02502
Image Analysis
Week 3 - Pixelwise operations and colour images

http://courses.compute.dtu.dk/02502/

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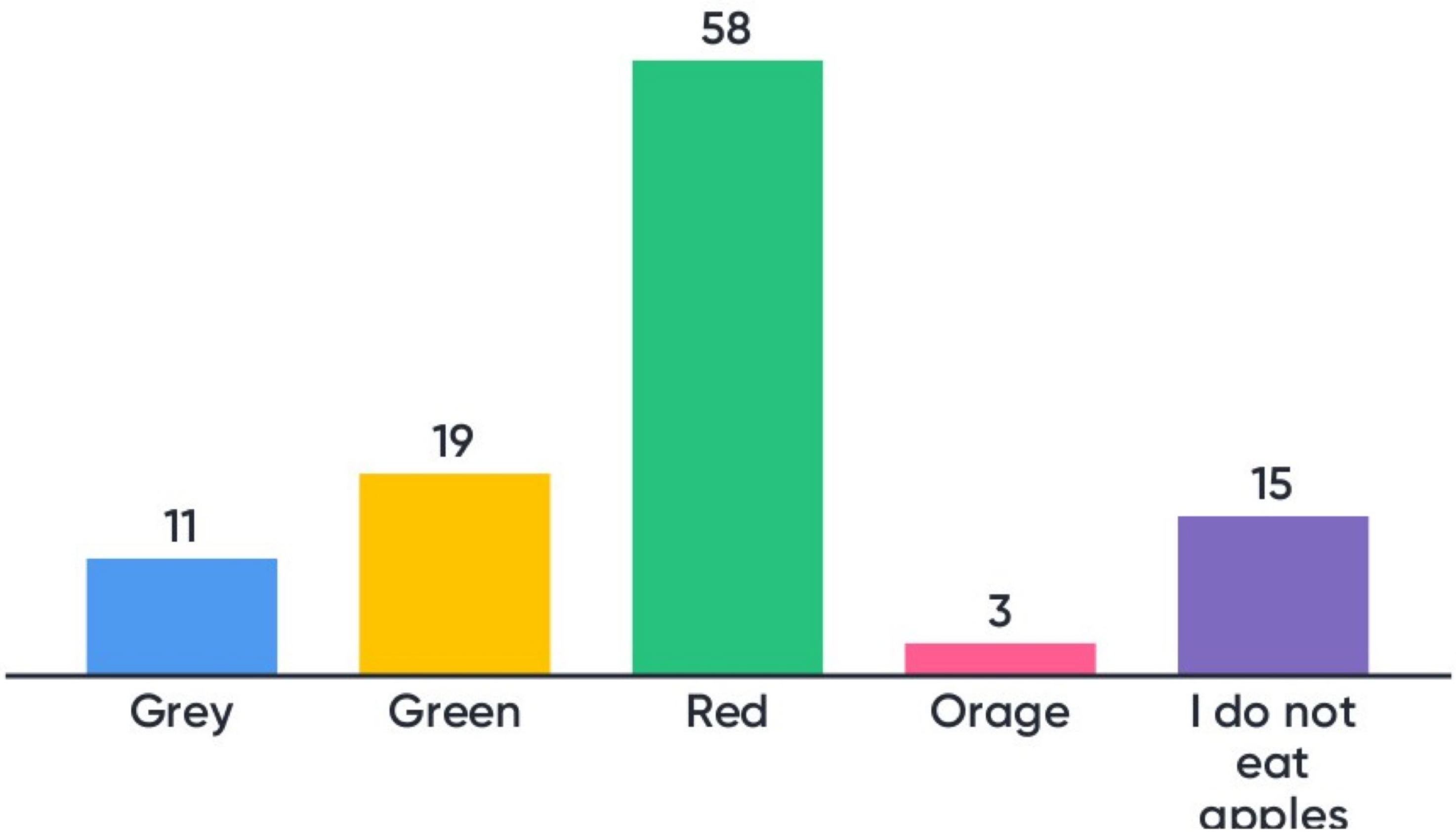
Quiz Testing - Pixel color of an apple?

Mentimeter



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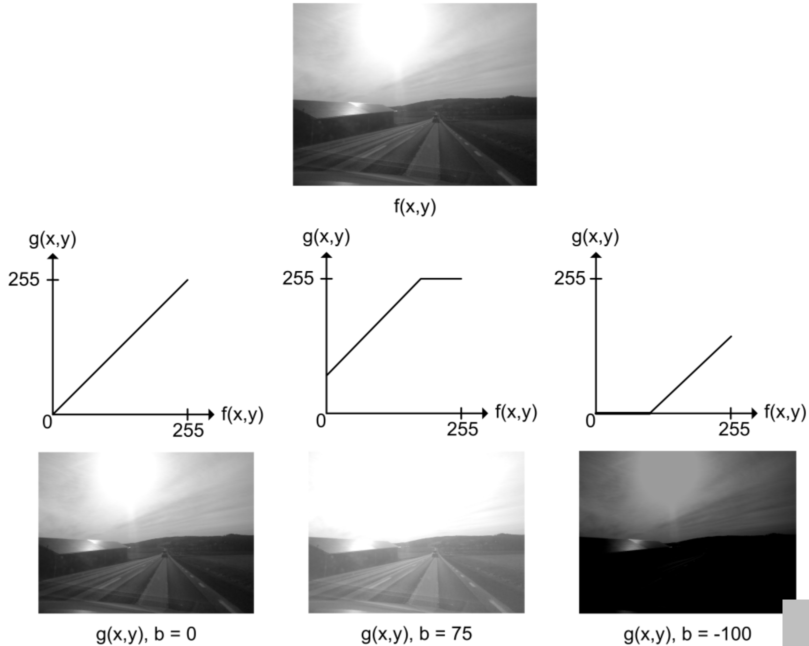
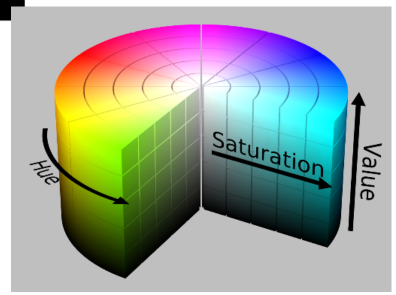
Week 3
Pixelwise operations and colour images



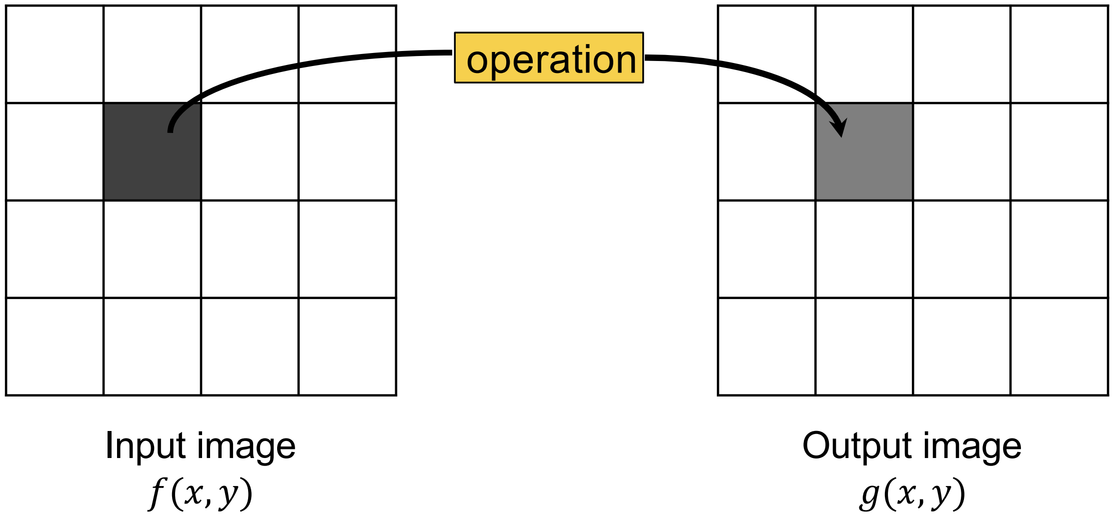


What can you do after today?

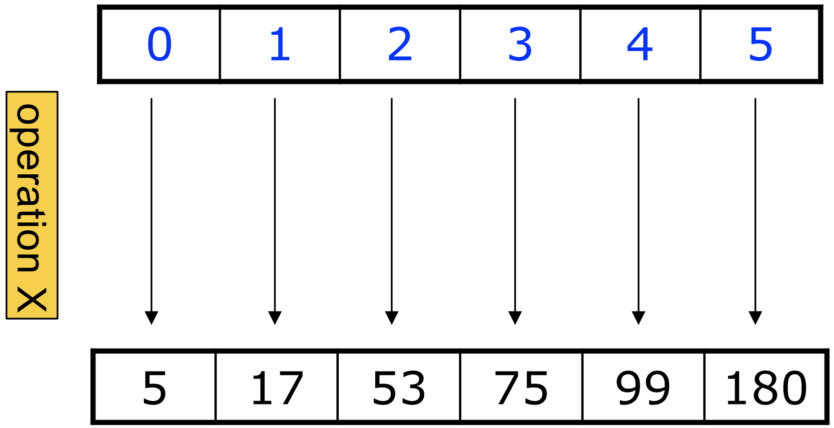
- Compute and apply a linear gray transformation
- Describe and compute the image histogram
- Implement and apply histogram stretching
- Implement and apply gamma transformation
- Implement and apply log and exp mappings
- Describe and use thresholding
- Describe and use automatic thresholding
- Perform conversions between bytes and doubles
- Use addition and subtraction of images
- Explain the benefits of bi-modal histograms
- Identify images where global thresholding can be used for object extraction

Pixelwise Operations

- The same operation is performed for all pixels
- We denote these operations for mappings: From one pixel to another
  - Both linear and non-linear mappings exists.



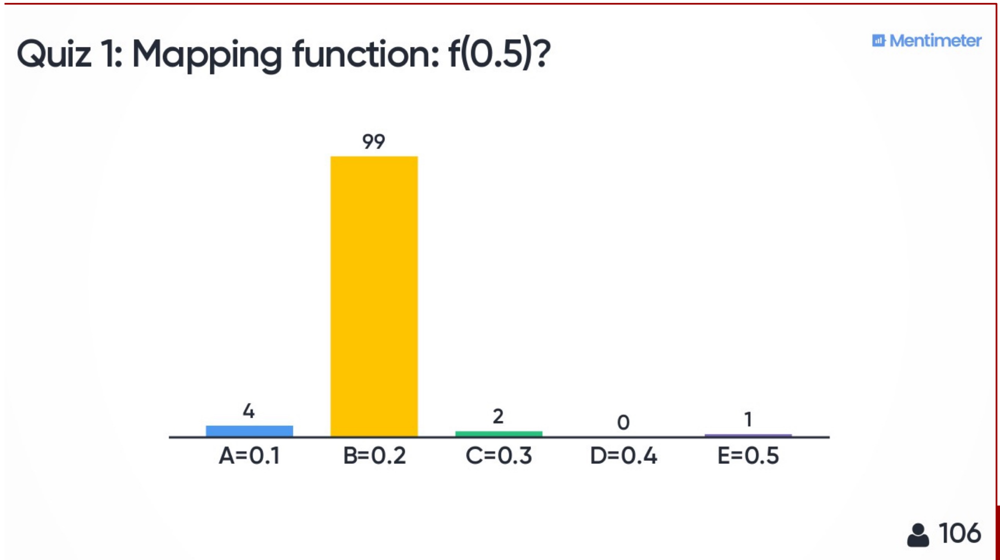
Gray value mappings



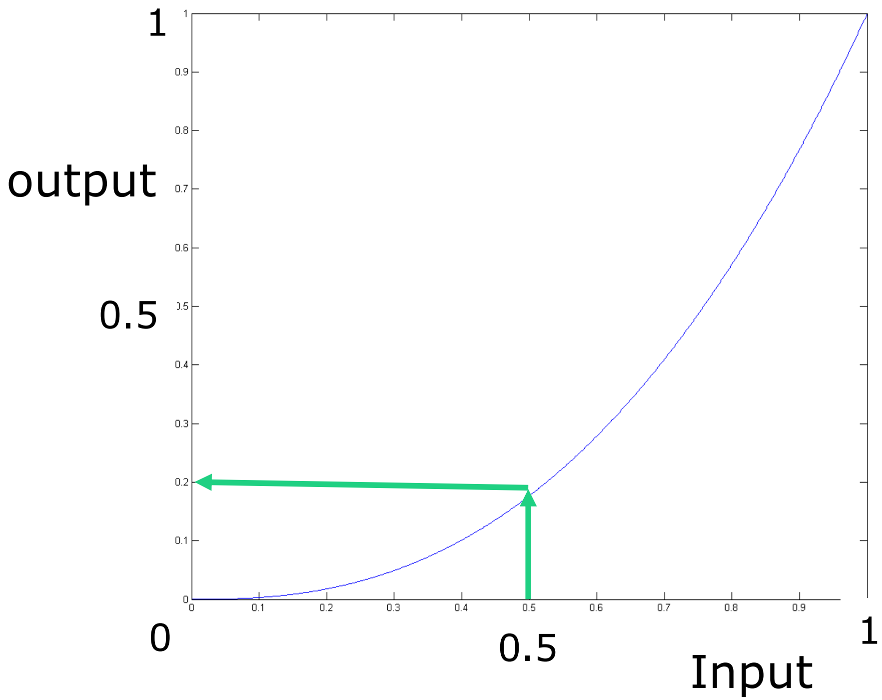
- Mapping
  - To make correspondence between two sets of values
- Look-up-table
  - A table of mappings

Quiz 1: Mapping function

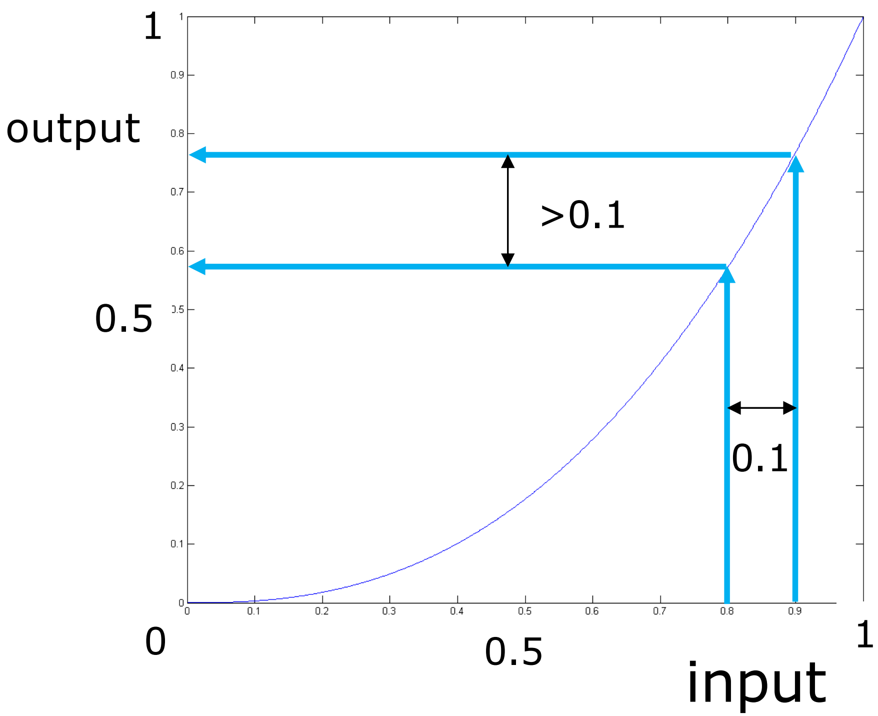
A) 0.1
B) 0.2
C) 0.3
D) 0.4
E) 0.5



f(0.5)?



Gray value mappings



- Mapping
  - To make correspondence between two sets of values
- Mapping function
  - out = f(in)
  - f(0.5)?
- What happens with the values?
  - Values with difference 0.1
  - Output values “spread out”

Why change gray level values

- When could it be good to change the gray level values?
  - Lack of contrast
  - Make the image lighter
  - Make the image darker

Point processing

Input

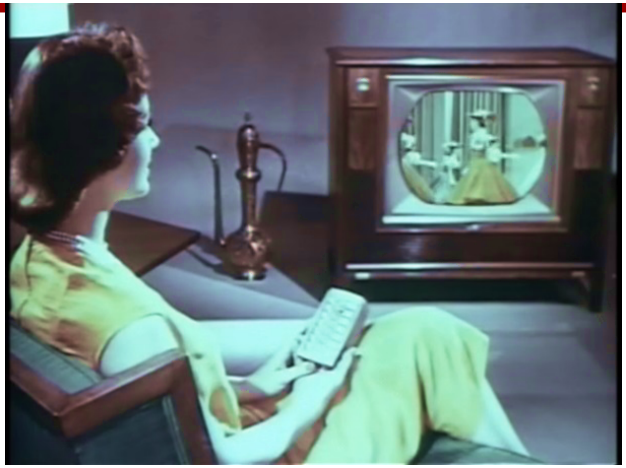
1	2	0	1	3	
2	1	4	2	2	
1	0	1	0	1	
1	2	1	0	2	
2	5	3	1	2	

Output

	$\frac{12}{9}$				

- The value of the output pixel is only dependent on the value of one input pixel
- A global operation – changes all pixels

Point processing

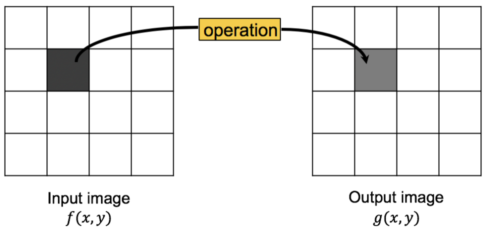


- Grey level enhancement
  - Process one pixel at a time independent of all other pixels
  - For example used to correct Brightness and Contrast
    - Known from the television remote control

Correct | Too high brightness | Too low brightness | Too high contrast | Too low contrast

Brightness

- The brightness is the intensity of the image
- Change brightness changes all intensities in the image:
  - To each pixel is added the value *b*
  - f(x, y) is the input image
  - g(x, y) is the (enhanced) output image
- If $b>0$: brighter image
- If $b<0$: less bright image

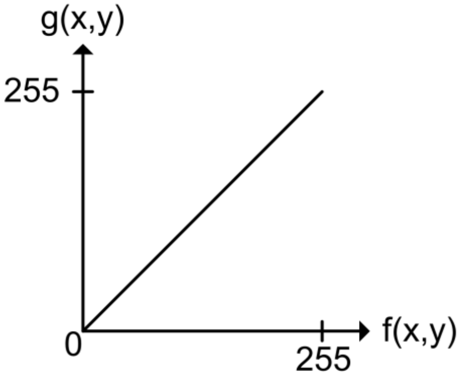


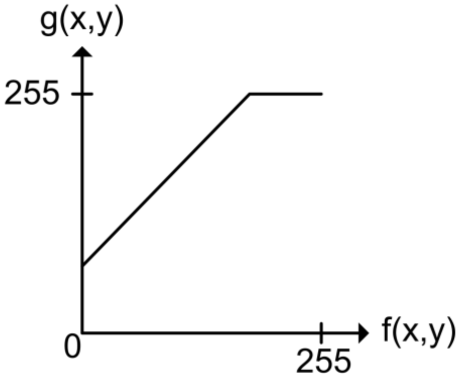
$$g(x,y) = f(x,y) + b$$

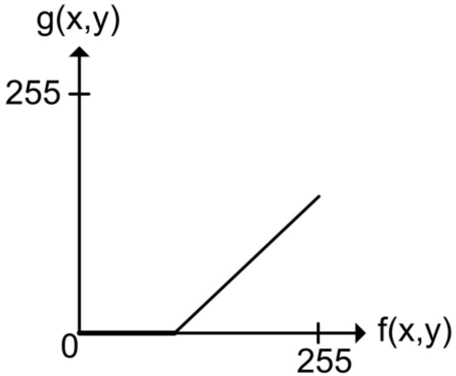
Brightness

$$g(x,y) = f(x,y) + b$$

f(x,y)

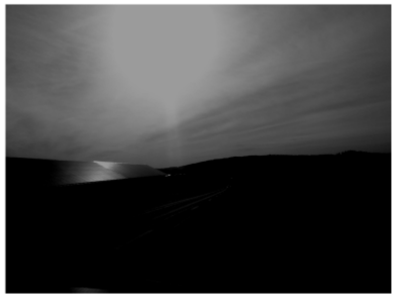






g(x,y), b = 0

g(x,y), b = 75



g(x,y), b = -100

Contrast

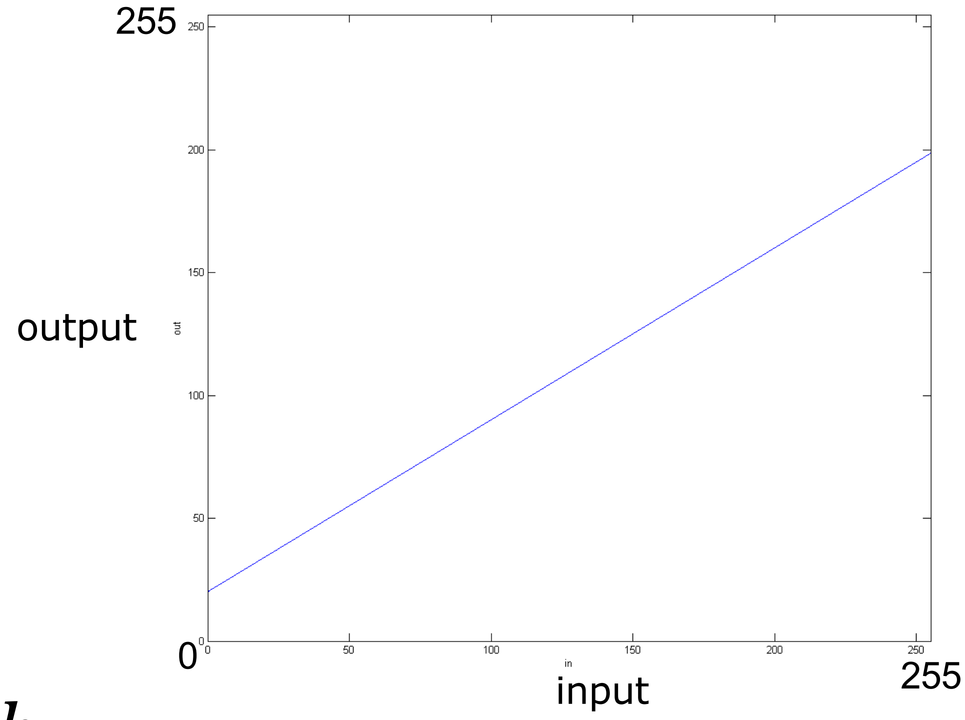
- The contrast describes the level of details we can see
- Change contrast:

  Each pixel is multiplied by *a*
  - f(x, y) is the input image
  - g(x, y) is the (enhanced) output image
- If *a*>1 => more contrast
- If *a*<1 => less contrast

$$g(x, y) = a * f(x, y)$$

Combining brightness and contrast

- A straight line
- Called a *linear transformation*
- Here a = 0.7 and b = 20

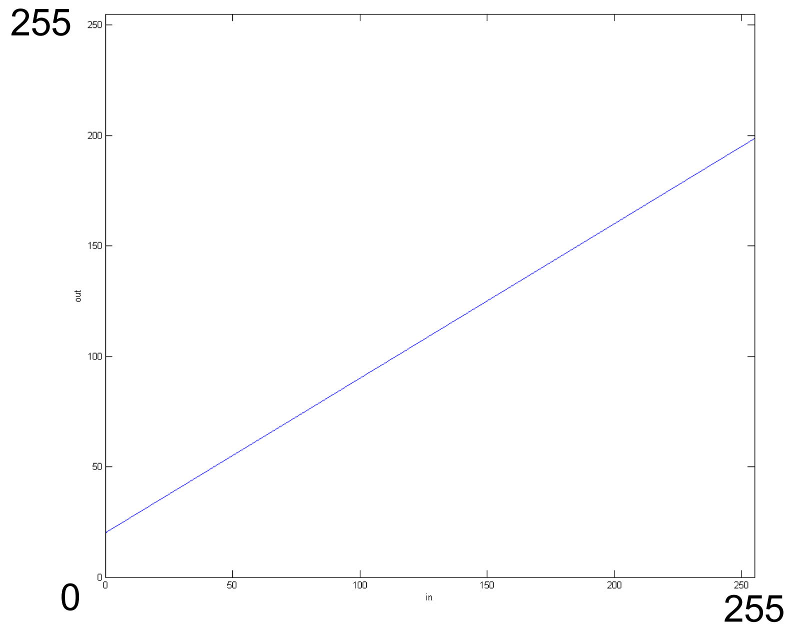


$$g(x, y) = a * f(x, y) + b$$

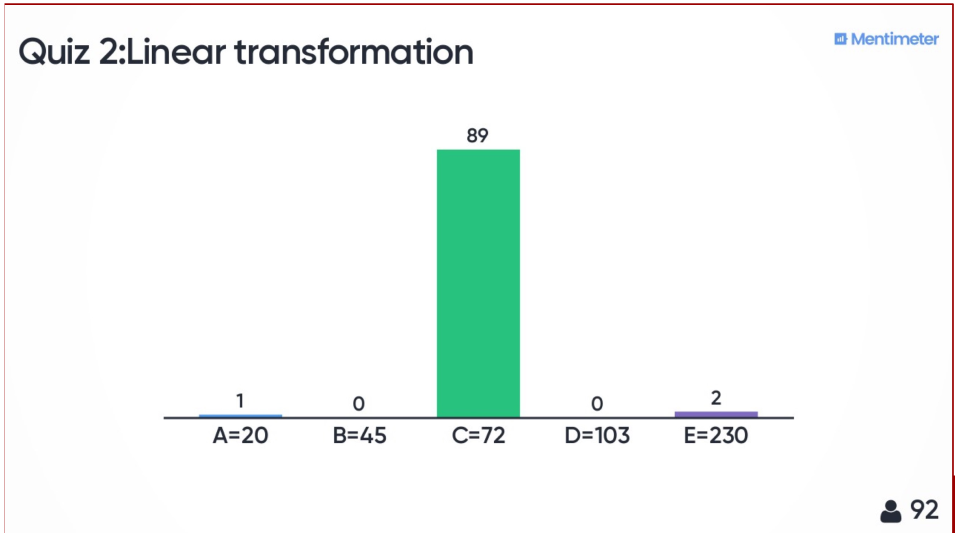
Quiz 2: Linear transformation

a = 0.7 and b = 20   Input = 75   Output = ?

A) 20
B) 45
C) 72
D) 103
E) 230



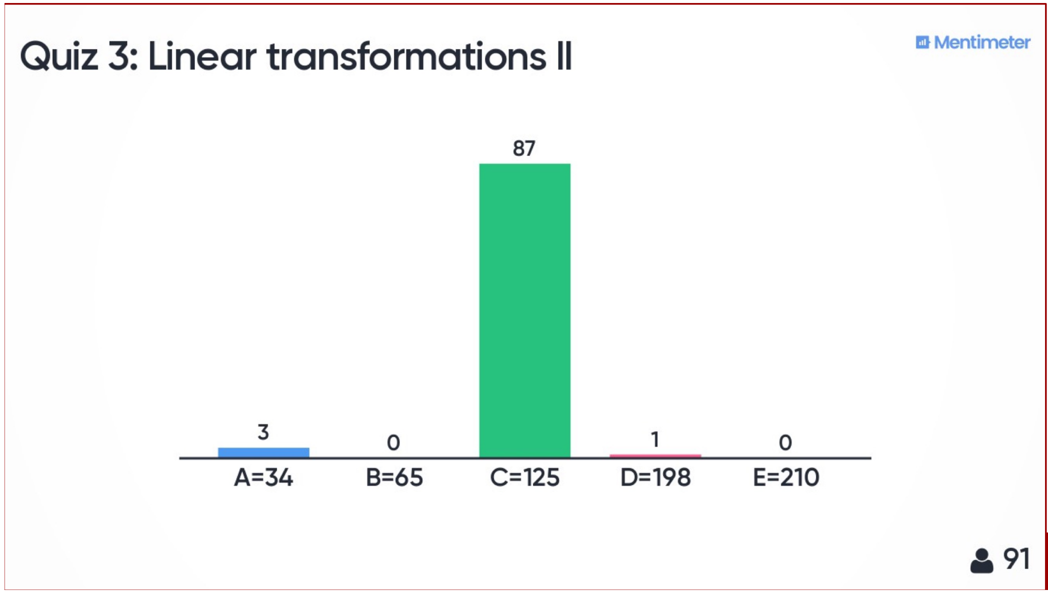
$$g(x, y) = a * f(x, y) + b$$

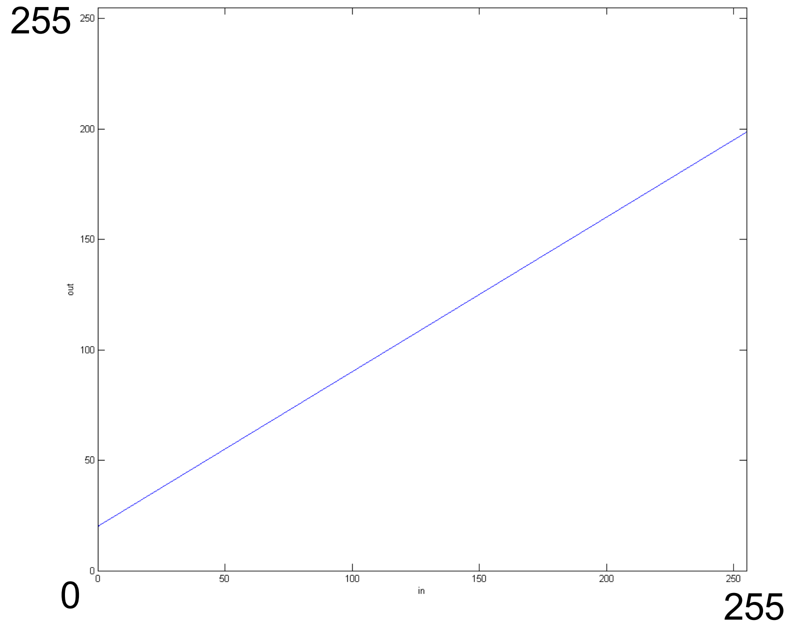


Quiz 3: Linear transformation II

a = 0.7 and b = 20   Input = 150   Output = ?

A) 34
B) 65
C) 125
D) 198
E) 210

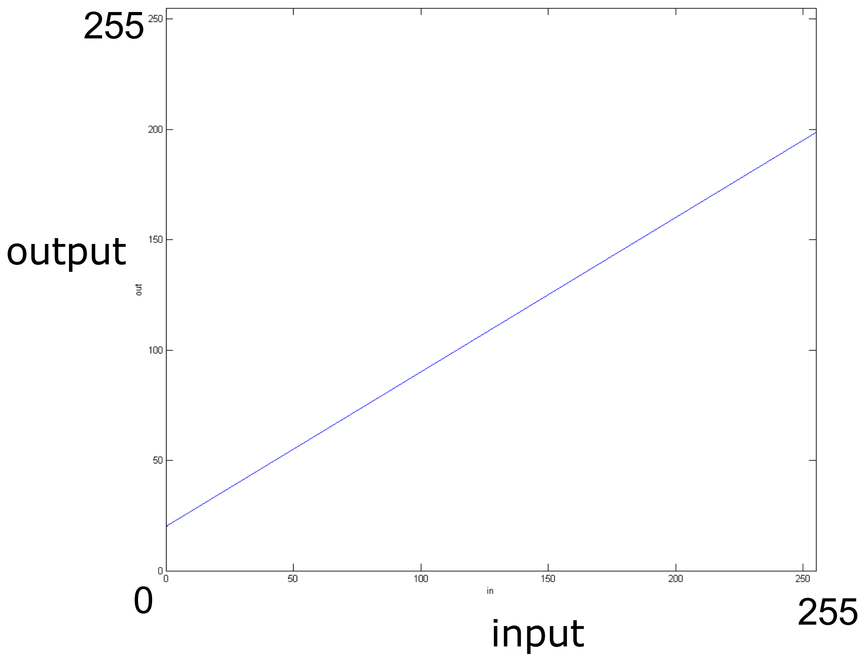




$$g(x, y) = a * f(x, y) + b$$

Combining brightness and contrast

- A straight line
- Called a *linear transformation*
- Here a = 0.7 and b = 20
- What will the result be on the output image?
  - More bright (b > 0)
  - Less contrast (a < 1)



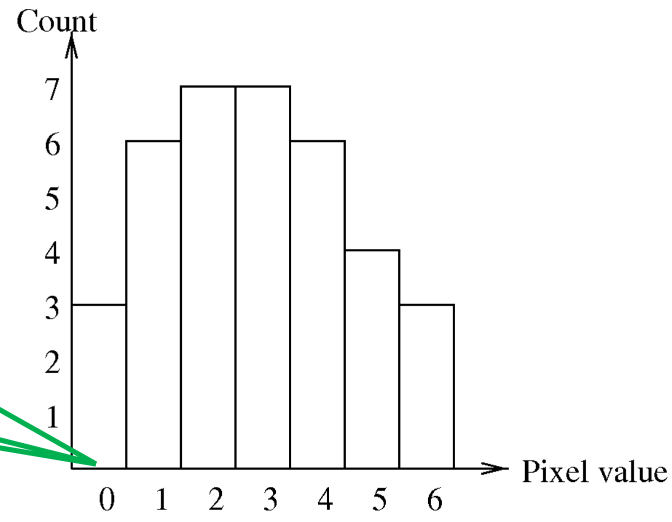
$$g(x, y) = a * f(x, y) + b$$

Histogram Reminder I

- A histogram normally contains the same number of “bins” as the possible pixel values
- A bin stores the number of pixel with that value

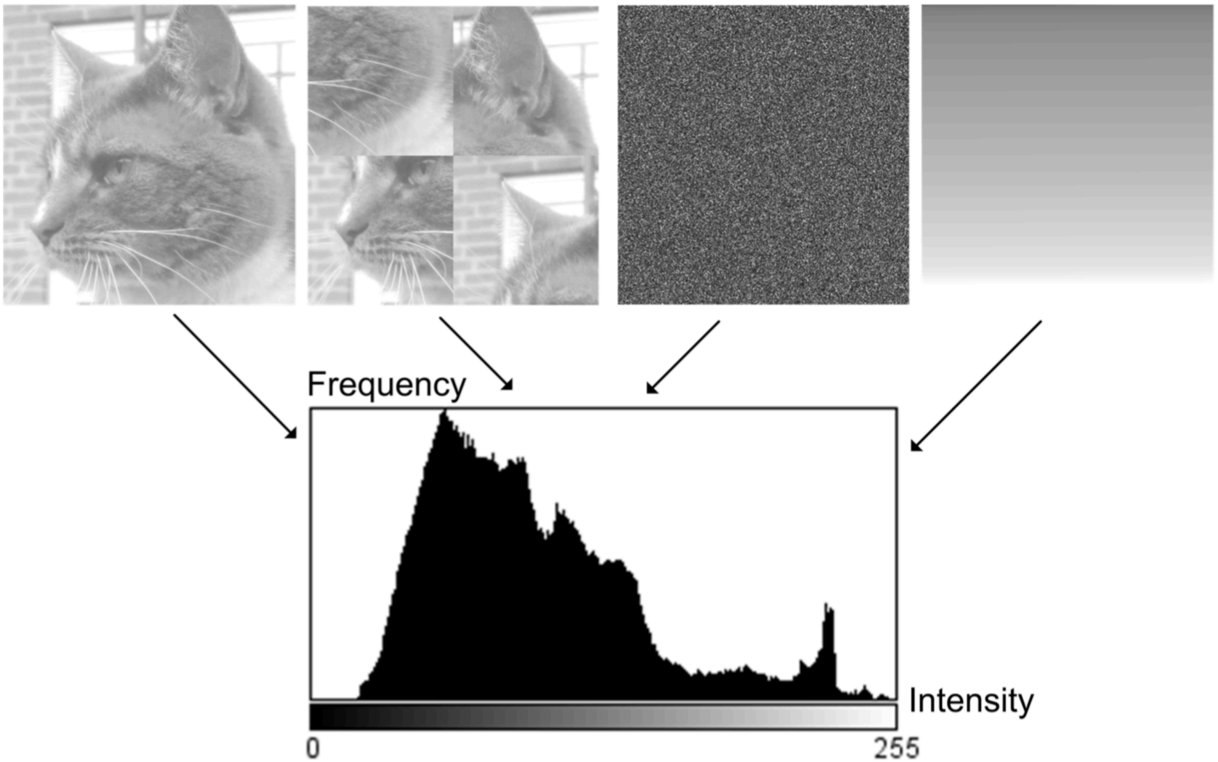
0	2	6	6	3	3
1	4	3	4	4	4
3	2	5	1	5	2
1	4	2	1	3	1
2	5	3	0	2	0
4	2	5	6	3	1

3



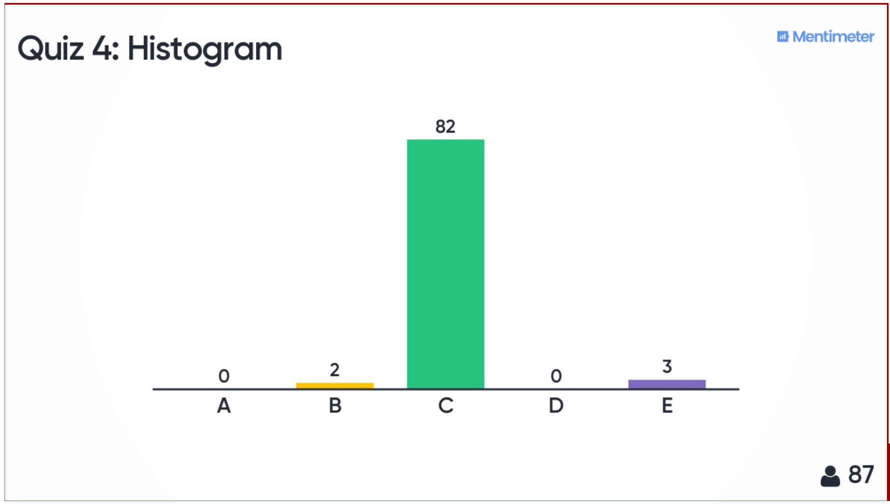
Histogram Reminder II

- It is your brain that needs a picture, not the histogram …
- Histogram: a pixel-wise operation, and pixels are spatial independent



Quiz 4: Histogram

A) A
B) B
C) C
D) D
E) E

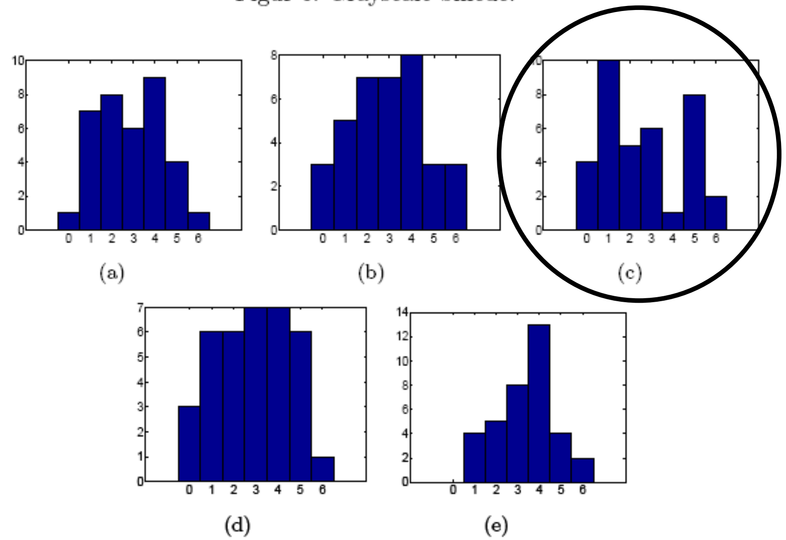


Opgave 10.8

Hvilket histogram hører til billedet, der ses i Figur 6:

0	5	3	5	2	1
3	5	5	3	3	1
1	1	1	3	2	3
6	2	2	1	0	0
0	2	1	5	1	5
5	5	1	4	1	6

Figur 6: Grayscale billede.

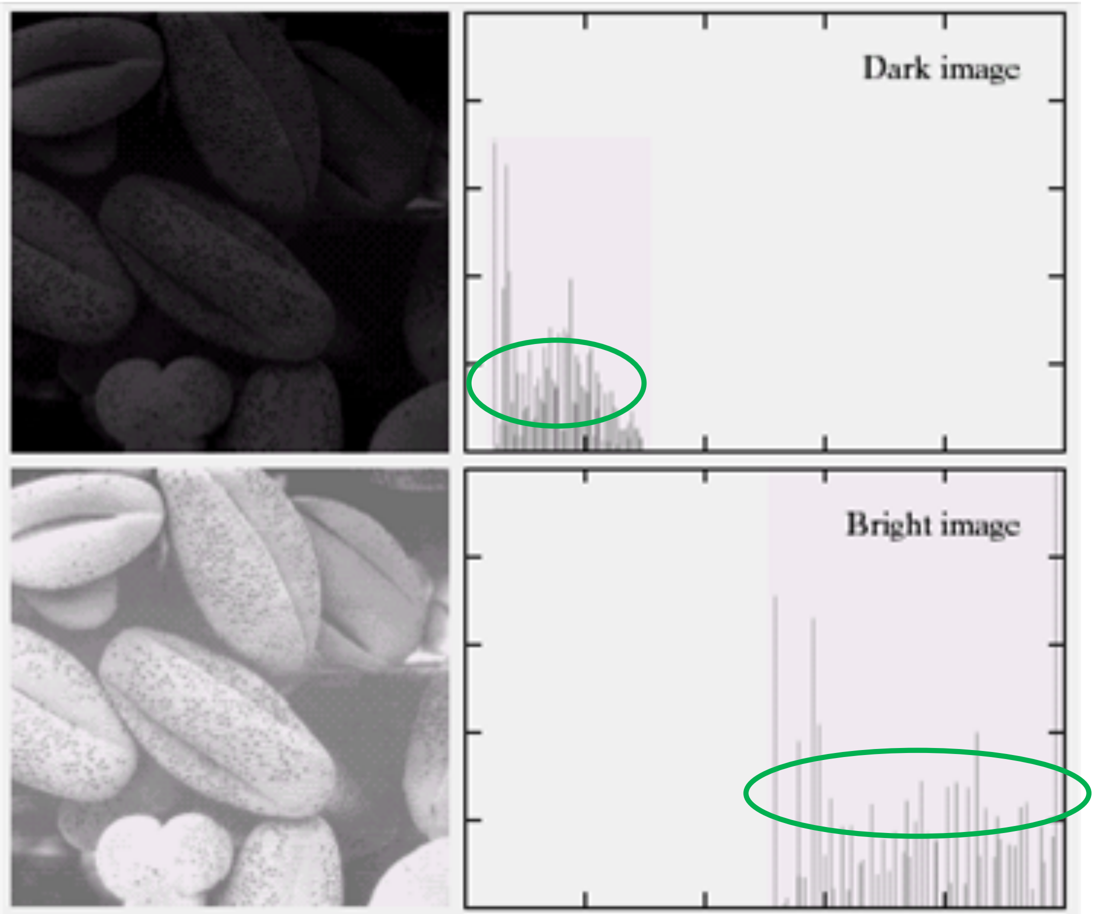


1. (a) 2. (b) 3. (c) 4. (d) 5. (e) 6. ved ikke.

Back to the histogram

- The shape of the histogram tells us a lot!

Histogram inspection



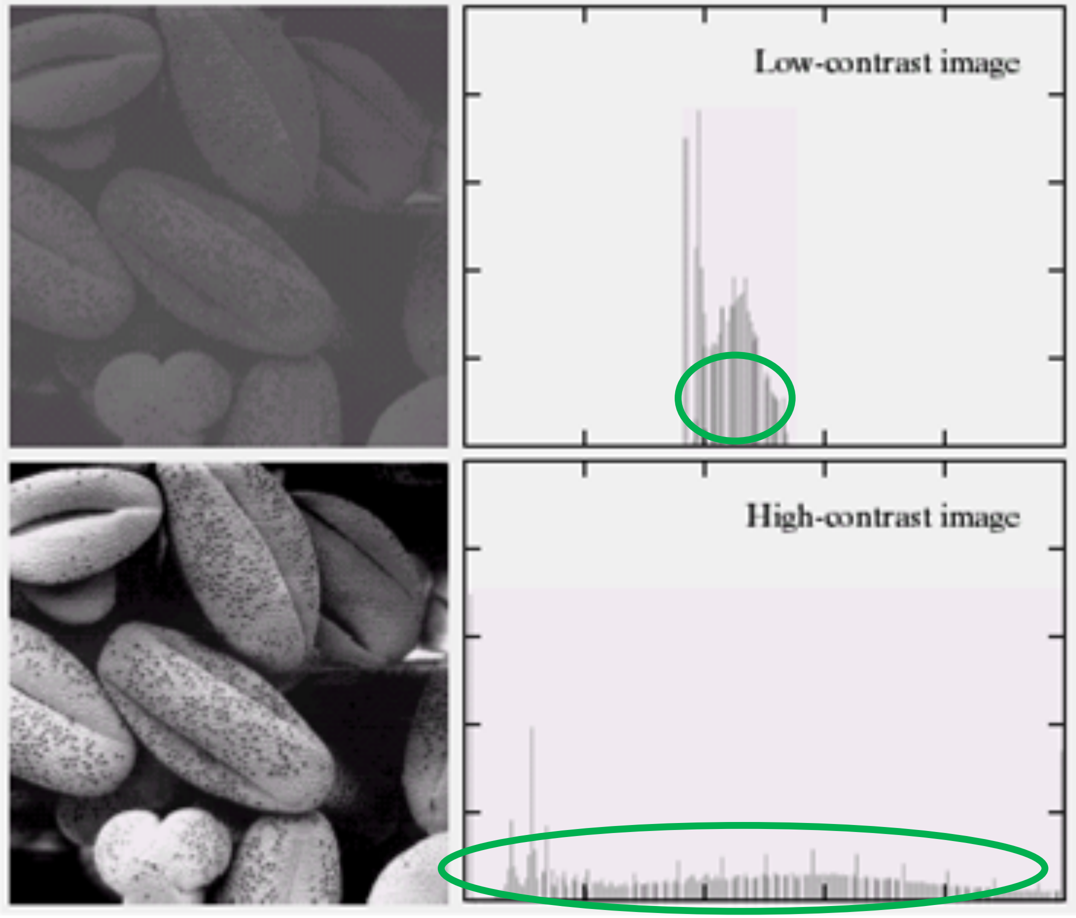
Brightness:

$$g(x, y) = f(x, y) + b$$

Dark image

Bright image

Histogram inspection



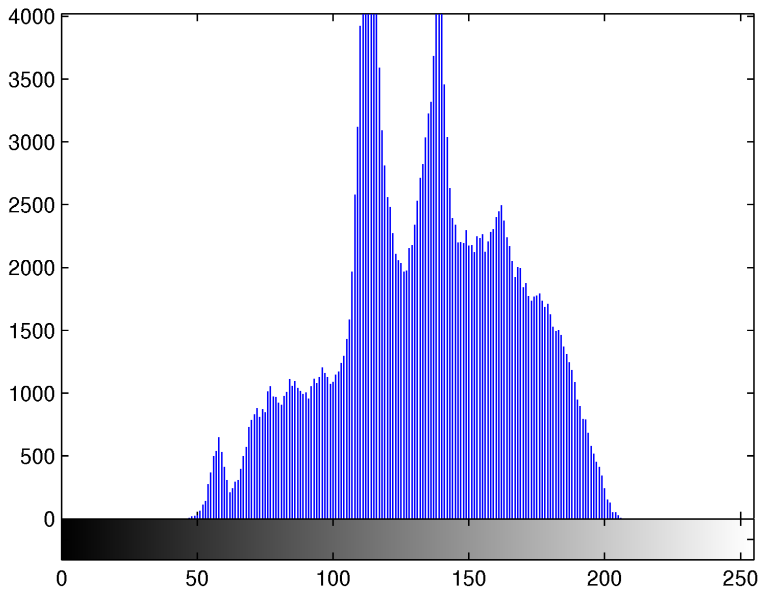
Contrast:

$$g(x,y) = a * f(x,y)$$

Low contrast

High contrast

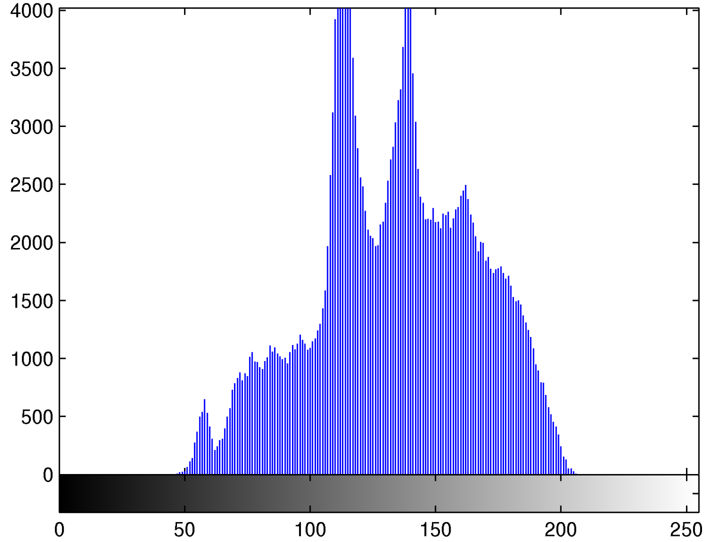
Histogram stretching



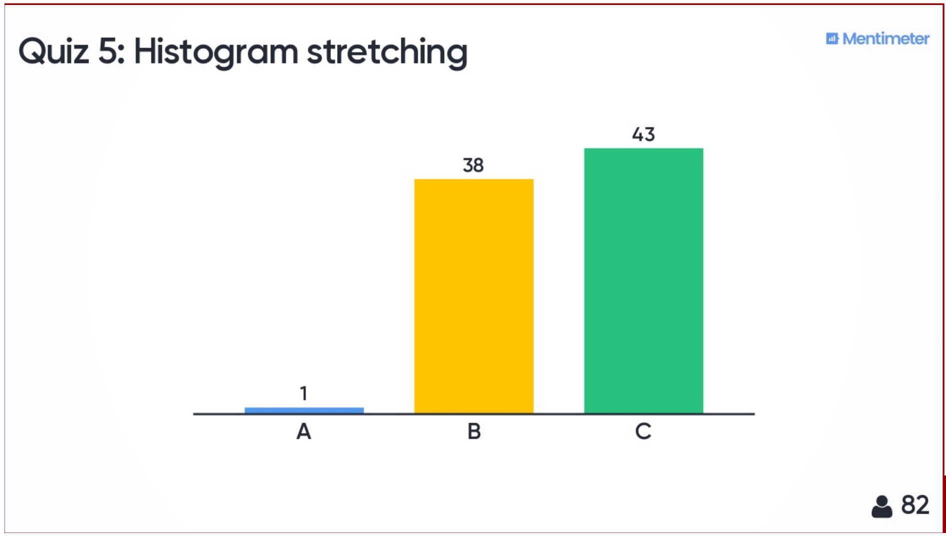
- How do we optimise the image intensities using the histogram?
  - Minimum and maximum values?
  - Stretch it so new minimum = 0 and new maximum = 255

Quiz 5: Histogram stretching

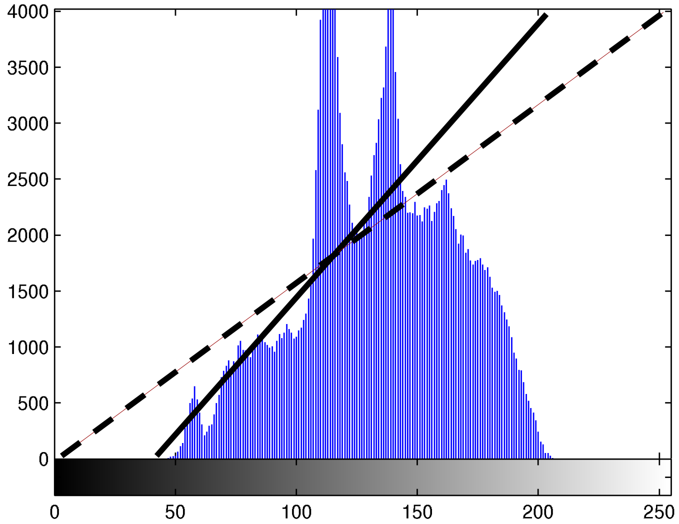
A) Using brightness
B) Using contrast
C) Using brightness and contrast



- We want
  - Min = 0
  - Max = 255
- We have
  - Min = 32
  - Max = 208



Quiz: Histogram stretching



- We want
  - Min = 0
  - Max = 255
- We have
  - Min = 32
  - Max = 208

… is brightness and Contrast:

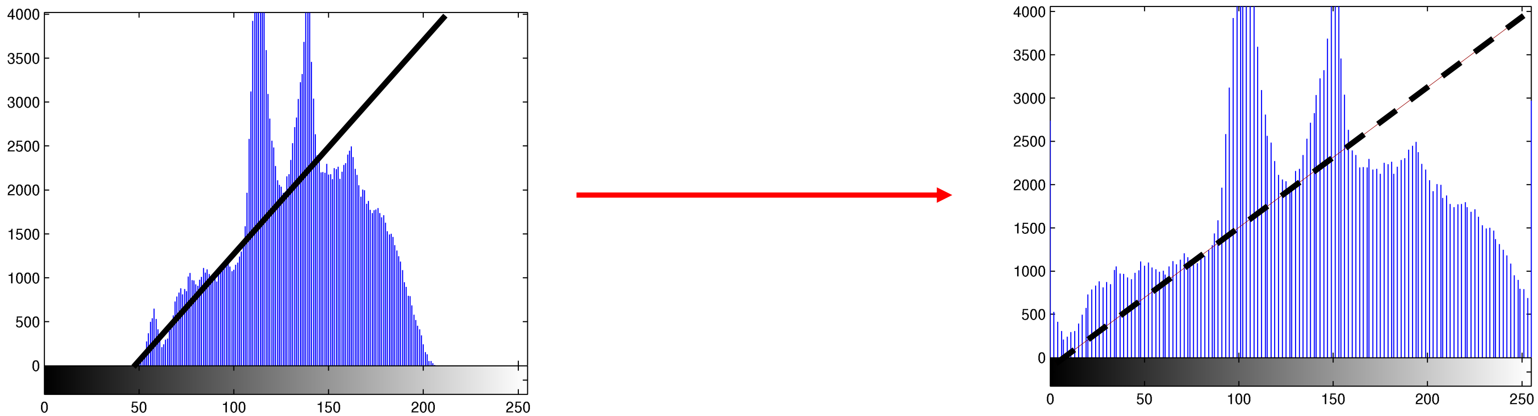
$$g(x,y) = a * f(x,y) + b$$

Histogram stretching formula

$$g(x,y) = \frac{v_{max,d} - v_{min,d}}{v_{max} - v_{min}}(f(x,y) - v_{min}) + v_{min,d}$$

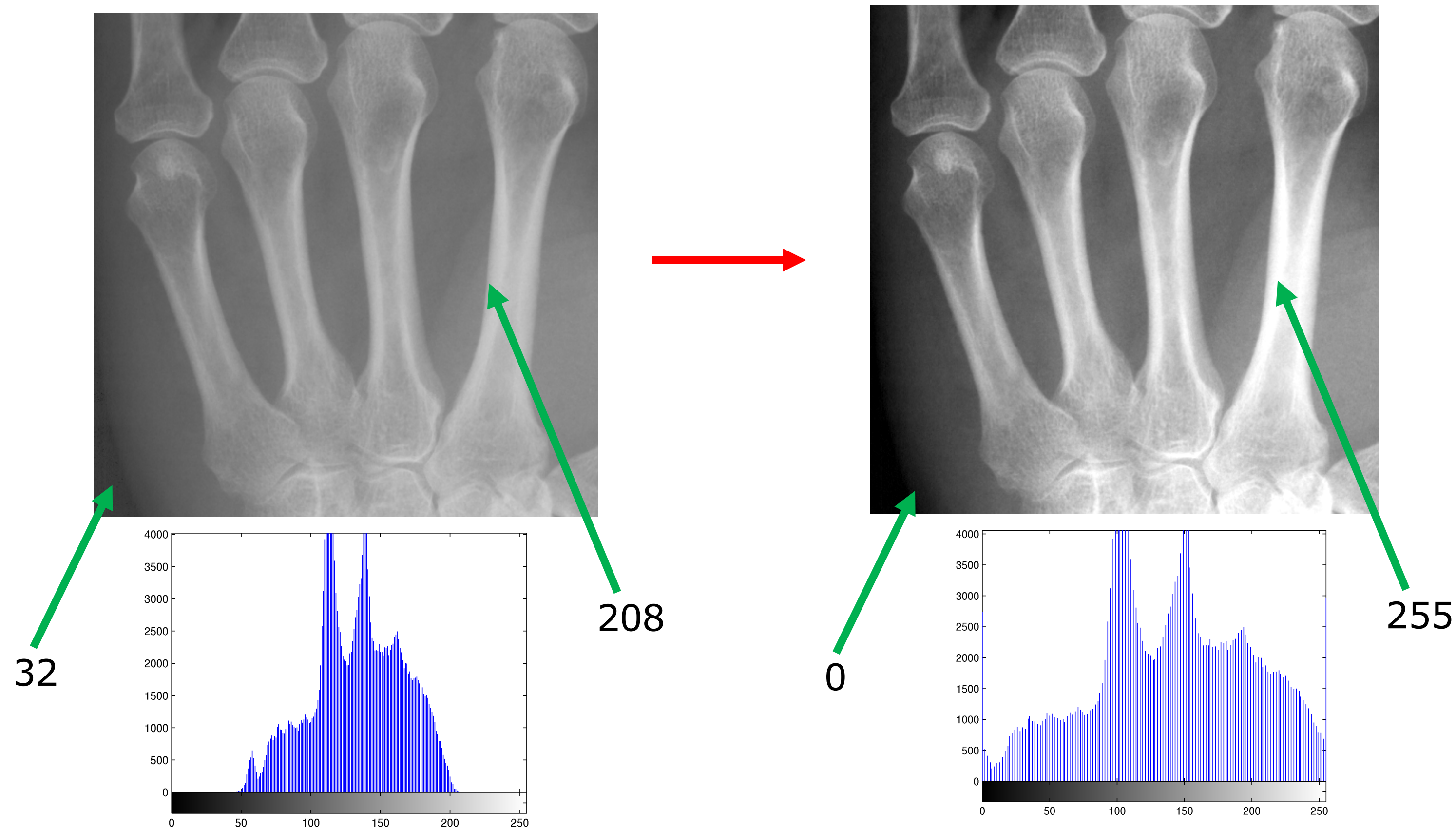
- Desired min value $V_{min,d} = 0$
- Desired max value $V_{max,d} = 255$
- Current min value $V_{min} = 32$
- Current max value $V_{max} = 208$

Histogram stretching



$$g(x,y) = \frac{255}{176}(f(x,y) - 32)$$

Effect of histogram stretching

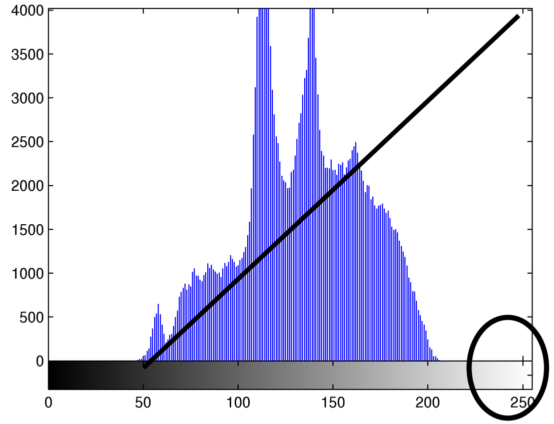


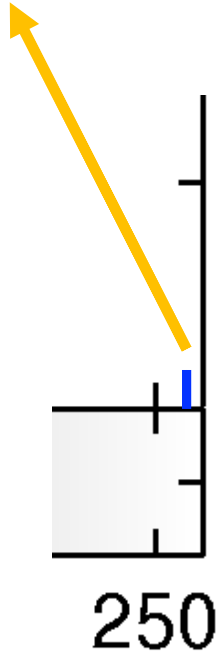
Histogram stretching – weaknesses

- A single pixel value of 0 or 255 ruins it (outlier)
- Sometimes you want
  - To stretch only the high pixel values
  - While "compressing" the low pixel values
  - Non-linear mapping

$$g(x,y) = \frac{v_{max,d} - v_{min,d}}{v_{max} - v_{min}}(f(x,y) - v_{min}) + v_{min,d}$$

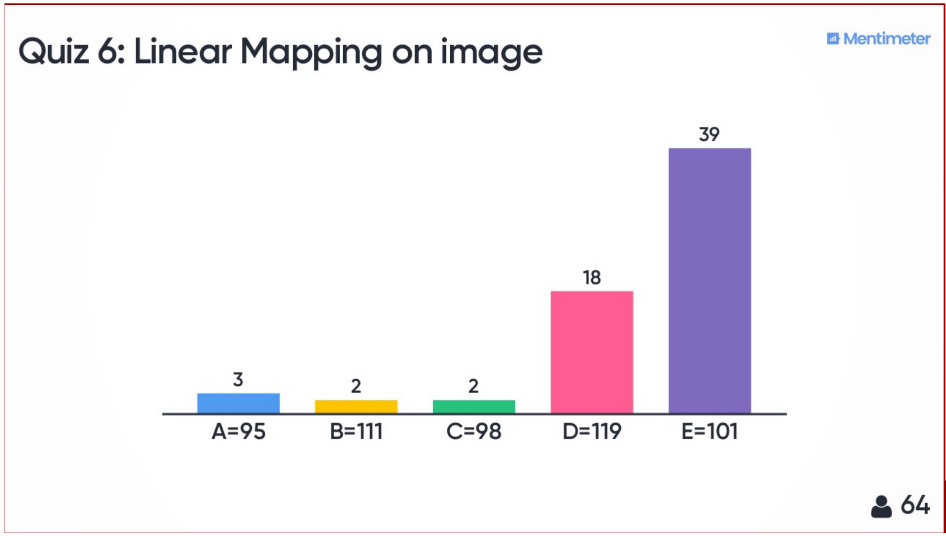
Outlier intensity pixel





Quiz 6: Linear mapping on image

A) 95
B) 111
C) 98
D) 119
E) 101



Opgave 10.25

Der udføres en *linear mapping* på billedet i Figur 16 hvor resultatet er et ny grayscale billede med maksimum værdi 255 og minimum værdi 0. Hvad er den nye værdi af den pixel, der har værdi 108?

1. 95
2. 111
3. 98
4. 119
5. 101
6. Ved ikke

$$g(x,y) = a * f(x,y) \frac{v_{max,d} - v_{min,d}}{v_{max} - v_{min}}(f(x,y) - v_{min}) + v_{min,d}$$

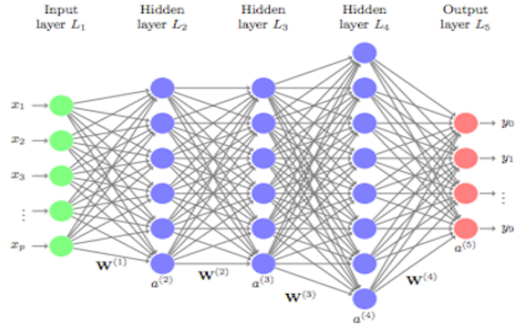
$$101 = \frac{255-0}{245-18}(108-18)+0$$

208	25	40	36	167
231	71	23	108	18
32	139	244	234	217
233	244	124	202	238
161	245	204	245	173

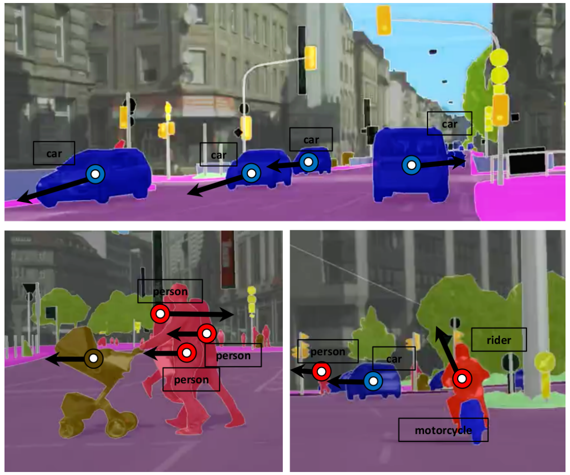
Figur 16: Grayscale billede.

Deep learning and color/gray scale transformations

- Deep learning needs training data
  - Input image
  - Ground truth labels or classes
- When you lack data you can *augment* your data
  - Create artificial versions
  - Adding variation
  - Changing gray / color levels in the image
  - Point wise operations



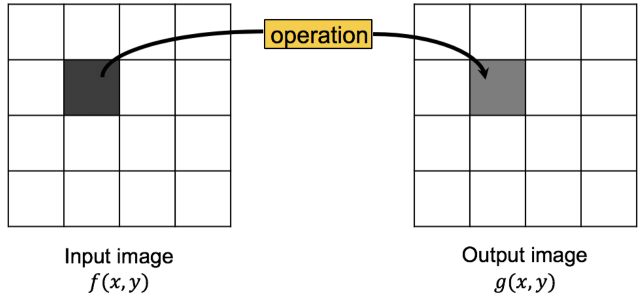
http://uc-r.github.io/feedforward_DNN



Luc, Pauline, et al. "Predicting deeper into the future of semantic segmentation." IEEE International Conference on Computer Vision (ICCV). Vol. 1. 2017.

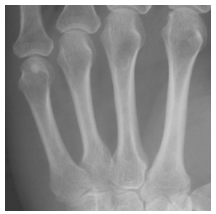
Input Segmentation [9]

https://www.quora.com/What-does-the-term-semantic-segmentation-mean-in-the-context-of-Deep-Learning



Other mappings

- Non-linear mappings
- Not always nice to work with byte images [0,255]
  - Better to work with image with values in floating point [0,1]



Byte image

Conversion to [0,1]

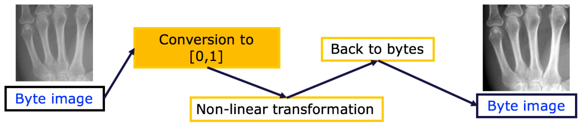
Non-linear transformation

Back to bytes

Byte image

Working with bytes and doubles

- A byte contains integer values [0,255]
  - A byte can not store 127.4232 i.e. floating point
- A value of type *double* is a floating point and can contain “all numbers”
- Why not use doubles always?
  - One double = 8 bytes in the memory
  - Images become very large!
  - Many things can be done with bytes



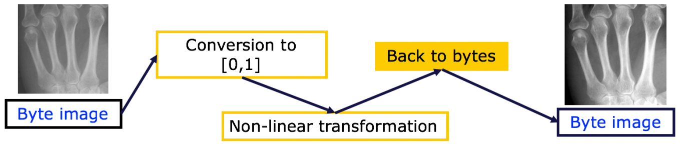
Map pixels into floating point [0,1]

- In Matlab it is easiest to create a new image of type double which allows floating-point format [0,1]
  - Itemp = double(I);

    (temp means temporary and is used by many programmers for variables that quickly are thrown away)

- Conversion to [0,1]:

$$g(x,y) = \frac{1}{255} f(x,y)$$

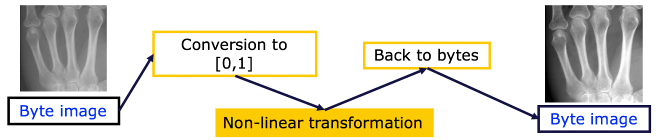


Pixels back to bytes [0,255]

- Input pixels are [0,1]
- We want them to be [0,255]
- Simple linear transformation equal to
  - Contrast?
  - Brightness?

- Back to bytes
  - Ifinal = uint8(Itemp);

$$g(x,y) = 255 * f(x,y)$$



Gamma mapping

- Gamma mapping is used in televisions and flat panels
- Can increase the contrast (dynamics) in more selected part of the histogram
- Many games have a possibility for a gamma correction

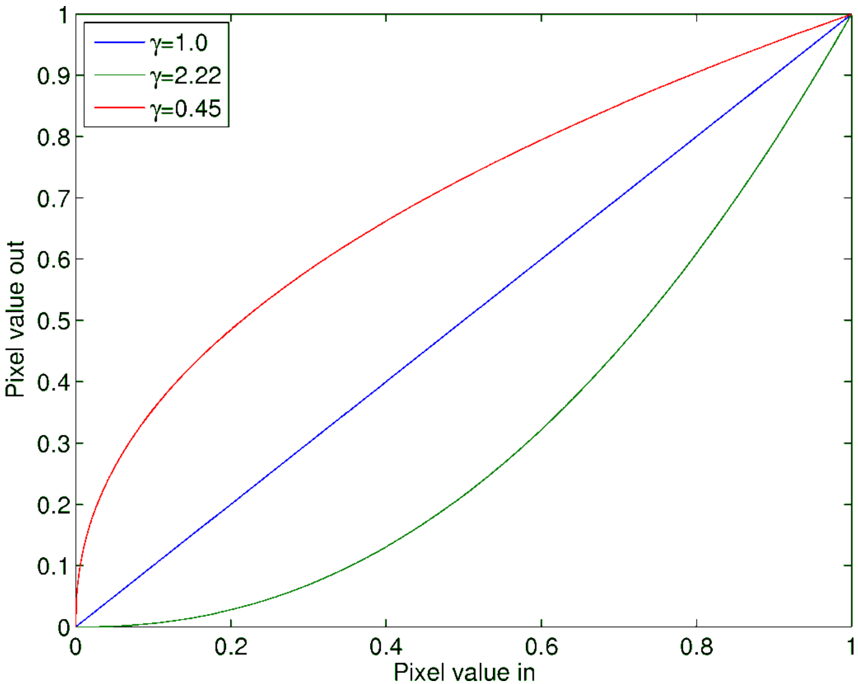
Gamma encoding of images is used to optimize the usage of bits when encoding an image, or bandwidth used to transport an image***, by taking advantage of the non-linear manner in which humans perceive light and color.***

$$g(x,y) = f(x,y)^{\gamma}$$

- Gamma value $\gamma<1$ is called an *encoding gamma*
- Gamma value $\gamma>1$ is called a *decoding*
- Only normalized intensities [0,1]

https://en.wikipedia.org/wiki/Gamma_correction

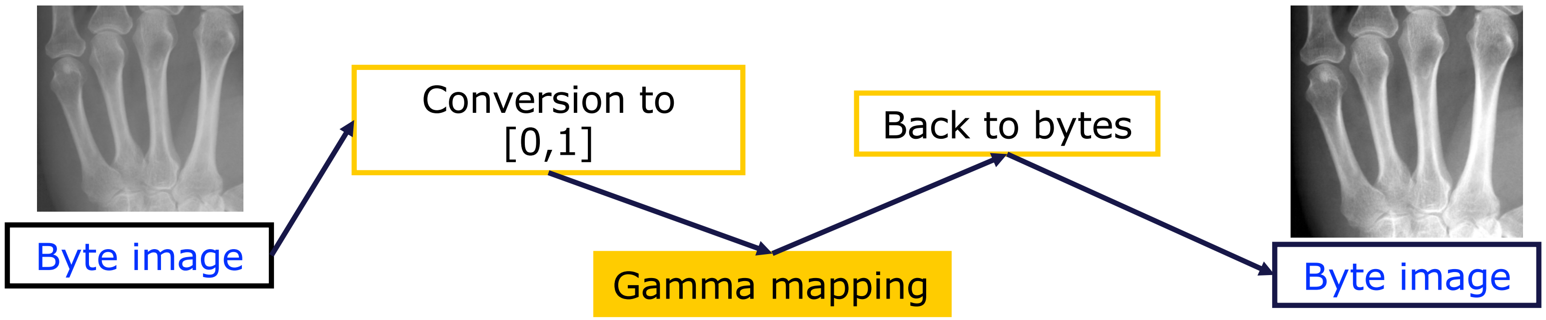
Gamma curves



- Names after the Greek letter gamma
- What happens to the dark areas
  - With 0.45?
  - With 2.22?

$$g(x, y) = f(x, y)^{\gamma}$$

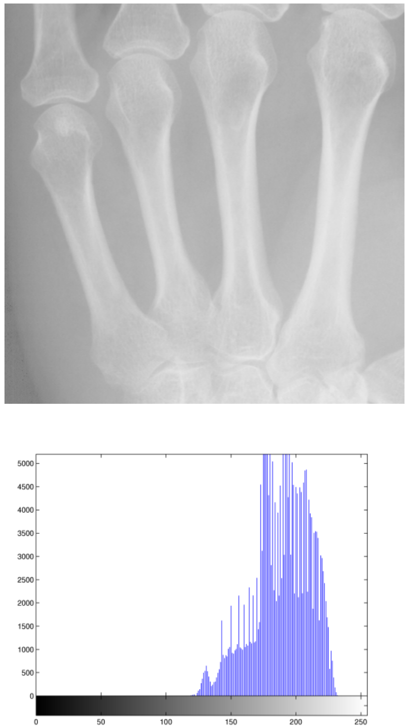
Perform the gamma mapping

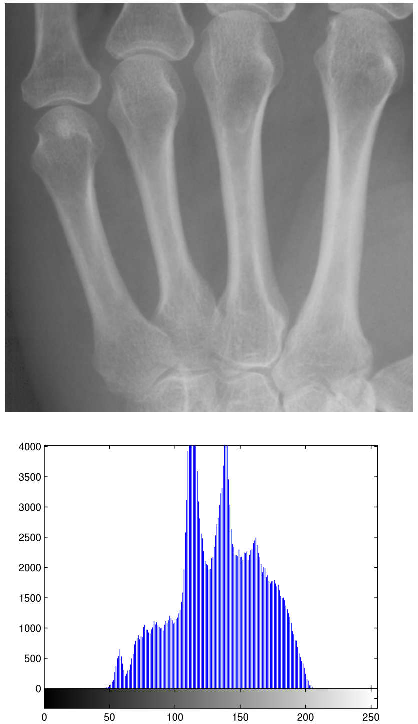


Results of gamma mapping

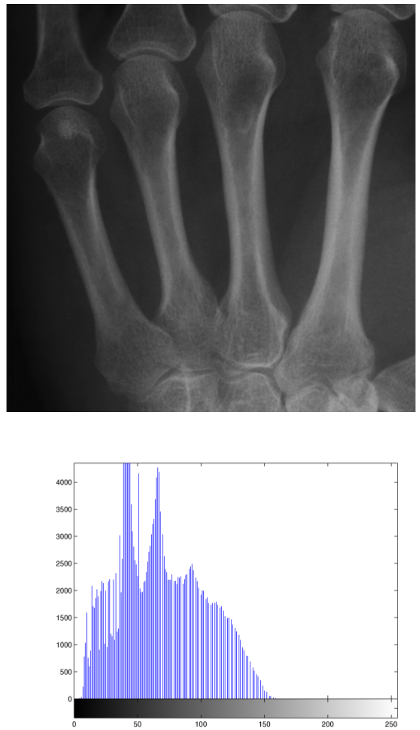
$$g(x,y) = f(x,y)^{\gamma}$$

$\gamma = 0.45$





$\gamma = 2.22$



Quiz 7: Gamma mapping

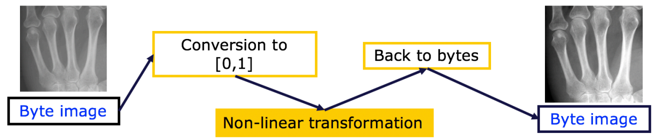
1. 0 og 255
2. 25 og 130
3. 8 og 242
4. 15 og 230
5. 37 og 219

Opgave 10.3

Der udføres en *gamma mapping* med $\gamma = 1.3$ på billedet i Figur 3. Resultatet er et ny grayscale billede. Hvad er den mindste og største pixelværdi i det nye billede?

1. 0 og 255
2. 25 og 130
3. 8 og 242
4. 15 og 230
5. 37 og 219
6. Ved ikke

$$g(x,y) = f(x,y)^{\gamma}$$



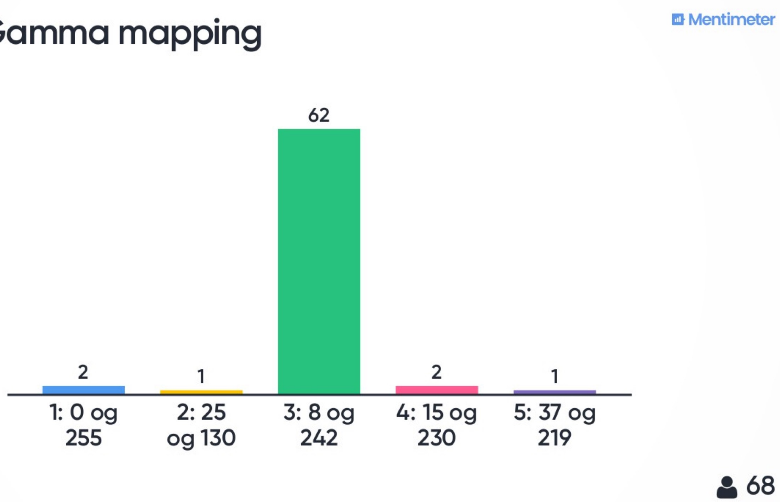
$$8 = 255\left(\frac{1}{255}18\right)^{1,3}$$

$$242 = 255\left(\frac{1}{255}245\right)^{1,3}$$

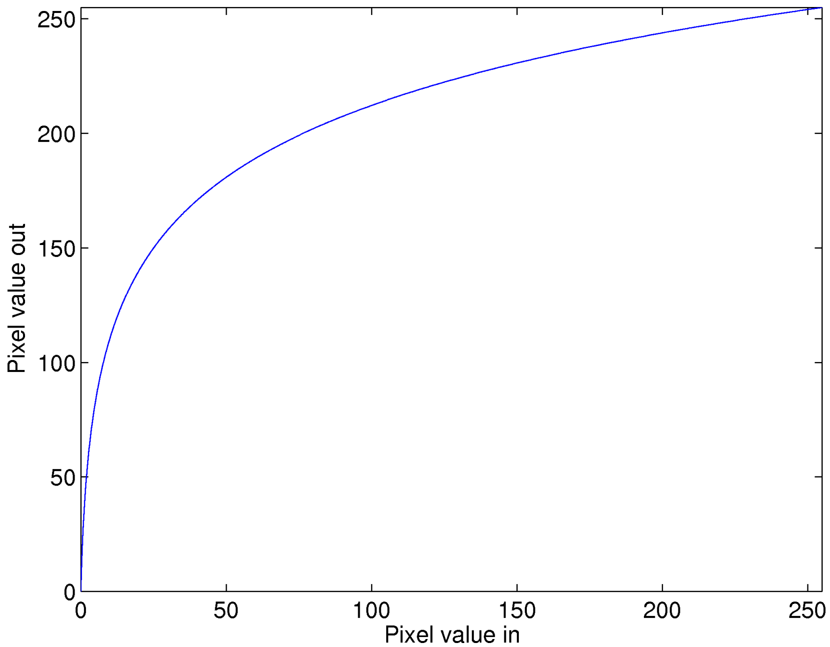
208	25	40	36	167
231	71	23	108	18
32	139	244	234	217
233	244	124	202	238
161	245	204	245	173

Figur 3: Grayscale billede.

Quiz 7: Gamma mapping



Logarithmic mapping



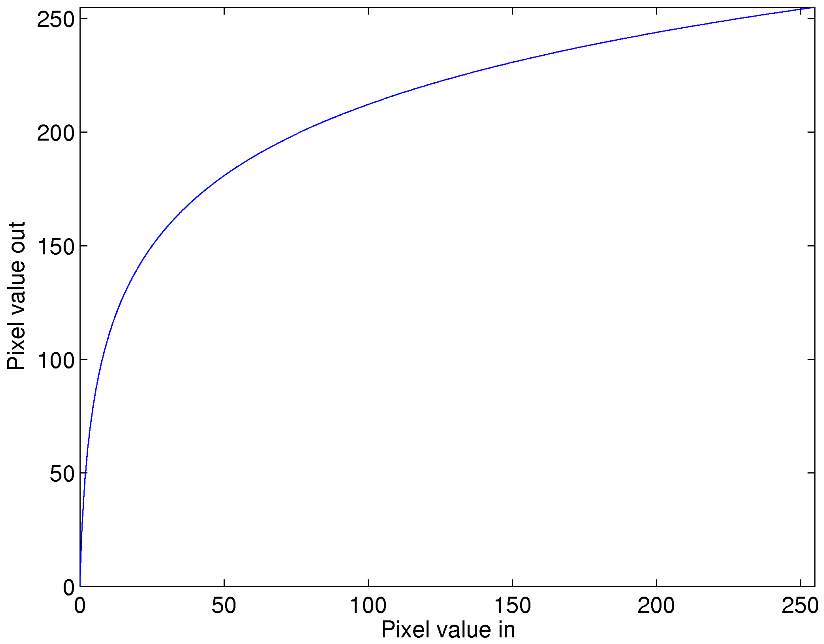
Maps from [0,255] to [0,255]

Why?

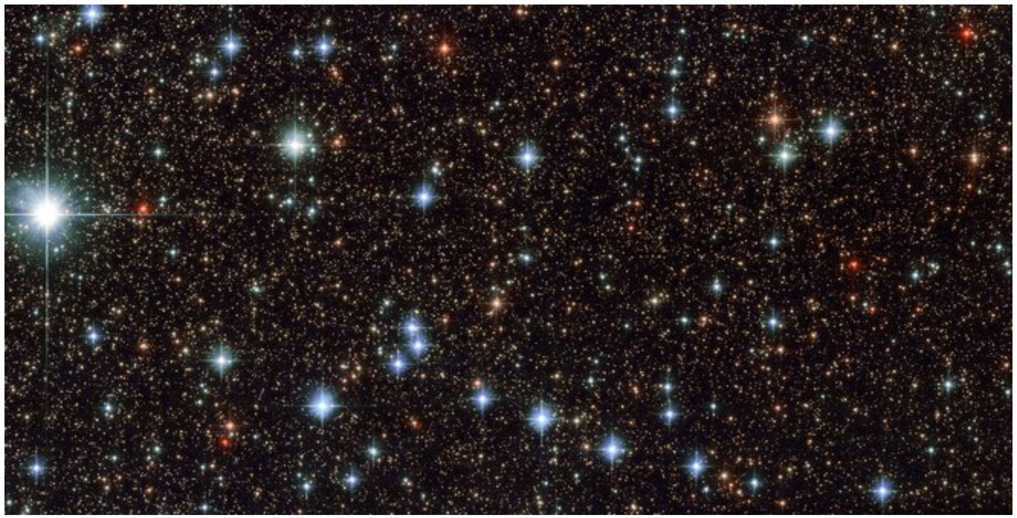
$$g(x,y) = c \log(1 + \mathrm{f}(x,y))$$

$$c = \frac{255}{\log(1 + v_{max})}$$

Logarithmic mapping – when?



- For images with very bright spots
- Low intensity pixel values are enhanced
- Similar effect as contrast but is non-linear



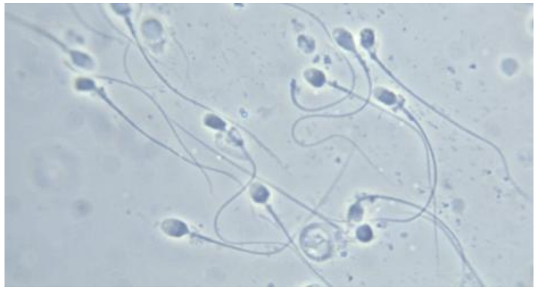
What do we get out of pixel mappings

- Spreading out or compressing pixel values
  - For your brain only(?): Better for humans to see
  - New image information – no!
  - Important tool: Normalising histogram as input to algorithms e.g. Neural Networks

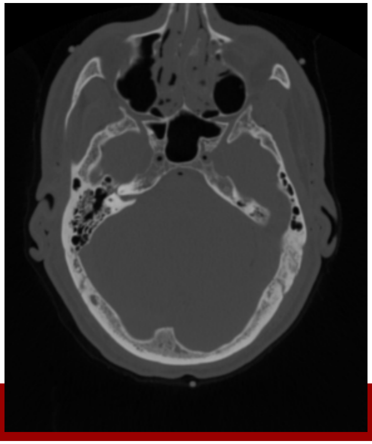
Now for something different

- Until now image processing
  - Input image transformed to output image
- Now for something more like image analysis
- Segmentation
  - Segment the image into regions
  - Background and objects for example

Light Microscopy



Computed tomography (CT)



Camara

Magnetic Resonance Imaging (MRI)

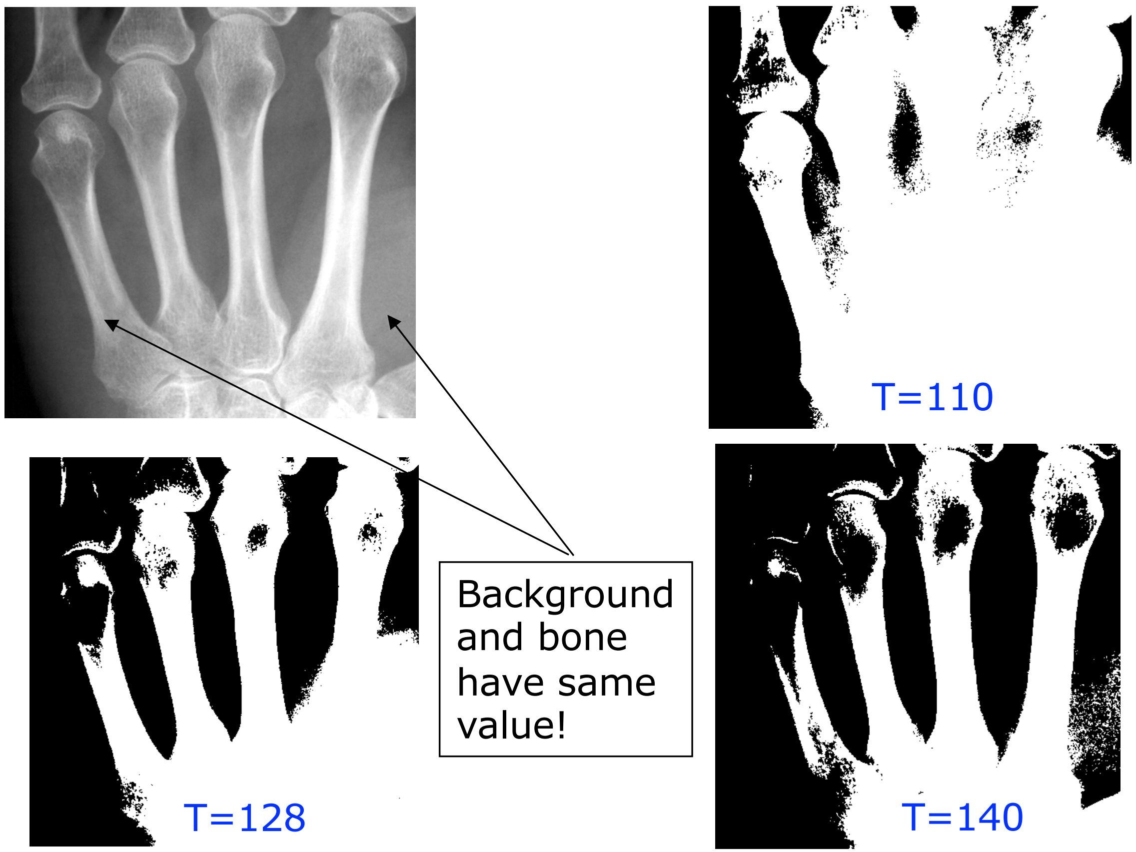
Thresholding

- A threshold *T* is a value
  - Pixels below that value is set to 0 (background)
  - Pixels equal or above is set to 1 (object)
- One threshold value for the entire image
  - Difficult to choose!

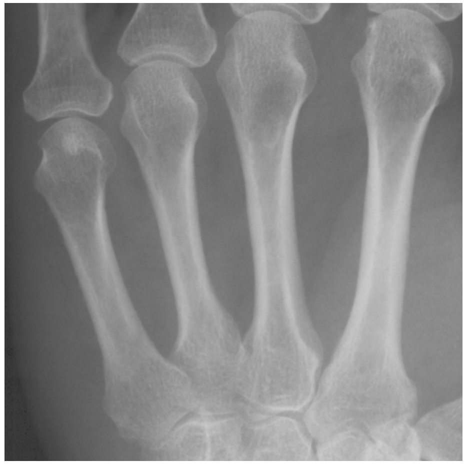
$$\text{if } f(x,y) \leq T \;\text{ then } g(x,y) = 0$$

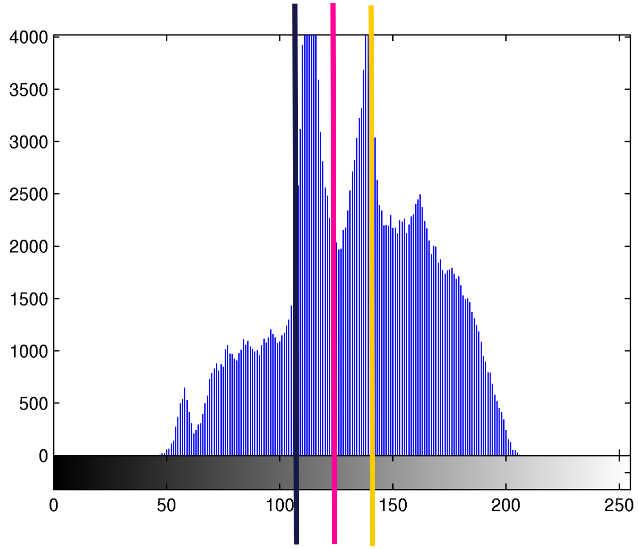
$$\text{if } f(x,y) > T \;\text{ then } g(x,y) = 255$$

Thresholding

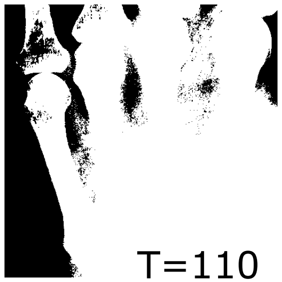


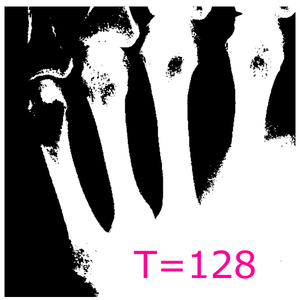
Thresholding based on the histogram

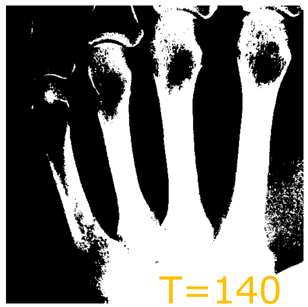




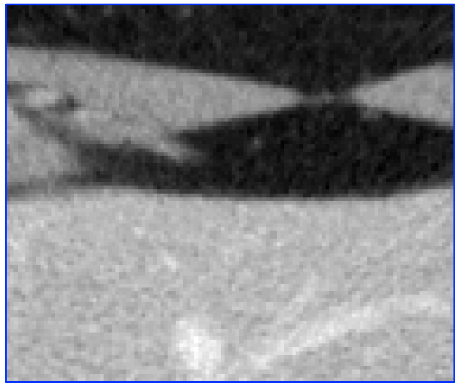
The bones are visible and some soft tissue in the histogram!

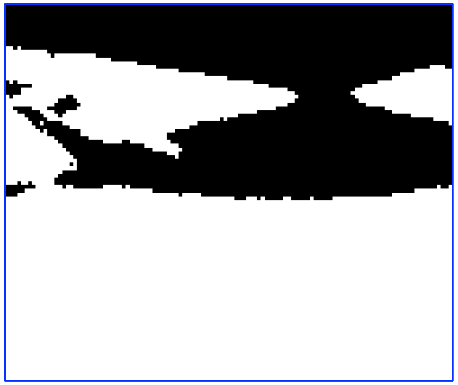


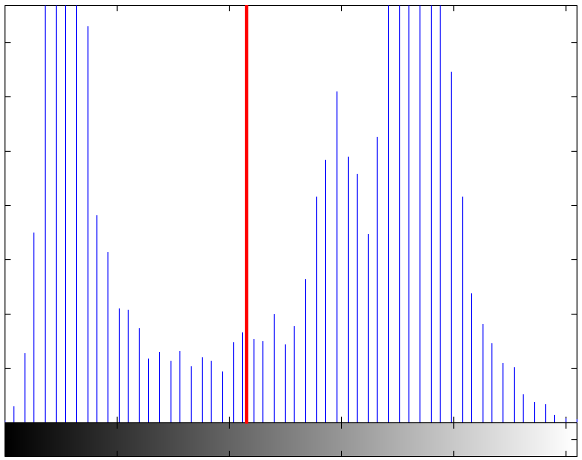




Automatic Thresholding

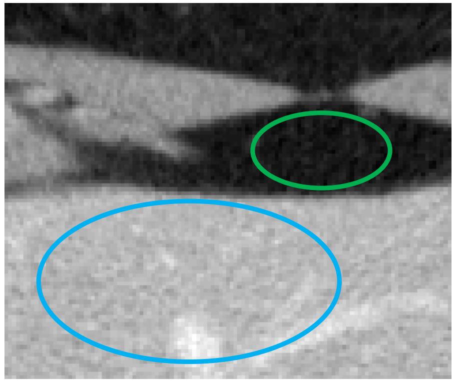




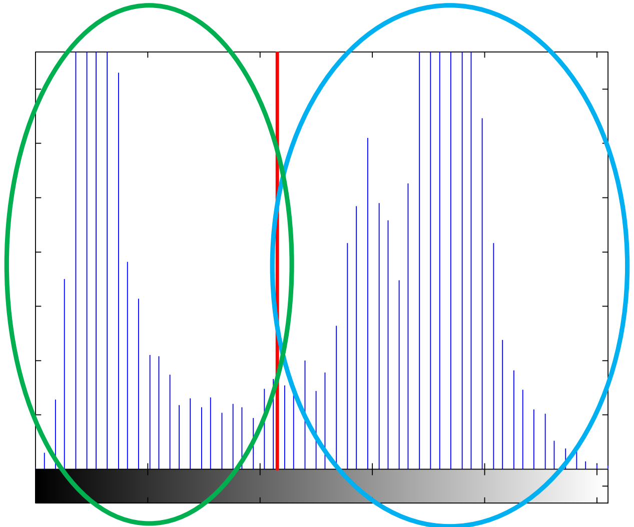


Otsu, Nobuyuki. "A threshold selection method from gray-level histograms." IEEE transactions on systems, man, and cybernetics 9.1 (1979): 62-66.

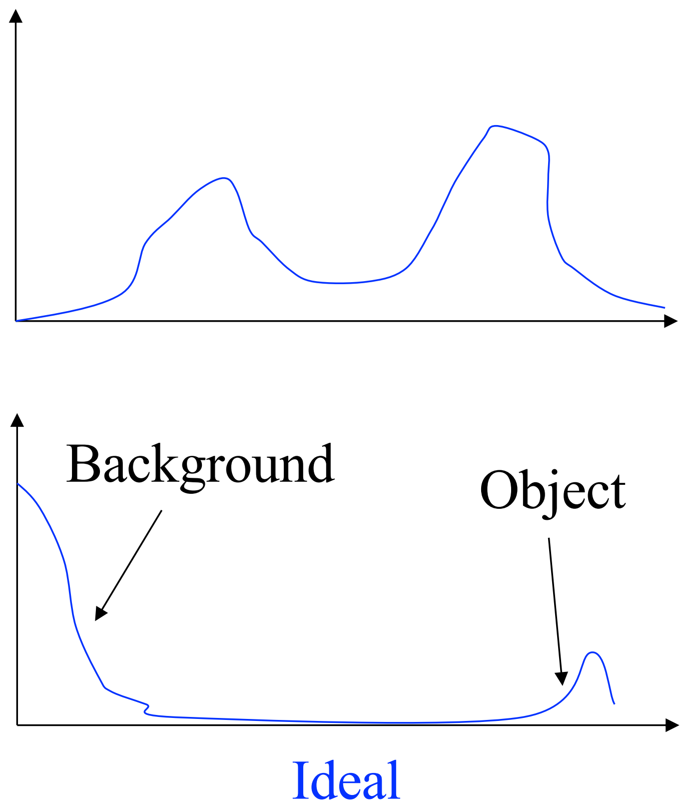
Automatic Tresholding
Otsu's method



- Two classes: background and object
- T divides pixels into object and background
- Compute pixel value variance in each class
- Find T that minimises combined variance

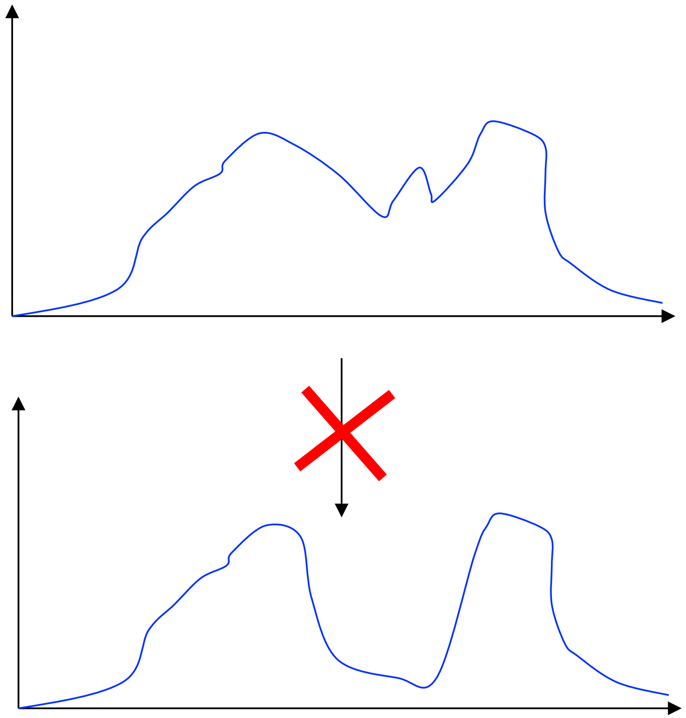


Segmentation – histogram shaping

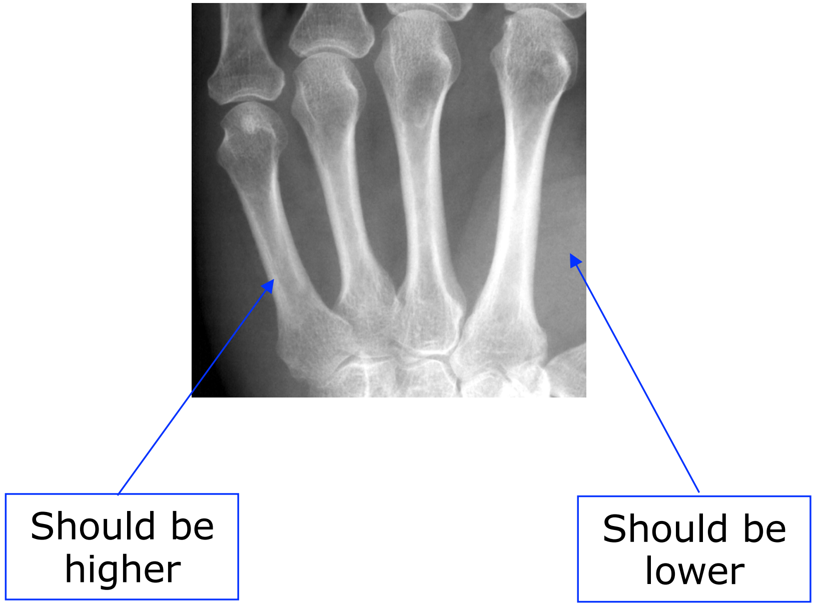


- With a threshold you want a histogram with two peaks
  - *Bimodal*
- An ideal histogram has well separated peaks
- Obtaining a bi-modal histogram is very important in the image acquisition

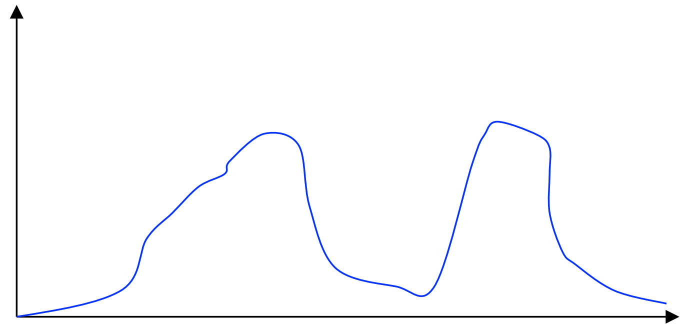
Histogram shaping



- It is not possible to "unmix" using gray level transformations



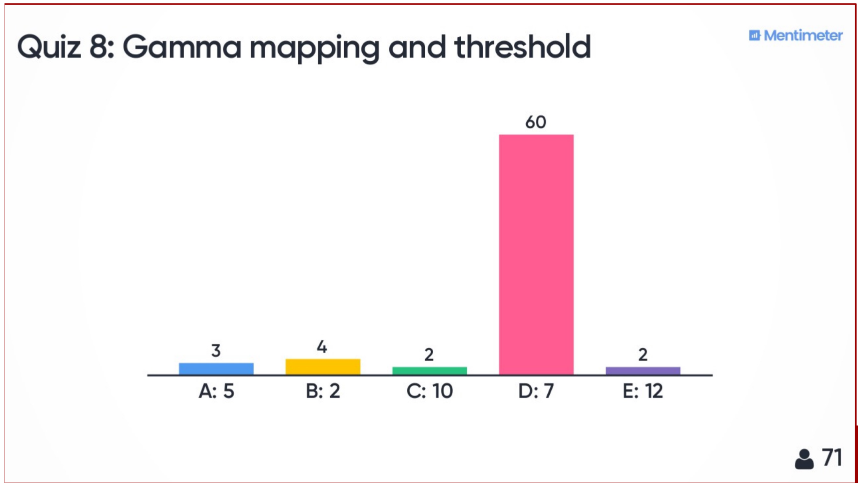
How to obtain good histograms



- With cameras
  - Light
  - Setup
  - Camera
  - Lens
  - Backlight?

Quiz 8: Gamma mapping and threshold

A) 5
B) 2
C) 10
D) 7
E) 12

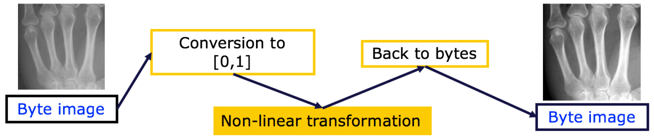


Opgave 14.19

Der udføres en *gamma mapping* med $\gamma = 1.1$ på billedet i Figur 12 og herefter sættes der et *threshold* på 120. Pixels over threshold sættes til forgrund og resten til baggrund. Hvor mange forgrundspixler er der i resultatbilledet?

1. 5
2. 2
3. 10
4. 7
5. 12
6. Ved ikke

$$g(x, y) = f(x, y)^{\gamma}$$



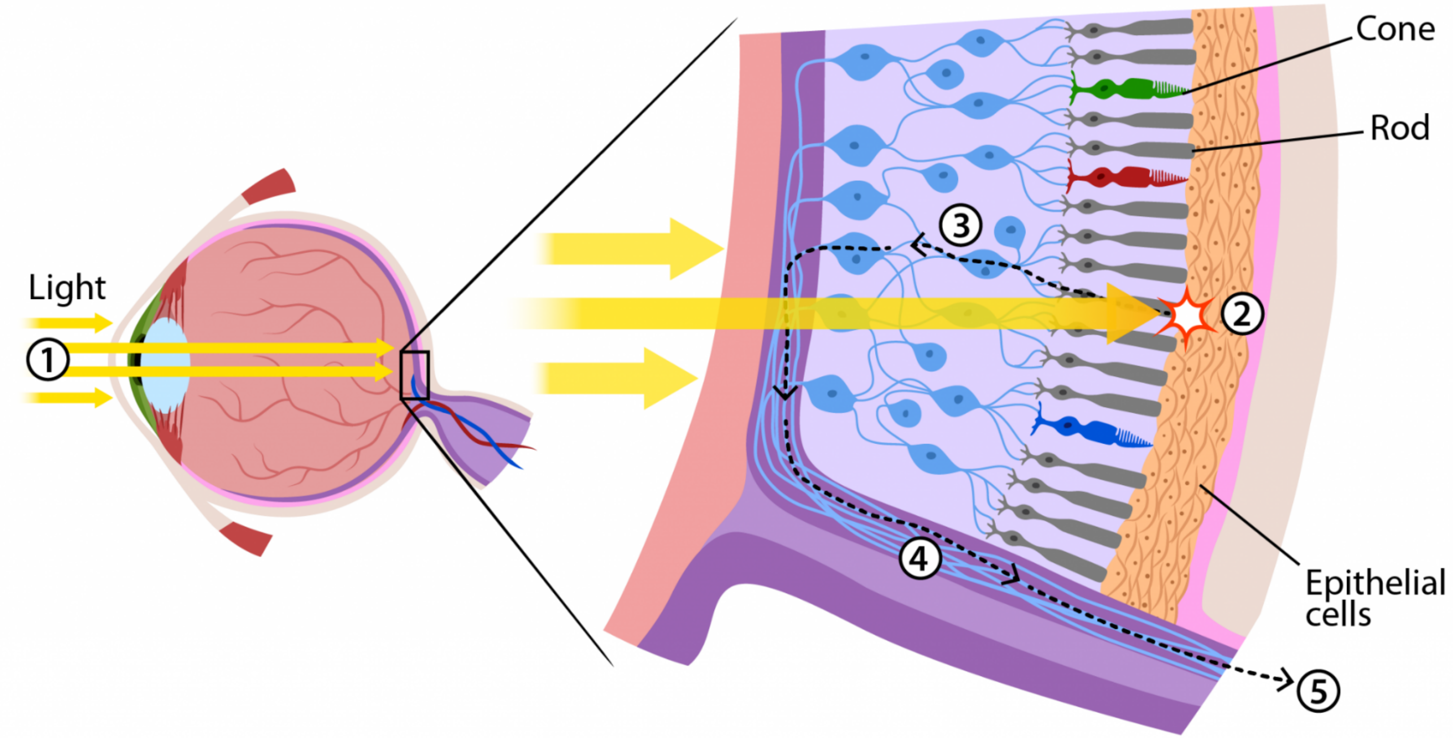
108	245	217
234	167	238
202	9	173

Figur 12: Grayscale billede.

$$160 = 255\left(\frac{1}{255}167\right)^{1,1}$$

$$99 = 255\left(\frac{1}{255}108\right)^{1,1}$$

Colour images and colour perception
The Human Eye



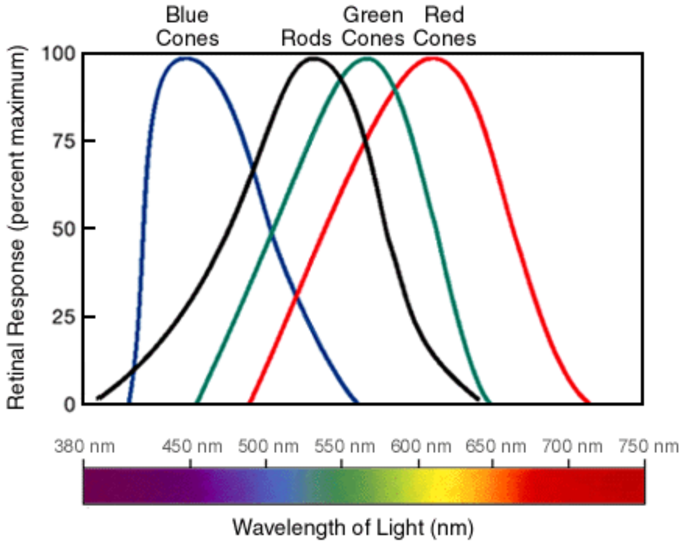
(Color, and need more light)

(Low level of light, no color)

https://askabiologist.asu.edu/rods-and-cones

Color sensitivity

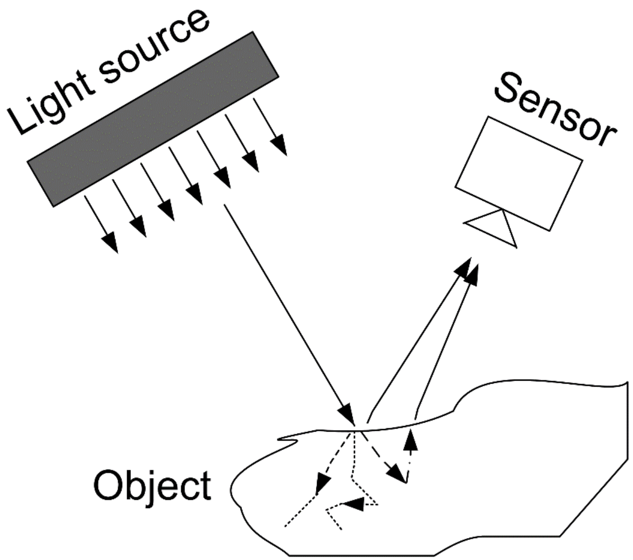
Photoreceptor cell	Wavelength in nanometers (nm)	Peak response in nanometer (nm)	Interpretation by the human brain
Cones (type L)	[400-680]	564	Red
Cones (type M)	[400-650]	534	Green
Cones (type S)	[370-530]	420	Blue
Rods	[400-600]	498	Shade of gray

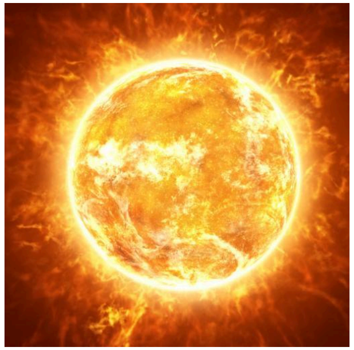


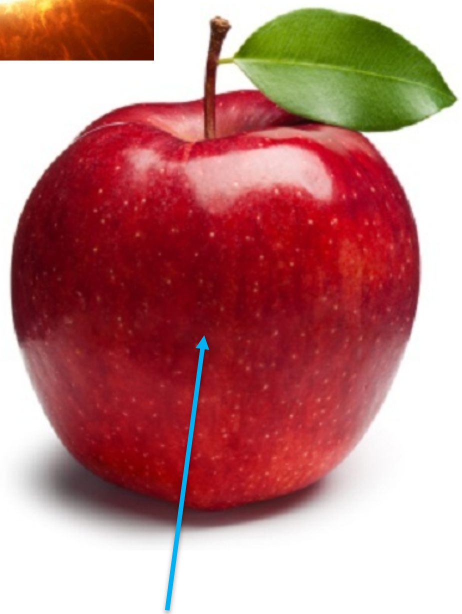
https://askabiologist.asu.edu/rods-and-cones

Object colors

Subtractive colors







All other colors than red absorbed

Object colors

Additive colors

- Additive colours: Final colour is made by mixing red, green, and blue
- RGB = Red, Green, and Blue
- Television, computers, digital cameras use the “RGB color space”
- Typically the values of R, G, and B lie between 0 and 255

RGB Colours

- When all three “Lamps” are turned of we get black
- When all three “lamps” are on what do we get?

RGB = (255,255,255)

RGB = (0,0,0)

(255,0,0)

(0,255,0)

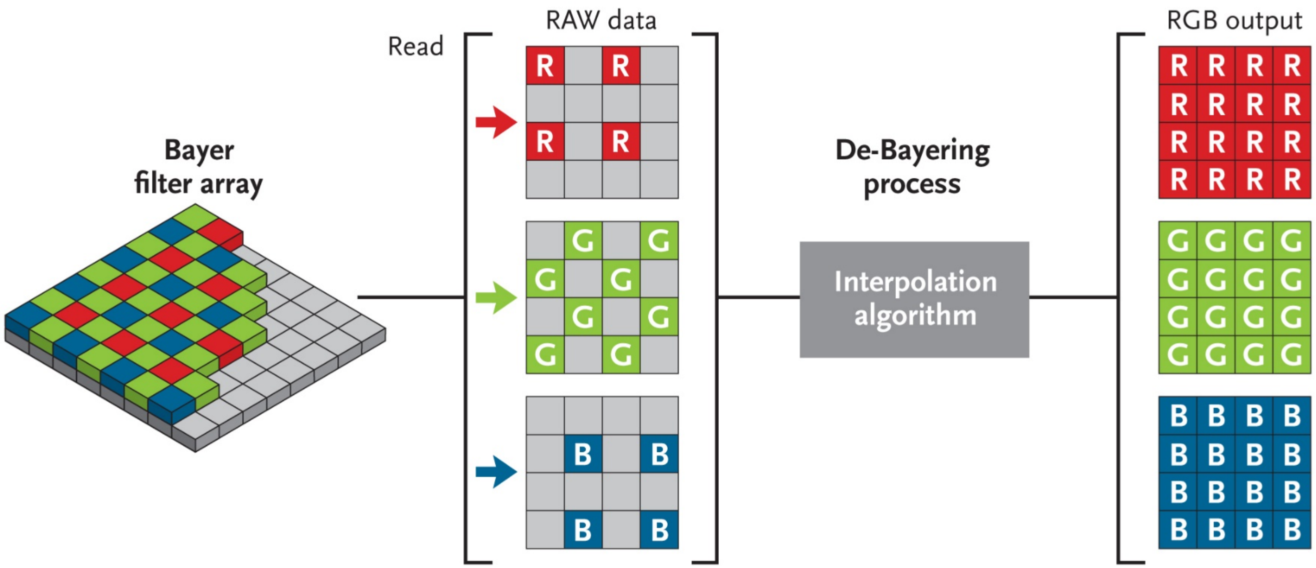
(0,0,255)

(255,255,0)

(0,255,255)

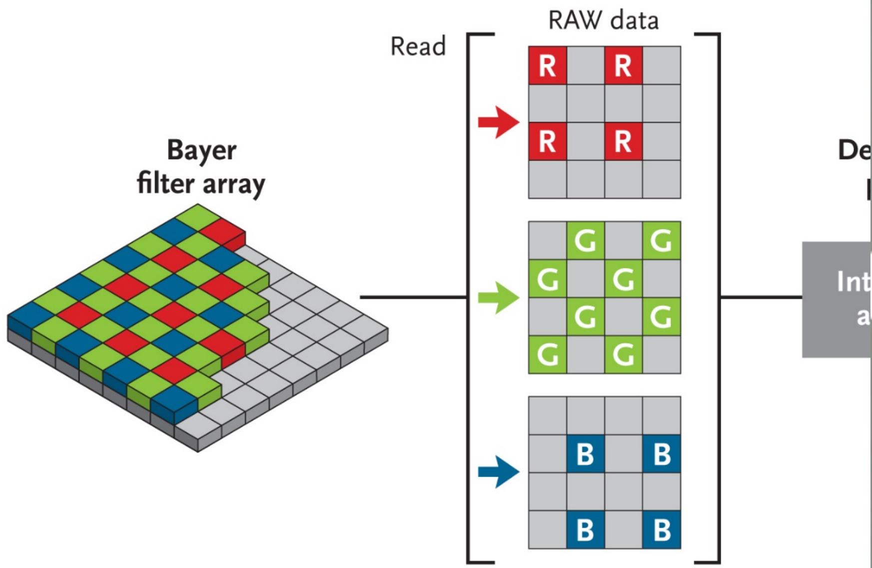
(255,0,255)

Color camera with one sensor

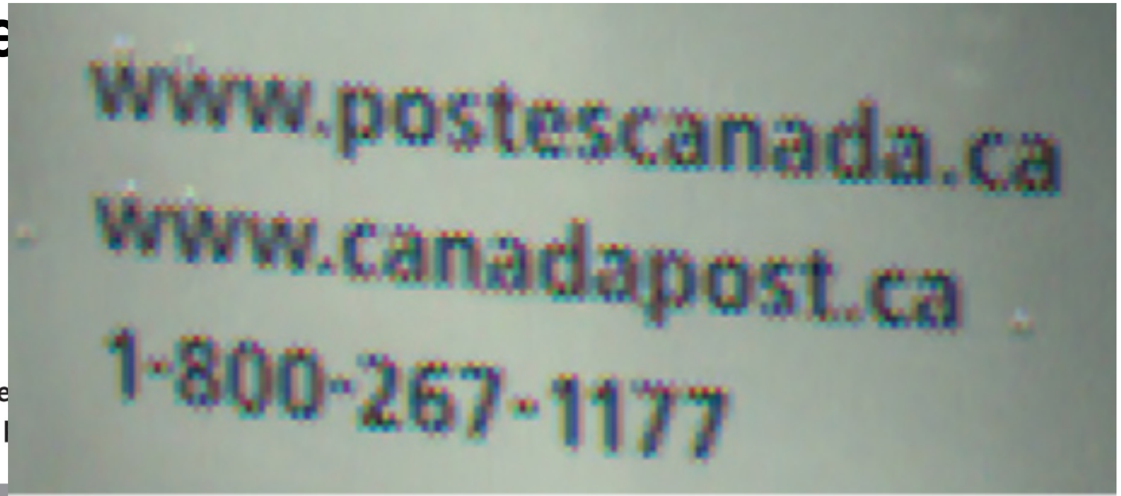


http://www.skyandtelescope.com/astronomy-resources/astrophotography-tips/redeeming-color-planetary-cameras/

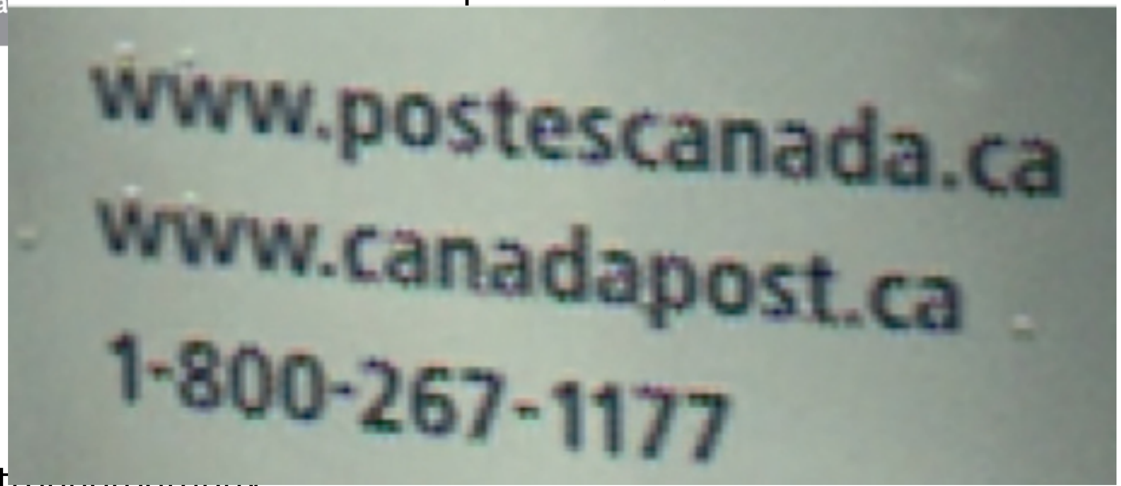
Color camera with one



Simple interpolation: Nearest Neighbourhood

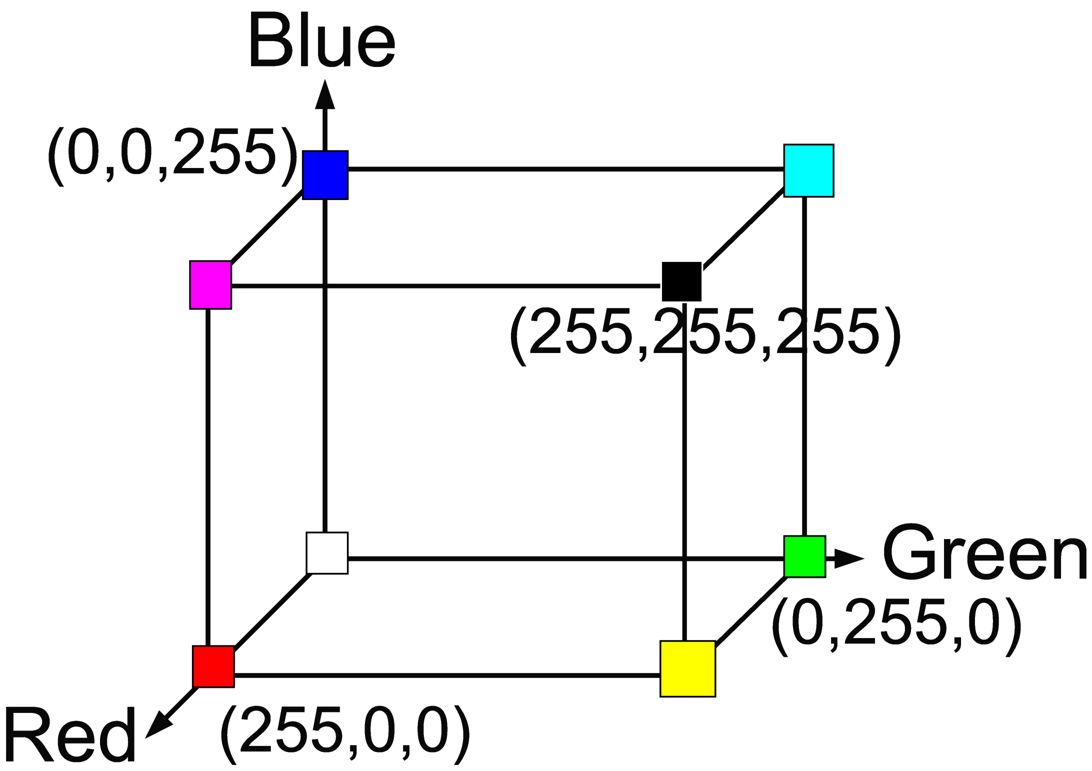


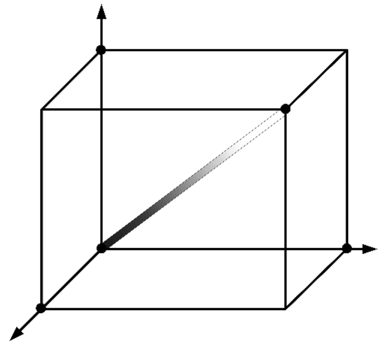
Advanced interpolation: HQ Linear



http://www.skyandtelescope.com/astronomy-resources/ast
tips/redeeming-color-planetary-cameras/

RGB color space





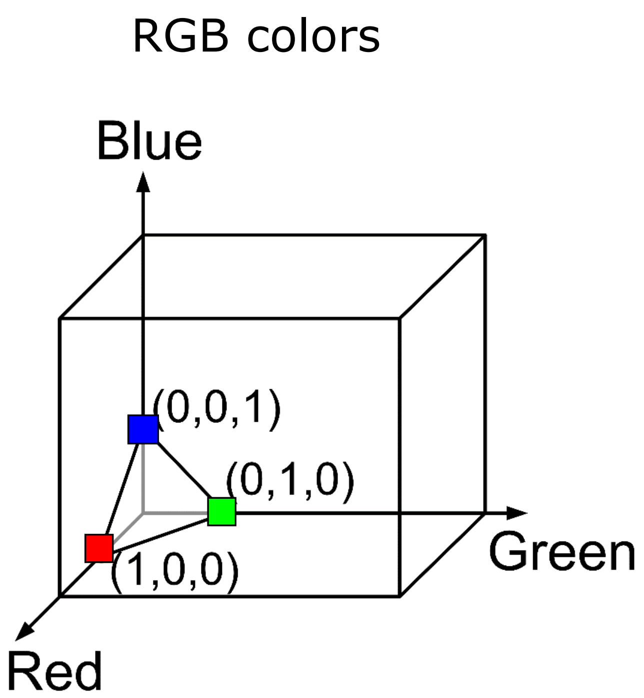
Processing RGB images

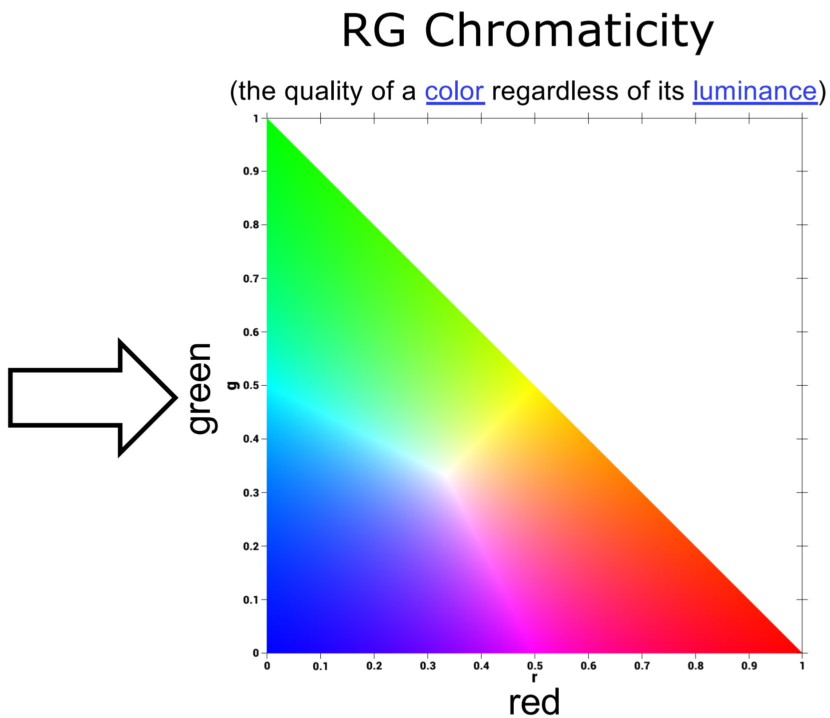
- Each pixel in a colour image contains 3 values
- Equal to a “vector function” in mathematics
- Much more complicated to analyse
- Medical images are typically grayscale
- Therefore we convert from colours to grayscale before the analysis

Converting colour to grayscale

$$v = 0.2989 * R + 0.5870 * G + 0.1140 * B$$

Normalised RGB colors



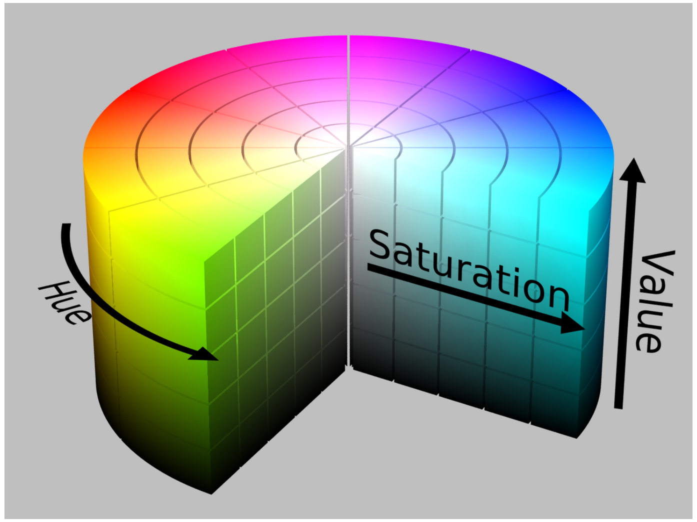


$$(r, g, b) = \left(\frac{R}{R+G+B}, \frac{G}{R+G+B}, \frac{B}{R+G+B}\right)$$

Where r+g+b=1

HSI Color Representation

- **Hue** – the dominant wave length in the perceived light (the pure color)
- **Saturation** – the purity of the color
- **Intensity** – the brightness of the color (sometimes called the value)



Converting between RGB and HSI

- You have an RGB value
- You want the corresponding HSI value

$$H = \begin{cases} \cos^{-1}\left(1/2 \cdot \frac{(R-G)+(R-B)}{\sqrt{(R-G)(R-G)+(R-B)(G-B)}}\right), & \text{if } G \geq B; \\ 360^\circ - \cos^{-1}\left(1/2 \cdot \frac{(R-G)+(R-B)}{\sqrt{(R-G)(R-G)+(R-B)(G-B)}}\right), & \text{Otherwise.} \end{cases} \quad (8.8)$$

$$H \in [0, 360[$$

$$S = 1 - 3 \cdot \frac{\min\{R, G, B\}}{R+G+B} \qquad S \in [0, 1] \quad (8.9)$$

$$I = \frac{R+G+B}{3} \qquad I \in [0, 255] \ , \quad (8.10)$$

Where:

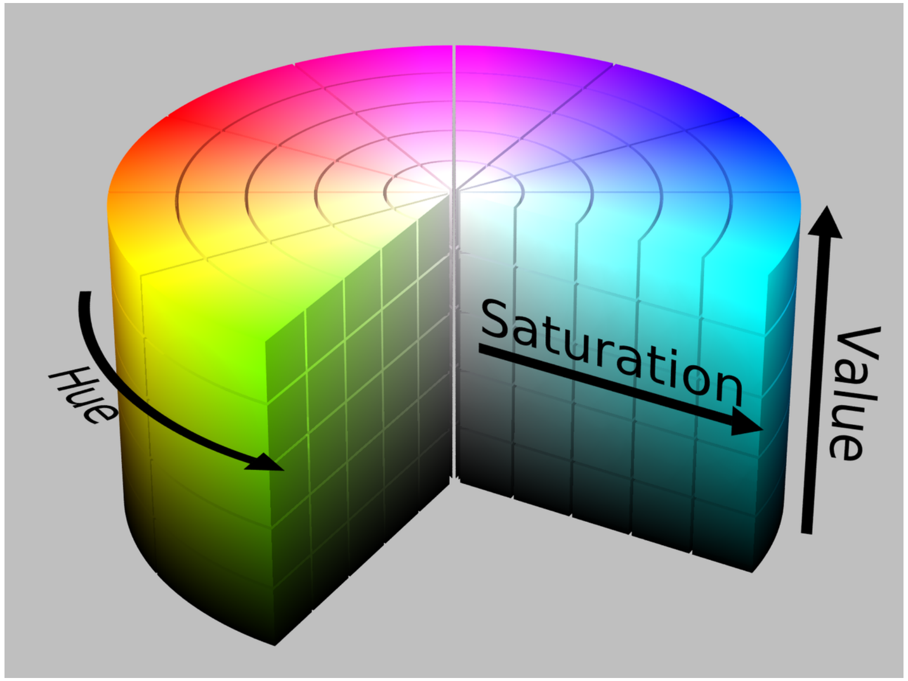
H: Hue

S: Saturation

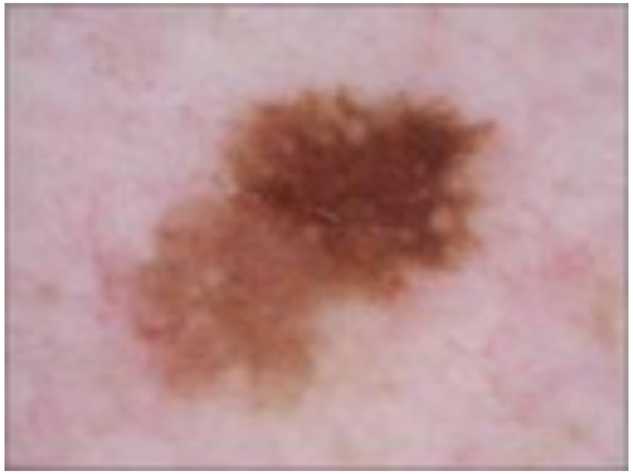
I: Intensity

Why other colorspaces

- Why should we use for example HSI ?



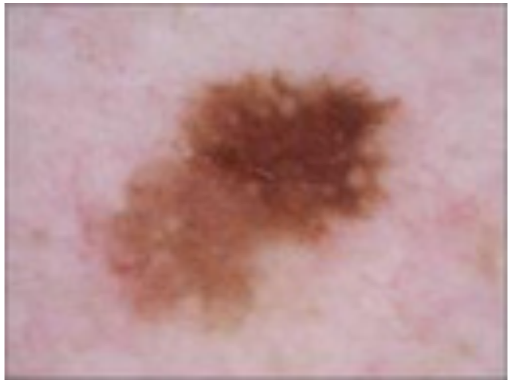
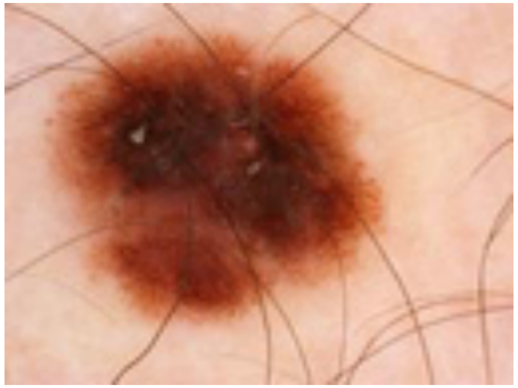
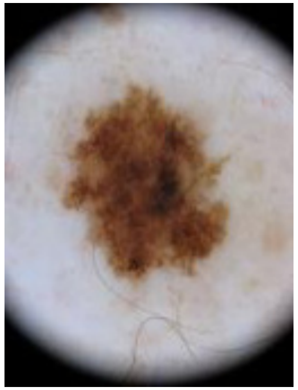
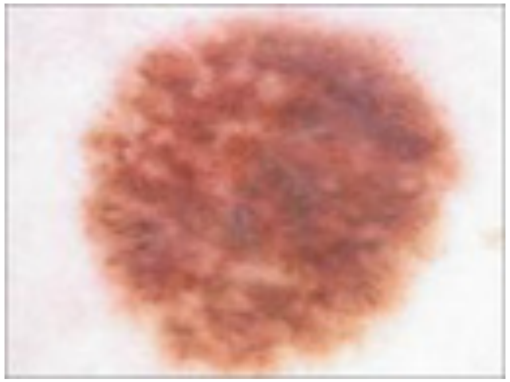
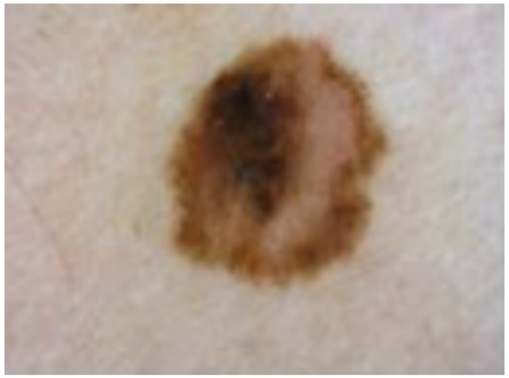
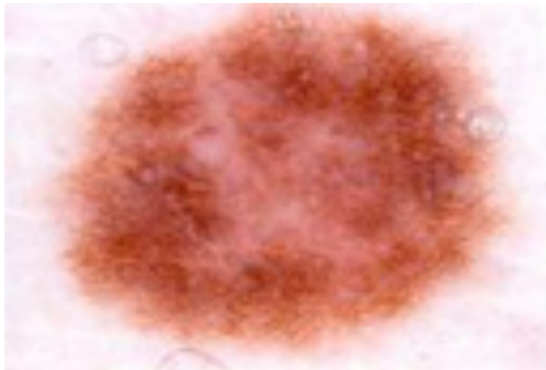
Melanoma segmentation



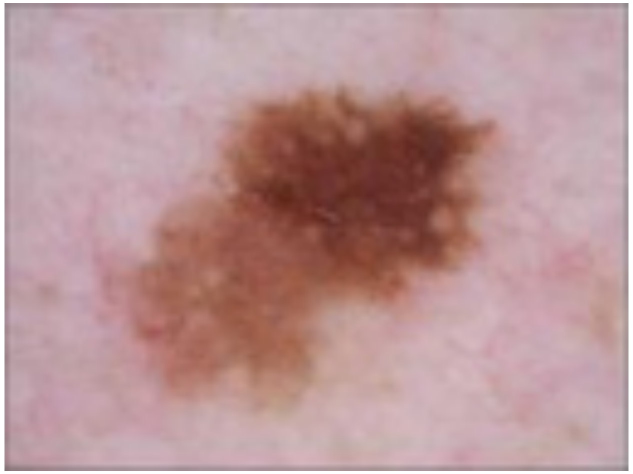
- An algorithm that can do pixelwise classification
  - Background / skin
  - Melanoma

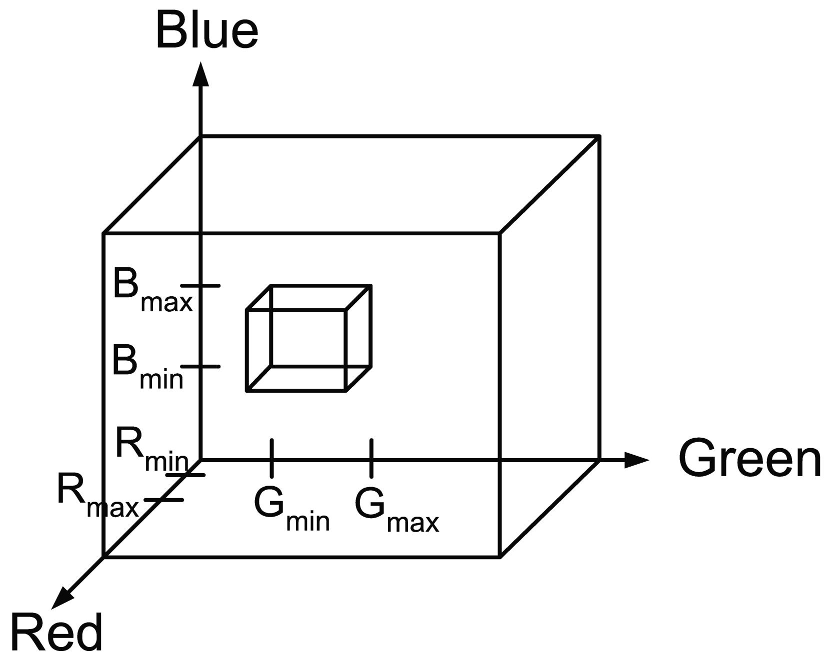
- Use the colors

Melanoma segmentation – color variation

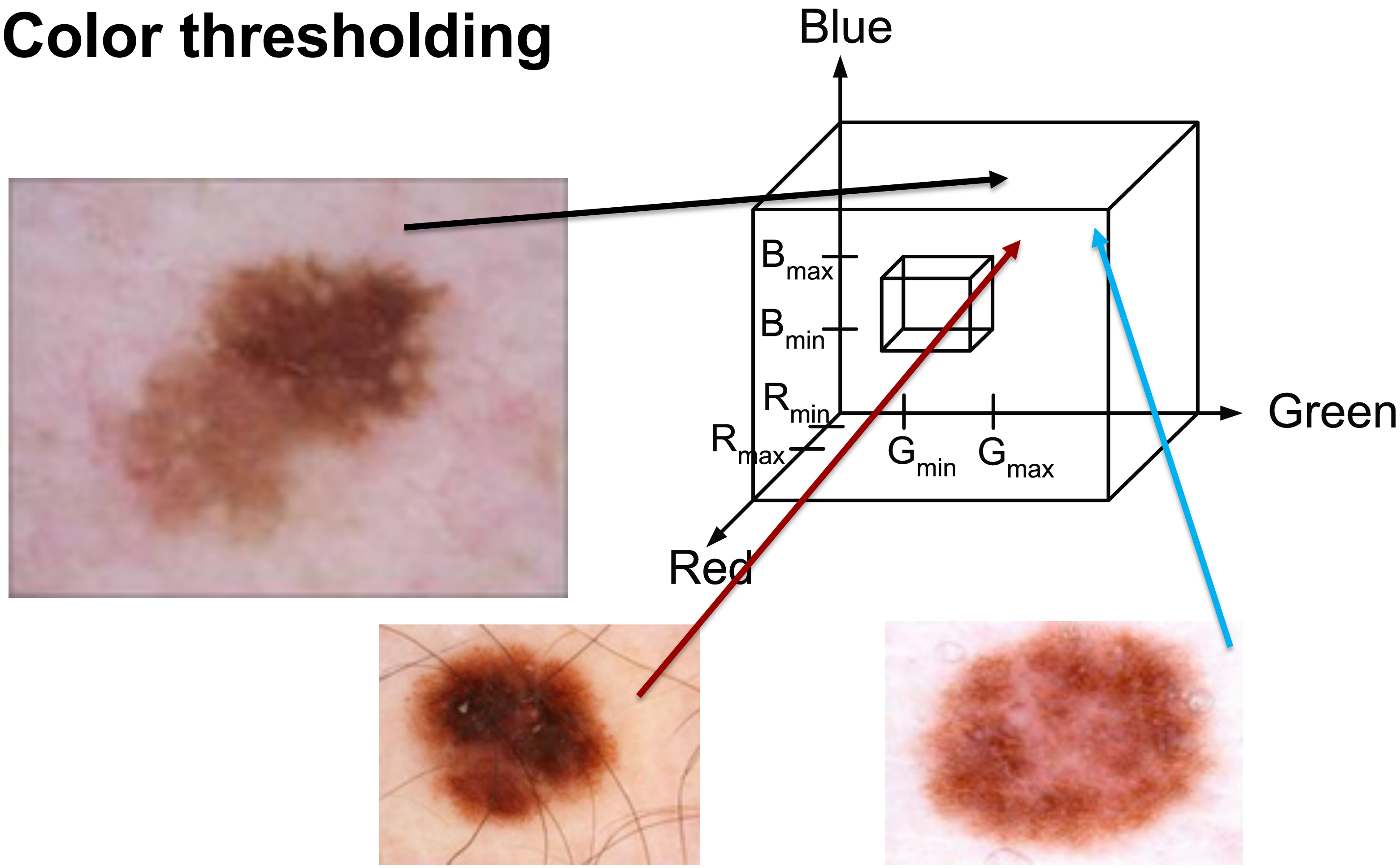
Color thresholding





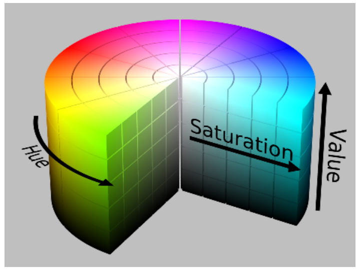
If
$R > R_{min}$ and $R < R_{max}$ and
$G > G_{min}$ and $G < G_{max}$ and
$B > B_{min}$ and $B < B_{max}$
Then $g(x, y) = 255$
Else $g(x, y) = 0$

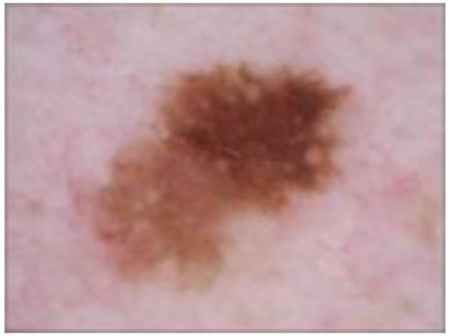
Color thresholding

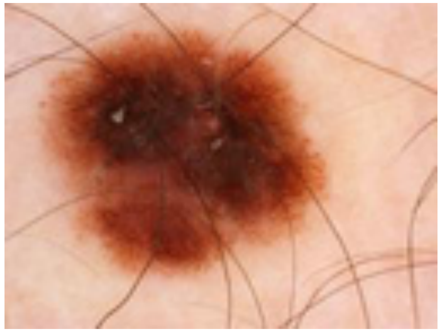


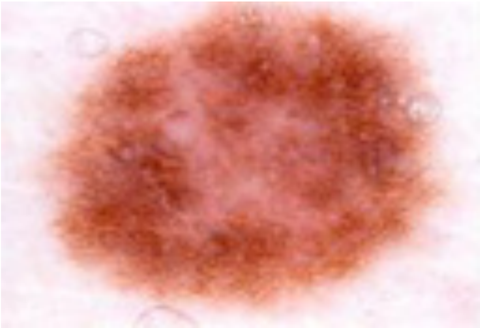
Color variation

- The major variation is in the brightness
  - This will spread out the values in RGB space
- The Hue is rather constant
- HSI Space
  - HUE and saturation rather stable
  - Only variation in intensity / value

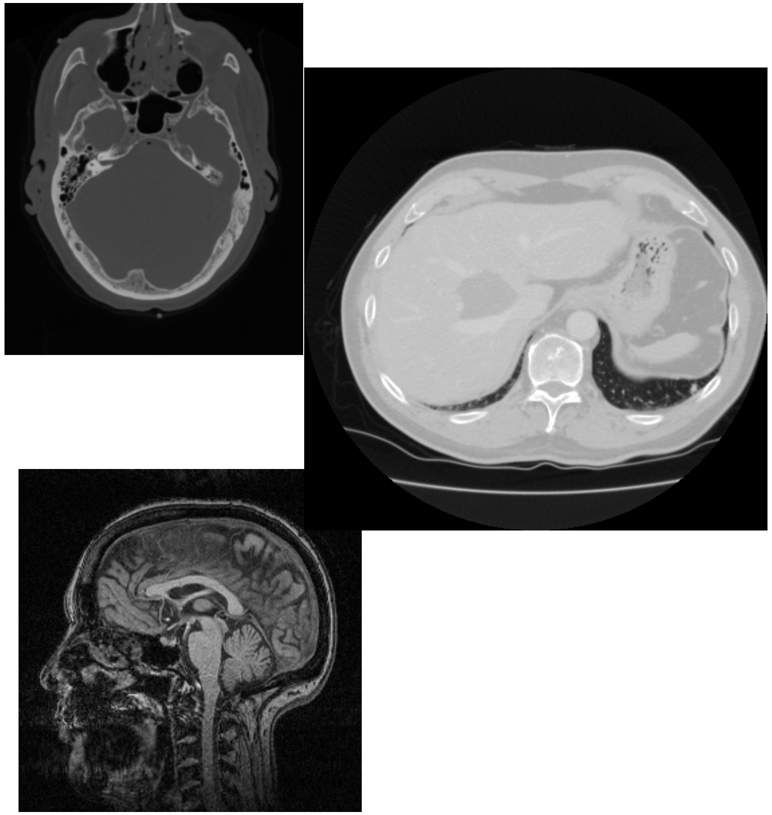






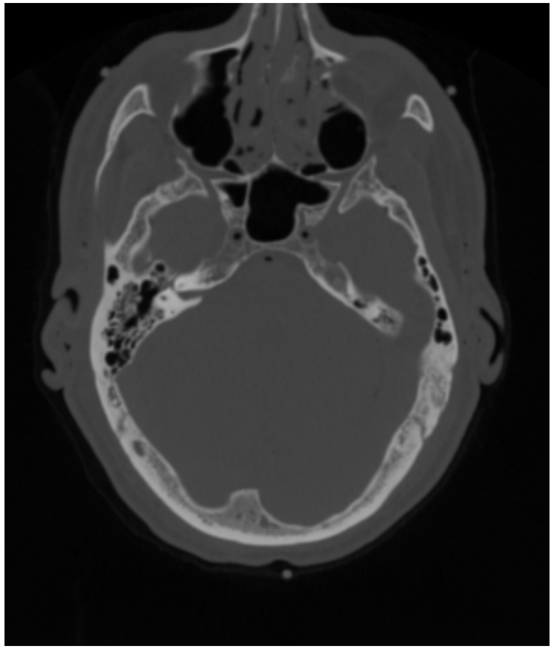


Contrast in medical images



- How do we optimise image acquisition when we want to look at
  - Bones
  - Brain structures
  - Cancer

Image acquisition - bone



- X-rays
  - goes through soft tissue with little loss
  - are attenuated in bone
- CT scanners use X-rays
  - Good for imaging bones
- A simple threshold can often extract the bones
- Areas with only bone and soft-tissue will have a bimodal histogram

Attenuation - the gradual loss in intensity

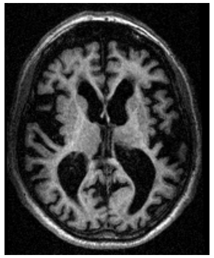
Image acquisition – brain structures

- Magnetic Resonance Imaging (MRI) is often used
- Much more difficult to explain!
  - Based on very powerful magnetic fields and radio waves
- Needs water molecules!
- Bone is black!

Histogram stretching in multi-centre MRI

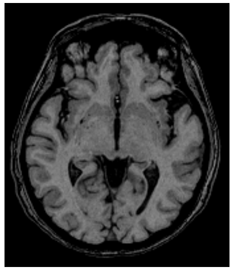
Harmonising data before using it as input to an algorithm

Different MR scanner: Different brightness and contrast



Center A

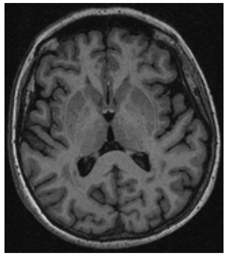
Center B



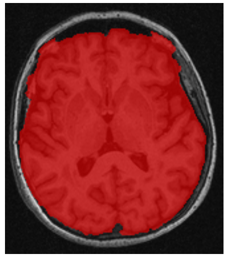
Center C

Data processing

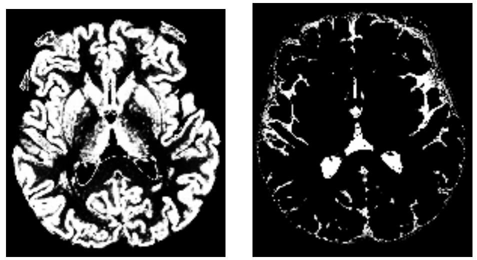
Subject



Brain mask

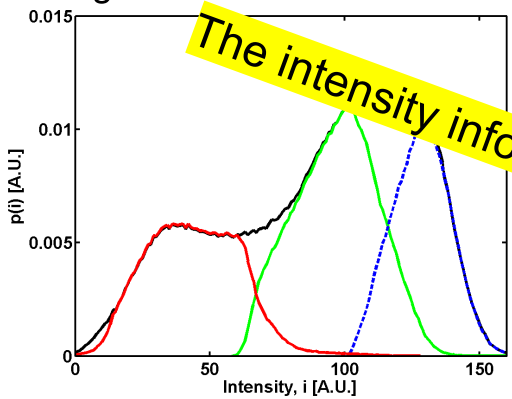


Segmentation: Thresholding



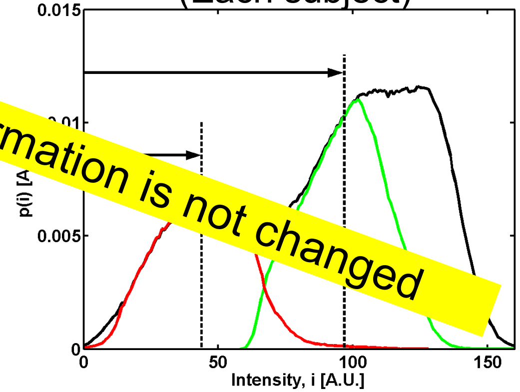
Histograms

Segmented tissue classes



Histogram stretching

(Each subject)

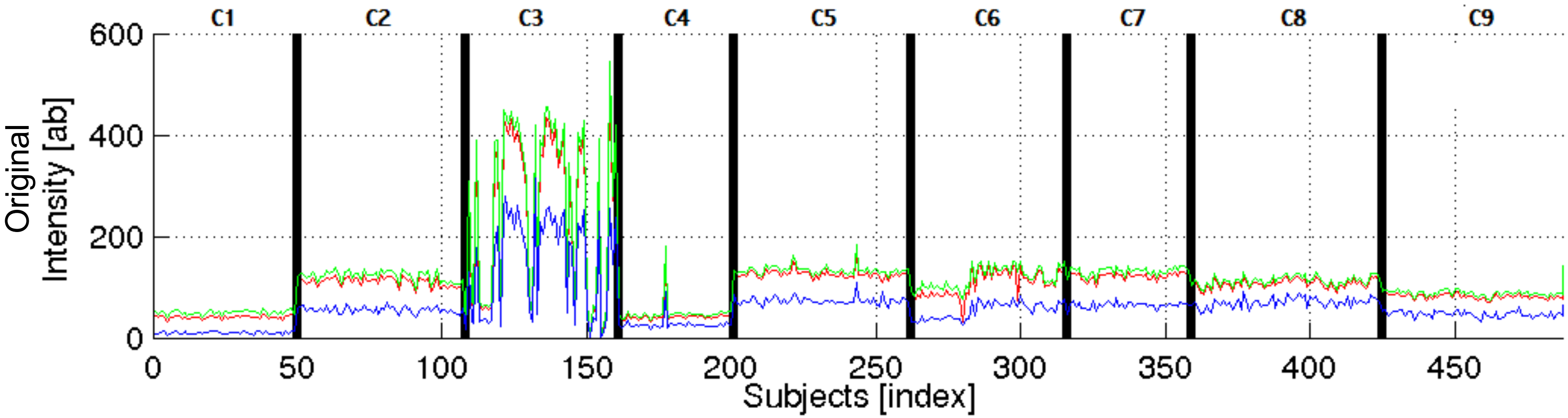


The intensity information is not changed

$$g(x,y) = \frac{v_{max,d} - v_{min,d}}{v_{max} - v_{min}}(f(x,y) - v_{min}) + v_{min,d}$$

Histogram stretching in multi-centre MRI

- 9 MR centre
- 500 subjects



Point processing: Pixel wise arithmetic

- Blending two figures (alpha blending)

$$g(x,y) = \boldsymbol{\alpha} f_1(x,y) + (1-\boldsymbol{\alpha}) f_2(x,y)$$

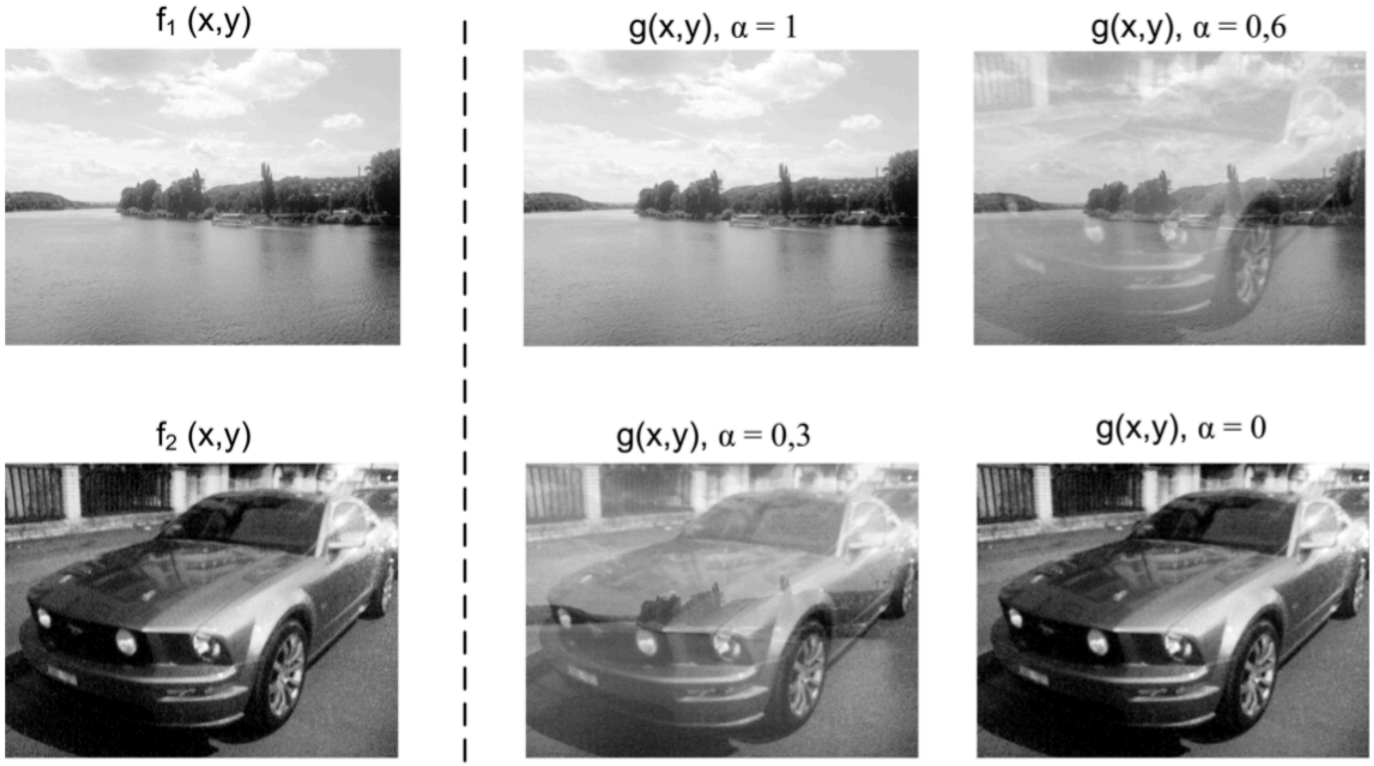
f_1 (x,y)

f_2 (x,y)

Point processing: Pixel wise arithmetic

- Blending two figures (alpha blending)

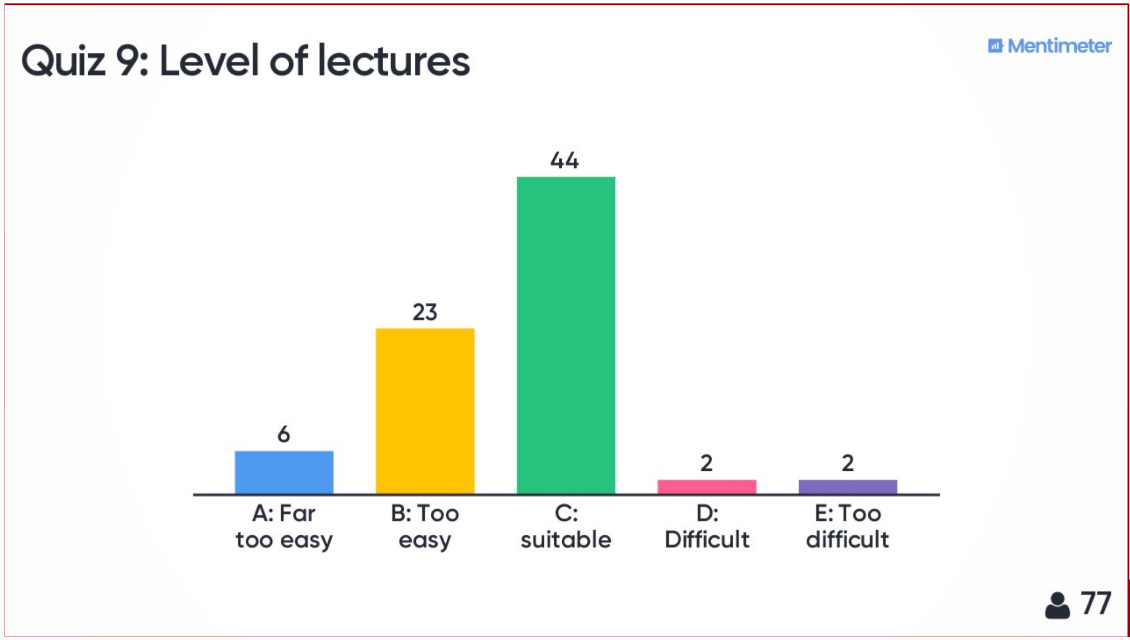
$$g(x,y) = \boldsymbol{\alpha} f_1(x,y) + (1 - \boldsymbol{\alpha}) f_2(x,y)$$



Pixelwise Operations (Chapter 4)

- Learning objectives:
  - Compute and apply a linear gray transformation
  - Describe and compute the image histogram
  - Implement and apply histogram stretching
  - Implement and apply gamma transformation
  - Implement and apply log and exp mappings
  - Describe and use thresholding
  - Describe and use automatic thresholding
  - Perform conversions between bytes and doubles
  - Use addition and subtraction of images
  - Explain the benefits of bi-modal histograms
  - Identify images where global thresholding can be used for object extraction

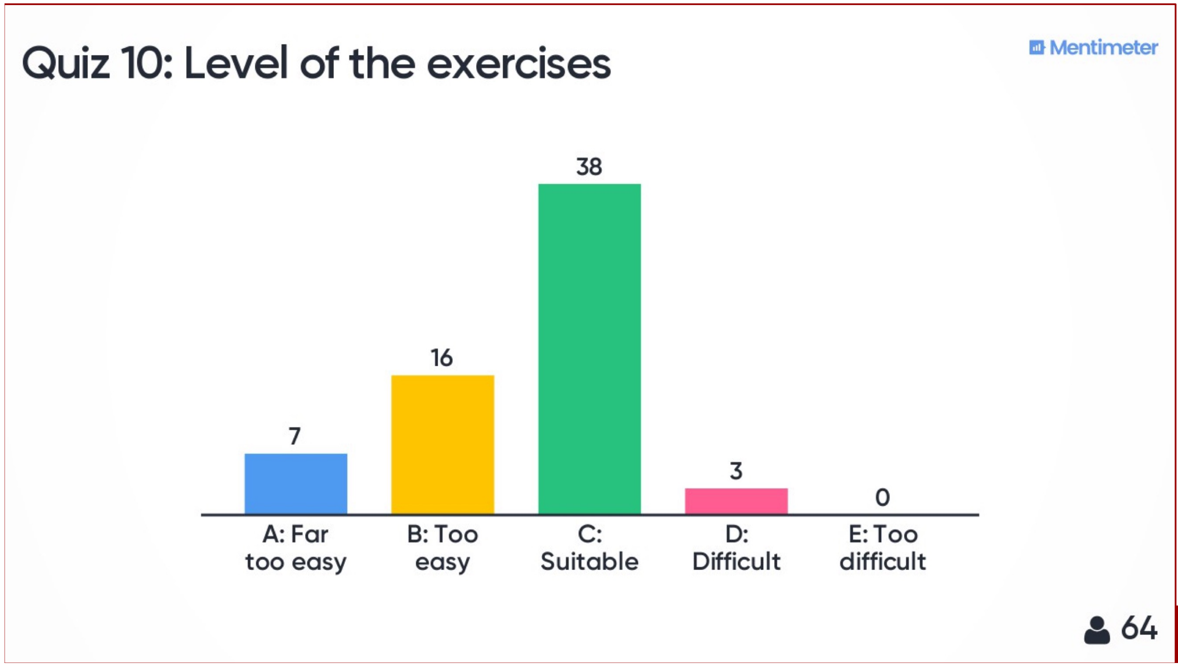
Quiz 9: Level of the lectures



A) Far too easy
B) Too easy
C) Suitable
D) Difficult
E) Too difficult

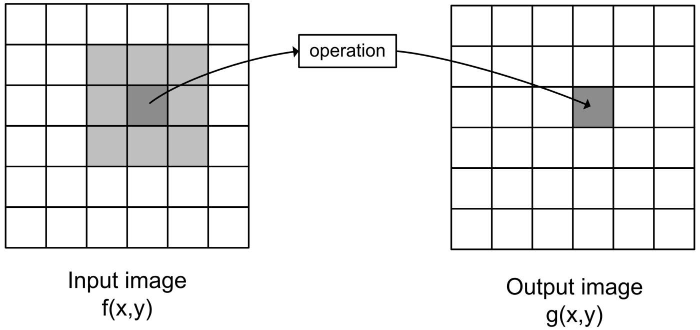
Quiz 10: Level of the exercises

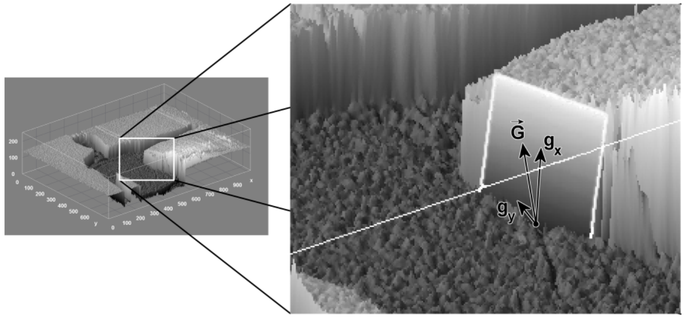
A) Far too easy
B) Too easy
C) Suitable
D) Too difficult
E) Far too difficult

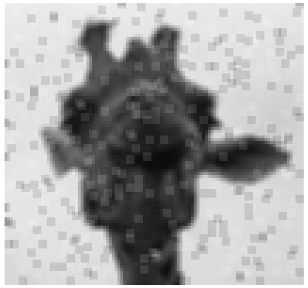


Next week

- Neighbourhood processing
  - Filtering







Mean filtered

Median filtered