02502
Image Analysis
Week 5 - Morphology

http://courses.compute.dtu.dk/02502/

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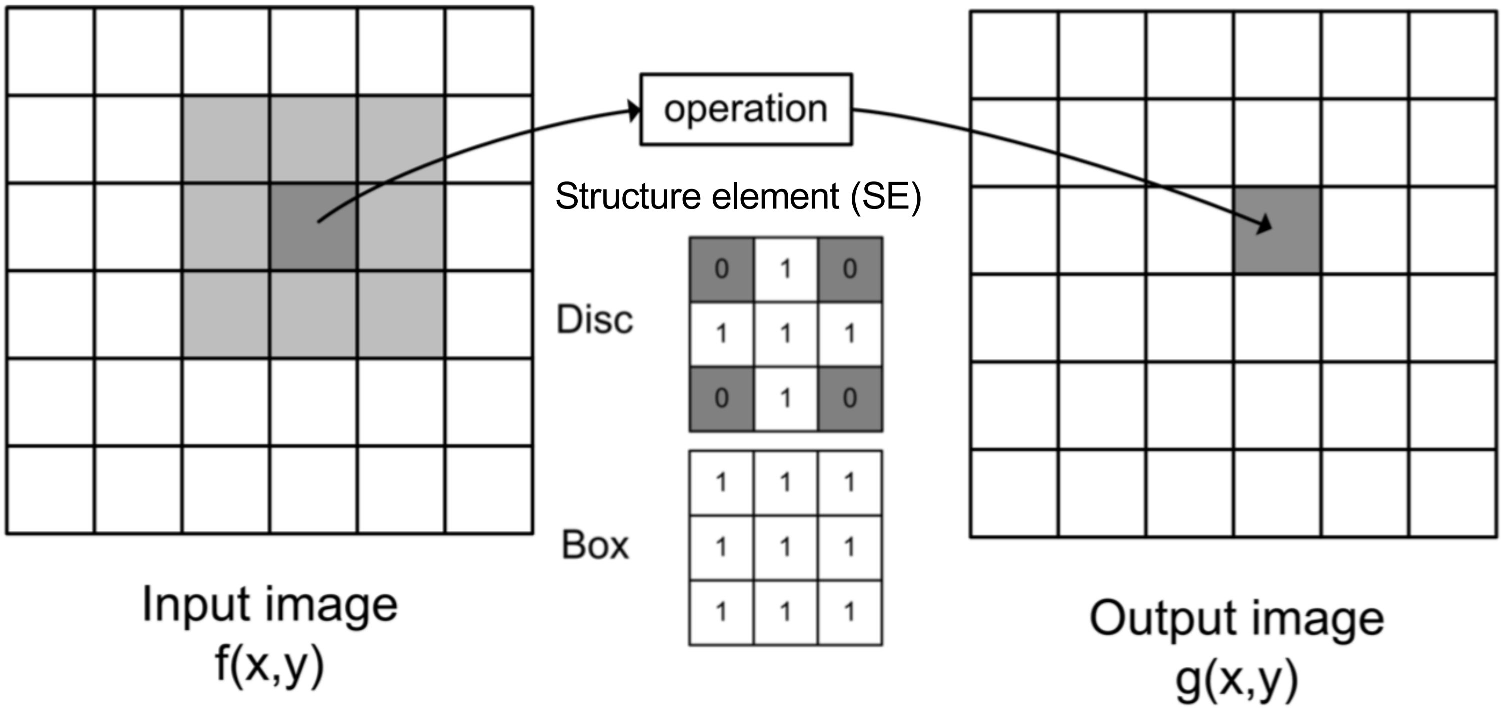
Plenty of slides adapted from Thomas Moeslunds lectures

Lecture 5 - What can you do after today?

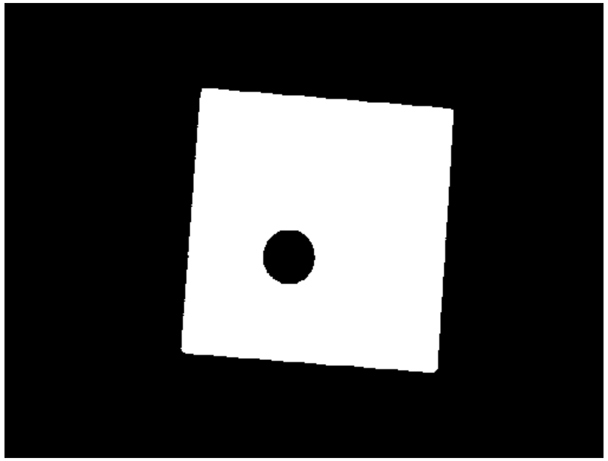
- Describe the similarity between filtering and morphology
- Describe a structuring element
- Compute the dilation of a binary image
- Compute the erosion of a binary image
- Compute the opening of a binary image
- Compute the closing of a binary image
- Apply compound morphological operations to binary images
- Describe typical examples where morphology is suitable
- Remove unwanted elements from binary images using morphology
- Choose appropriate structuring elements and morphological operations based on image content

Lecture 5 - What can you do after today?

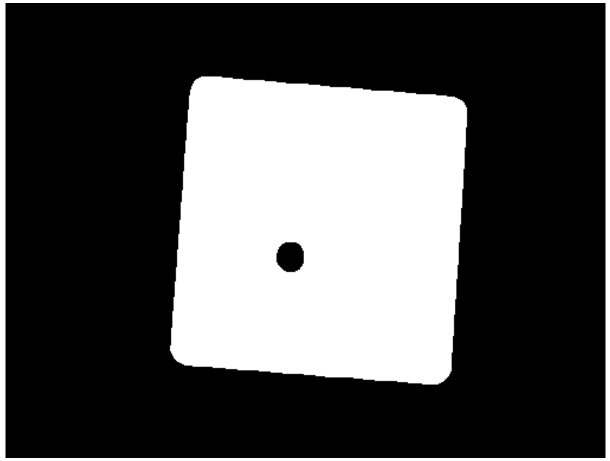
- Describe th
- Describe a
- Compute th
- Compute th
- Compute th
- Compute th
- Apply comp
- Describe ty
- Remove ur
- Choose ap ... age content



Lecture 5 - What can you do after today?

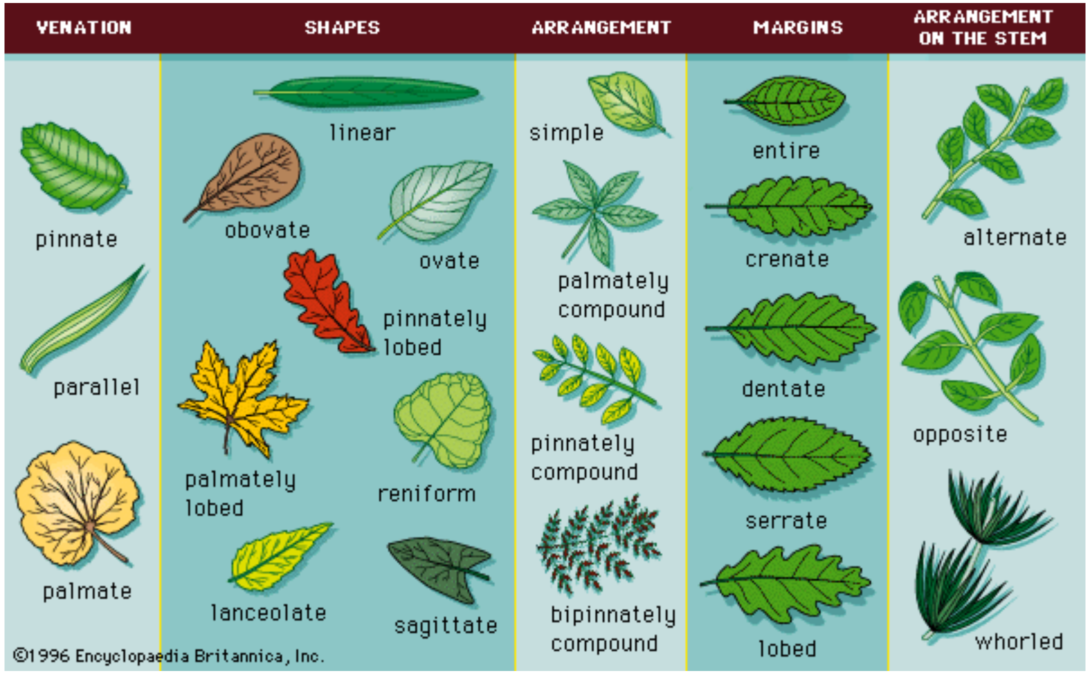


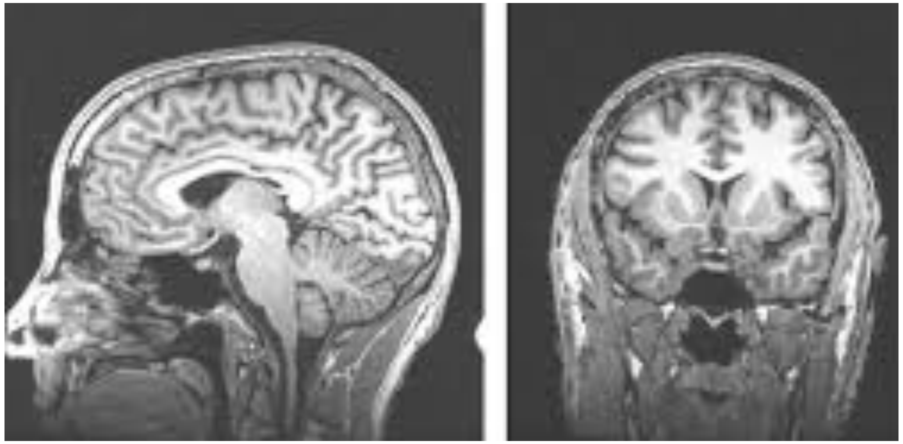
0	0	1	1	1	0	0
0	1	1	1	1	1	0
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
0	1	1	1	1	1	0
0	0	1	1	1	0	0



Morphology

- The science of *form, shape* and *structure*
- In biology: The form and structure of animals and plants





Common leaf morphologies

Mathematical morphology

Theorem 4.10
$$\begin{cases} \psi_m = \widetilde{\varphi}\,\widetilde{\gamma} = \widetilde{\gamma}\,\widetilde{\varphi}\,\widetilde{\gamma} = \psi\widetilde{\gamma} & , \\ \psi_M = \widetilde{\gamma}\widetilde{\varphi} = \widetilde{\varphi}\,\widetilde{\gamma}\,\widetilde{\varphi} = \psi\widetilde{\varphi} & , \\ \psi = \widetilde{\gamma}\,\psi = \widetilde{\varphi}\,\psi, & \\ \widetilde{\gamma} \leq \psi_m \leq \psi \leq \psi_M \leq \widetilde{\varphi} & . \end{cases}$$

The same theorem may be restated in another way. If $\mathcal{J}\mathrm{d}(\mathcal{B}) \neq \varnothing$ then let B_i be a family of elements of $\mathcal{B}$. We have $\vee\, B_i \in \sim B$, and thus $\widetilde{\gamma}(\vee\, B_i) = \vee B_i$. From the first relation above, it follows for any $\psi \in \mathcal{J}\mathrm{d}(\mathcal{B})$, that

$$\psi\,(\vee\, B_i) = \psi\widetilde{\gamma}\,(\vee\, B_i) = \widetilde{\varphi}\,\widetilde{\gamma}\,(\vee\, B_i)\,.$$

But $\widetilde{\gamma}\,(\vee\, B_i) = \vee\, B_i$, so that

$$\widetilde{\varphi}\,(\vee B_i) = \psi\,(\vee B_i) \in \mathcal{B}.$$

In the same way, we also obtain

$$\widetilde{\gamma}\widetilde{\varphi}\,(\wedge\, B_i) = \widetilde{\gamma}\,(\wedge\, B_i) = \psi\,(\wedge\, B_i) \in \mathcal{B}.$$

In other words, $\mathcal{B}$ is a *complete lattice* with respect to the ordering on $\mathcal{B}$ induced by $\leq$, i.e. any family B_i in $\mathcal{B}$ has a smallest upper bound $\widetilde{\varphi}\,(\vee\, B_i)\,\mathcal{B}$ and a greatest lower bound $\widetilde{\gamma}\,(\wedge\, B_i) \in \mathcal{B}$.

Conversely, let us assume that $\mathcal{B}$ is a complete lattice. Thus, for any $A \in \mathcal{L}$, the family $\{B : B \in \mathcal{B}, B \geq A\}$ has in $\mathcal{B}$ a greatest lower bound, which is

$$\widetilde{\gamma}\,(\wedge\,\{B : B \in \mathcal{B}, B \geq A\}) = \widetilde{\gamma}\,\widetilde{\varphi}\,(A) \in \mathcal{B}.$$

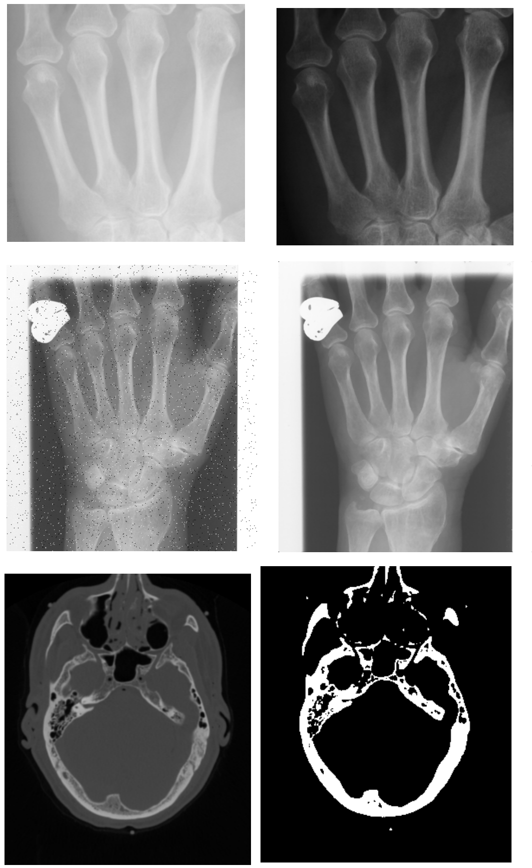
But this implies $\mathcal{B}_{\psi_M} \subseteq \mathcal{B}$ for the filter $\psi_M = \widetilde{\gamma}\,\widetilde{\varphi}$. Conversely, for any

https://en.wikipedia.org/wiki/Mathematical_morphology

- Developed in 1964
- Theoretical work done in Paris
- Used for classification of minerals in cut stone
- Initially used for binary images

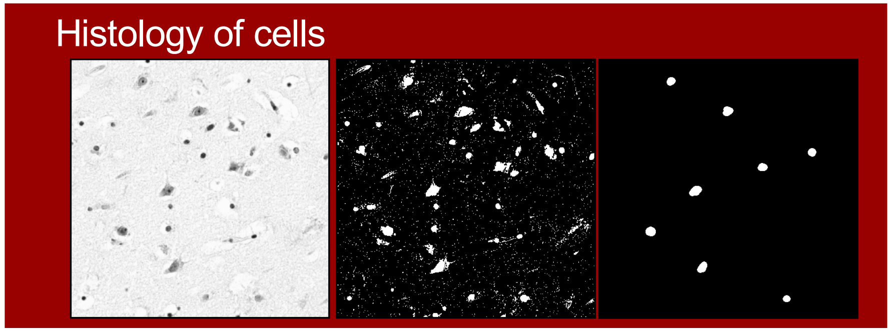
Do not worry! We use a much less theoretical approach!

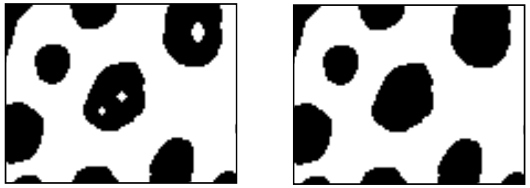
Relevance?

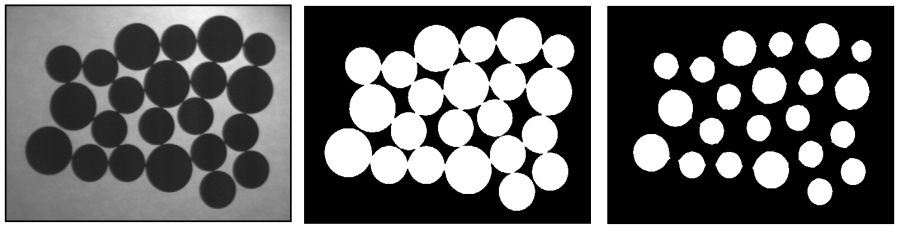


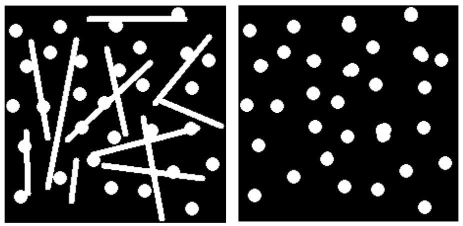
- Point wise operations (histogram)
- Filtering
- Thresholding
  - Gives us objects that are separated by the background
- Morphology
  - Manipulate and enhance binary objects

What can it be used for?



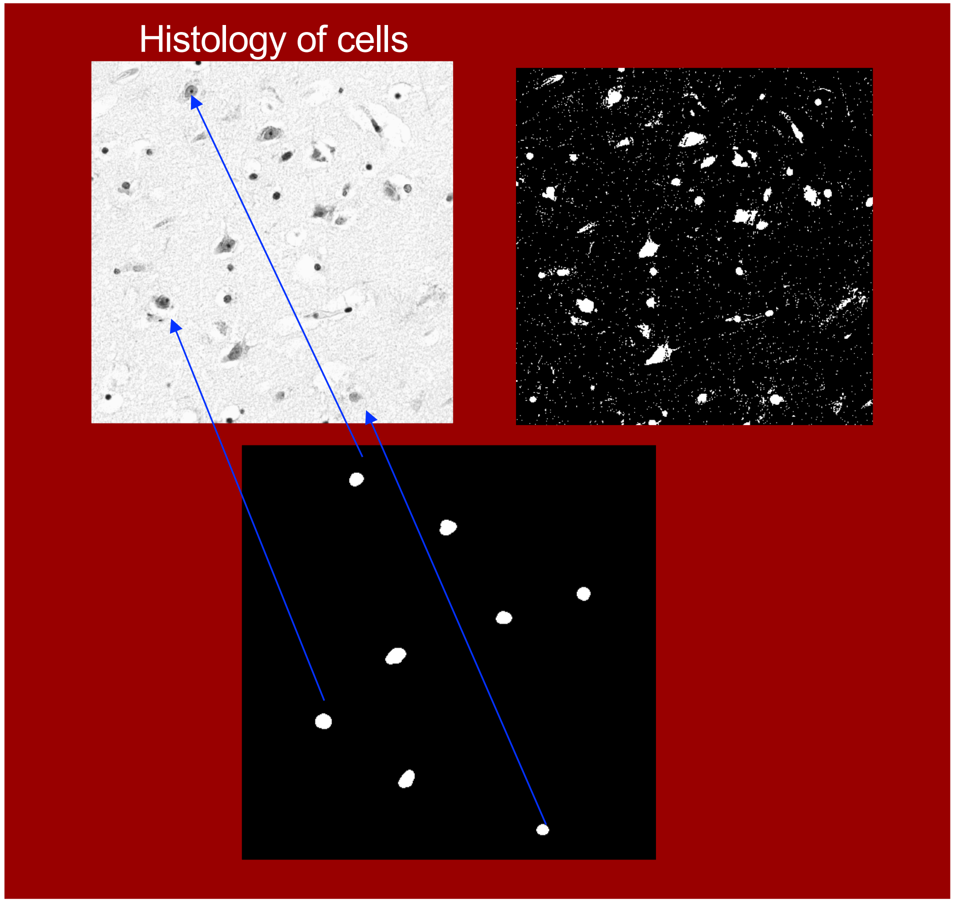






- Remove noise
  - Small objects
  - Fill holes
- Isolate objects
- Customized to specific shapes

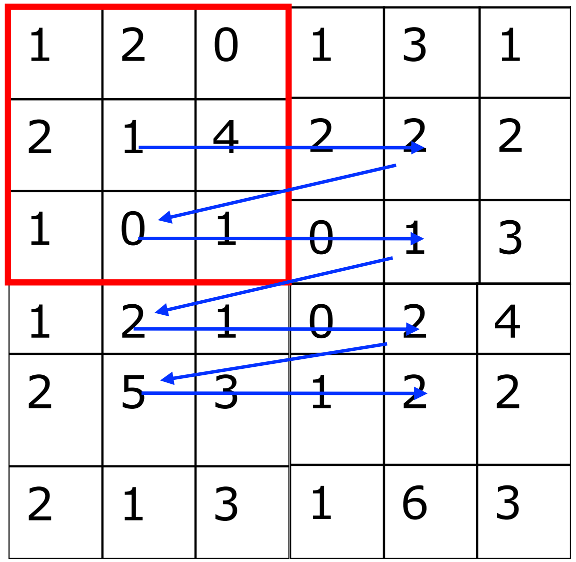
How does it work?



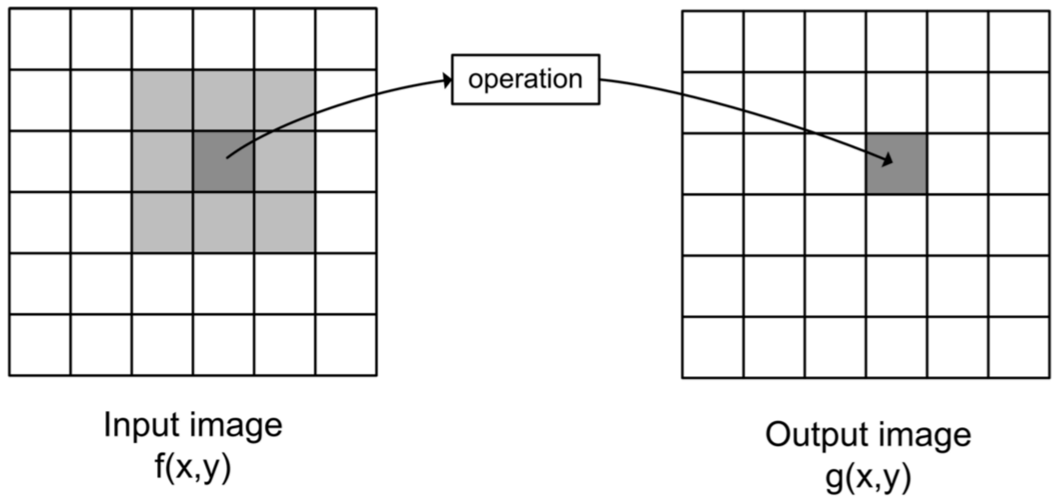
The processing pipeline:

- Grayscale image
- Preprocessing
  - Inversion
- Threshold => Binary image
- Morphology extraction

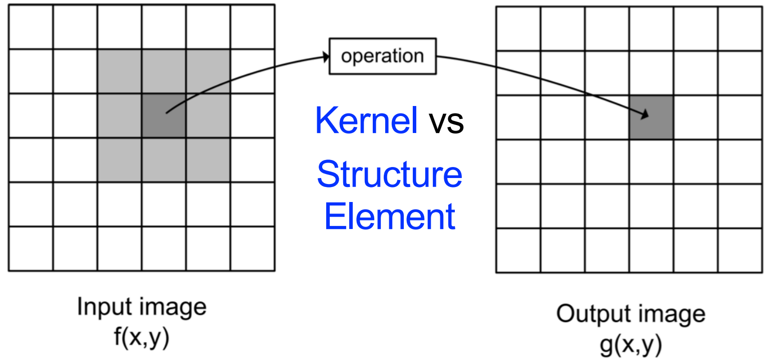
Filtering and morphology



- Filtering
  - Gray level images
  - Kernel
  - Moves it over the input image
  - Creates a new output image



Filtering and morphology



3x3

Disk

0	1	0
1	1	1
0	1	0

Box

1	1	1
1	1	1
1	1	1

- Filtering
  - Gray level images
  - Kernel
  - Moves it over the input image
  - Creates a new output image

- Morphology
  - Binary images
  - Structuring element (SE)
  - Moves the SE over the input image
  - Creates a new binary output image

1D Morphology

Input image

1	0	0	0	1	1	1	0	1	1

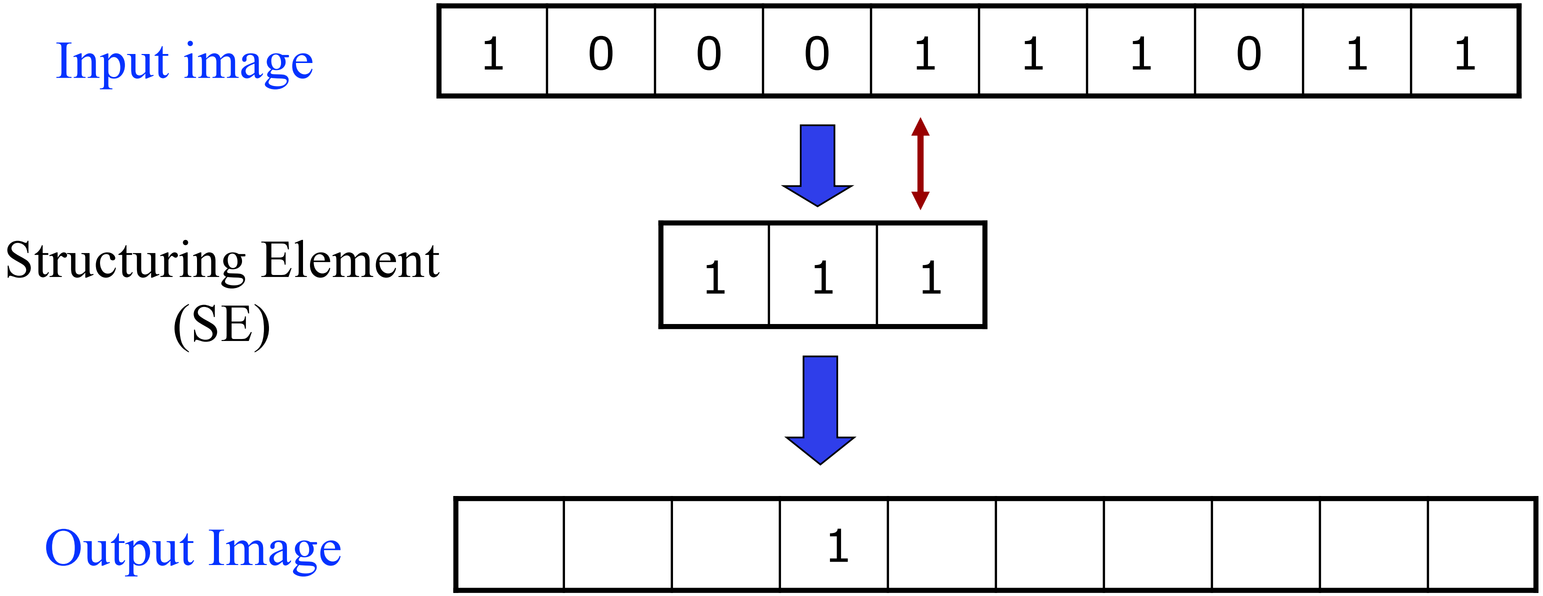
Structuring Element (SE)

1	1	1

Output Image

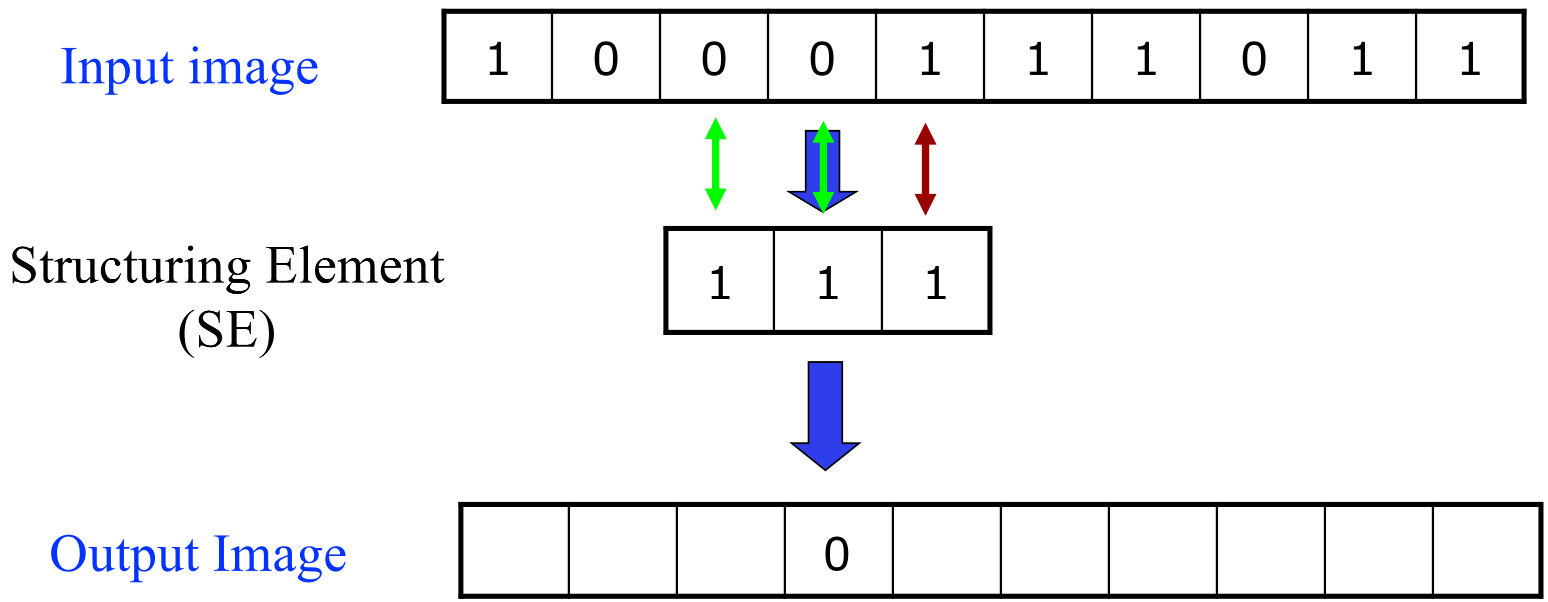
			?						

1D Morphology : The *hit* operation



- If <u>just one</u> 1 in the SE match with the input
    - output 1
- else
    - output 0

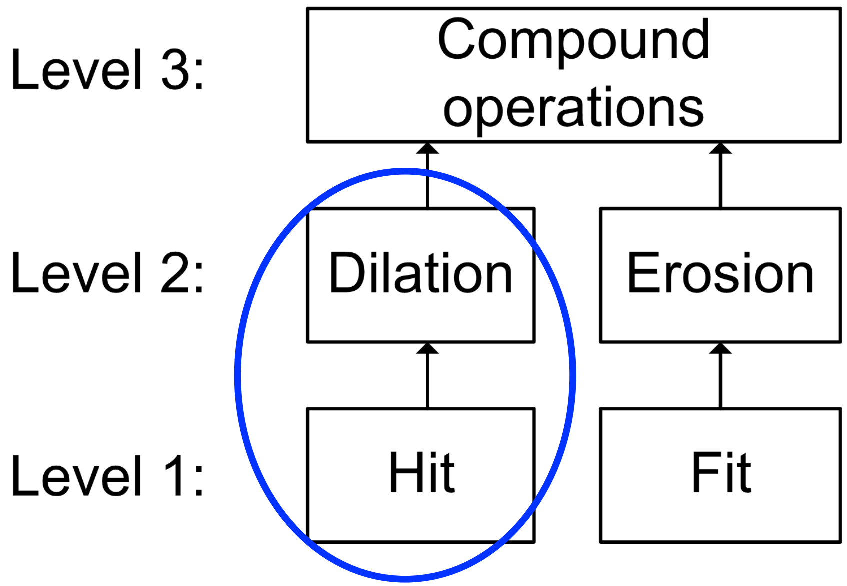
1D Morphology : The *fit* operation



- If <u>all</u> 1 in the SE match with the input
  - output 1
- else
  - output 0

1D Morphology : Dilation

- Dilate : To make wider or larger
- Based on the *hit* operation



1D Dilation example

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element (SE)

1	1	1

$$g(x) = f(x) \oplus SE$$

to make bigger

Output Image

	1								

The Hit operation:

- If just one 1 in the SE match with the input
  - output 1
- else
  - output 0

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element (SE)

1	1	1

Output Image

	1	0							

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element (SE)

1	1	1

Output Image

	1	0	1						

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element (SE)

1	1	1

Output Image

	1	0	1	1					

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element (SE)

1	1	1

Output Image

	1	0	1	1	1				

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element (SE)

1	1	1

Output Image

	1	0	1	1	1	1			

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

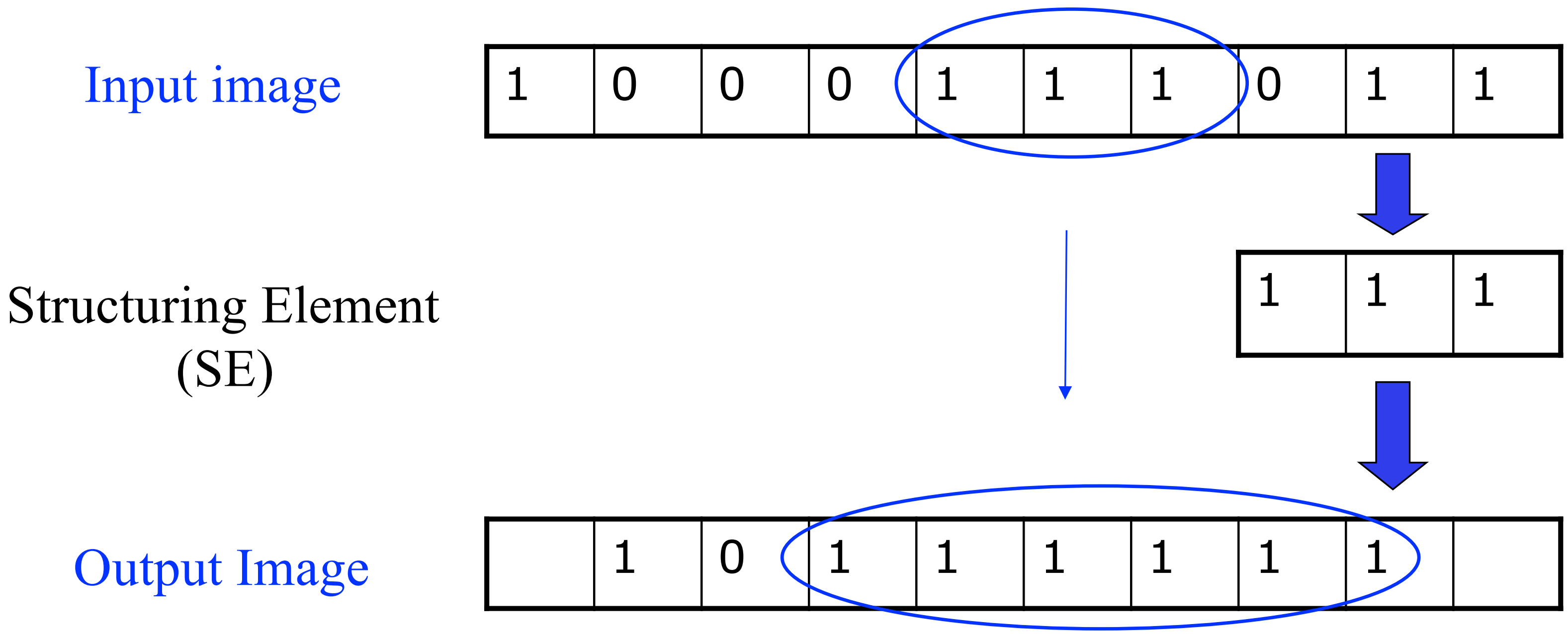
Structuring Element (SE)

1	1	1

Output Image

	1	0	1	1	1	1	1		

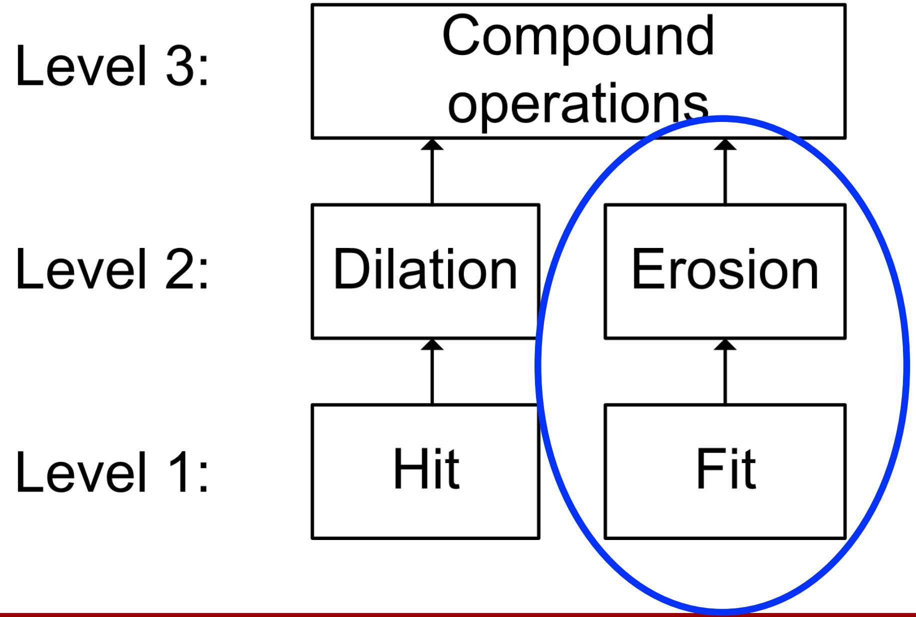
Example for Dilation



The object gets bigger and holes are filled!

1D Morphology : Erosion

- Erode : To wear down (*Waves eroded the shore*)
- Based on the *fit* operation



Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element (SE)

1	1	1

$$g(x) = f(x) \ominus SE$$

to make smaller

Output Image

	0								

The Fit operation:

- If all 1 in the SE match with the input
  - output 1
- else
  - output 0

Quiz 1: Erosion

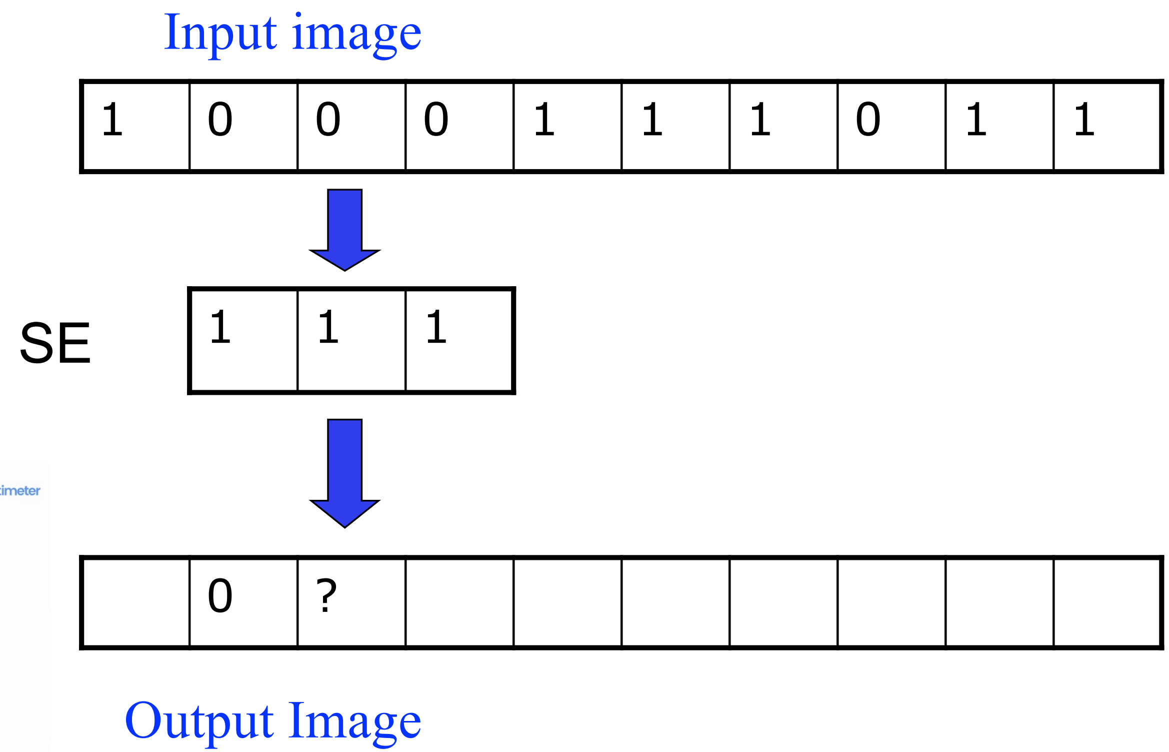
A) 0 1 0 0 1 1 0 0

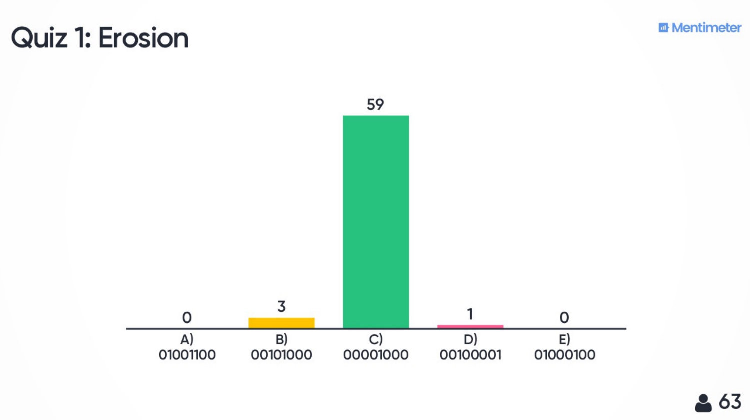
B) 0 0 1 0 1 0 0 0

C) 0 0 0 0 1 0 0 0

D) 0 0 1 0 0 0 0 1

E) 0 1 0 0 0 1 0 0





Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0						

Solution: C) 0 0 0 0 1 0 0 0

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0					

Solution: C) 0 0 0 0 1 0 0 0

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1				

Solution: C) 0 0 0 0 1 0 0 0

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1	0			

Solution: C) 0 0 0 0 1 0 0 0

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1	0	0		

Solution: C) 0 0 0 0 1 0 0 0

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1	0	0	0	

Solution: C) 0 0 0 0 1 0 0 0

The object gets smaller !!

Structuring Element

3x3

0	1	0
1	1	1
0	1	0

Disk

1	1	1
1	1	1
1	1	1

Box

7x7

		1	1	1		
	1	1	1	1	1	
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
	1	1	1	1	1	
		1	1	1		

- Structuring Elements can have varying sizes
- Usually, element values are 0 or 1, but other values are possible (including none!)
- Structural Elements have an origin
- Empty spots in the Structuring Elements are ***don't cares***!

Structuring Element Origin

0	1	0
1	1	1
0	1	0

1	1	1
1	1	1
1	1	1

- The origin is not always the center of the SE

Special structuring elements

- Structuring elements can be customized to a specific problem

0	0	0	1	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
1	1	1	1	1	1	1
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	1	0	0	0

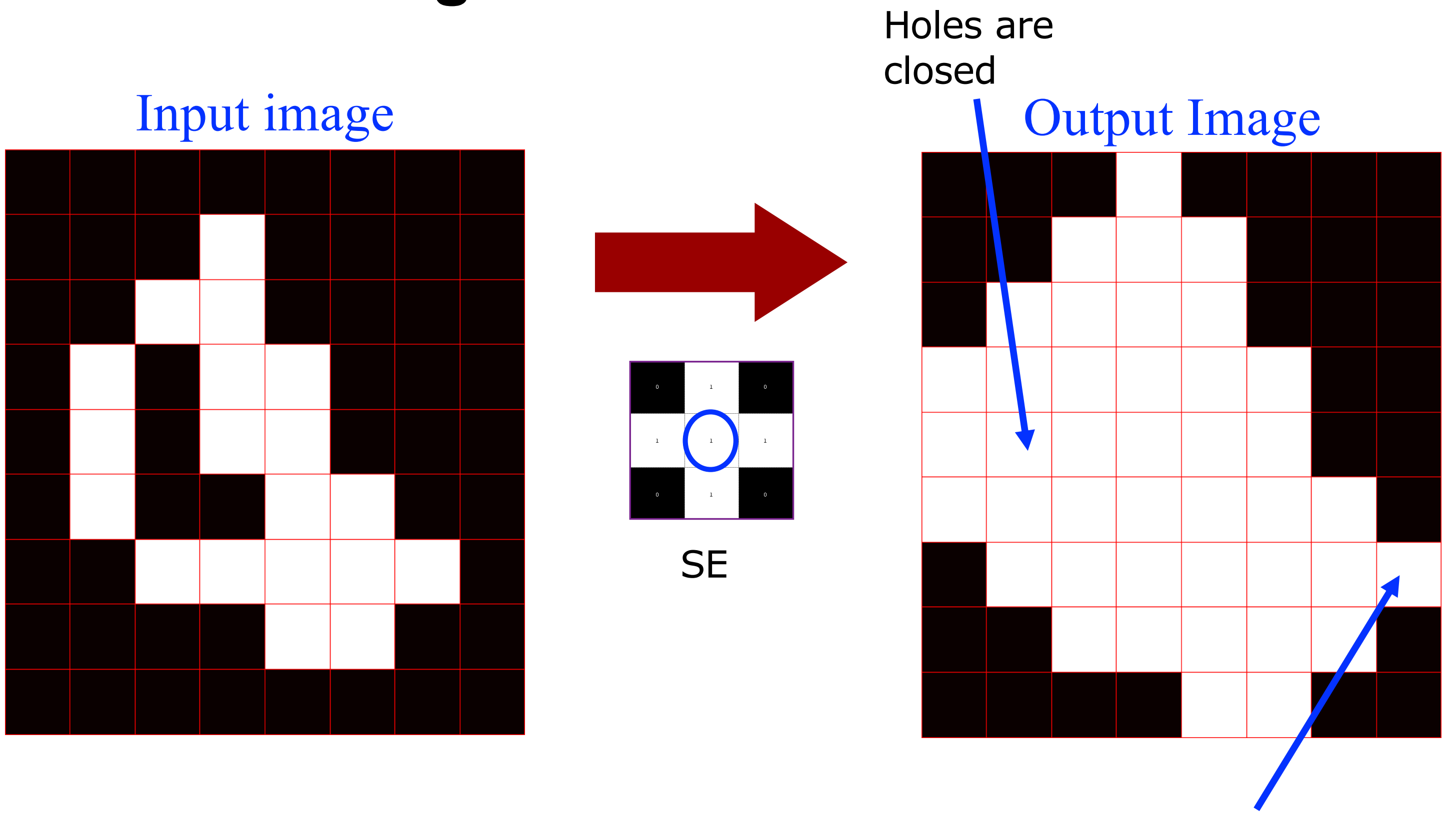
Diamond

0	0	0	0	0	1	1
0	0	1	1	1	0	0
1	1	0	0	0	0	0

Line

Note: In case of a boundary effect as for the SE in below example then extend the input f(x,y) image with zero-padding.

Dilation on images - disk



$$g(x,y) = f(x,y) \oplus SE$$

Object is bigger

Boundary effects?

Quiz 2: Dilation on images – box

1 2 3 4

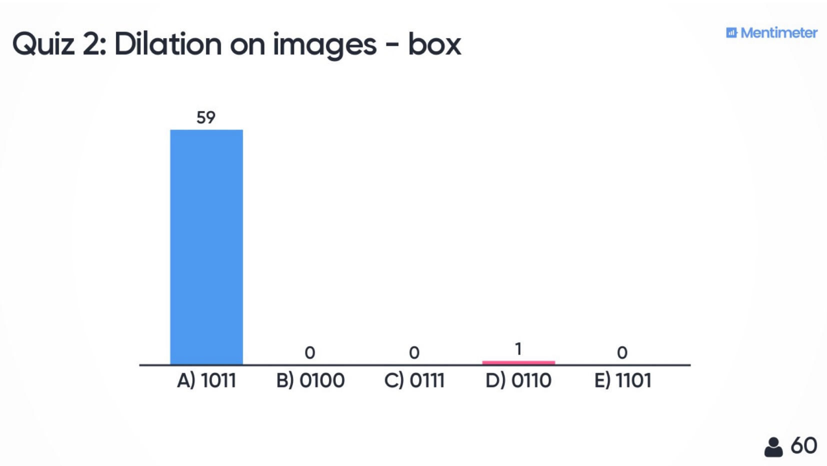
A) 1 0 1 1

B) 0 1 0 0

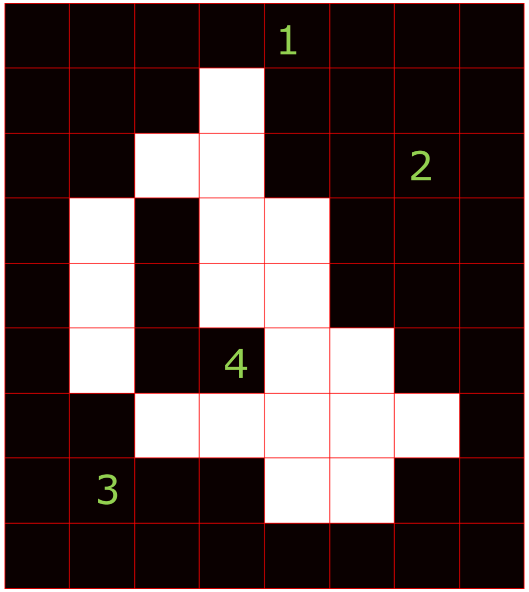
C) 0 1 1 1

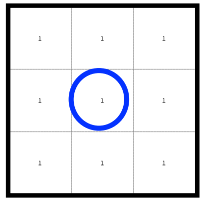
D) 0 1 1 0

E) 1 1 0 1



- Which operation to use: Hit or Fit?

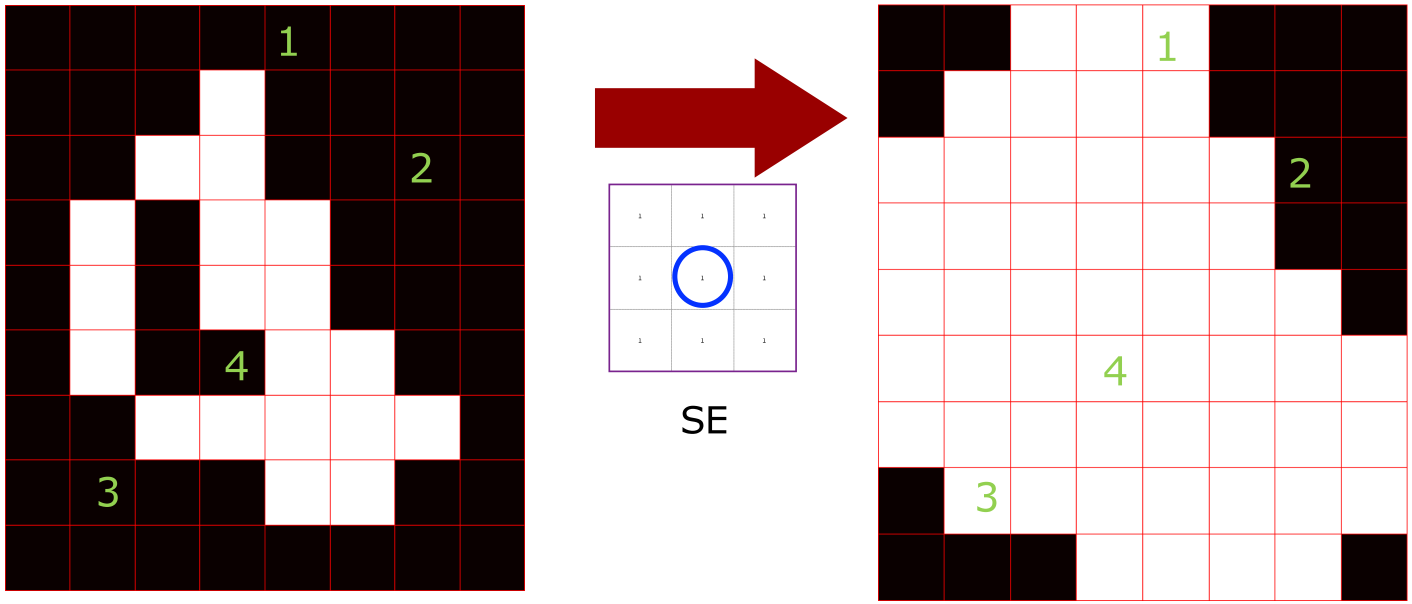




SE

$$g(x,y) = f(x,y) \oplus SE$$

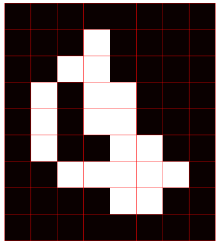
Quiz 2: Dilation on images – box



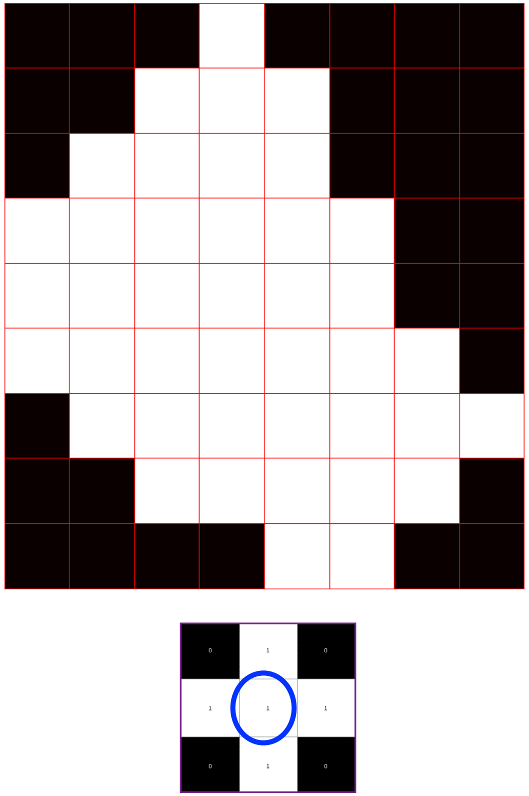
$$g(x, y) = f(x, y) \oplus SE$$

Dilation – the effect of the SE

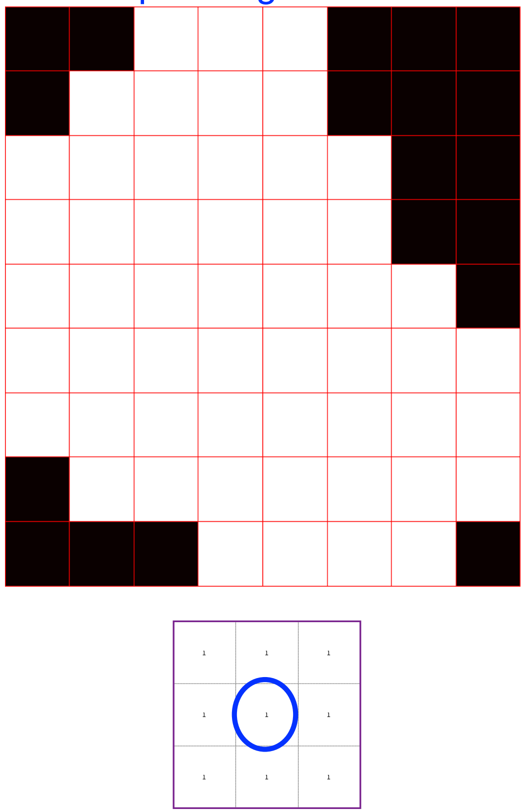
Input image



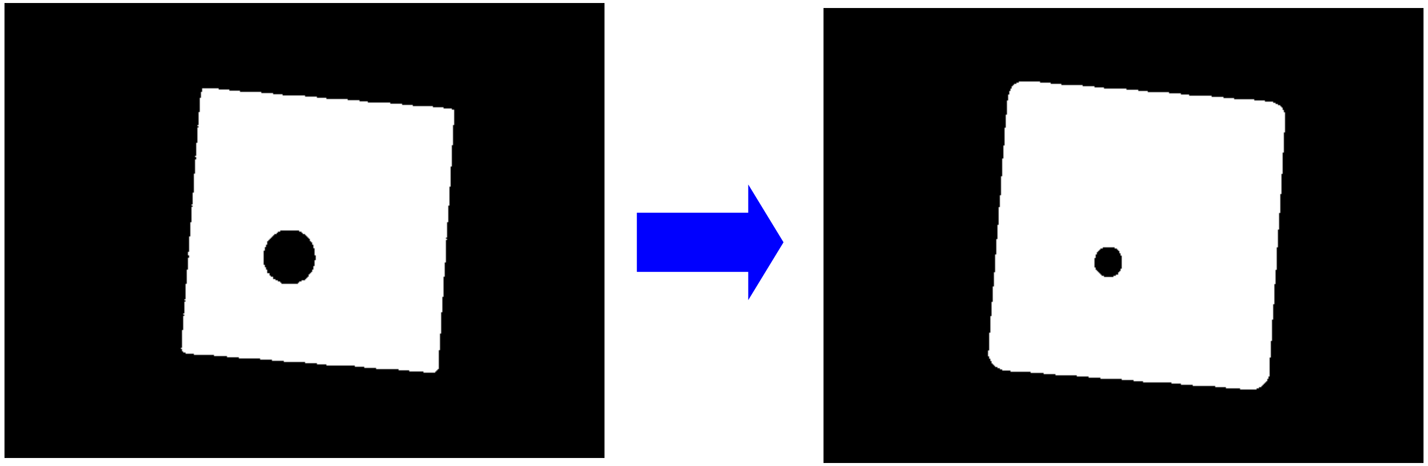
output image - disc



output image - box



Dilation Example



0	0	1	1	1	0	0
0	1	1	1	1	1	0
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
0	1	1	1	1	1	0
0	0	1	1	1	0	0

- Round structuring element (disk)
- Creates round corners

Quiz 3: Threshold and dilation

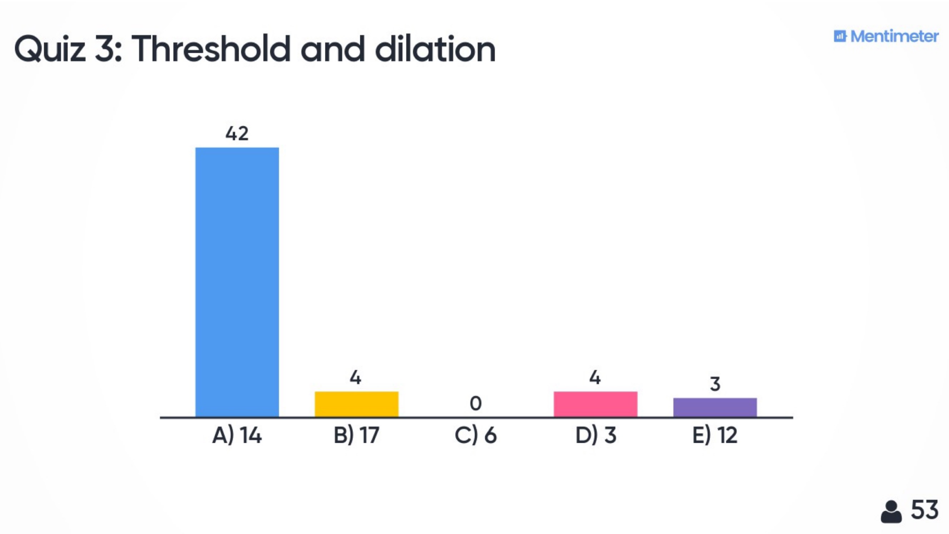
A) 14

B) 17

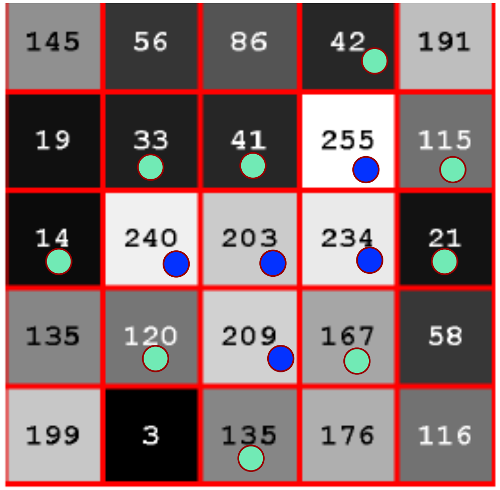
C) 6

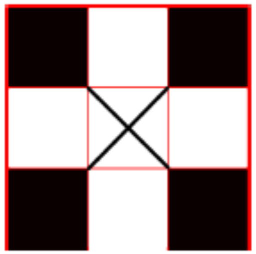
D) 3

E) 12

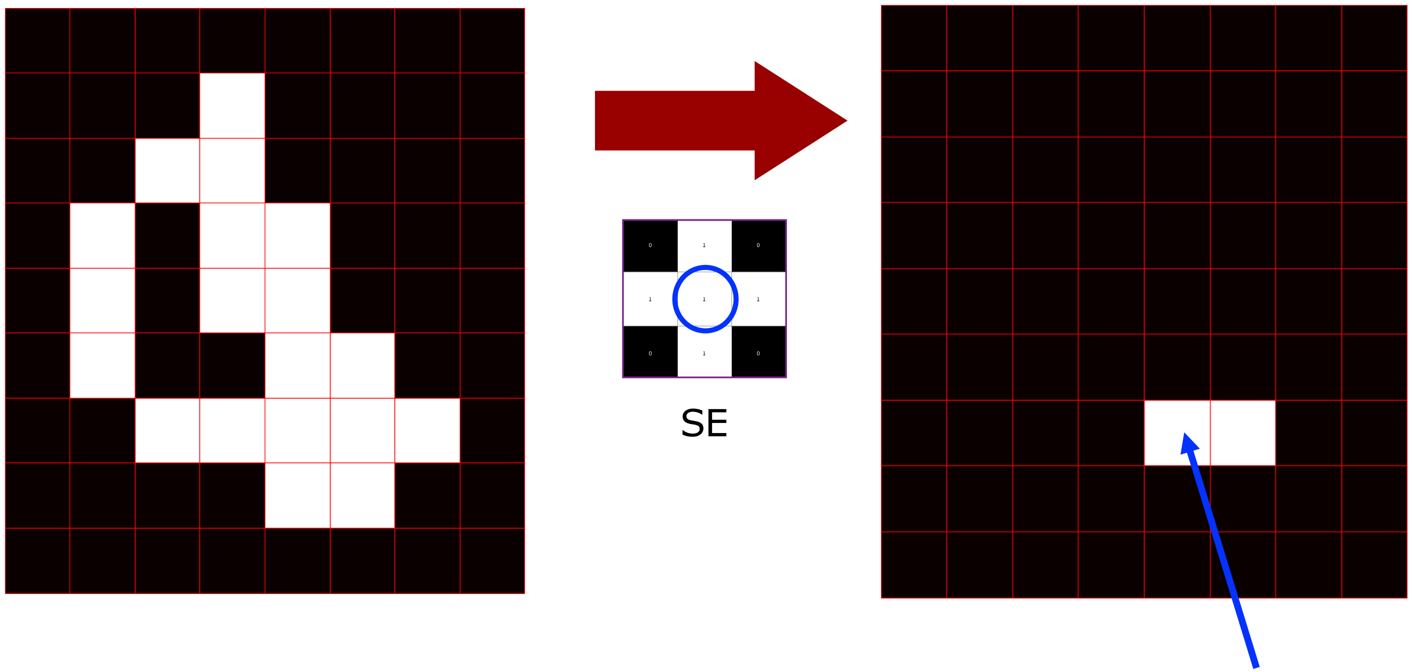


A threshold of 200 is applied to the image and the result is a binary image. Now a dilation is performed with the structuring element below. How many foreground pixels are there in the resulting image?





Erosion on images - disk



$$g(x,y) = f(x,y) \ominus SE$$

Object is smaller

Quiz 4: Erosion on images – box

1 2 3 4

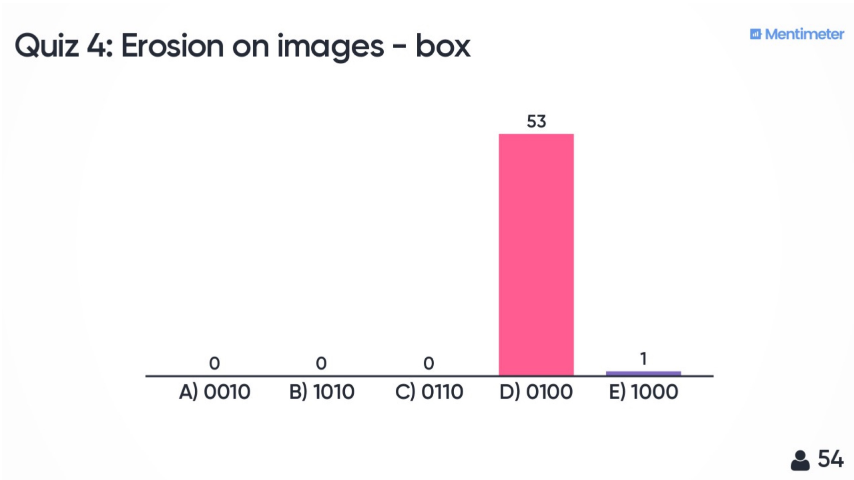
A) 0 0 1 0

B) 1 0 1 0

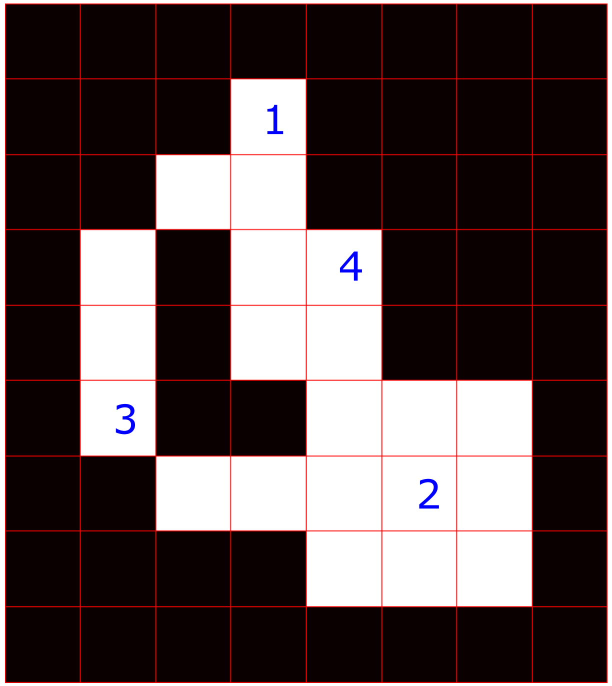
C) 0 1 1 0

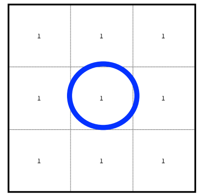
D) 0 1 0 0

E) 1 0 0 0



- Which operation to use: Hit or Fit?

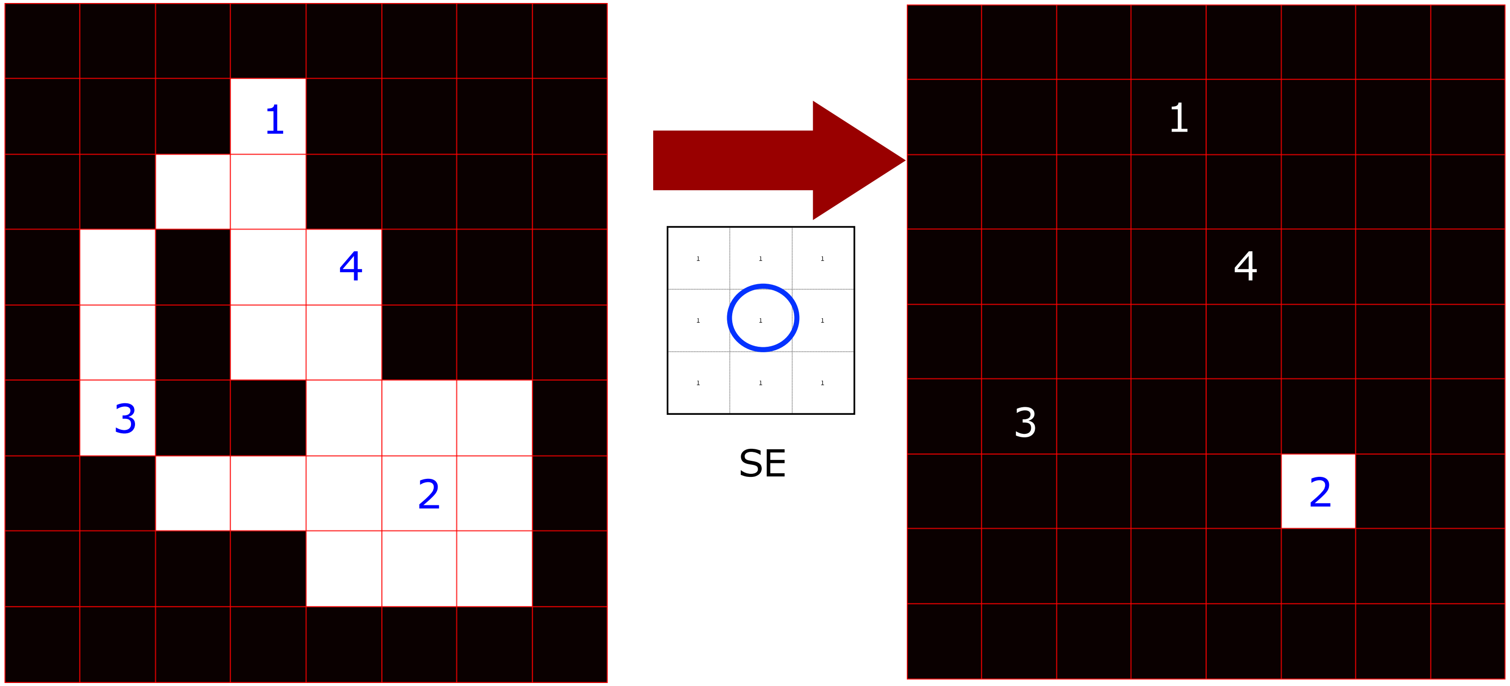




SE

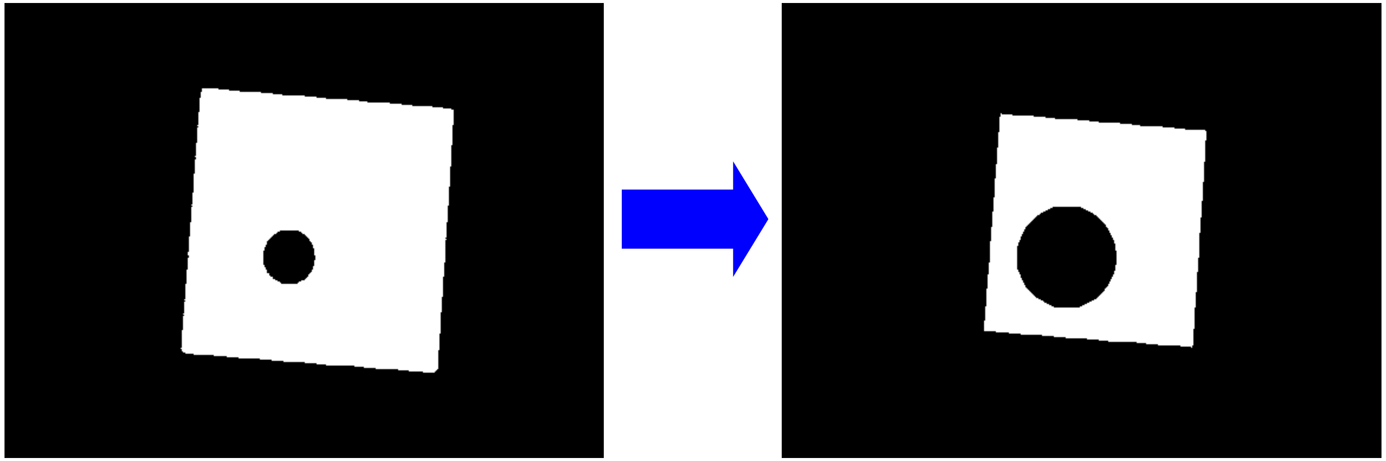
$$g(x,y) = f(x,y) \ominus SE$$

Quiz 4: Erosion on images – box



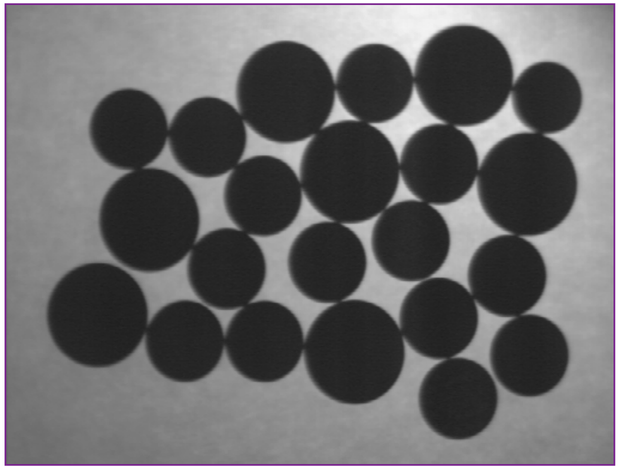
$$g(x,y) = f(x,y) \ominus SE$$

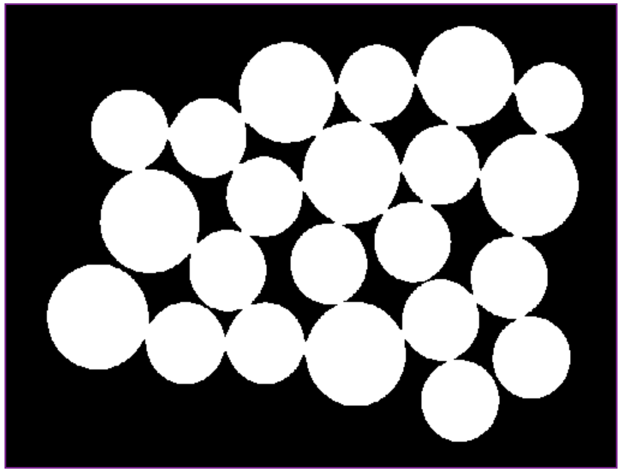
Erosion example

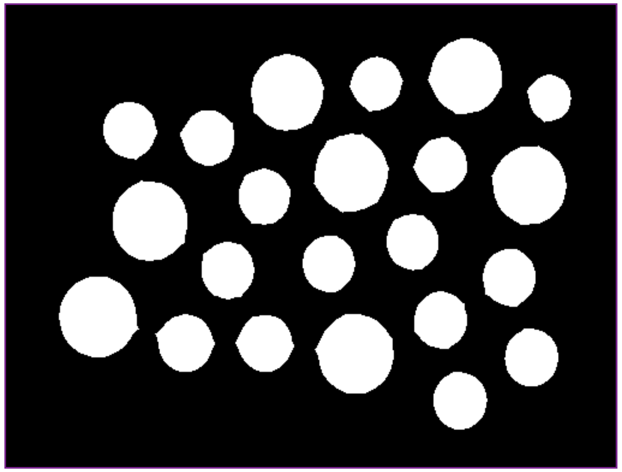


Counting Coins

- Counting these coins is difficult because they touch each other!
- Solution: Threshold and Erosion separates them!
- More on counting next time!

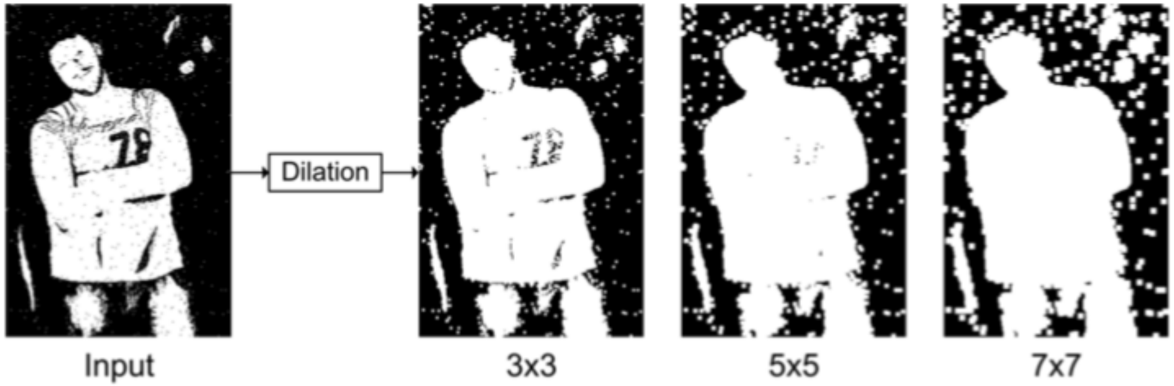




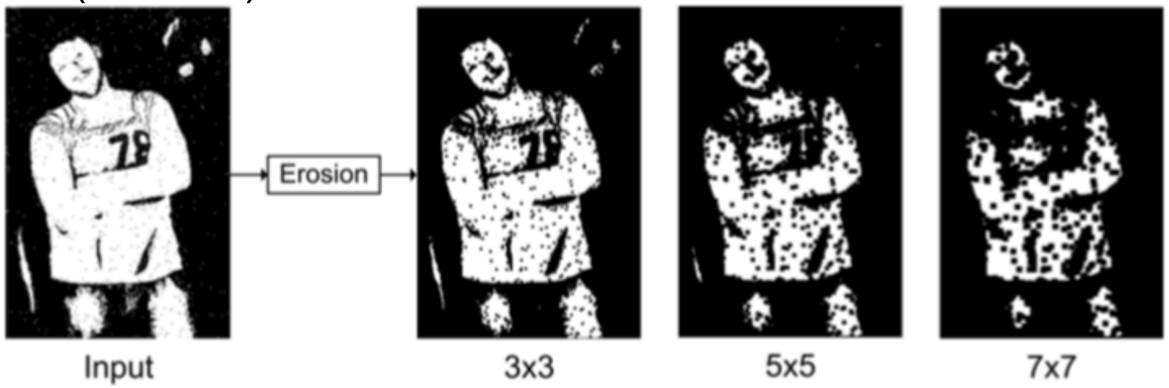


The size of structuring elements matters!

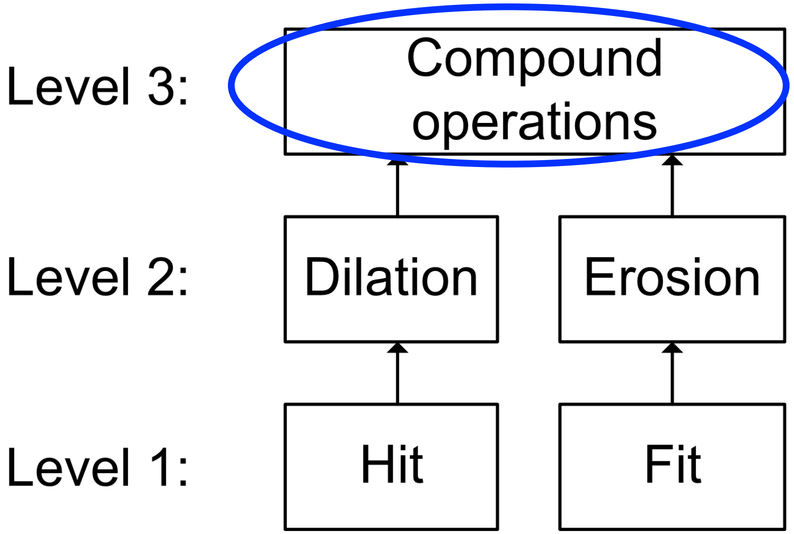
- Dilation (SE: box)



- Erosion (SE: box)



Compound operations

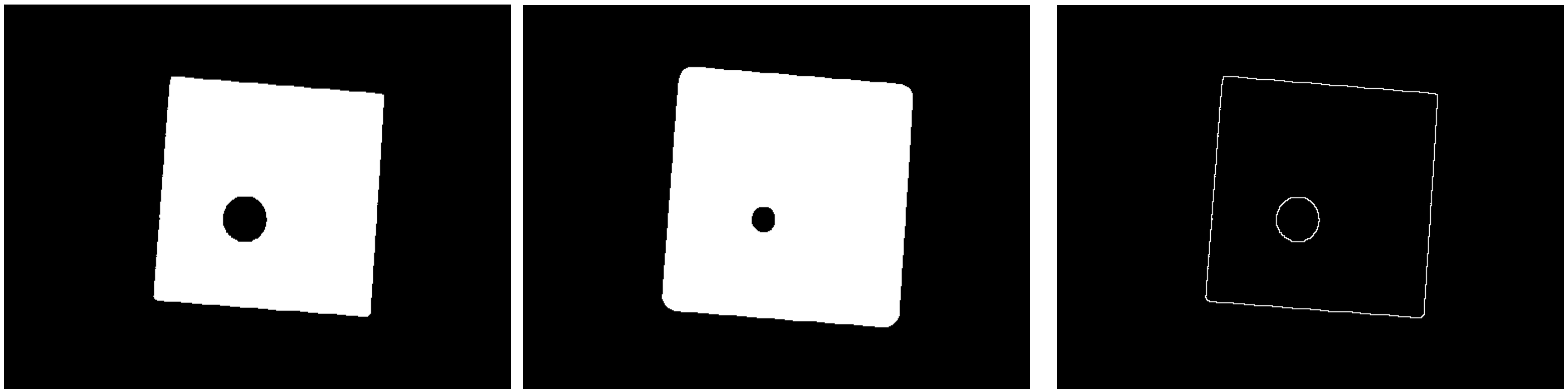


- Compound
  - made of two or more separate parts or elements
- Combining Erosion and Dilation into more advanced operations
  - Finding the outline
  - Opening
    - Isolate objects and remove small objects (better than Erosion)
  - Closing
    - Fill holes (better than Dilation)

Finding the outline

1. Dilate input image (object gets bigger)
2. Subtract input image from dilated image
3. The outline remains!

$$g(x,y) = (f(x,y) \oplus SE) - f(x,y)$$



Opening

- Motivation: Remove small objects BUT keep original size (and shape)
- Opening = Erosion + Dilation
  - Use the same structuring element!
  - Similar to erosion but less destructive
- Math:

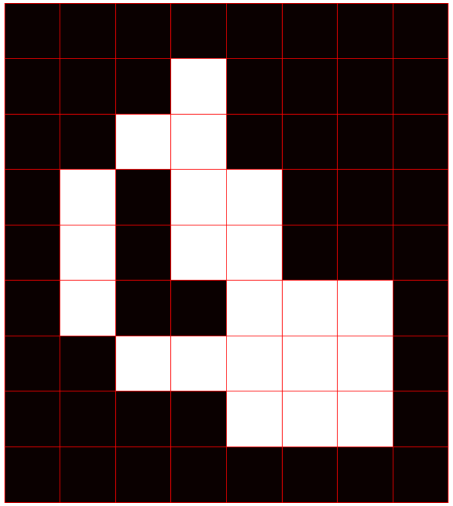
$$g(x,y) = f(x,y) \circ SE = (f(x,y) \ominus SE) \oplus SE$$

- Opening is **idempotent:** Repeated operations has no further effects!

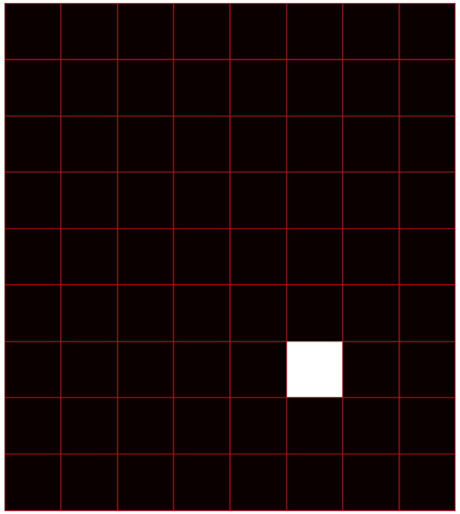
$$f(x,y) \circ SE = (f(x,y) \circ SE) \circ SE$$

Opening

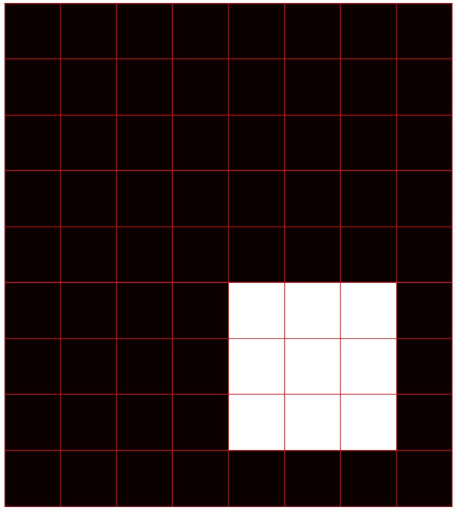
$$g(x,y) = \ (f(x,y) \ominus SE) \oplus SE$$



Original

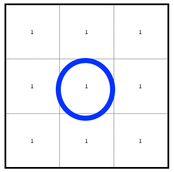


Eroded



Dilated

Opening = erosion+dilation



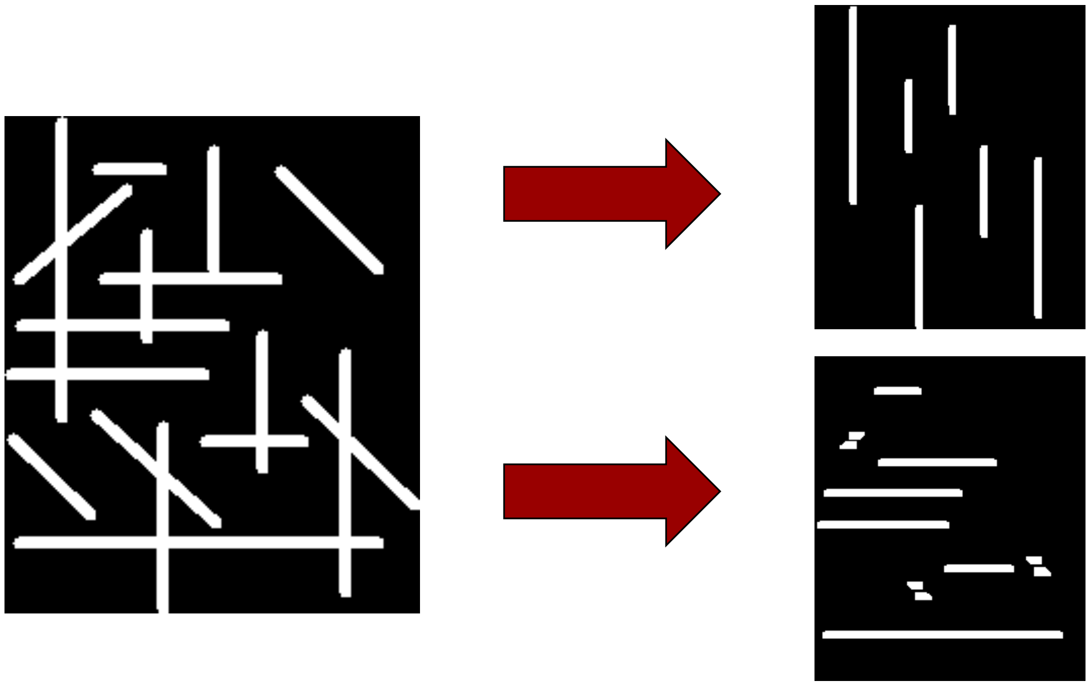
SE

Note:

- Same size and shape
- Idempotent

Opening example

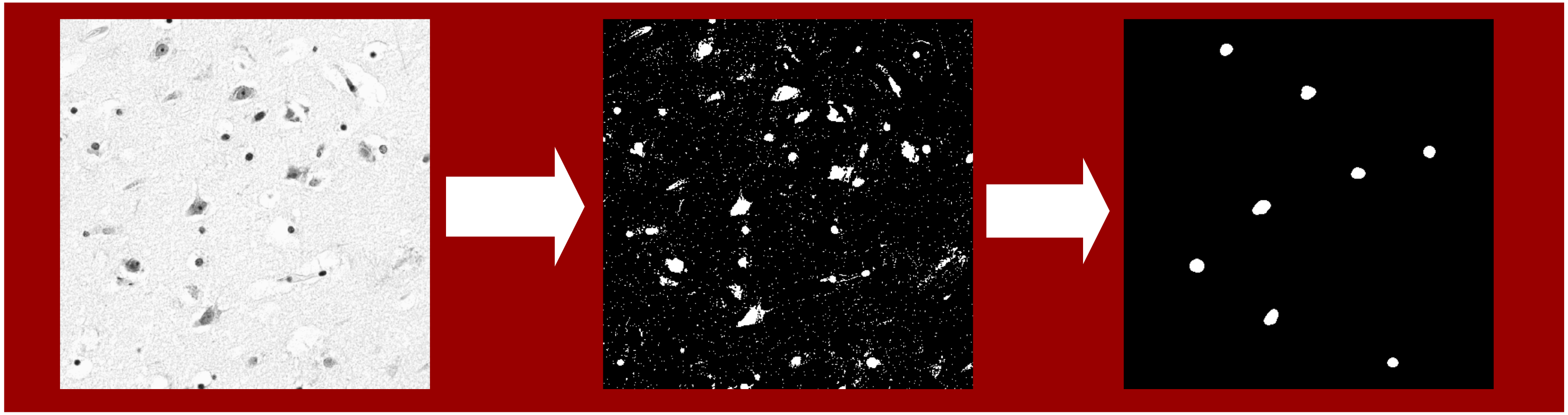
- 9x3 and 3x9 Structuring Elements



Opening example

- Size of structuring element should fit into the smallest object to keep
- Structuring Element: 11 pixel disc

Histology image: Cells



Quiz 5: Compound operations

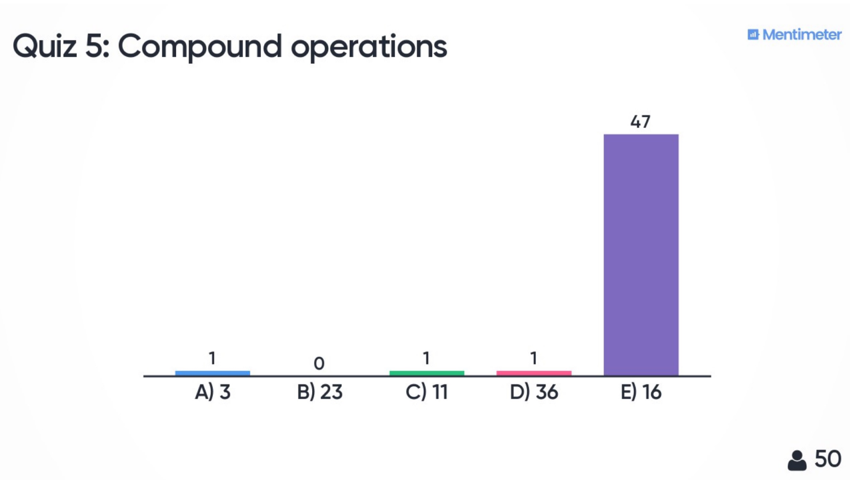
A) 3

B) 23

C) 11

D) 36

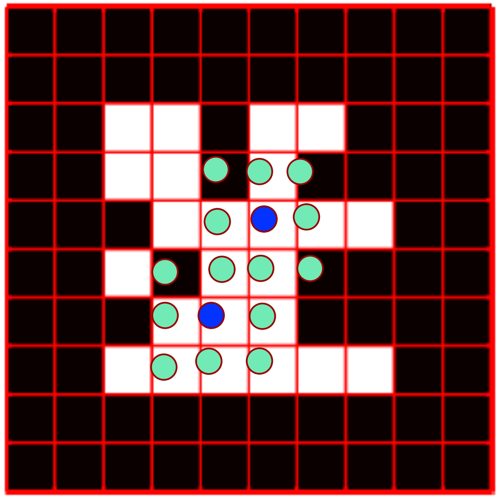
E) 16

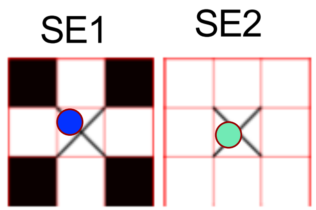


The compound morphological operation seen below is applied to the image.

How many foreground pixels are there in the resulting image?

$$(\mathrm{I} \ominus \mathrm{SE1}) \oplus \mathrm{SE2},$$





Closing

- Motivation: Fill holes BUT keep original size (and shape)
- Closing = Dilation + Erosion
  - Use the same structuring element!
  - Similar to dilation but less destructive
- Math:

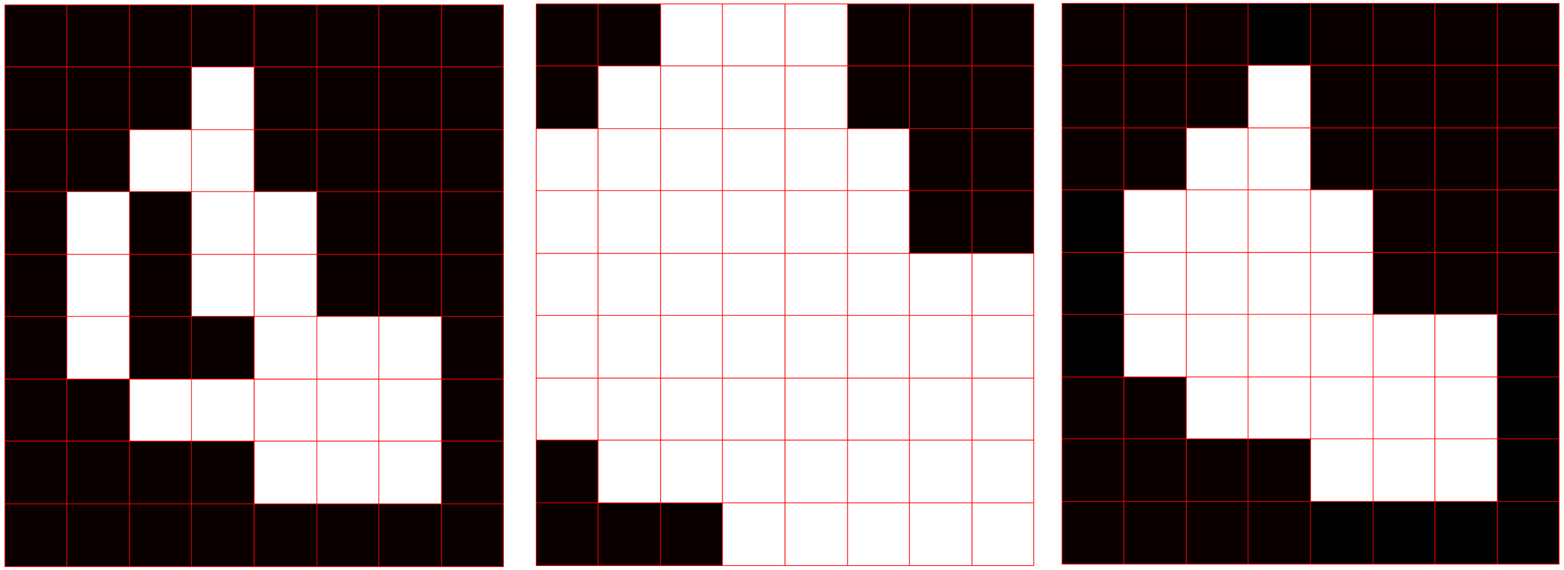
$$g(x,y) = f(x,y) \bullet SE = (f(x,y) \oplus SE) \ominus SE$$

- Closing is **idempotent:** Repeated operations has no further effects!

$$f(x,y) \bullet SE = (f(x,y) \bullet SE) \bullet SE$$

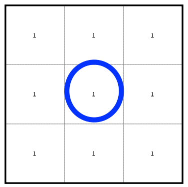
Closing

$$g(x,y) = (f(x,y) \oplus SE) \ominus SE$$



Original Dilated Eroded

Closing = dilation + erosion



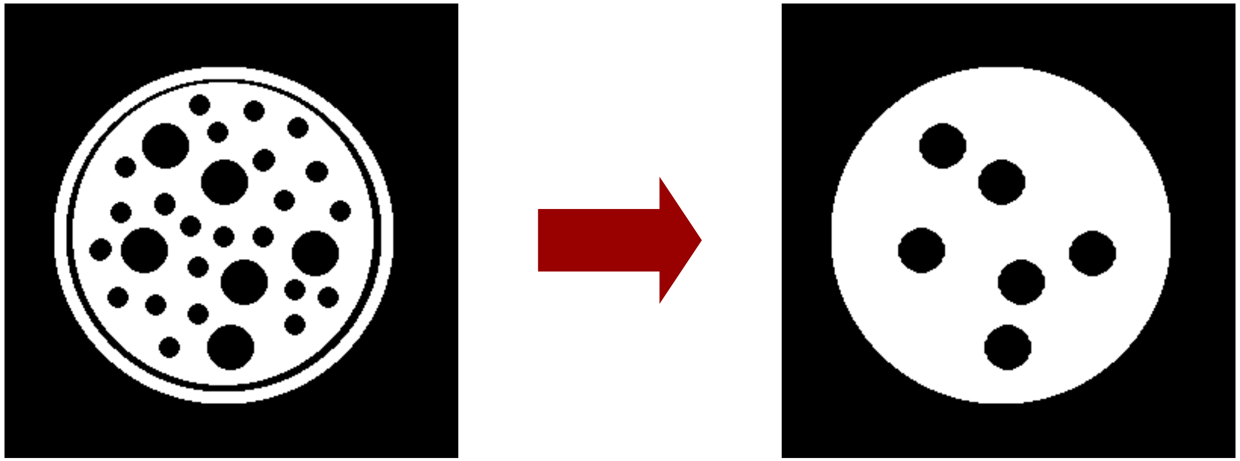
SE

Note:

- Same size and shape
- Idempotent

Closing example

- Closing operation with a 22 pixel disc
- Closes small holes



Quiz 6: Closing

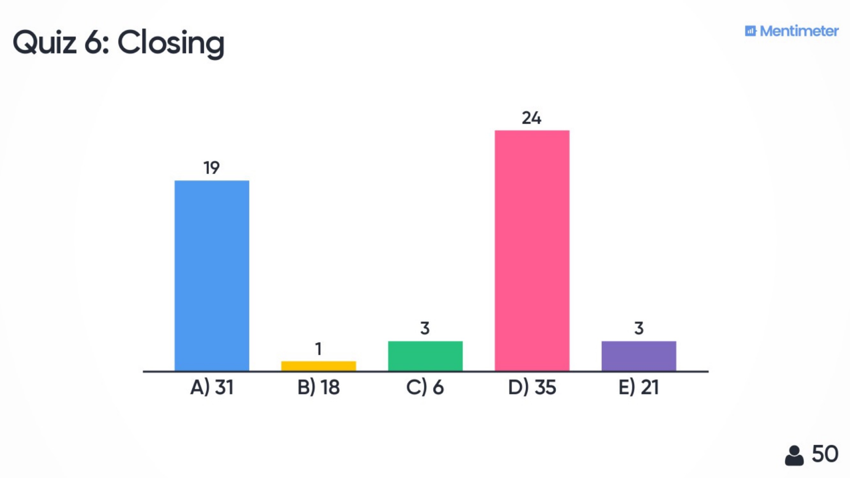
A) 31

B) 18

C) 6

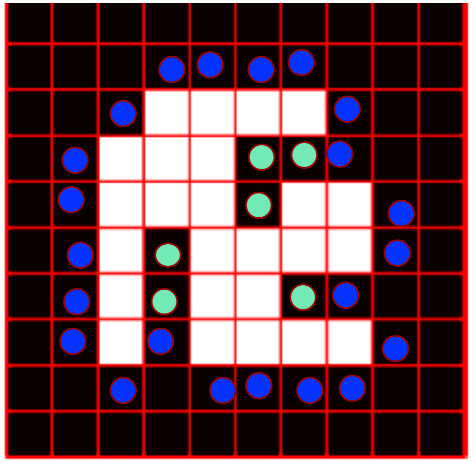
D) 35

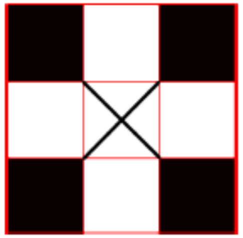
E) 21



Morphological closing is applied to the image using the structuring element below.

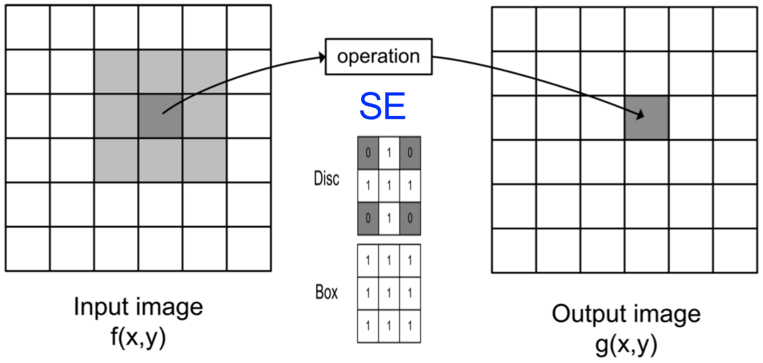
How many foregrounds pixels are there in the resulting image?





What did we learn today?

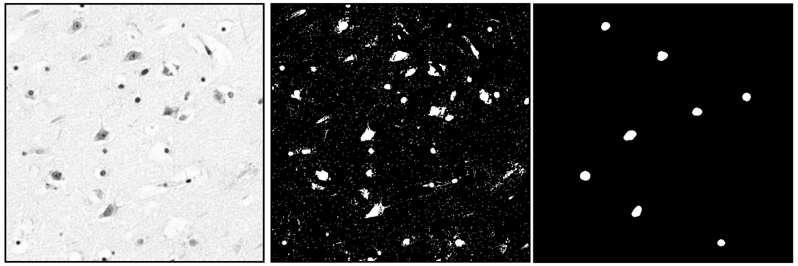
Morphology of binary images - (Chapter 6)

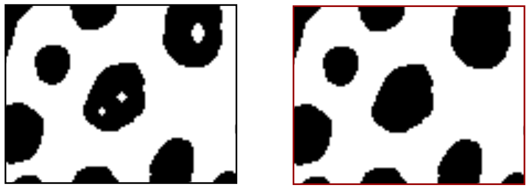


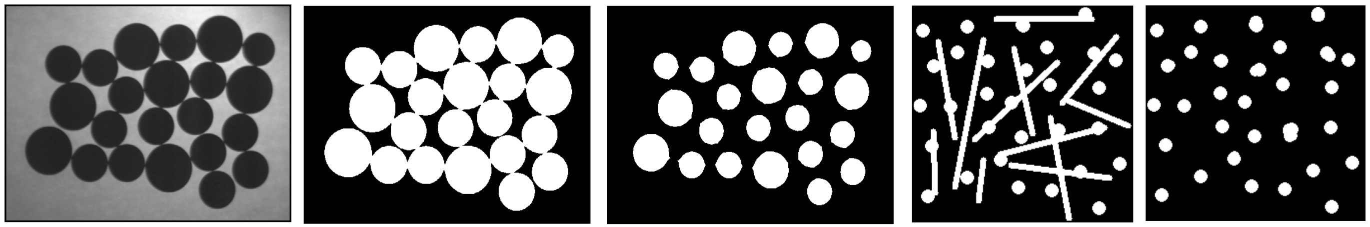
Level 3: Compound operations

Level 2: Dilation | Erosion

Level 1: Hit | Fit







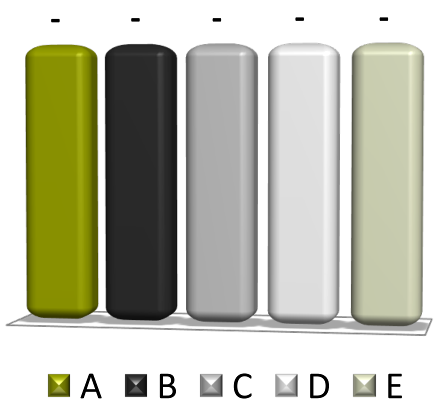
- Remove noise
  - Small objects
  - Fill holes
- Isolate objects
- Customized to specific shapes
- Size of SE matter

How do you like the book?

A) Very bad book
B) Bad book
C) Ok book
D) Good book
E) Really good book

Flipped classroom
TA 8-10, Lecture 10-12

A) It really does not work
B) It is not optimal
C) It is ok
D) It is fine
E) It works very well



How much time do I spend on preparing every week?

A) 0 minutes
B) 0-15 minutes
C) 15-30 minutes
D) 30-60 minutes
E) 1-2 hours
F) 2-4 hours
G) More than 4 hours

How do I feel about Matlab

A) I simply do not get it
B) I find it hard
C) We are ok friends
D) I feel confident in Matlab
E) I write Matlab scripts even when I sleep

Next week: Blob Analysis