Modeling Wealth Inequality

How Talent and Luck Shape Wealth

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1 Introduction

The World Inequality Report (Piketty et al., 2022) highlights the extreme concentration of capital for countries worldwide. While the top 10% of the population own around 70% of the total wealth, the bottom 50% have a share of less than 5% across the different continents. This wealth gap has widened over the last decades, especially in the United States, where inequality is even higher than in other advanced economies, which can be, for example, quantified by the Gini coefficient, a commonly used measure of inequality (Horowitz et al., 2020). This inequality not only has a detrimental impact on individuals, who are more likely to experience depression (Patel et al., 2018) or show an increased anxiety about their social status (Buttrick & Oishi, 2017), but also society as a whole can be affected. Unequal societies, for example, show higher crime rates (Pratt & Cullen, 2005) or a higher affective polarization (Gidron et al., 2018). Therefore, understanding which factors play a role in generating this inequality becomes more and more important.

One interesting avenue is to look at inequality from a micro level and determine the individual-level factors that influence how people's wealth evolves and what helps people rise to the top of the wealth distribution. The meritocratic principle proposes one factor, as it says that individuals rise to wealth and power based on their abilities and achievements. However, recent research suggests that seemingly random factors such as your name (Laham et al., 2012) or your birth month (Deaner et al., 2013) can influence people's success.

These two factors are summarized by Pluchino et al. (2018) broadly as talent versus luck. By adopting an agent-based modeling approach, they try to quantify the respective roles of these two factors in shaping individual life outcomes to get a better understanding of how the differences between people regarding wealth can be explained. In the following, I will present the model with its assumptions and results along with my own implementation. Furthermore, I will analyze the resulting wealth distribution generated by the simulation, an aspect only briefly discussed by the original authors. Finally, I will assess the model's strengths and limitations and explore its broader implications for understanding inequality. The goal of this project is to gain a deeper understanding of the mechanisms that shape the development and structure of wealth distributions.

2 Talent vs. Luck Model

2.1 Model Description and Assumptions

The Talent vs. Luck Model by Pluchino et al. (2018) assumes that there are N individuals modeled as agents with a fixed talent t_i which is drawn from a normal distribution with a given mean m_T and standard deviation σ_T . The agents are placed randomly within a squared grid. Additionally, the grid contains so-called lucky and unlucky events which are in the beginning of the simulation uniformly distributed in space. One simulation covers a period of 40 years with time steps equal to six months. In each time step, the events change their location randomly while the individuals do not move. The goal of the simulation is to simulate the success I wealth of the individuals over time. In order to do this, the model assumes that each individual has an equal starting capital I00 representing the wealth of an individual. This capital can change if the agent intersects with an event, meaning that an event is placed within a radius of one cell centered around the agent, otherwise, the agent's capital does not change in this timestep. There are two different scenarios depending on the kind of event the agent intersects with: In case of an intersection

with an unlucky event, the agent always halves its capital:

$$C_k(t) = \frac{C_k(t-1)}{2}$$

In case of an intersection with a lucky event, the agent doubles its capital proportional to its talent to reflect that more talented people can make better use of their opportunities.

$$C_k(t) = \begin{cases} 2C_k(t-1) & \text{if } rand[0,1] < T_k \\ C_k(t-1) & \text{else} \end{cases}$$

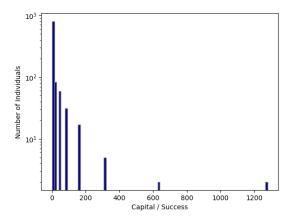
The authors justify their decision to multiply or divide capital by a factor of 2 as a way to keep the model simple while still capturing the possibility of rapid increases or decreases in success. However, this assumption of rapid change is supported only by "common sense evidence" (Pluchino et al., 2018, p. 6) and is not empirically validated. While the use of multiplicative growth is consistent with research showing that past success can increase the likelihood of future success, commonly referred to as the Matthew Effect (Merton, 1968), the specific choice of 2 as a growth or decay factor remains debatable. In particular, halving one's wealth or success in a single timestep appears to be a rather extreme scenario and is unlikely to reflect typical real-world dynamics.

Another assumption made by Pluchino et al. (2018) is that all agents begin with the same initial capital, thereby avoiding any inherent advantage or disadvantage among individuals. This view does not take into consideration that children are born into very different socio-economic circumstances which are shown to relate to occupational status and earnings in later life (Duta et al., 2020). To better reflect these real-world disparities, I modified the model to draw each individual's starting capital from a normal distribution with mean m_C and standard deviation σ_C .

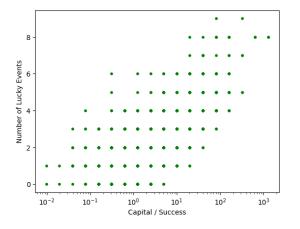
2.2 Simulation Results

The results presented first are based on a simulation conducted by Pluchino et al. (2018) with N=1000 agents whose talent is drawn from a normal distribution $N\sim (0.6,0.1)$. The simulation includes 500 event points with 50% of lucky events. Their model produces an unequal distribution of capital which is described in the paper as following a power law. As I will focus specifically on the resulting distributions in part 3, I will not go into detail here. For now, I will concentrate on the results regarding the influences of talent and luck on the final outcome. When looking at the most successful individuals at the end of the simulation, the authors report that they are not the ones having the most talent; the most successful agent has a talent of 0.61 while the most talented individual has a final capital of lower than one unit. On the other hand, there is a clear relationship with the number of lucky and unlucky events experienced by the agent. Agents being more successful encountered more lucky events and fewer unlucky events. This is further supported when looking at the results of the authors' multiple simulation runs. Extracting the talent of the agent with the highest final capital for each simulation results in a normal distribution with a mean that is slightly higher than the average talent of agents ($T_{av}=0.66$).

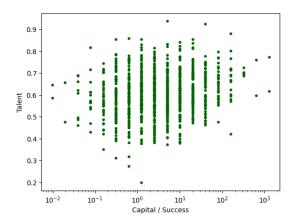
I implemented the Talent vs. Luck model in Python using the Mesa framework (Masad & Kazil, 2015), version 3.0, which provides a structured environment for agent-based modeling. The results are presented in Figure 1. Figure 1a displays the final distribution of capital among agents, closely resembling the distribution reported by Pluchino et al. (2018). Figure 1b illustrates the relationship between final capital and talent, which does not exhibit a linear trend as was also found in the paper. In contrast, Figures 1c and 1d reveal a strong linear relationship between final capital and the number of lucky and unlucky events,



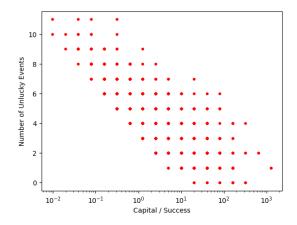
(a) Plot 1: Distribution of final capital after one model run.



(c) Plot 3: Relationship between final capital and number of lucky events experienced by agents during simulation.



(b) Plot 2: Relationship between final capital and talent of agents.



(d) Plot 4: Relationship between final capital and number of unlucky events experienced by agents during simulation.

Figure 1: Replicated results of the simulation using N = 1000 agents with a mean talent $m_t = 0.6$ and $\sigma = 0.1$. Simulation included 500 event points with an equal proportion of lucky and unlucky events.

respectively. All in all, the results of my own implementation look very similar to the ones reported by Pluchino et al. (2018).

2.3 Additional Results

I also wanted to quantify the impact each factor has on the final capital of an agent. For this analysis, I added a varying starting capital as another component by drawing the starting capital for each agent from a normal distribution with a mean $m_C = 2000$ and standard deviation $\sigma_C = 500$. I then tried to predict the final capital with the 4 predictors talent, starting capital, number of lucky, and number of unlucky events using a linear regression. The results indicated that talent is not a significant predictor (t = 0.963, p = 0.336) while the other predictors had a p-value of p < 0.001 indicating their significance.

To better compare the importance of the different predictors, I additionally used the feature importance scores of a random forest regressor which I also built with the same 4 features as were the predictors in the regression and with final capital as the outcome. The feature importance scores are averaging across the different trees in the forest how much each feature contributed to decreasing the prediction error (Breiman,

2001). To make them more interpretable, the scores for the different features are normalized and sum up to 1. Features with higher values are considered more influential in the model's predictions. Looking at the results, the number of unlucky events gets the highest importance score of around 0.6 which is followed by the number of lucky events with a score of around 0.4. As encountering an unlucky event always leads to halving the agent's capital, this has apparently a higher impact on the final capital than the number of lucky events where the outcome also depends on the talent of an agent. The other two predictors have an importance score which is slightly above 0 meaning that both starting capital and talent are playing only a small role in predicting the final outcome compared to the number of lucky and unlucky events.

3 Wealth Distribution

Besides quantifying the impact of talent vs. luck on the final capital of an individual, Pluchino et al. (2018) also looked at the resulting wealth distribution. Although the authors claim that the resulting distribution of the simulation follows the wealth distribution found in the real world, they only superficially investigate the underlying distribution. Therefore, I will focus on this topic in more detail in this section, looking at the underlying growth mechanisms and testing the simulated data against the candidate distributions.

3.1 Growth Mechanisms and Resulting Distributions

There is an ongoing debate in many fields whether a lognormal or a power law distribution is a better fit for empirically observed distributions (Mitzenmacher, 2004). A variable is following a power law distribution if

$$P[X \ge x] \sim cx^{-\alpha}$$

with α , c > 0. A variable is log-normally distributed if the random variable Y = lnX is normally distributed. The reason for this disagreement is that very similar generative models can either lead to a power law or a log-normal distribution, which is also true for the distribution of wealth (Mitzenmacher, 2004).

One of the most basic ideas for generating wealth distributions, formalized in the so-called Gibrat's rule (Gibrat, 1931), is that the wealth of an individual changes multiplicatively rather than additively, which can be written as

$$W_t = \Lambda_t W_{t-1}$$

where W_t is the wealth at timestep t and Λ_t is a random factor. If the factors are independent from each other and their probability distribution does not change, a log-normal distribution of wealth evolves over time (Burda et al., 2019). The distribution can become very broad with a standard deviation growing exponentially, resulting in a non-stationary distribution.

Another growth mechanism is known as preferential attachment, which models that the probability of gaining new units increases with the current size. This "rich-get-richer" rule is best known from network science when looking at the degree of nodes, introduced in the Barabási-Albert model (Barabási & Albert, 1999), where new vertices attach preferentially to those that already have many connections. In this model, another important ingredient is, that the network grows over time, meaning that more nodes are added in each step. Such dynamics then produce power-law (heavy-tailed) size distributions, in which very large values occur more frequently than in lognormal or other more thin-tailed distributions.

One special case of the power law distribution is the Pareto distribution (Pareto, 1897) which is used to describe the Pareto principle, often observed in the wealth and income distribution of societies (Mitzenmacher, 2004). This principle is a general observation that a large proportion of effects come from

a small proportion of causes. In the specific case of wealth it means that a small proportion of people owns a large share of the total wealth. The principle is also known as 80-20 rule because it can be observed that approximately 20% of the population own 80% of the total wealth.

While both mechanisms describe growth processes, they model differently how size influences growth: Gibrat's rule assumes size-independent proportional growth rates, whereas preferential attachment builds in a bias toward faster growth for larger entities. These mechanisms do not necessarily have to contradict each other as Gibrat's rule also favors larger entities through the multiplicative growth. Nevertheless, Gibrat's rule makes the strict assumption that the relative growth has to be independent from the current size, which may be violated by processes following preferential attachment.

Although Gibrat's rule normally leads to a log-normal distribution, only small variations in the process can generate power-law tails. One of these variations is the assumption, that the random factors Λ_t are contractive, meaning that their expected value is smaller than 1, which is one condition for so called Kesten processes besides an additive noise component (Burda et al., 2019). This again shows how closely related the two distributions are, making it necessary to consider both as candidates for the final wealth distribution of the simulation.

3.2 Wealth Distribution of the Talent vs. Luck Model

The growth mechanism in the Talent vs. Luck model follows a multiplicative process with random factors $\Lambda_t \in \{\frac{1}{2}, 1, 2\}$, chosen according to whether an agent intersects with an event. Because of the influence of talent in the choice of 1 vs. 2 in the case of the intersection with a lucky event, the factors are not completely independent of each other. Agents with high talent are more likely to repeatedly grow their wealth by a factor of 2. Another interesting observation is that, because an unlucky event always leads to a factor of $\frac{1}{2}$, $\frac{1}{2}$ is drawn more often than 2, for which it still depends on the talent. This asymmetry should lead to an expected value $E(\Lambda_t) < 1$, meaning that the random factors are contractive.

Pluchino et al. (2018) investigate the underlying distribution only by plotting the final capital in log-log scale. They find that the tail of the distribution can be well fitted with a straight line, indicating a power law distribution. This method is error-prone because also other distributions, like the log-normal can approximate a power law closely over many orders of magnitude and can thus be incorrectly interpreted as a power law (Clauset et al., 2009).

A more accurate approach is to compare the power law with the alternative hypothesis of a log-normal distribution via a likelihood ratio test (Clauset et al., 2009). For this analysis, I used the powerlaw package (Alstott et al., 2014), which provides methods for fitting distributions to empirical data and performing statistical model comparison. To increase statistical power, I aggregated results from ten independent simulations, each run with the same parameters as reported in the results section, but with unequal starting capital values to ensure a continuous range of final wealth values.

I first fitted both a lognormal and a power-law distribution to the aggregated simulation data to examine the estimated parameters. For the power-law fit, the scaling exponent was $\alpha = 2.75$ with a lower bound of $x_{min} = 29449.54$. An exponent $\alpha > 2$ means that both the mean and the variance of the distribution are finite. For the lognormal fit, the parameter estimates were $\mu = 6.33$ and $\sigma = 1.69$, suggesting a very broad distribution. I then compared the fit of the two distributions with a likelihood ratio test. The test revealed a nonsignificant result with a p-value of p = 0.50 but the negative test statistic of R = -0.70 indicates a slight preference for the lognormal distribution.

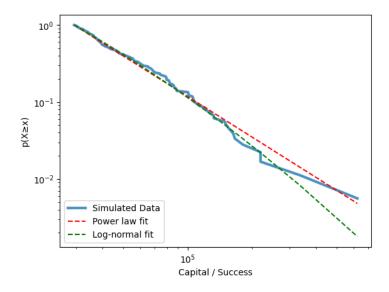


Figure 2: Complementary cumulative distribution function (CCDF) of the simulated data, along with the fitted lognormal and power-law distributions. Both axes are shown on a logarithmic scale.

I also visually assessed the fits of the two candidate distributions using the complementary cumulative distribution function (CCDF) shown in Figure 2 to better compare the differences between the distributions. Both models closely match the simulated final capital for lower values. At higher values, the lognormal fit declines more steeply, initially aligning well with the simulated distribution. However, in the extreme upper tail, the simulated distribution again approaches the slope of the power-law fit.

The results suggest that, in this case, it is not possible to clearly distinguish between the two distributions, as both the lognormal and the power-law models provide a good fit to the simulated data especially in the mid-tail region.

4 Discussion

Pluchino et al. (2018) tried to assess the role of talent vs. luck in individuals' accumulation of capital by implementing a simple agent-based model. The model produces an unequal distribution of wealth, with the most successful individuals not being the most talented ones. The final wealth produced by the simulation can be best predicted by the number of unlucky events an agent encounters; the number of lucky events has a slightly lower influence on the final outcome. The role of talent, on the other hand, has a negligible influence, and also changing the starting capital of individuals does not alter the outcomes notably.

4.1 Implications

These results challenge the meritocratic assumption that individuals rise to wealth and power solely based on their abilities and hard work. The simulations suggest that especially the role of the absence of unlucky events is underestimated when it comes to evaluating a person's success. Instead of perceiving highly capitalized individuals as highly talented, the findings imply that such individuals may simply have experienced fewer unlucky events and more lucky events influencing their capital than the average person. These findings might have implications for how we design education or social mobility policies, suggesting a greater need to account for randomness in outcomes.

However, as Pluchino et al. (2018) emphasizes, a distinction is needed between micro and macro level interpretations. At the individual level, a higher degree of talent slightly increases the probability of becoming wealthy, as talented individuals are more capable of exploiting the opportunities presented by lucky events. At the societal level, however, due to the greater number of individuals with medium talent, it is more likely that those at the top of the wealth distribution are not the most talented, but rather those who benefited from a favorable sequence of events.

Nevertheless, the results should be interpreted with caution, as they are derived from simulations based on a highly simplified model and assumptions not grounded in empirical data. The following section will therefore examine the limitations of this model in greater detail.

4.2 Limitations

Besides the already criticized assumptions of the model, Pluchino et al. (2018) do not provide a clear definition of what they subsume under the terms lucky and unlucky events. It seems necessary to me to distinguish between two conceptually different forms of luck. First, there are enduring, birth-related characteristics, such as name or birth month, that previous research has shown to influence life outcomes (Deaner et al., 2013; Laham et al., 2012). These factors exert a continuous effect across an individual's lifespan. Second, there are chance events that occur independently of any personal characteristics and cannot be anticipated in advance. While the Talent vs. Luck model mainly captures the second type, the first concept could be incorporated by introducing an agent-specific baseline luck factor. This randomly assigned factor could then act as a bias parameter that increases or decreases the probability of experiencing favorable events throughout the agent's lifetime.

Additionally, the model does not include interactions between agents and instead concentrates exclusively on the growth component of wealth dynamics. By contrast, Boltzmann-type wealth models emphasize the exchange of money between agents under the assumption that the total money supply in the system remains constant (Dragulescu & Yakovenko, 2000). Only a limited number of studies combine both exchange and growth mechanisms (Forbes & Grosskinsky, 2022). Therefore, it would be interesting to compare the two mechanisms in a combined model and assess their contributions to the resulting wealth distribution.

Finally, Pluchino et al. (2018) did not investigate the wealth distribution produced by their simulations in detail but claimed to have found a power-law distribution. My own contribution, therefore, was to examine the growth mechanisms and the resulting distribution more closely. My results suggest that distinguishing between a power-law and a lognormal distribution is challenging. This aligns with current research in econophysics and income distribution modeling, which supports a hybrid view: a mixture distribution consisting of a lognormal body and a power-law tail for the wealthiest individuals appears to provide the best fit (Hajargasht & Reed, 2013; Mitzenmacher, 2004).

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5 Data and Code Availability

 $All\ code\ and\ simulation\ data\ is\ uploaded\ on\ Github\ (https://github.com/hanna27m/TalentLuckModel).$