#share}$　　$P_{ex} = P - DPS$

Reasons for Dividend policy

1. Residual Theory { when there are good investment opportunities they pay lower dividends.
2. Resolve Uncertainty: current certain div payout worth more than future uncertain div payout
3. Tax consideration: 税率 -dividend 低; 低- CG 低
4. Agency Theory: waste less if there isn't extra cash
5. Trading Costs: sell/buy shares; issue shares; issue dividends / received cost.
6. Investment policy rules: policy ⇒ SS & DD
7. Signaling or imperfect information

Dividend reinvestment plan
Share repurchase: pay out earning to CSH

Pros:
- Positive signal of CF (managers think undervalued)
- Benefit high income SH (lower tax on CG) (defer)
- not expect higher future CF

Cons:
- negative signal of CF (no good project?)
- done regularly → tax them as dividends

methods { Tendor offer; open market purchase

Stock dividend & split

(taxed)　(not tax)

share↑, share price↓
RE → CS

Post-split stock price = P(old shares) × $\frac{\text{\# of old shares}}{\text{\# New shares}}$

Post-split # shares = # old shares × $\frac{\text{\# New shares}}{\text{\# old shares}}$

(equity account 不变)

	cash dividends	Share repurchase	stock dividends	stock split	reverse →
MV of equity	↓	↓	—	—	—
Stock price	↓	no direct ↑ bc Market	↓	↓	↑
# shares outstanding	—	↓	↑	↑	↓
SH total W	—	—			
E/S	—	↑			
P/E	↓	↓			

days of credit = 30 − 10 = 20

2/10 net 30 with 1% per month for overdue.
10天内 pay 只收 98% (2% discount); 30天后 late charge

eg $-1300 \times 425 + \frac{1300 \times 500 \times 0.95}{(1.1)^{30/365}}$ ← Prob. of payment

total cost　　credit term: net 30

Financial Planning

Sustainable Growth rate:

$\frac{\Delta S}{S_0} = \frac{P(1-d)(1+\frac{D}{E})}{T - [P \times (1-d)(1+\frac{D}{E})]}$

T: ratio of total assets to sales
p: net profit margin on sales

1. Issue new shares
2. Increase its reliance on debt
3. Increase profit margins
4. Reduce the dividends payout ratio
5. Decrease its asset requirement ratio

Credit Management:

1. character: willingness to meet obligation
2. Capacity: ability to pay out of operating CF
3. collecteral: pledged assets in case of default
4. conditions: general business condition that affect
5. Capital: financial reserves or net worth

Liquidity − ST
Solvency − LT

NI = P × new sales
addition to RE = NI(1−d)
RE = RE before + Addition

EFN = Total Assets − Total Liabilities & Equity
(projected)　(projected)　profit margin　dividend payout ratio

$= \frac{Assets}{sales} \times \Delta Sales - \frac{Debt}{sales} \times \Delta Sales - (P \times \text{projected sales}) \times (1-d)$

ΔAsset　ΔDebt　$\frac{NI}{Sales}$　ΔRE　$\frac{div}{NI}$

how to meet grows objective:
- Improve asset utilization ratio eg efficient machinery, turnover sales quicker
- Improve profit margin eg cut cost, ↑price
- Increase D/E → leverage → rel

Short term financing

Finance: current Asset
Trade credit:
① 2/10 net 40: without compounding
Per annum cost = $\frac{365}{30} \times 2\% = 24.33\%$
② 8% bank loan: 2% discount

cost of forgoing discount

$k = \left[1 + \frac{\text{n-Day financing cost}}{\text{Selling price - discount}}\right]^{365/n} - 1$

$= [1 + \frac{2}{98}]^{365/30} - 1 = 27.86\%$ ✓

compounding.

Another Idea: "Pecking Order Theory" of Capital Structure

- Firms have a "pecking order" of financing of new projects due to the desire to not divulge information, for instance, to competitors
  - Financing new projects with internal funds, retained earnings, involves the most privacy so this is the first choice
  - Next comes debt financing, with only privileged disclosures to lenders
  - Last comes public issues of equity, with full disclosure of investment plans
  - Not a bad fit with actual Canadian financing behavior for new projects!
- We would usually assume firms keep their capital structure weights as they invest in further projects but the pecking order theory assumes there being a preference for particular financing sources
  - Capital structure would change after new investments, according to this theory