



Image Analysis

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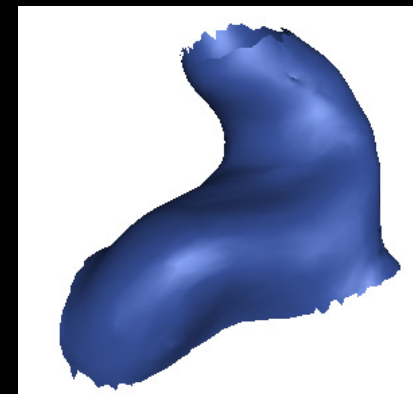
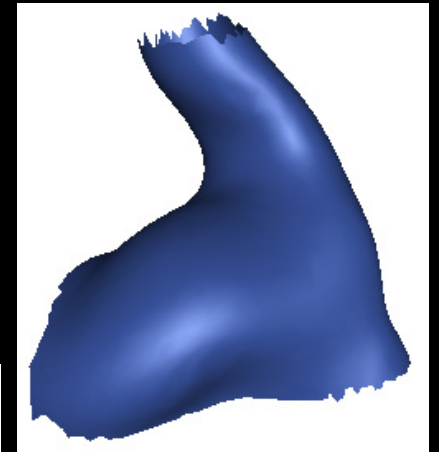
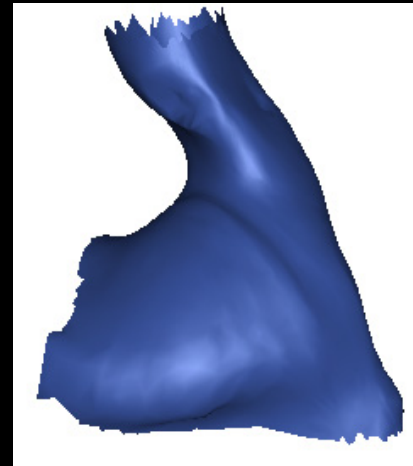
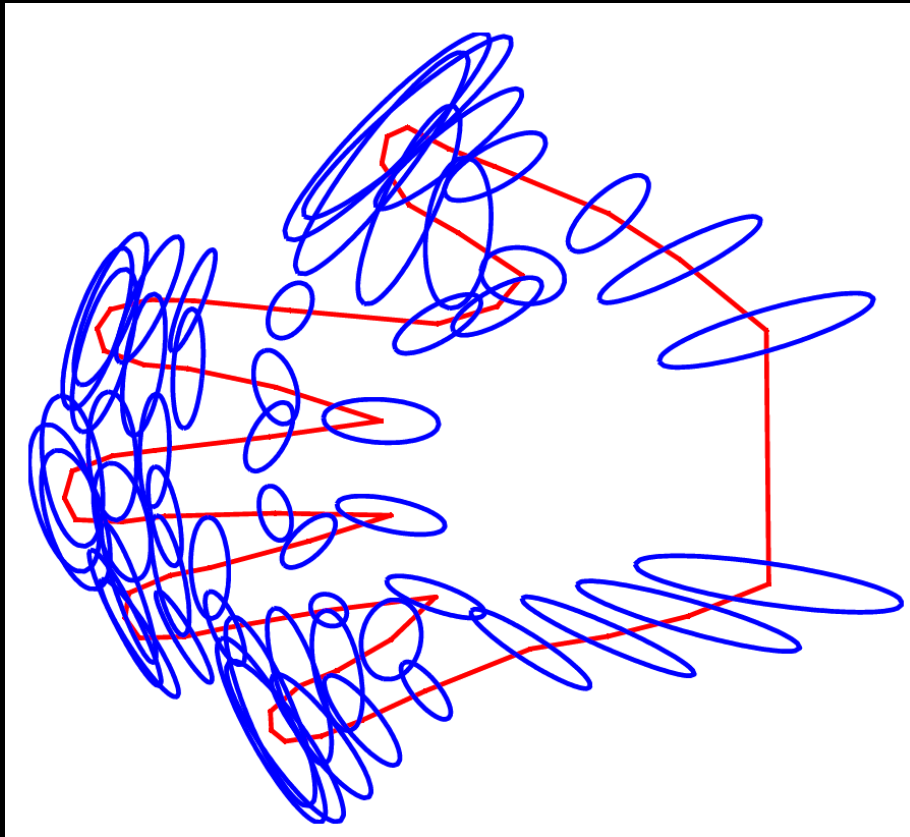
<http://courses.compute.dtu.dk/02502>



Course evaluation

- Please all use CampusNet to evaluate the course!
- What did you like and what worked well
- What do you think can be improved
 - Perhaps how?

Lecture 12 – Shape Analysis





Today's Learning Objectives

- Describe the concept of shape models
- Define the shape of an object using landmarks
- Describe point correspondence
- Describe and use the vector representation of a shape
- Describe how a shape can be seen as a point in high-dimensional space
- Explain how shapes can be aligned
- Describe how principal component analysis can be used to model shape variation

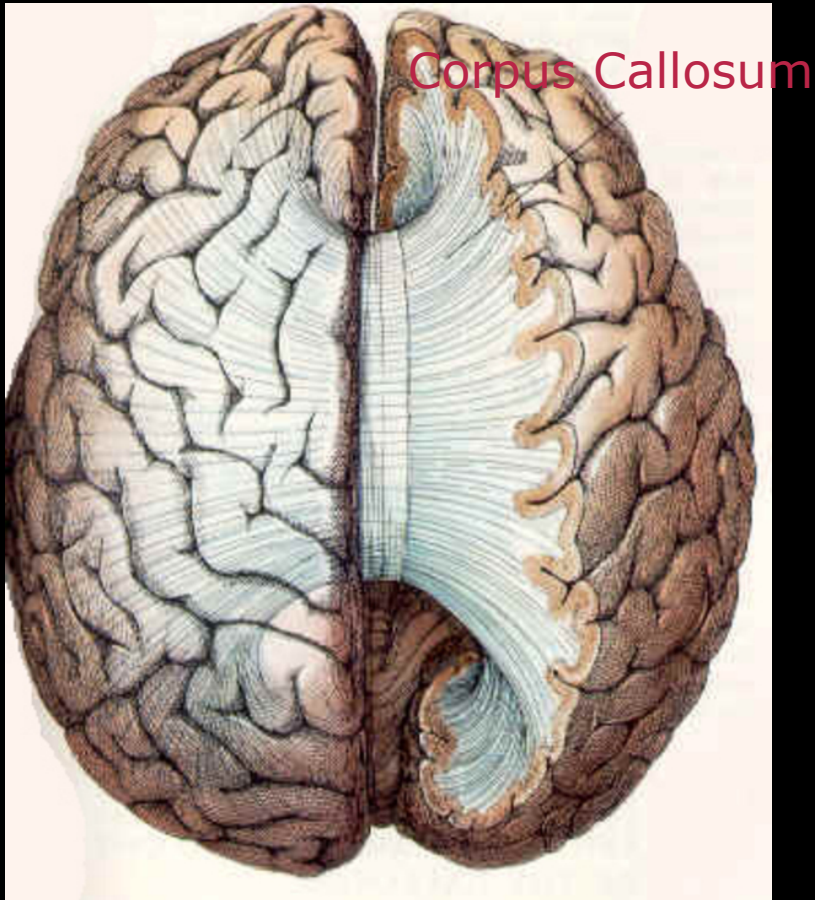
A typical scenario



- Doctor X believes that he can “see” on a hand X-ray if the patient is in risk of arthritis!
- Specifically Doctor X is sure that the *shape of the joints* is an estimator for arthritis!

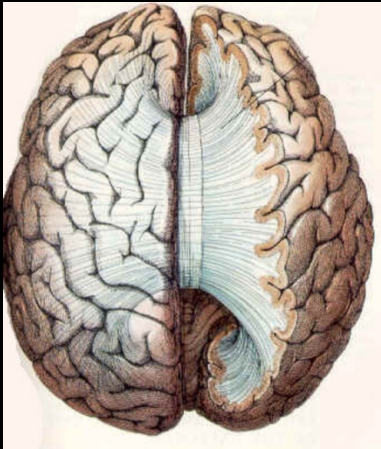
Can we verify that?

Scenario II



- MR images have been captured of a large group of people
- Cognitive abilities measured as well
- Is there a correlation between *how the brain looks* and how we behave?
- Does the shape of corpus callosum tell us something?

Scenario II



Corpus Callosum

- We can get the MR slice with the corpus callosum from all the patients

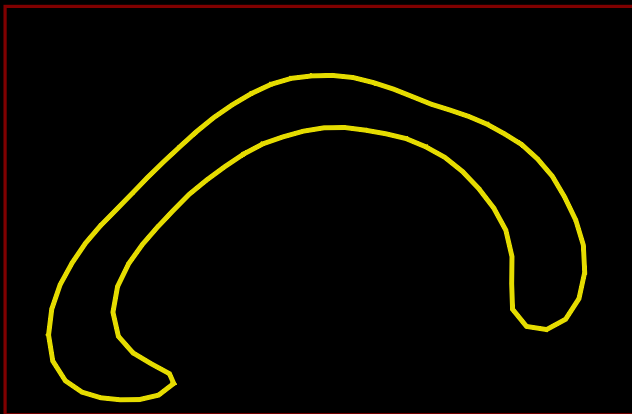


Image Analysis

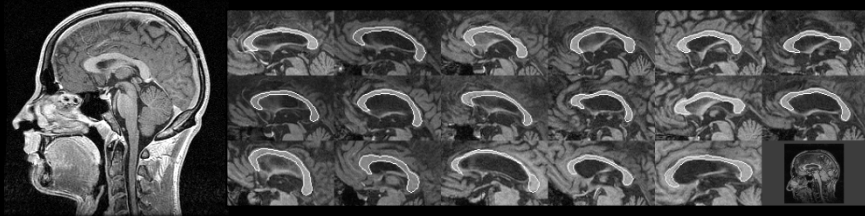
Scenario III



- An experienced hearing aid fitter has seen a lot of ears!
- Some hearing aid users are very difficult to fit. Why?
- Large variation in the shape of ears
- Ear canals change shape when people chew
- Is it possible to learn about the shape and use it?



Shape Analysis



600 MR scans and
behavioural data

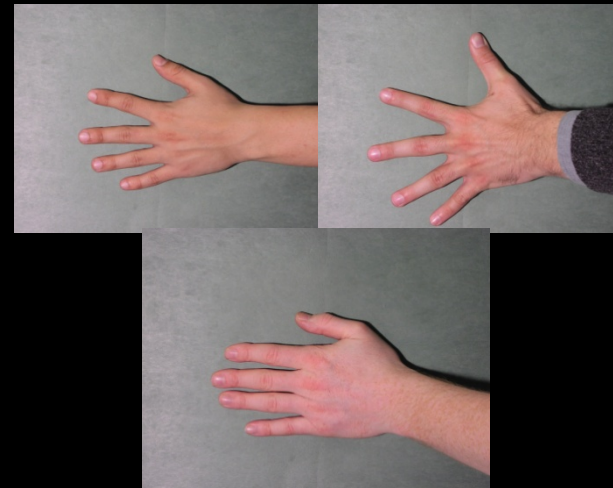
- What can we learn from shape?
- What can we use it for?
- How do we do it?



1000 historical X-rays

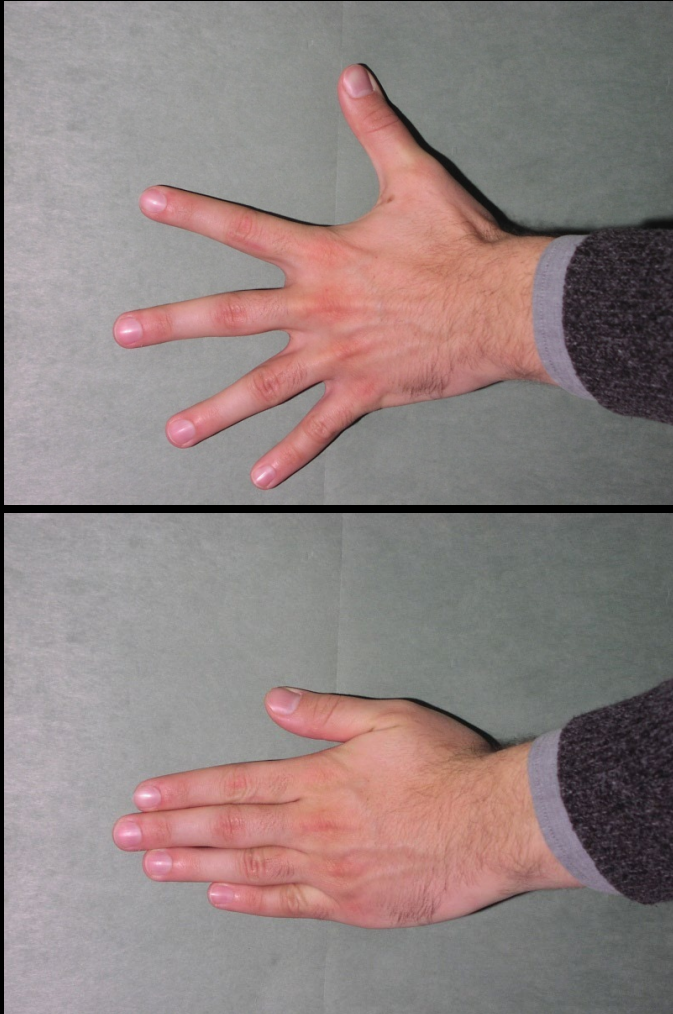


A boxful of something
that look like ear canals



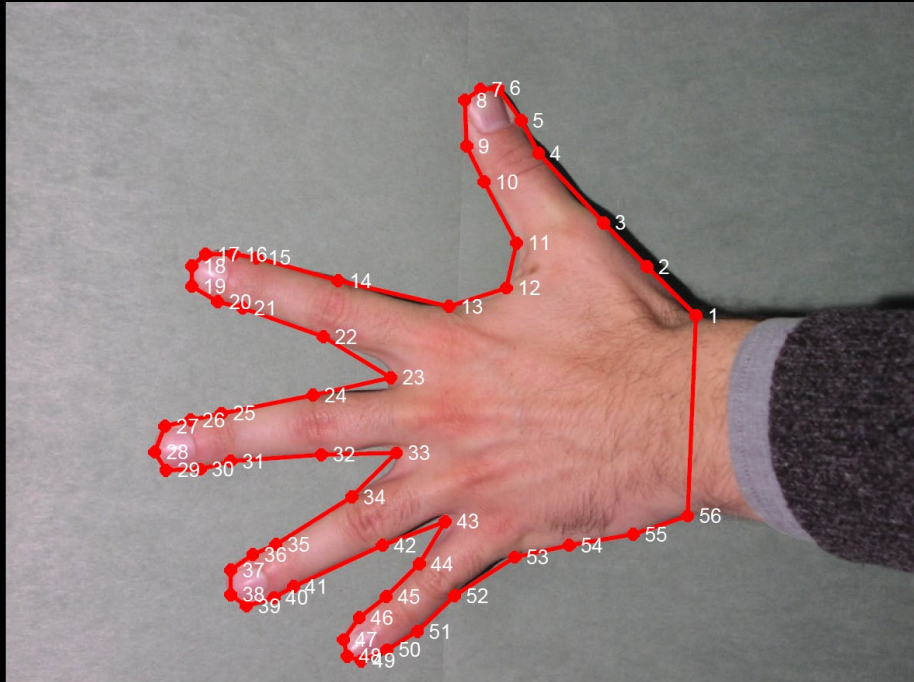
A set of hand
photographs

What is shape?



- How do we define the *shape* of this hand?
- What is the shape difference between the two hands?

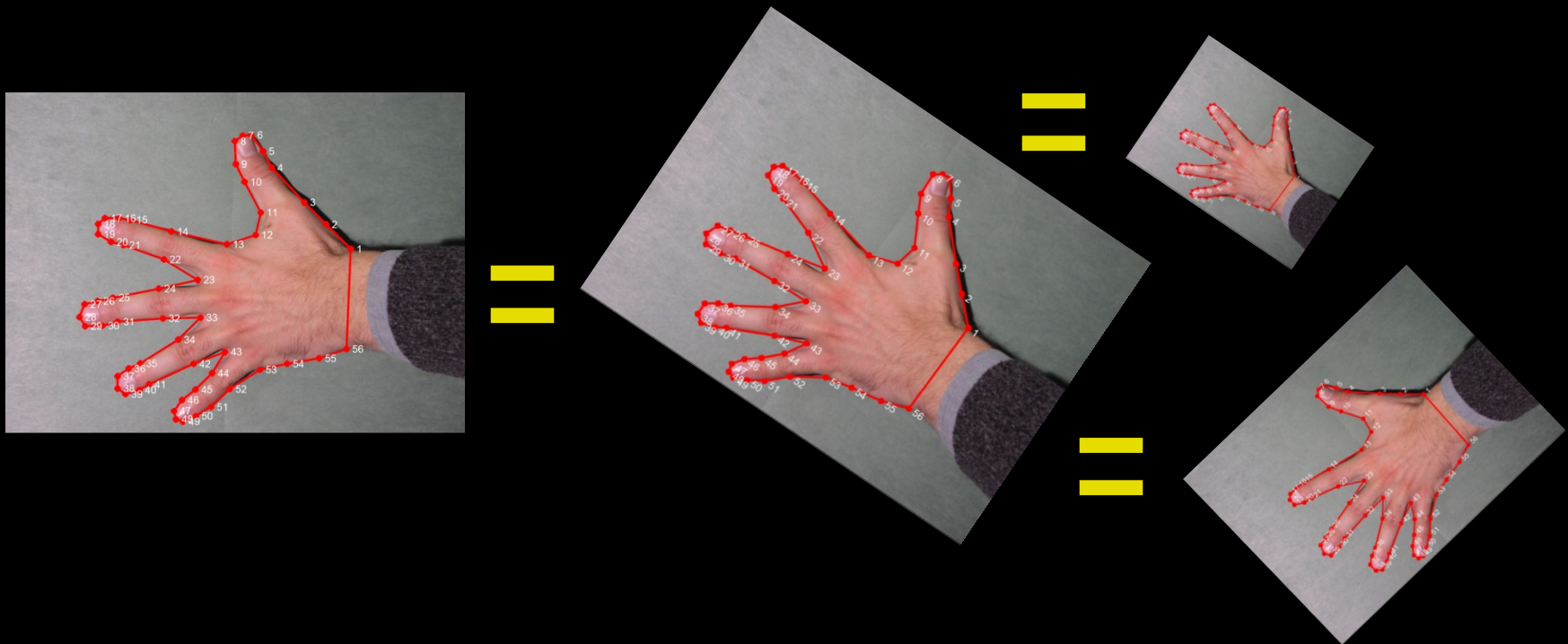
Shape definition



- Shape is defined using landmarks
 - Placed by an expert
- In this case the outer contour of the hand
- Just one of many ways!

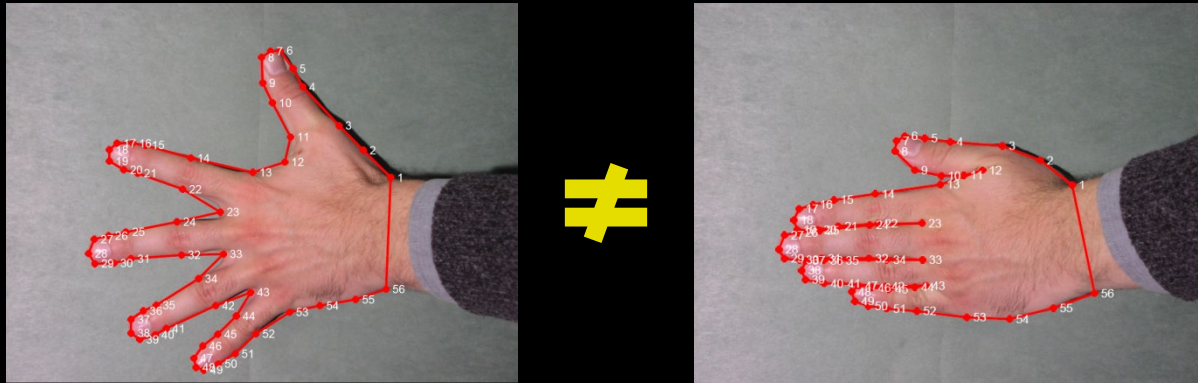
Shape definition

Shape is all geometrical information that remains when location, scale, and rotational effects are removed



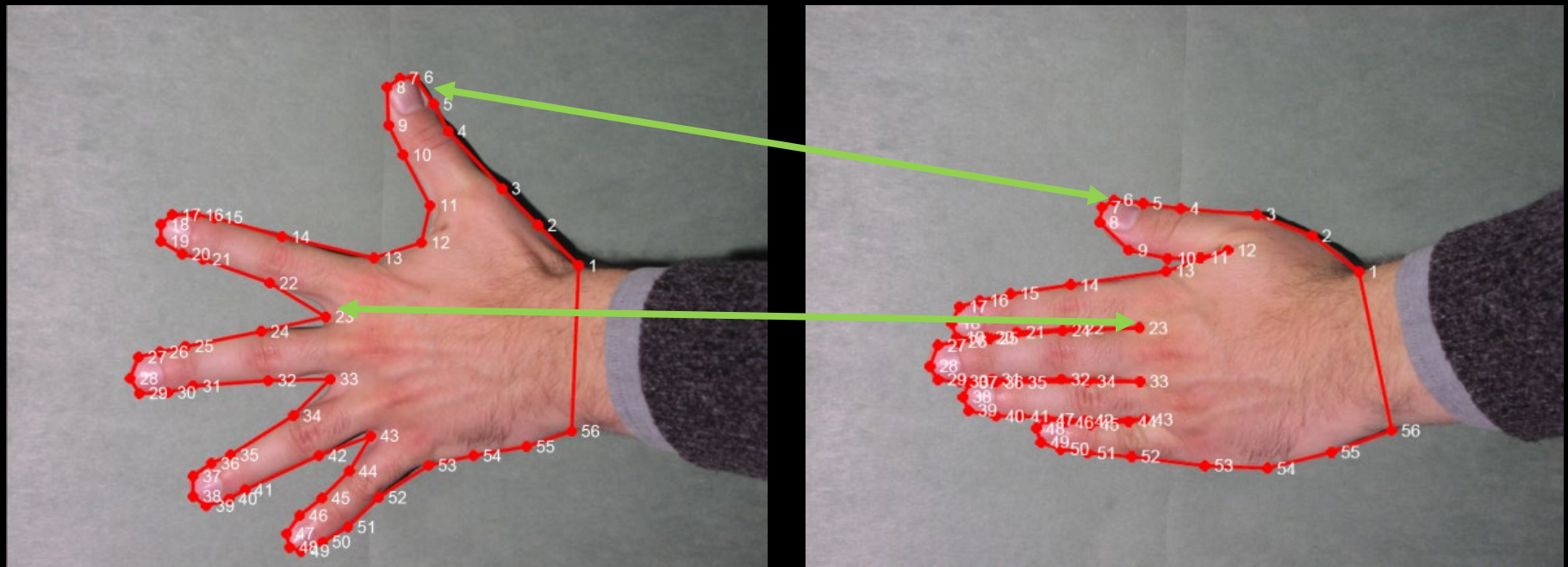
Shape definition

Shape is all geometrical information that remains when location, scale, and rotational effects are removed



Landmarks and point correspondence

Landmarks are placed on the same place on all shapes in the training set



Shape as a vector

1 : (x_1, y_1)

2 : (x_2, y_2)

⋮

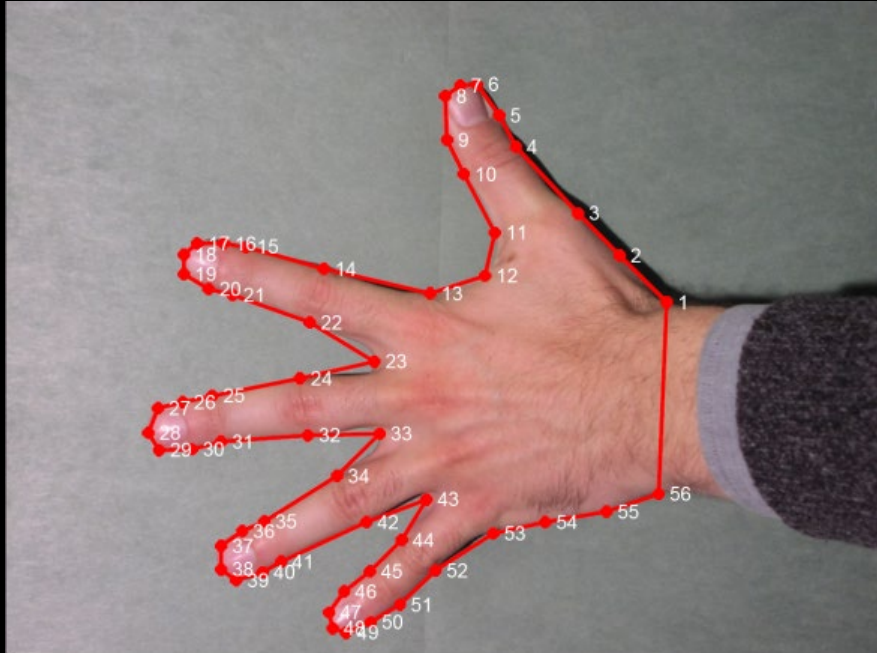
$N : (x_n, y_n)$

- The shape is represented as an array of (x,y) coordinates
- Trick number one!
All coordinates are put into one vector!
- $n=56$ points
 - Vector with 112 elements!



$$\mathbf{x} = [x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n]^T$$

Shapes in high-dimensional space

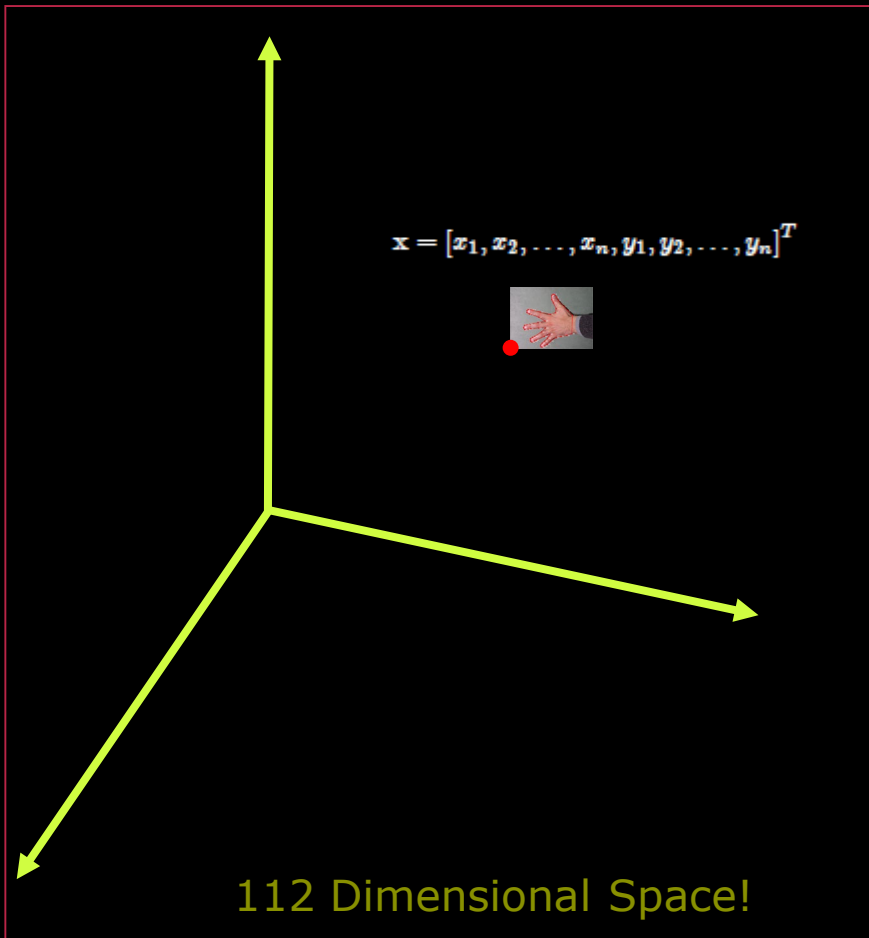


- One hand is now described using one vector
- A vector can also be seen as a point in space!

$$\mathbf{X} = [x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n]^T$$

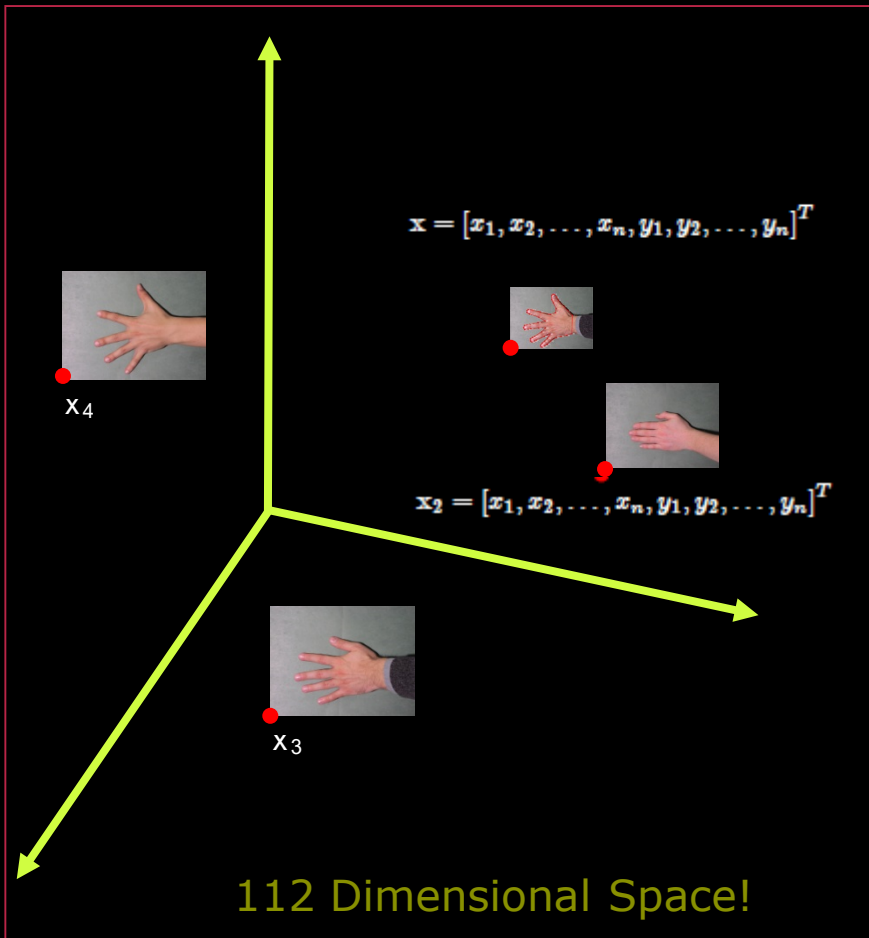
Trick number
two!

Coordinates in space



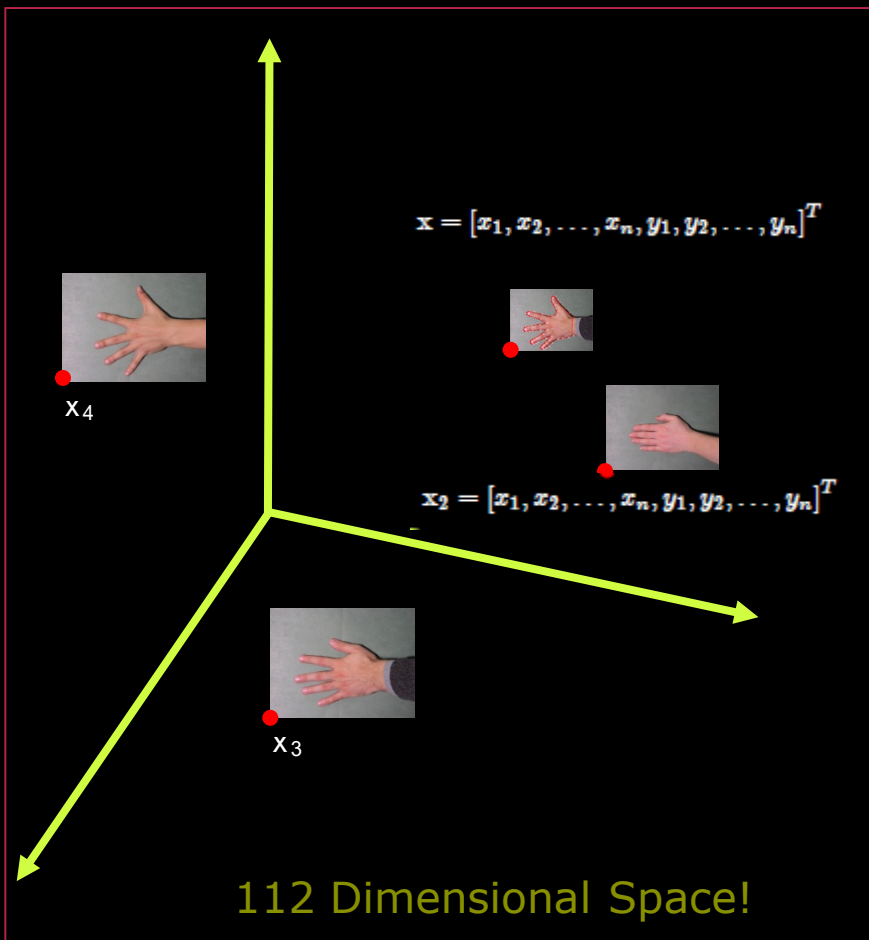
- On hand is now described using one vector
- A vector can also be seen as a coordinate in space!
- Not 2D space, not 3D space, not 4D space...
- 112 Dimensional Space!
- A hand has a position in this space!

Hands in Space



- A hand has a position in space!
- Another hand appears
 - in the same space
 - different position = different shape
- All hands have a place in this space!

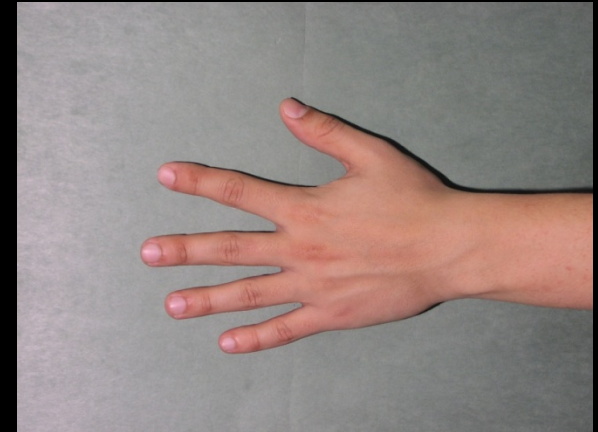
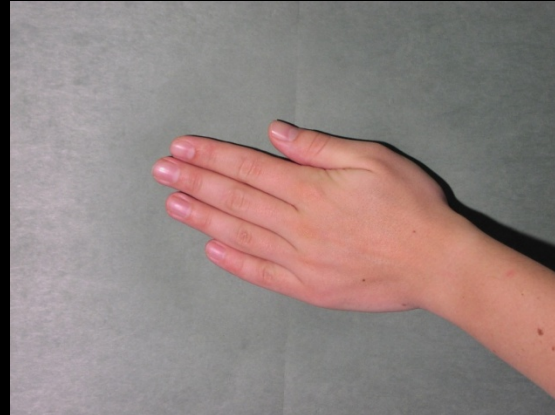
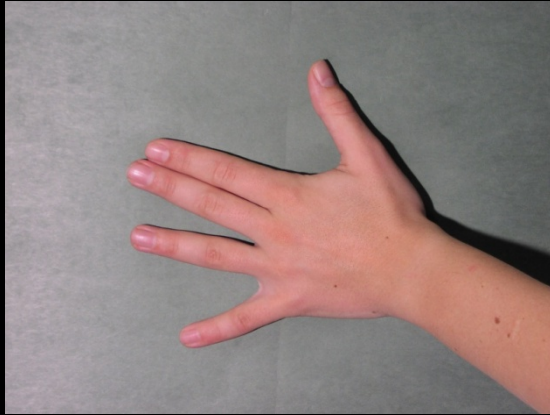
Shape Analysis



■ Shape analysis

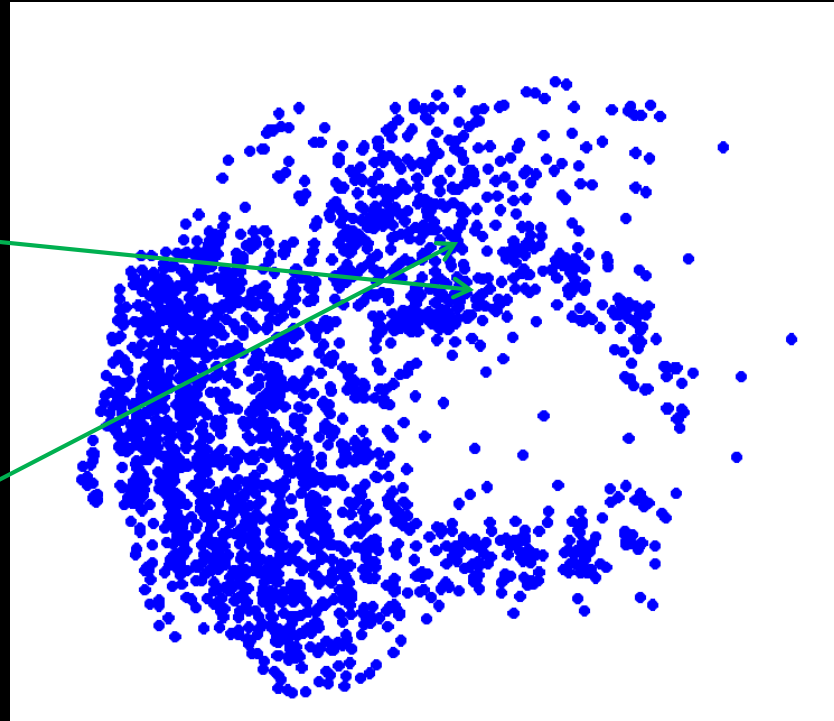
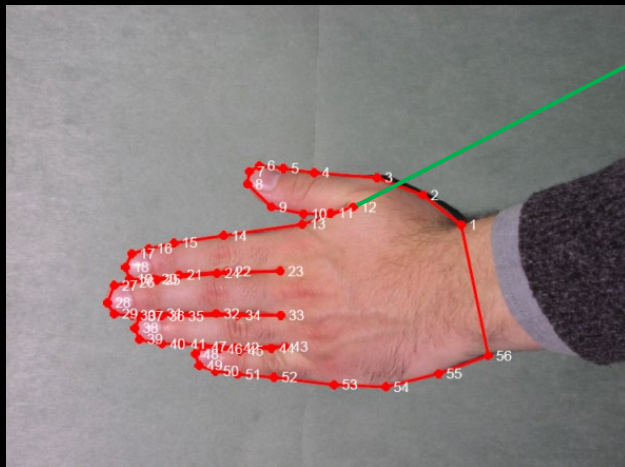
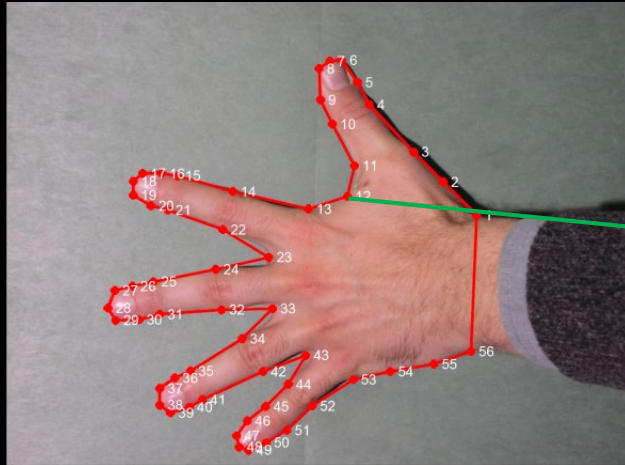
- Similar shapes are placed on “planes” in the shape-space
- Also called a manifold

Shape alignment



- 40 training images of hands
- 56 landmarks on each
- Placed in random location (translation+rotation)

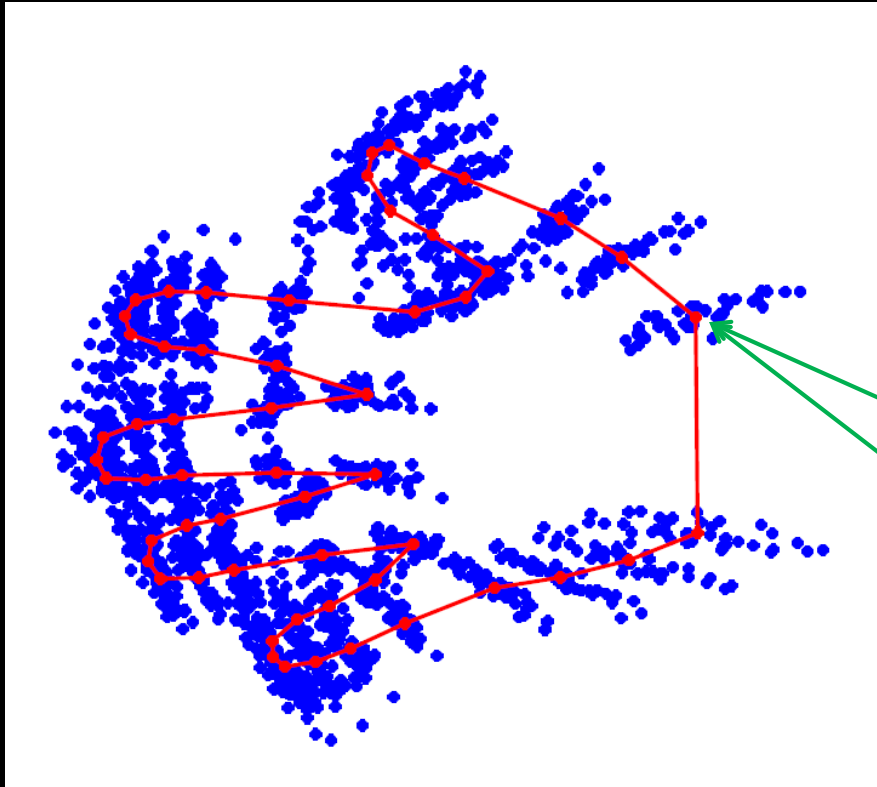
Shape alignment



Landmarks from all hands

Needs alignment!

What is alignment?



Average shape

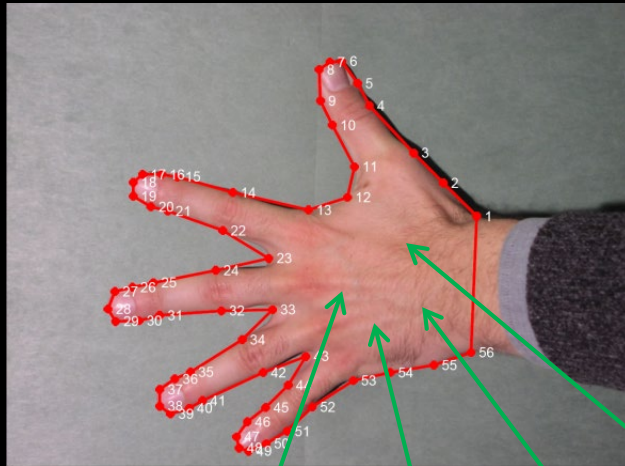
- Group wise registration
 - Not one-to-one
 - All to the average shape

$$\bar{\mathbf{x}} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i$$

$$\bar{\mathbf{x}} = [\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n, \bar{y}_1, \bar{y}_2, \dots, \bar{y}_n]^T$$

But hey! We do not have an average shape?

Procrustes Analysis (alignment)



"Average shape"

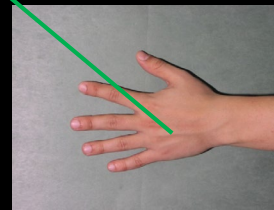
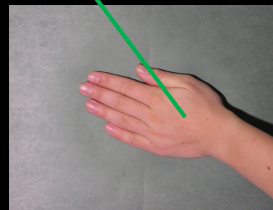
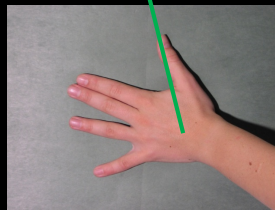
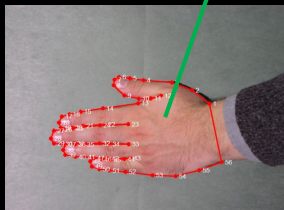
- We start by defining
 - Average shape = Shape #1
- Align shape #2 to shape #1
- Align all shapes to shape #1

Registration

Registration

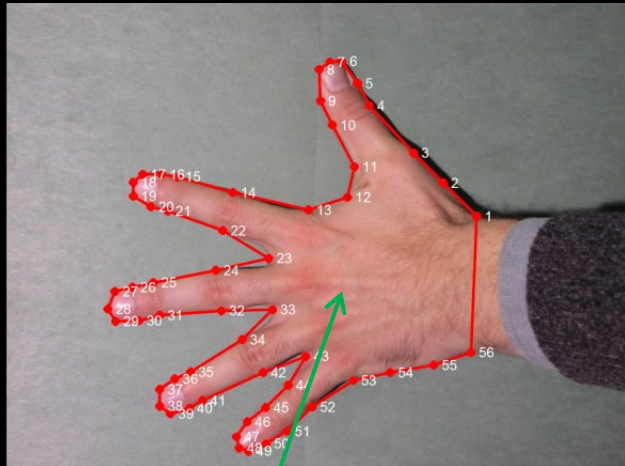
Registration

Registration



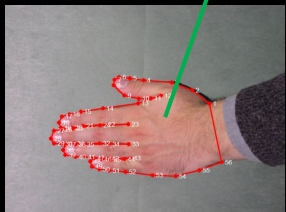
Shape #2

Landmark based registration



“Average shape”

Registration

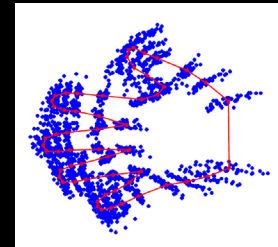
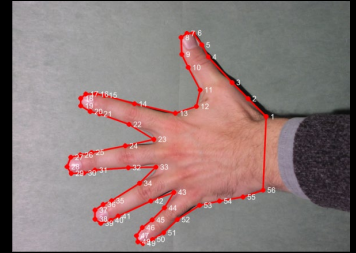


Shape #2

- Shape #2 is transformed to fit the average shape
 - Translation
 - Rotation
 - Scaling
 - = Similarity Transform
- Result
 - Shape #2 is placed *on top of* the average shape

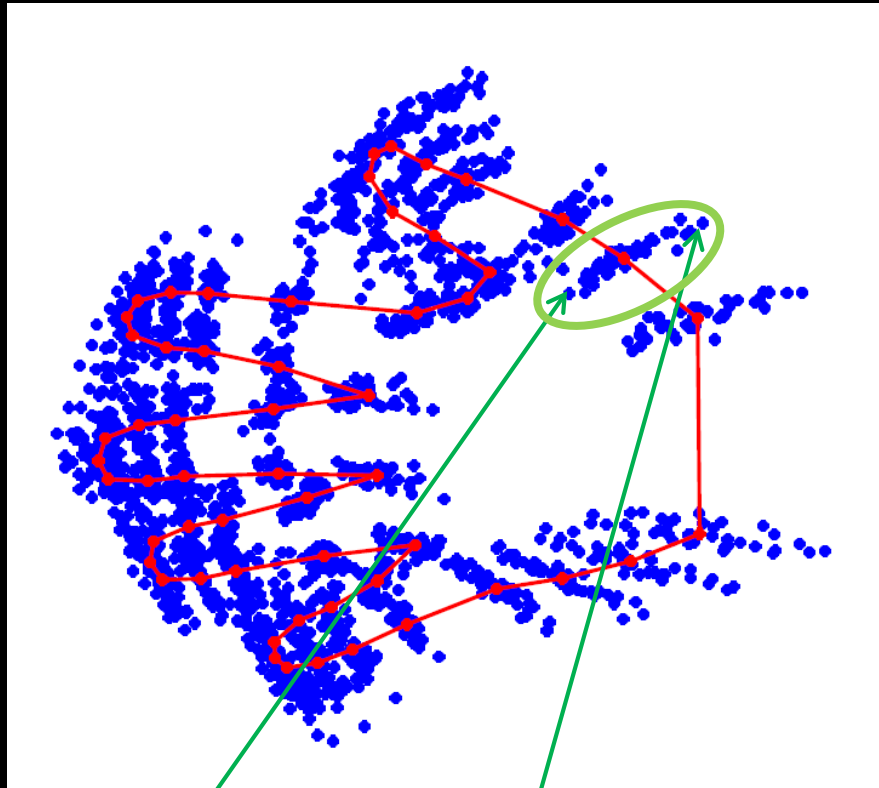
Procrustes Analysis

1. Average shape is set to shape #1
2. Register all shapes to the average shape
 - Landmark based registration
3. Recompute the average shape
4. If average shape changed return to step 2.



$$\bar{\mathbf{x}} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i$$

Aligned shapes – what now

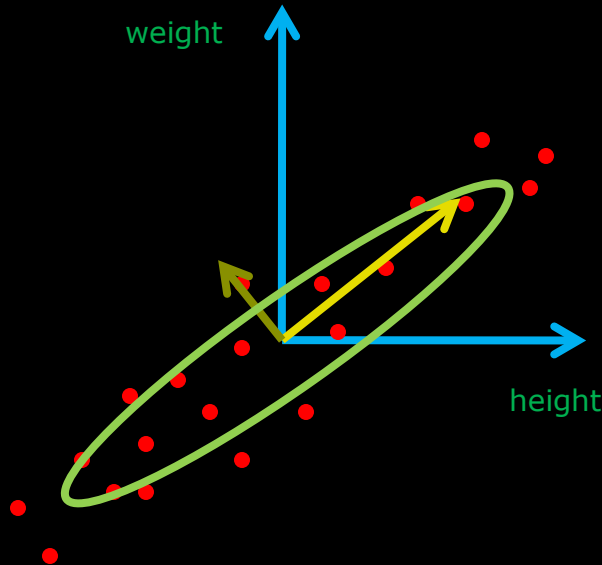


Shape #16

Shape #27

- Individual landmark variation
 - Over the training set
- What shape is the variation?

Principal Component Analysis (PCA)

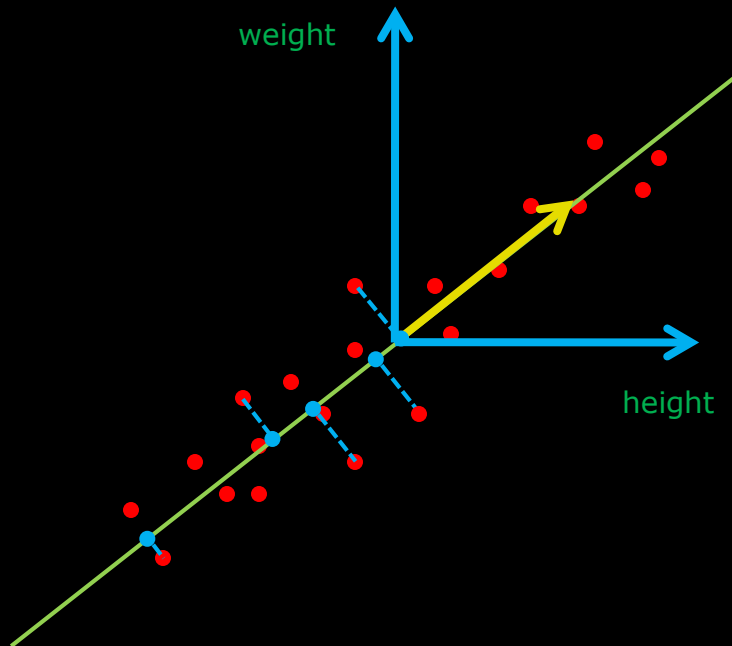


■ PCA

- Main axis in data
- Eigenvectors
- Eigenvalues

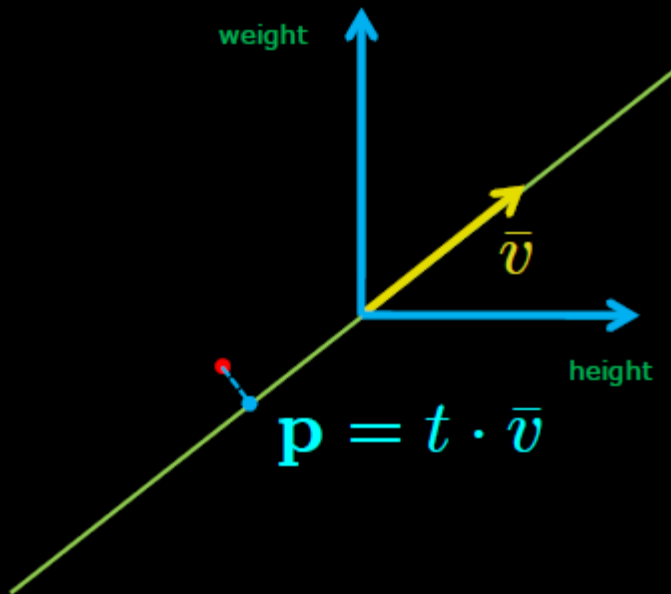
■ Size of Eigenvalues describe explained variance

Principal Component Analysis (PCA)



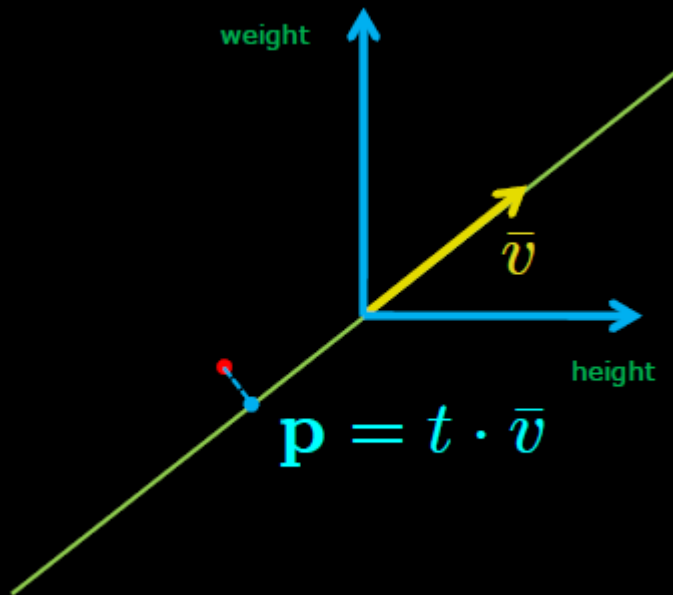
- We throw away the *noise dimensions*
- Points projected to the line

Principal Component Analysis (PCA)



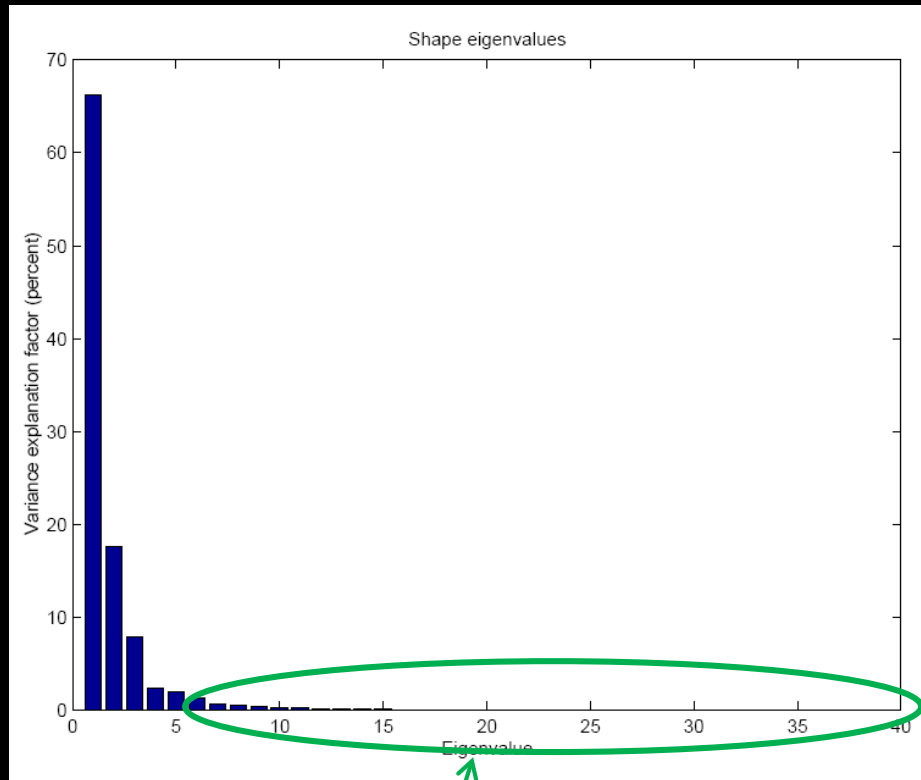
- We throw away the *noise dimensions*
- Points projected to the line
- A point can now be described by one parameter t
- We have reduced the number of dimensions

Principal Component Analysis (PCA)



- Used in statistics
- Measured 100 different things
 - Height, weight, color of eyes, age, smoking...
 - Which combination of measurements explain the most?

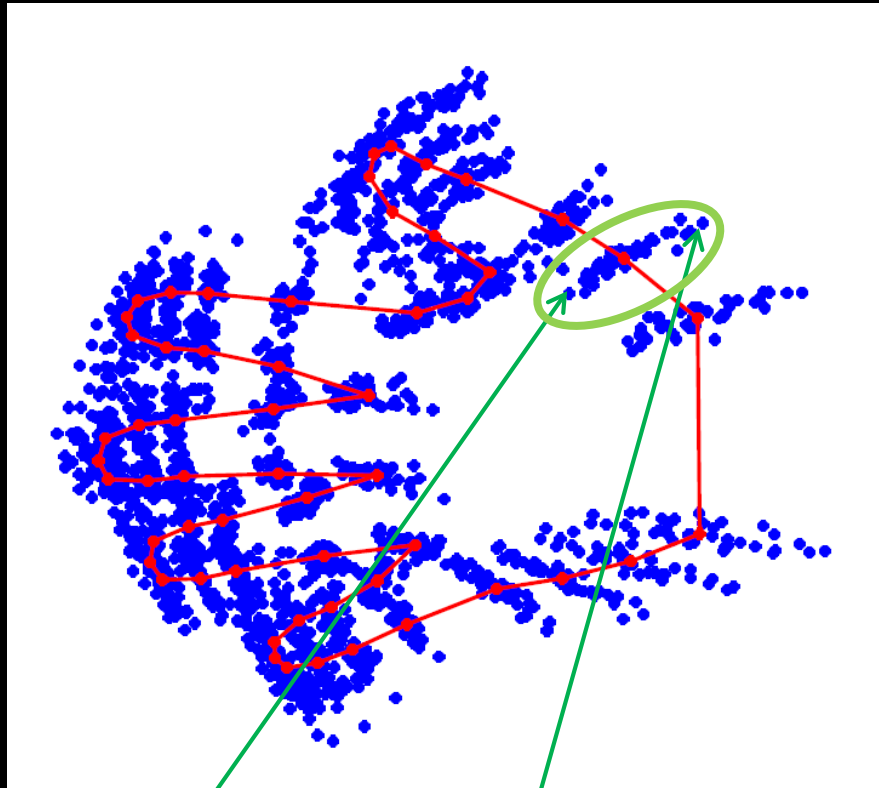
How many dimensions should we keep?



Noise dimensions

- Plot the Eigenvalues
- Explains how *important* each dimension is
- Cut away noise dimensions

Aligned shapes – what now

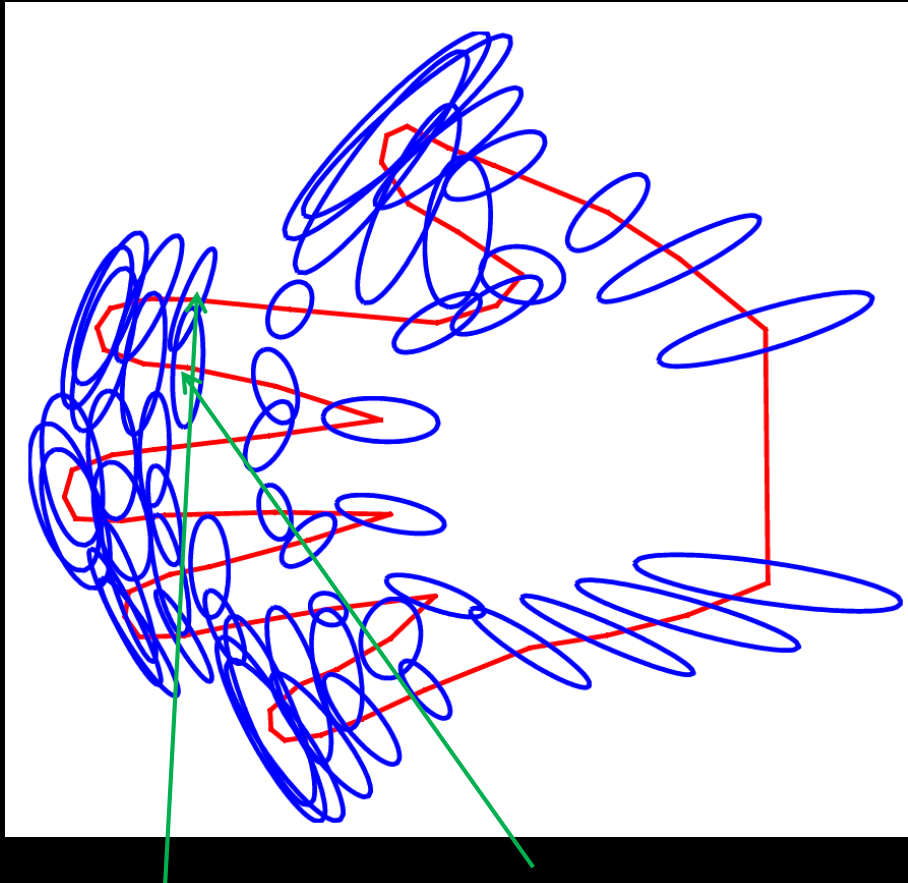


Shape #16

Shape #27

- Individual landmark variation
 - Over the training set
- What shape is the variation?

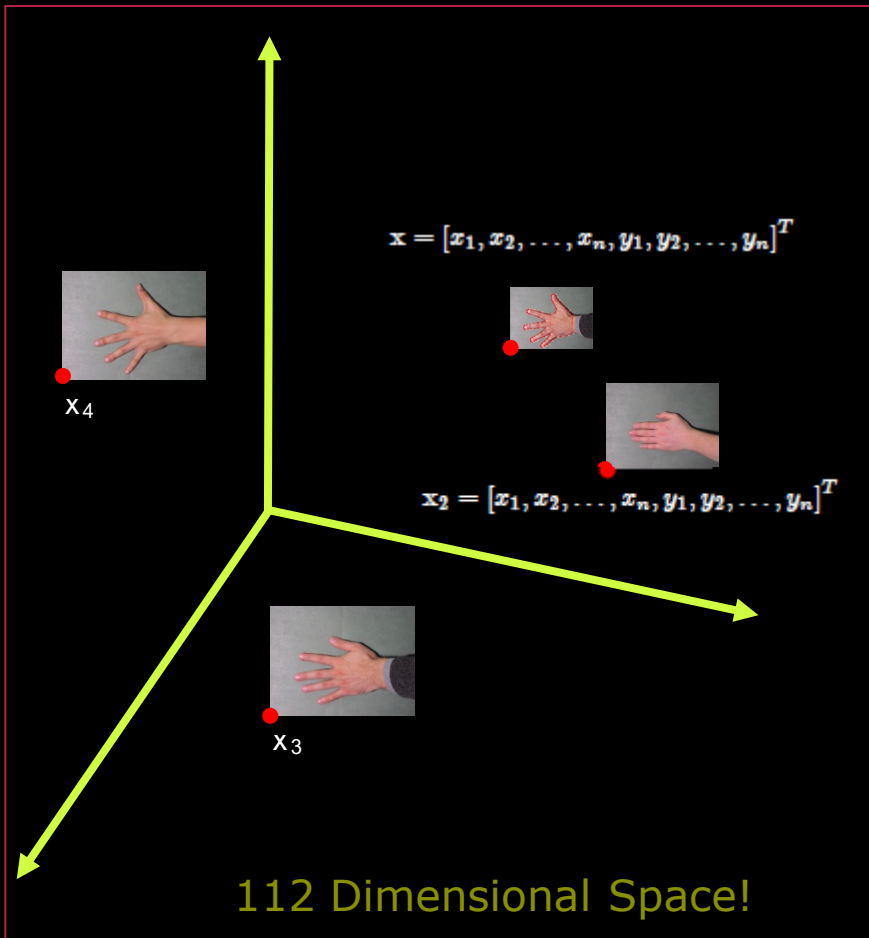
PCA Analysis



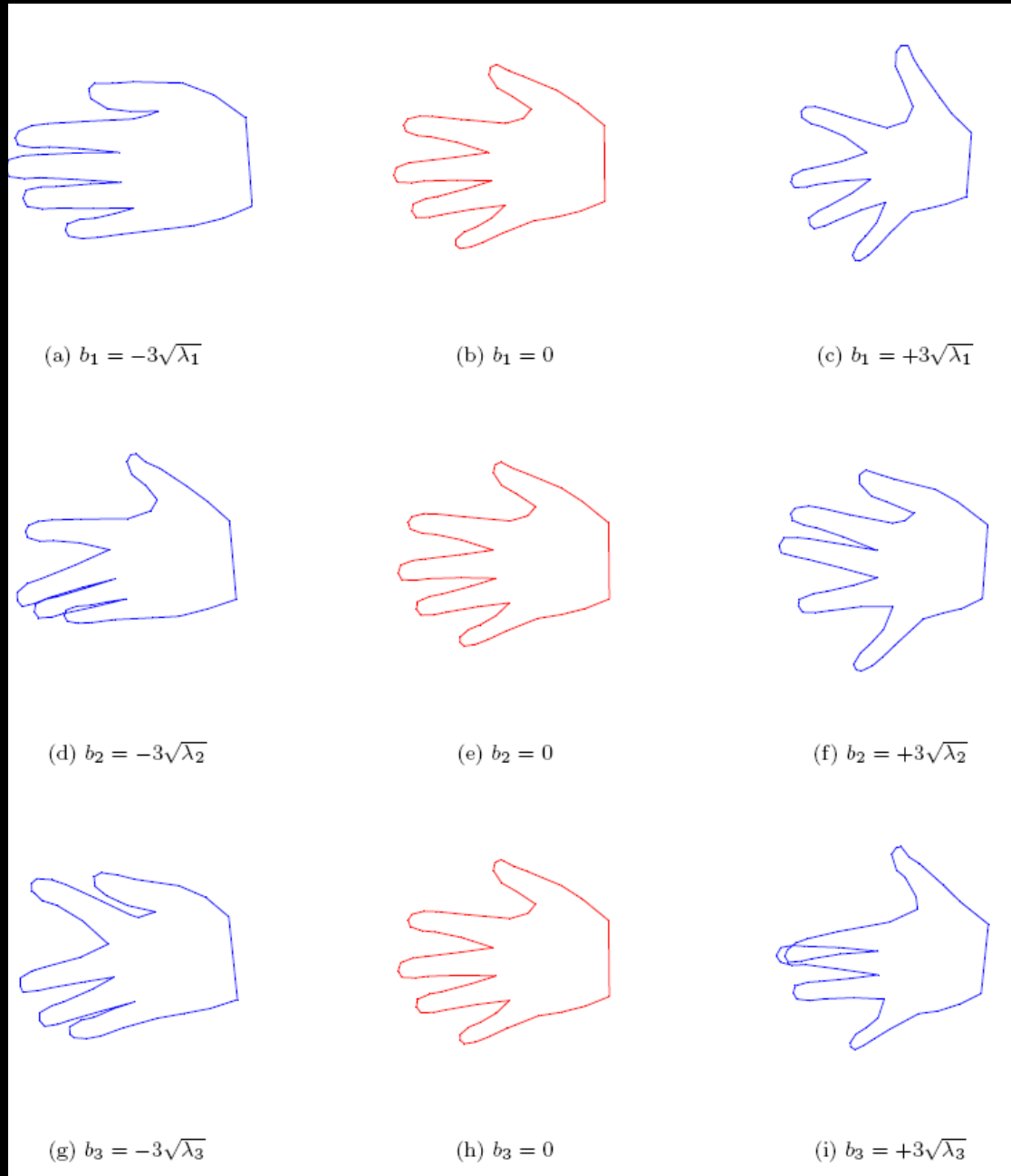
Landmark #14 Landmark #22

- PCA analysis on individual landmarks
- Describes the major direction of variation
- Landmarks are correlated!
- The movement over the shape is connected
- Return to shape space

PCA in shape space

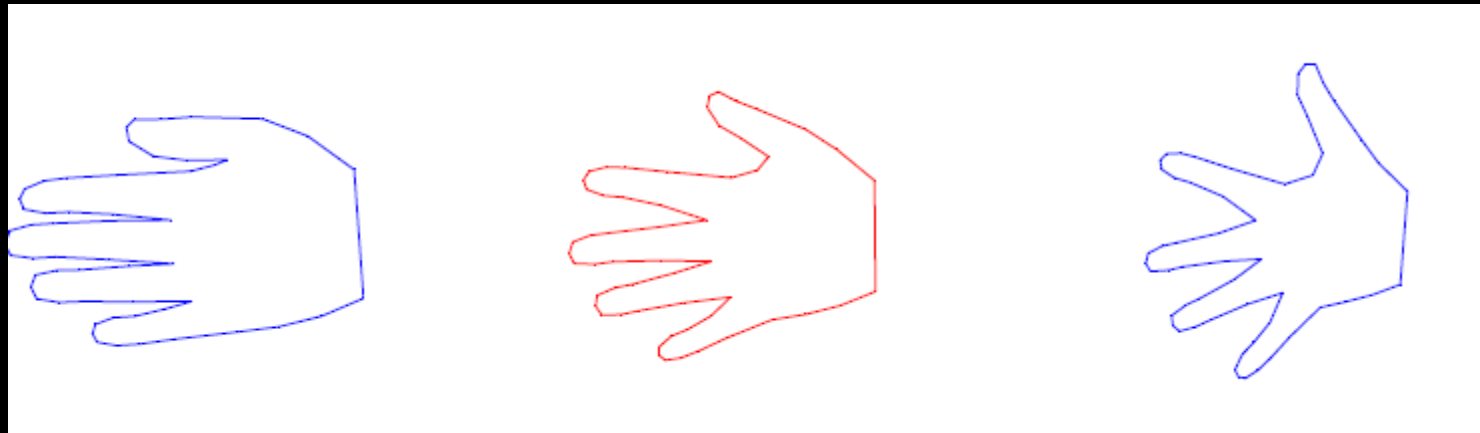


- Instead of doing PCA on 2D points we do it on 112D points
- Examine if our 40 *shapes is lying on a plane* in 112D space
- We find the directions that spans the maximum variation in shape space



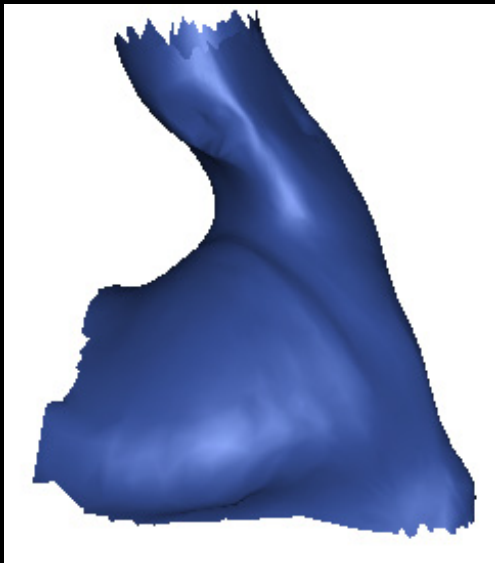
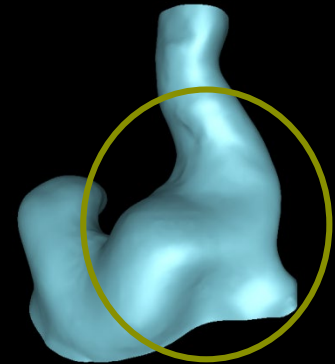
Results of Shape Analysis

- Visualisation of the major variation of the shape over a population

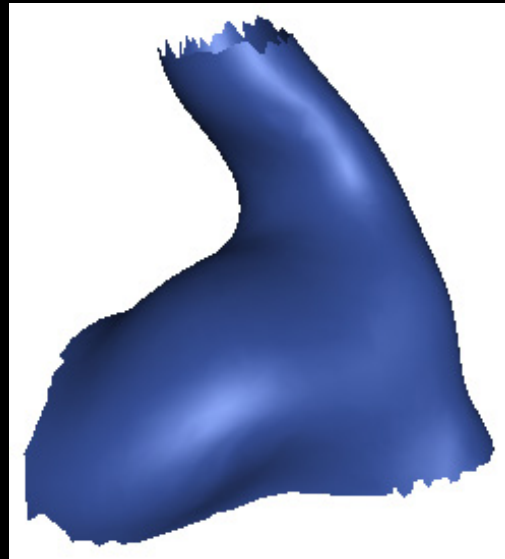


Hearing Aid Design

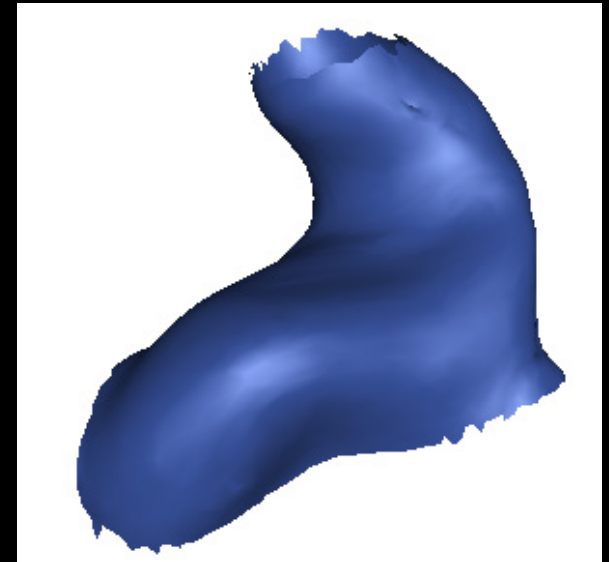
- Main variation of the shape of the ear canal
- Found using principal component analysis
- First mode of variation
- 7 modes explain 95% of the total variation



Average-1. mode



Average



Average+1. mode



Facial Analysis

- Demo of AAM explorer



Next week

- Exam questions
- Rasmus on 3D Shape Analysis and deep learning