Image Analysis

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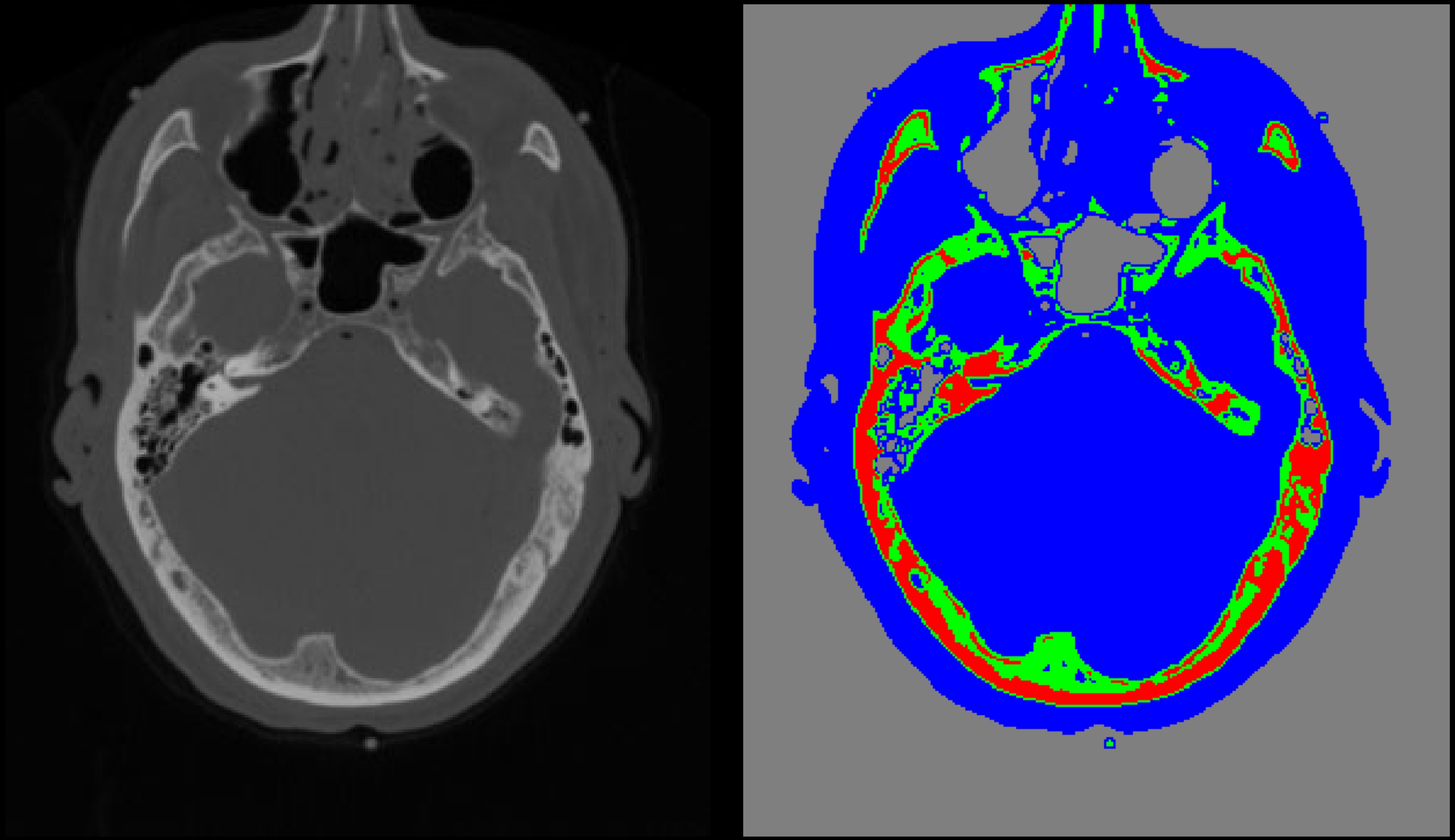
Lecture 7 – Pixel Classification



What can you do after today?

- Describe the concept of pixel classification
- Use Matlab to select pixel training data
- Compute the pixel value ranges in a minimum distance classifier
- Implement and use a minimum distance classifier
- Approximate a pixel value histogram using a Gaussian distribution
- Select pixel value ranges can in a parametric classifier
- Implement and use a parametric classifier
- Decide if a minimum distance or a parametric classifier is appropriate based on the training data
- Explain the concept of Bayesian classification
- Decide if a Bayesian classifier is useful given a set of training data

Classification

- Take a measurement and put it into a class

Measurement

Wheels: 2

HP: 50

Weight: 200

Classifier

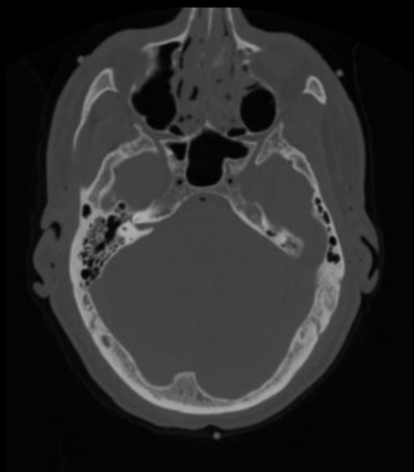
Classes

- Bike
- Truck
- Car
- Motorbike
- Train
- Bus

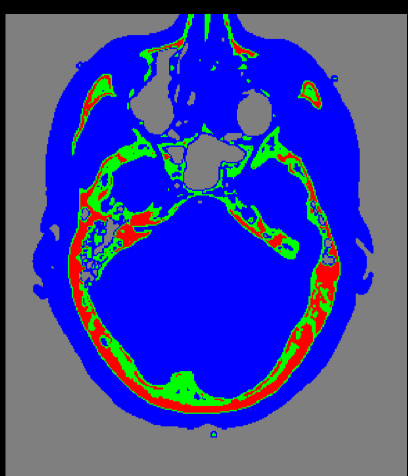
General Classification

- Multi-dimensional measurement
- Pre-defined classes
  - Can also be found automatically – can be very difficult!

Pixel Classification



CT scan of human head

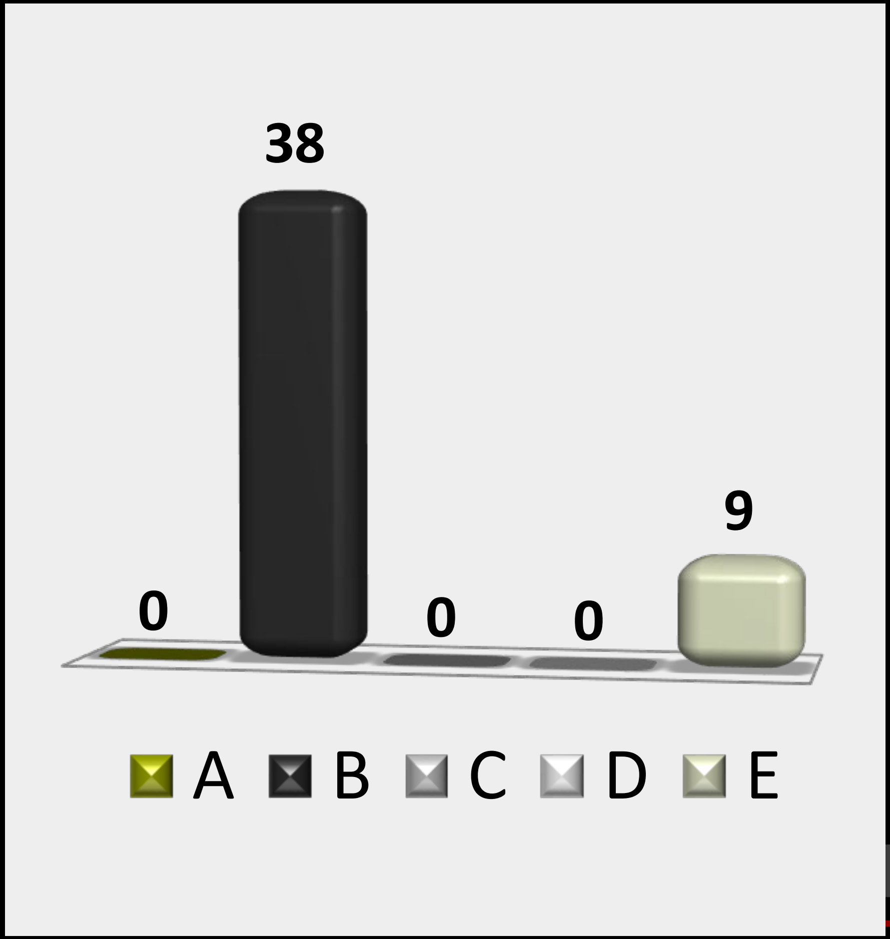


- Classify each pixel
  - Independent of neighbours
- Also called labelling
  - Put a label on each pixel
- We look at the pixel value and assign them a label
- Labels already defined

Quiz: Two class pixel classification?

Background and object

A) Median filter
B) Threshold
C) Brightness
D) Morphological Erosion
E) BLOB analysis



Pixel Classification – formal definition

Pixel value (the measurement) $v \in R$

k classes $C = c_1, \dots, c_k$

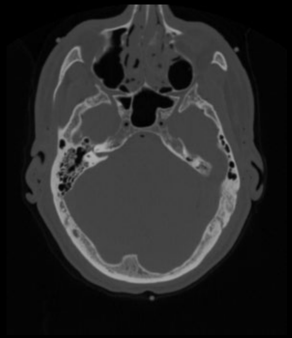
Classification rule $c: R \longrightarrow \{c_1, \dots, c_k\}$

Pixel Classification – example

Pixel value $v \in [0,255]$

Set of 4 classes C={background, soft-tissue, trabeculae, bone}

Classification rule $c: v \longrightarrow \{\text{background}, \text{soft} - \text{tissue}, \text{trabeculae}, \text{bone}\}$



How do we construct a classification rule?

Pixel classification rule

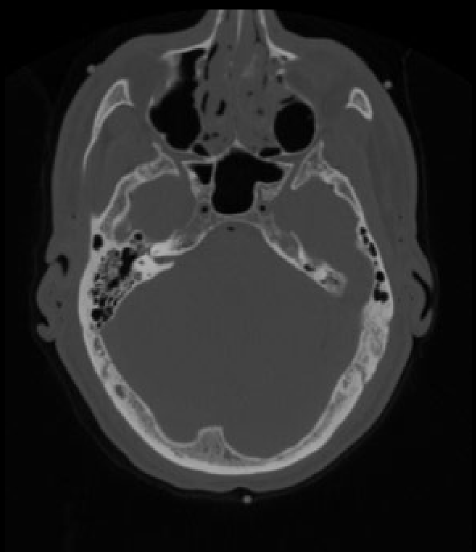
$$c: v \longrightarrow \{\text{background}, \text{soft} - \text{tissue}, \text{trabeculae}, \text{bone}\}$$

background

trabeculae

soft-tissue

bone

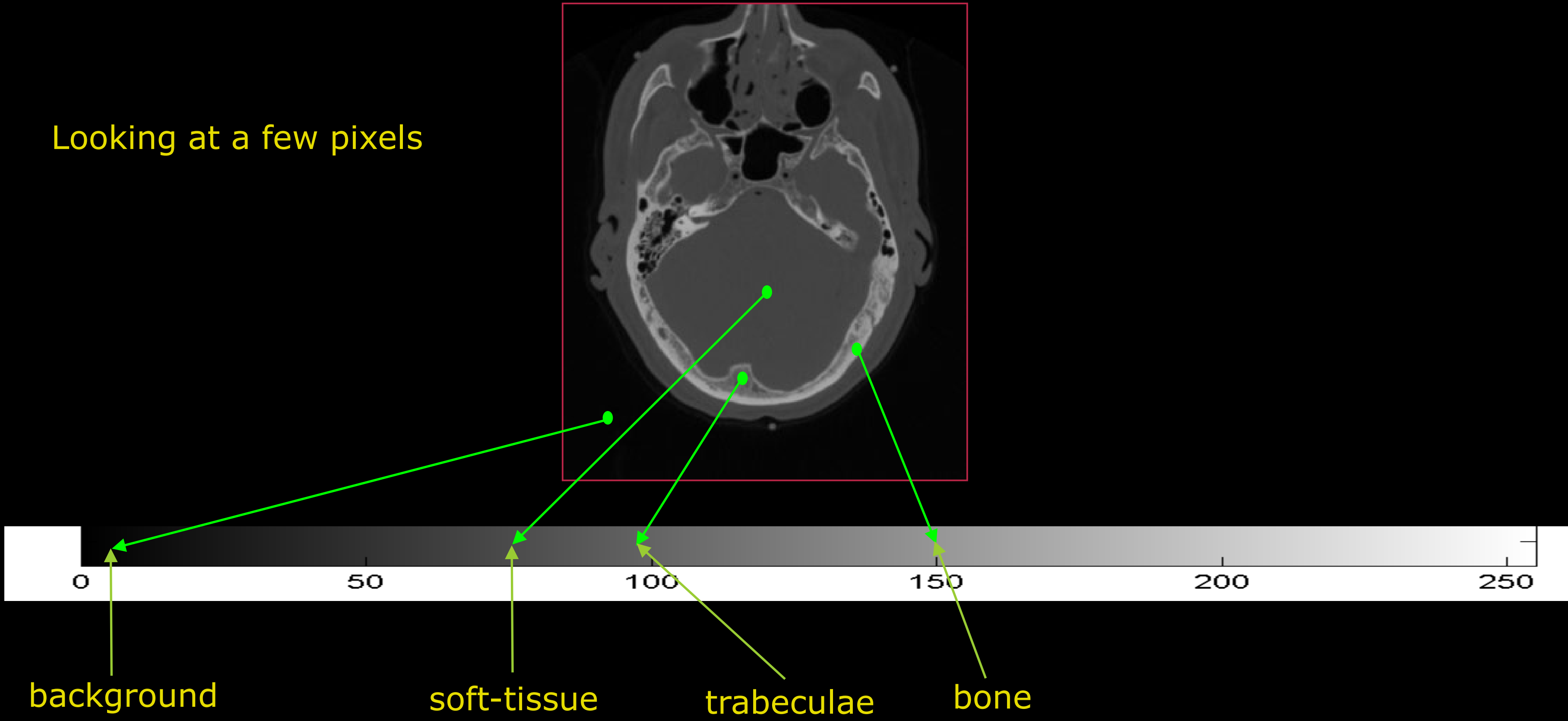


How do we do this?

Pixel classification rule – manual inspection

$$c: v \longrightarrow \{\text{background}, \text{soft} - \text{tissue}, \text{trabeculae}, \text{bone}\}$$

Looking at a few pixels

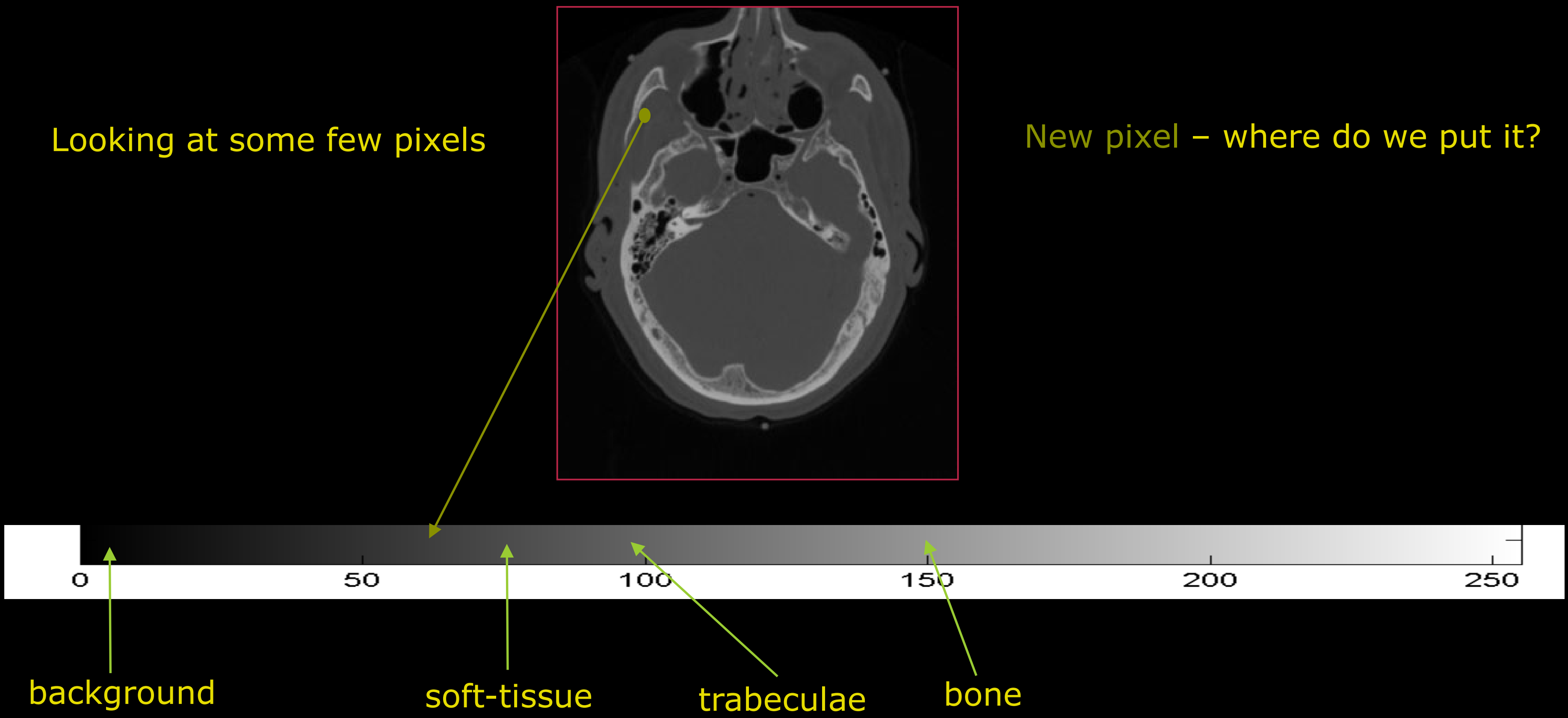


Pixel classification rule – manual inspection

$$c: v \longrightarrow \{\text{background}, \text{soft} - \text{tissue}, \text{trabeculae}, \text{bone}\}$$

Looking at some few pixels

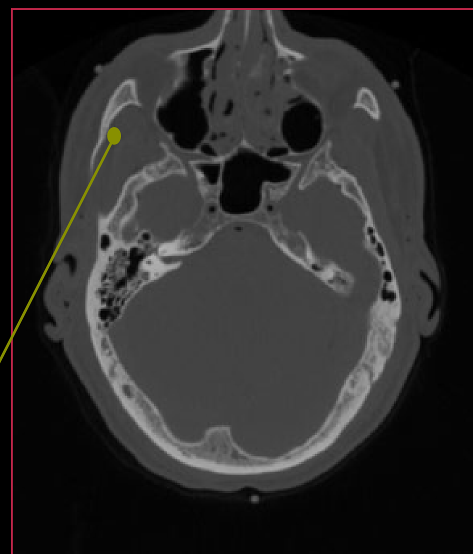
New pixel – where do we put it?



Pixel classification rule – manual inspection

$$c: v \longrightarrow \{\text{background}, \text{soft} - \text{tissue}, \text{trabeculae}, \text{bone}\}$$

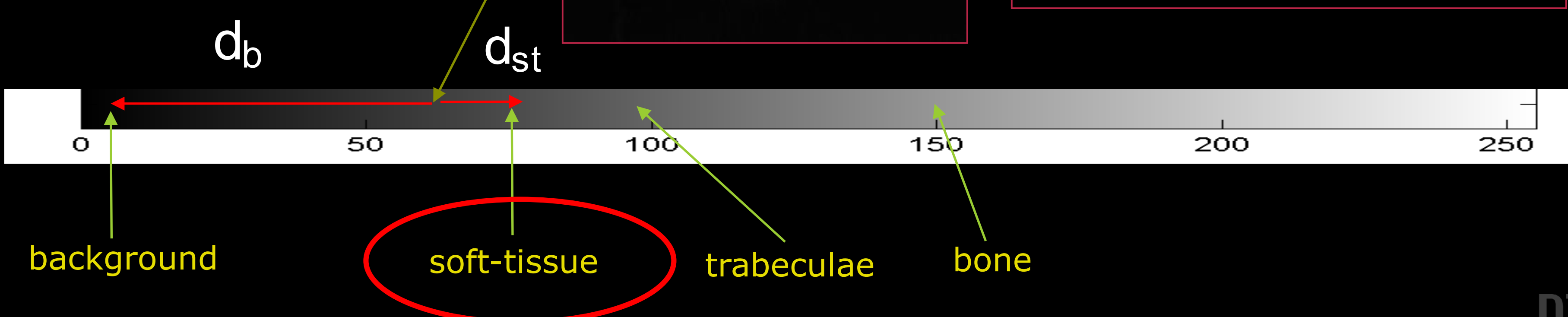
Looking at some few pixels



New pixel – where do we put it?

- Measure the "distance" to the other classes
- Select the closest class

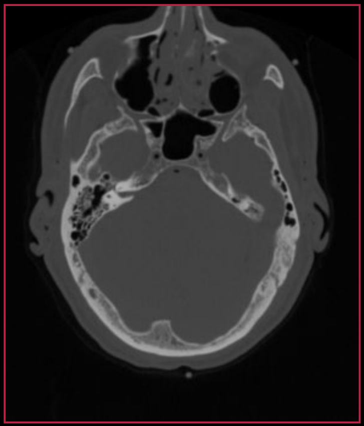
Minimum distance classification

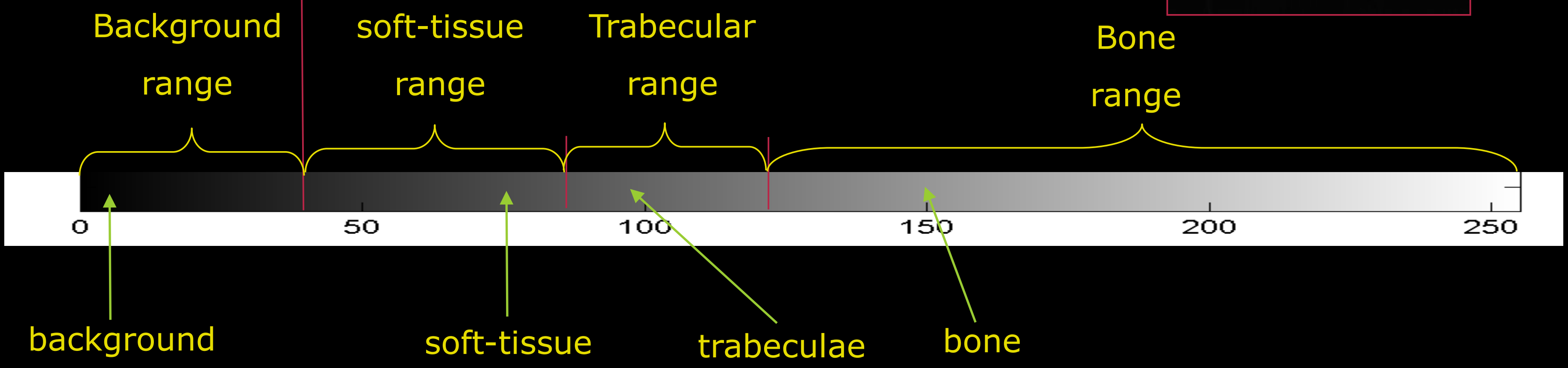


Pixel classification rule
Minimum Distance Classification

The possible pixel values are divided into ranges

Here the distance to "background" is equal to "soft-tissue"

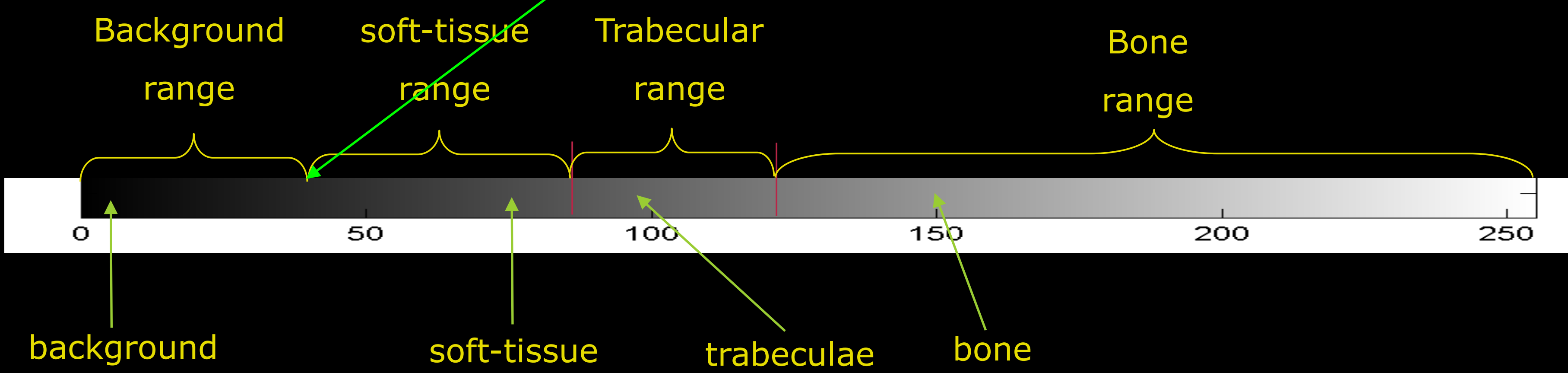




Pixel classification rule

Minimum Distance Classification

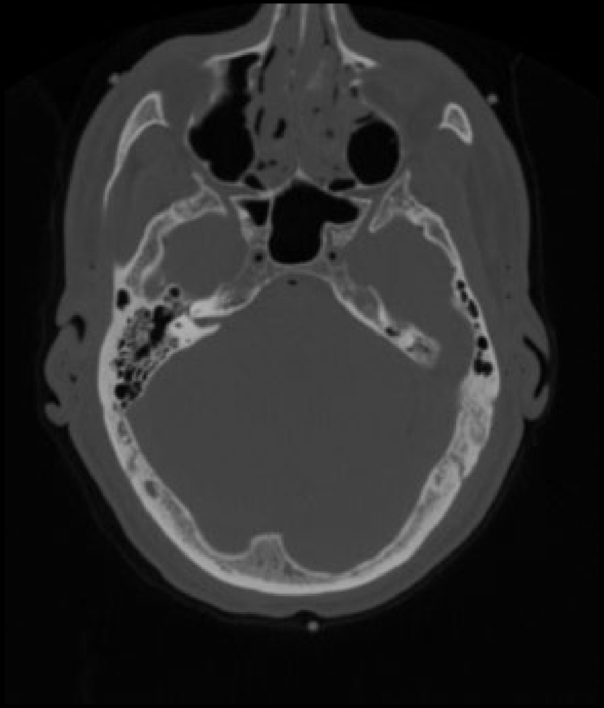
$$c(v) = \begin{cases} \text{background, if } v \le (4+67)/2 \\ \text{soft}-\text{tissue, if } \dfrac{(4+67)}{2} < v \le \dfrac{67+95}{2} \\ \text{trabeculae, if } \dfrac{67+95}{2} < v \le \dfrac{95+150}{2} \\ \text{bone, if } v > \dfrac{95+150}{2} \end{cases}$$



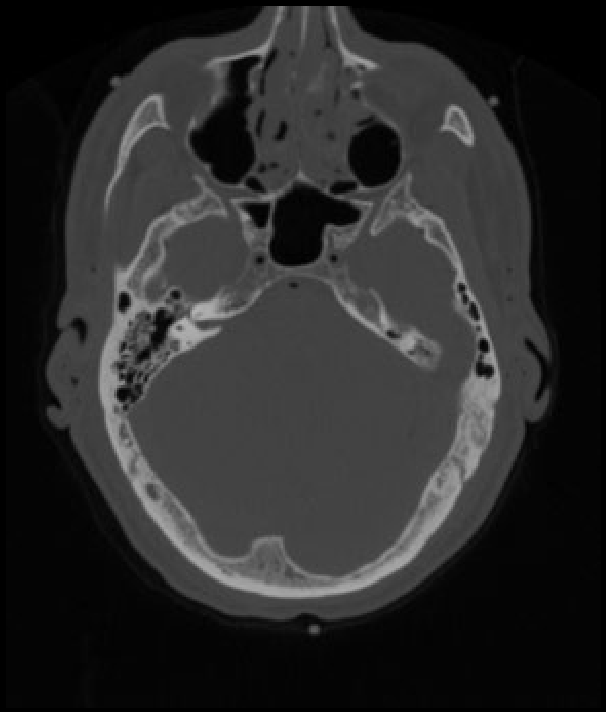
Pixel classification rule

- For all pixel in the image do

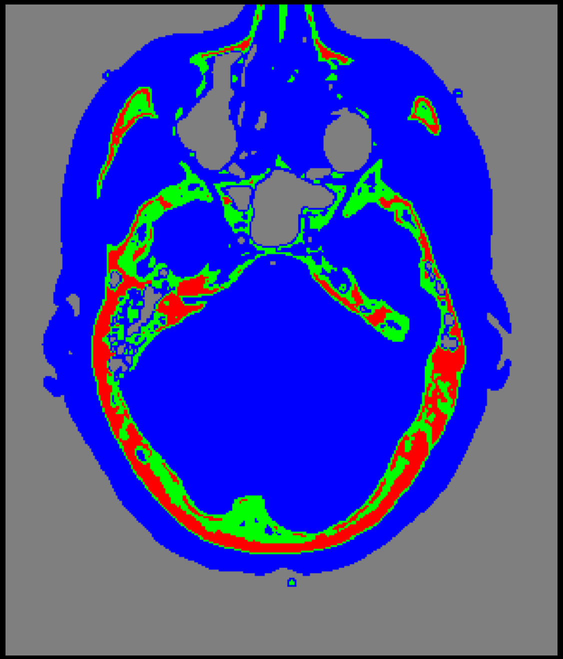
$$c(v) = \begin{cases} \text{background, if } \mathrm{v} \leq (4+67)/2 \\ \text{soft} - \text{tissue, if} \dfrac{(4+67)}{2} < \mathrm{v} \leq \dfrac{67+95}{2} \\ \text{trabeculae, if} \dfrac{67+95}{2} < \mathrm{v} \leq \dfrac{95+150}{2} \\ \text{bone, if } \mathrm{v} > \dfrac{95+150}{2} \end{cases}$$



Pixel Classification example

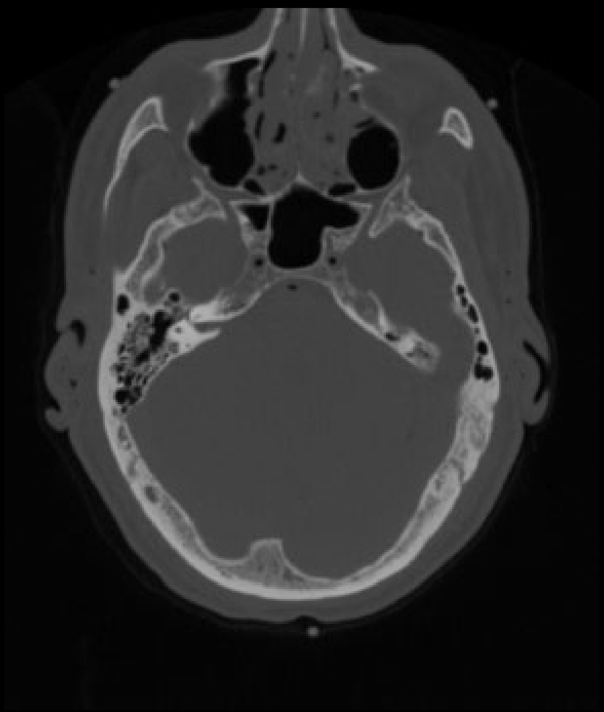


CT scan of human head



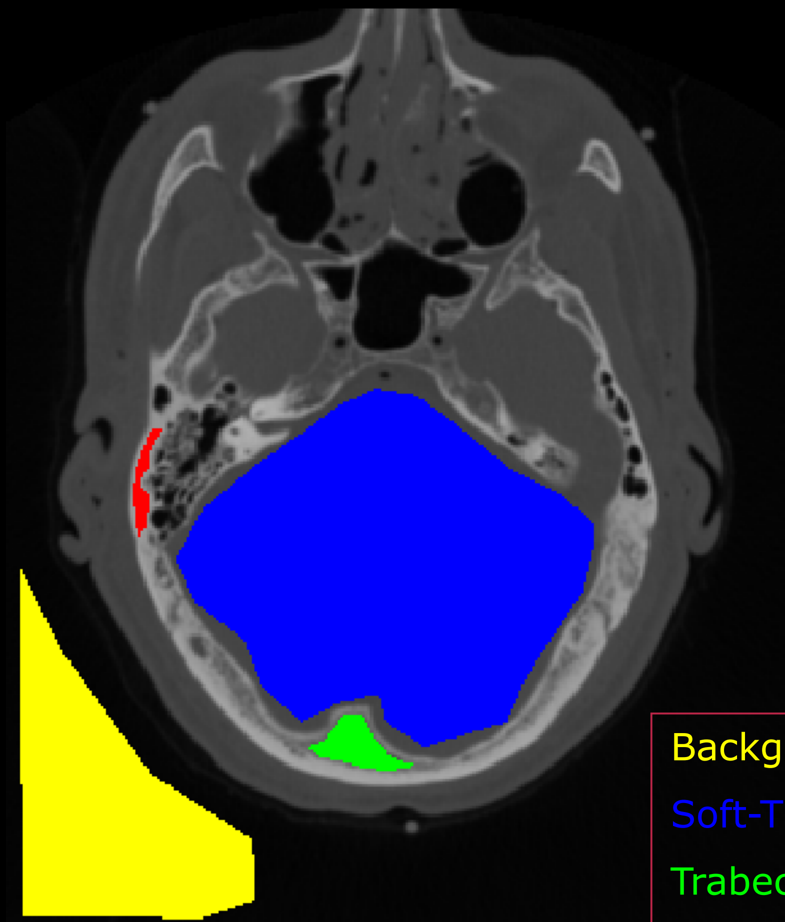
Background
Soft-Tissue
Trabecular Bone
Hard Bone

Better range selection



- Guessing range values is not a good idea
- Better to use "training data"
- Start by selecting representative regions from an image
- *Annotation*
  - To mark points, regions, lines or other significant structures

Classifier training - annotation



Background

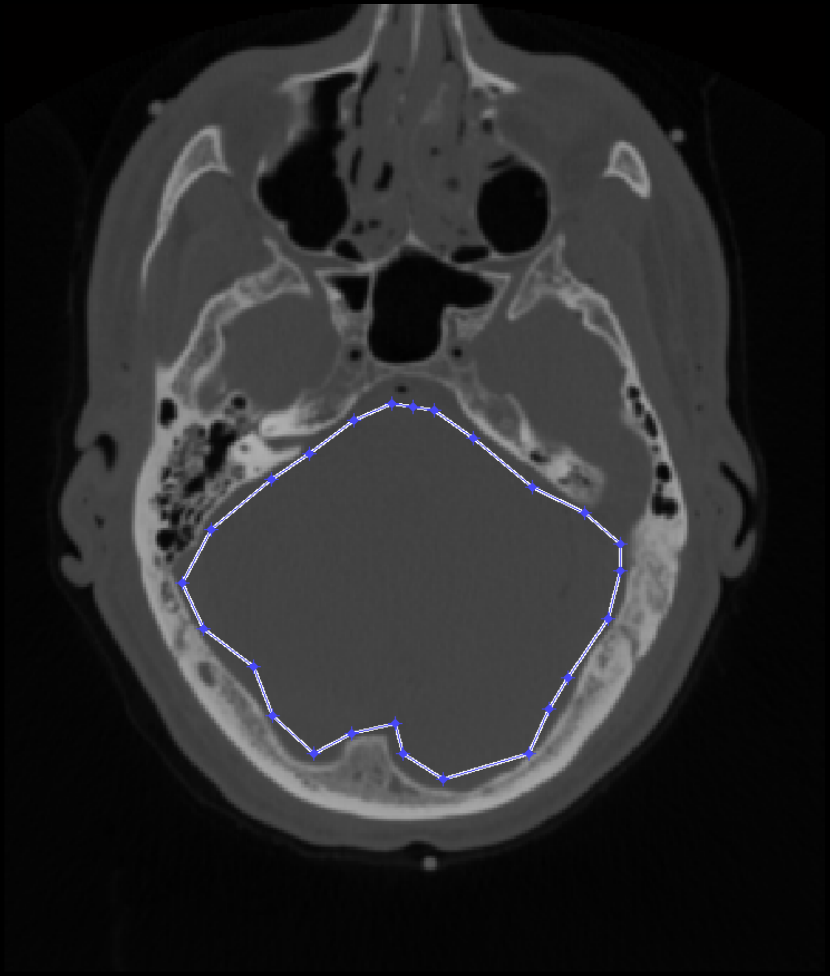
Soft-Tissue

Trabecular Bone

Hard Bone

- An “expert” is asked how many different tissue types that are possible
- Then the expert is asked to mark representative regions of the selected tissue types

Classifier training – region selection

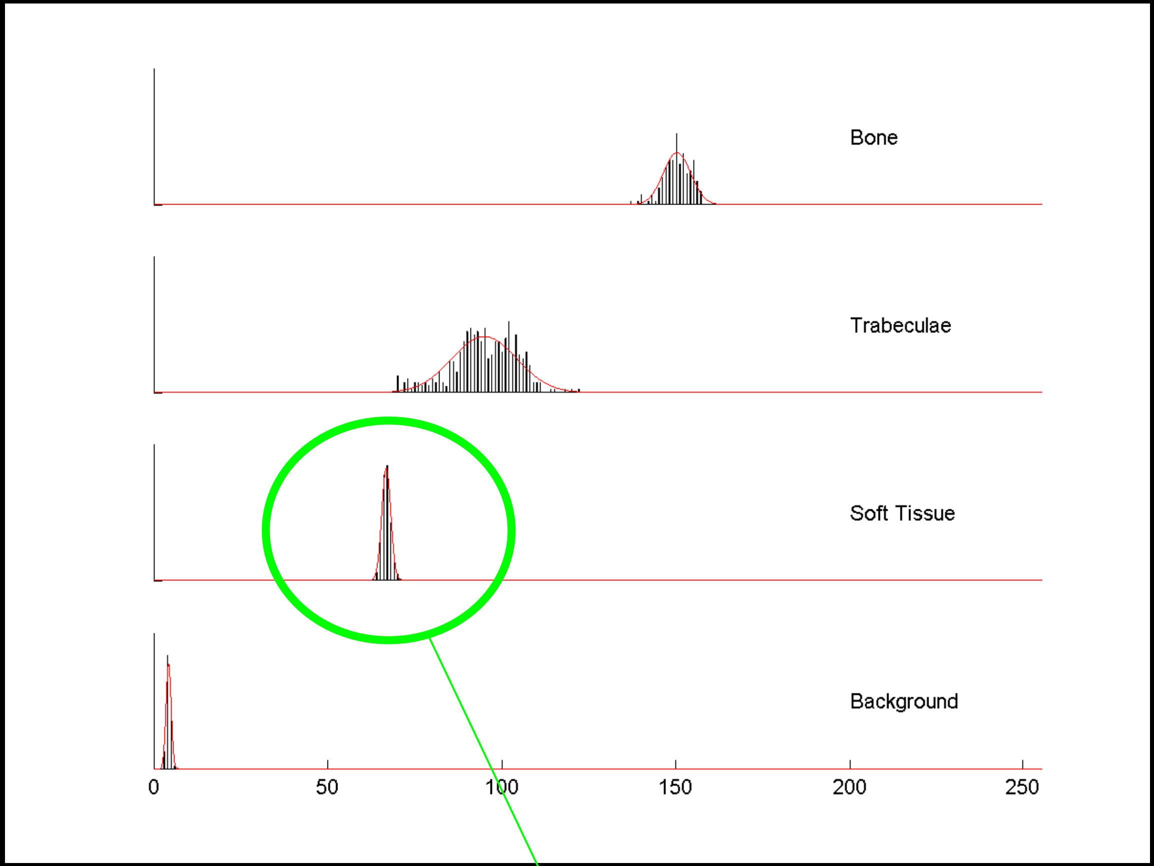


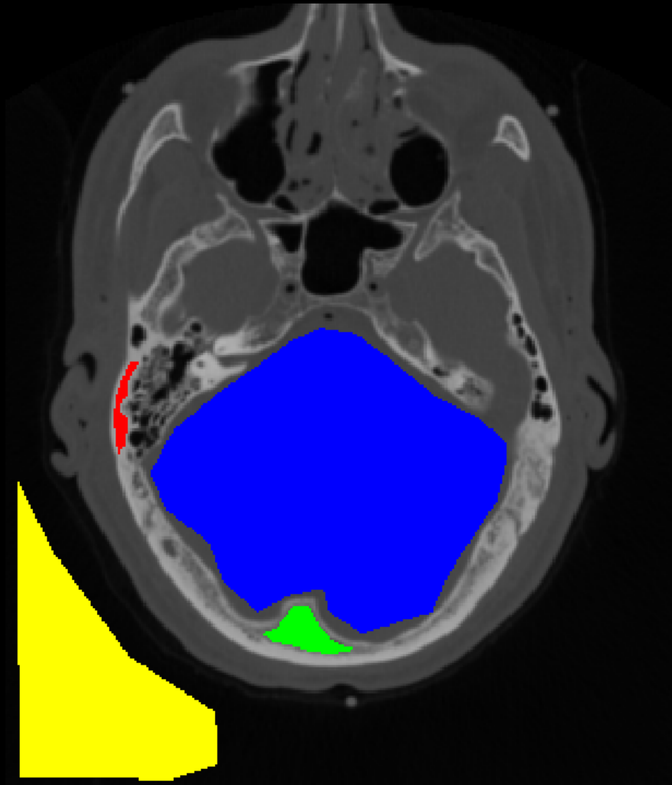
- Many tools exist
- Matlab tool roipoly
  - Select closed regions using a piecewise polygon

Training is only done once!

Optimally, the training can be used on many pictures that contains the same tissue types

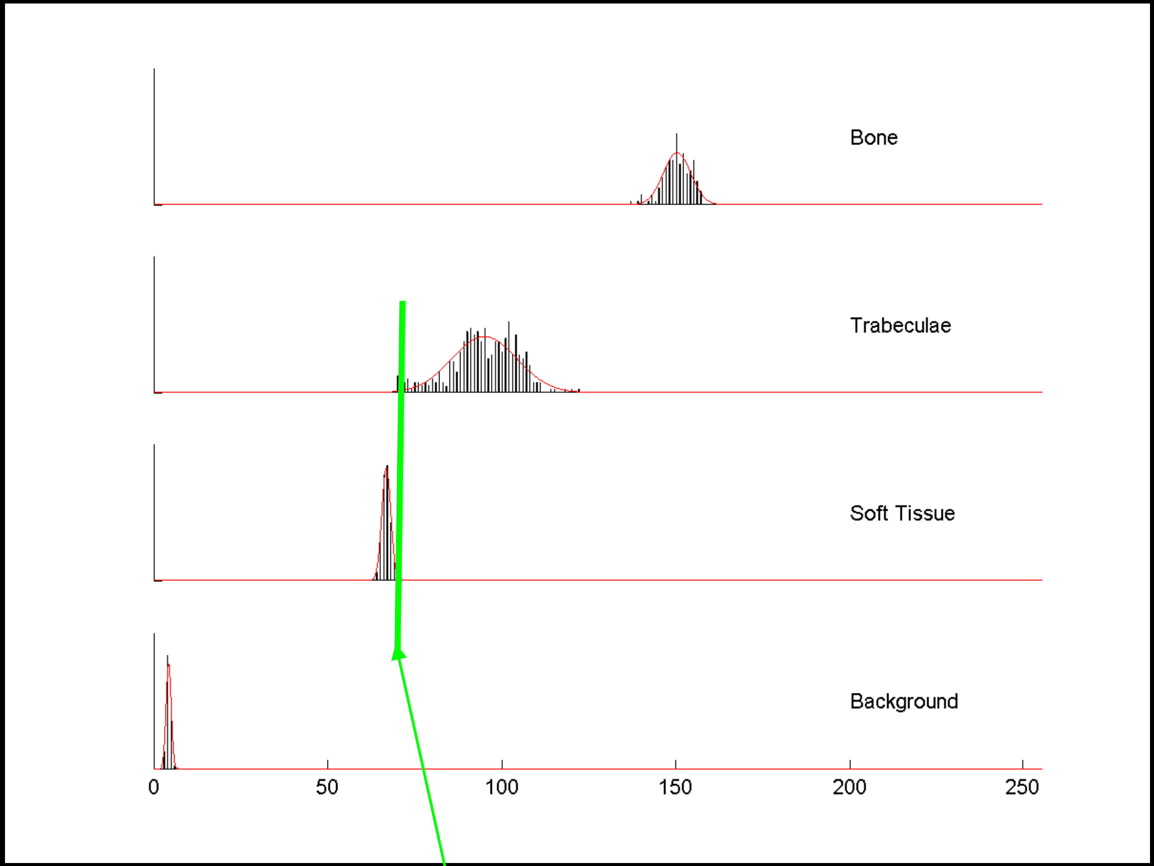
Initial analysis - histograms

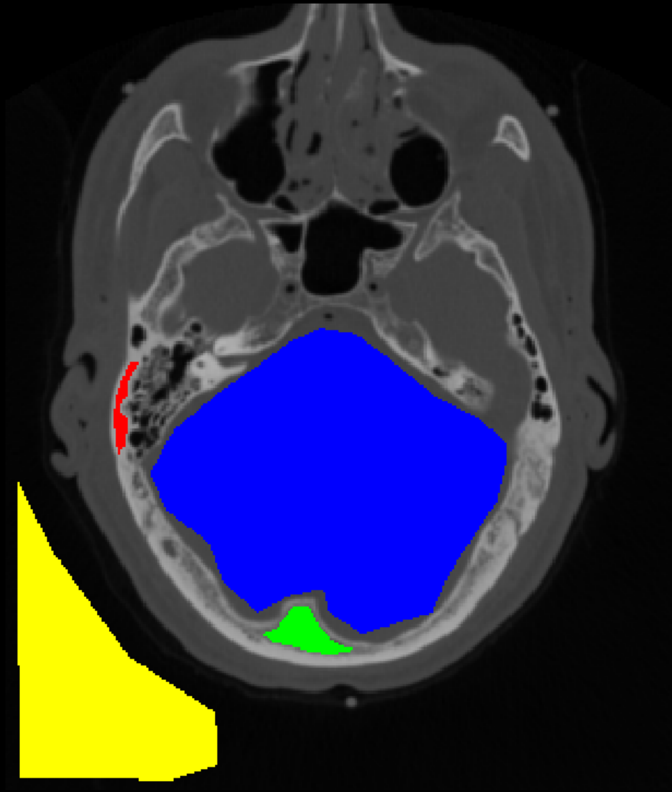




Gaussian

Initial analysis - histograms

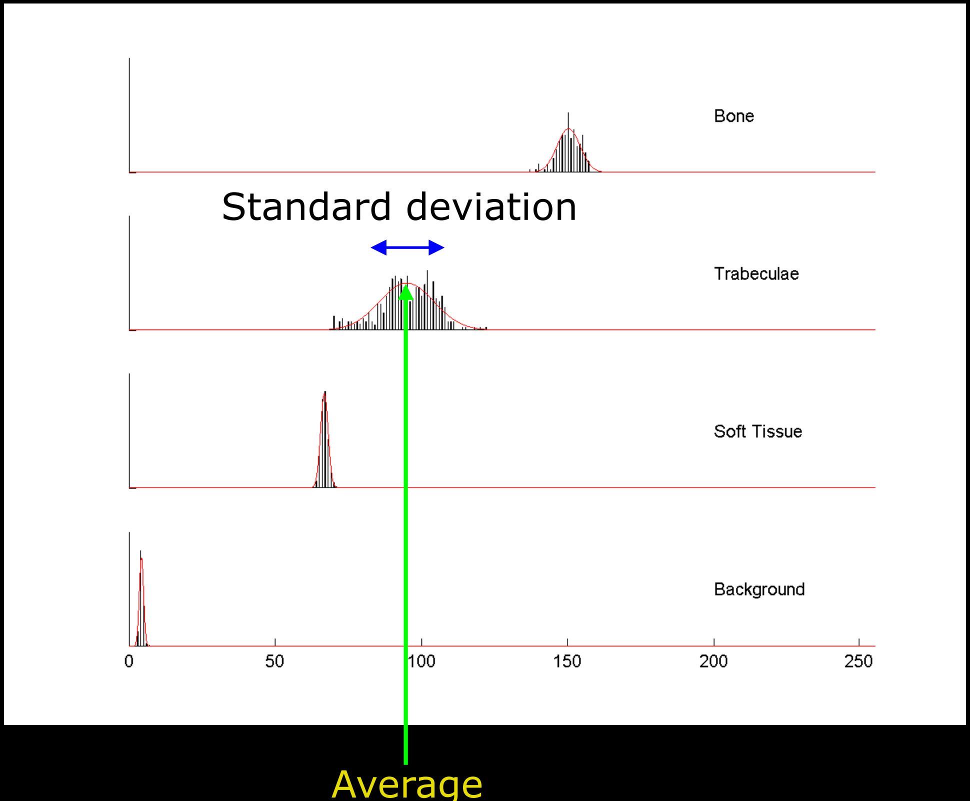


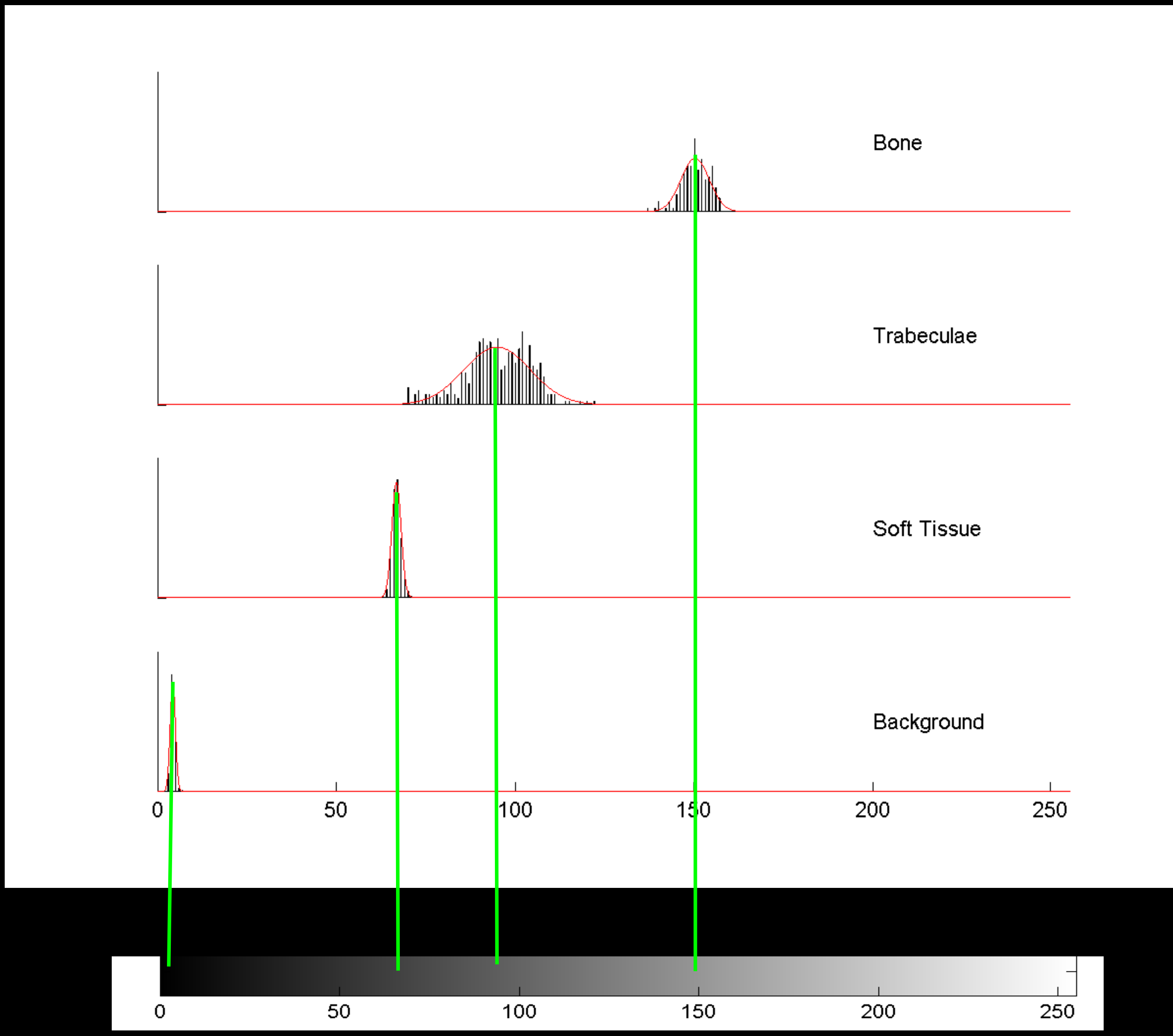


Class separation

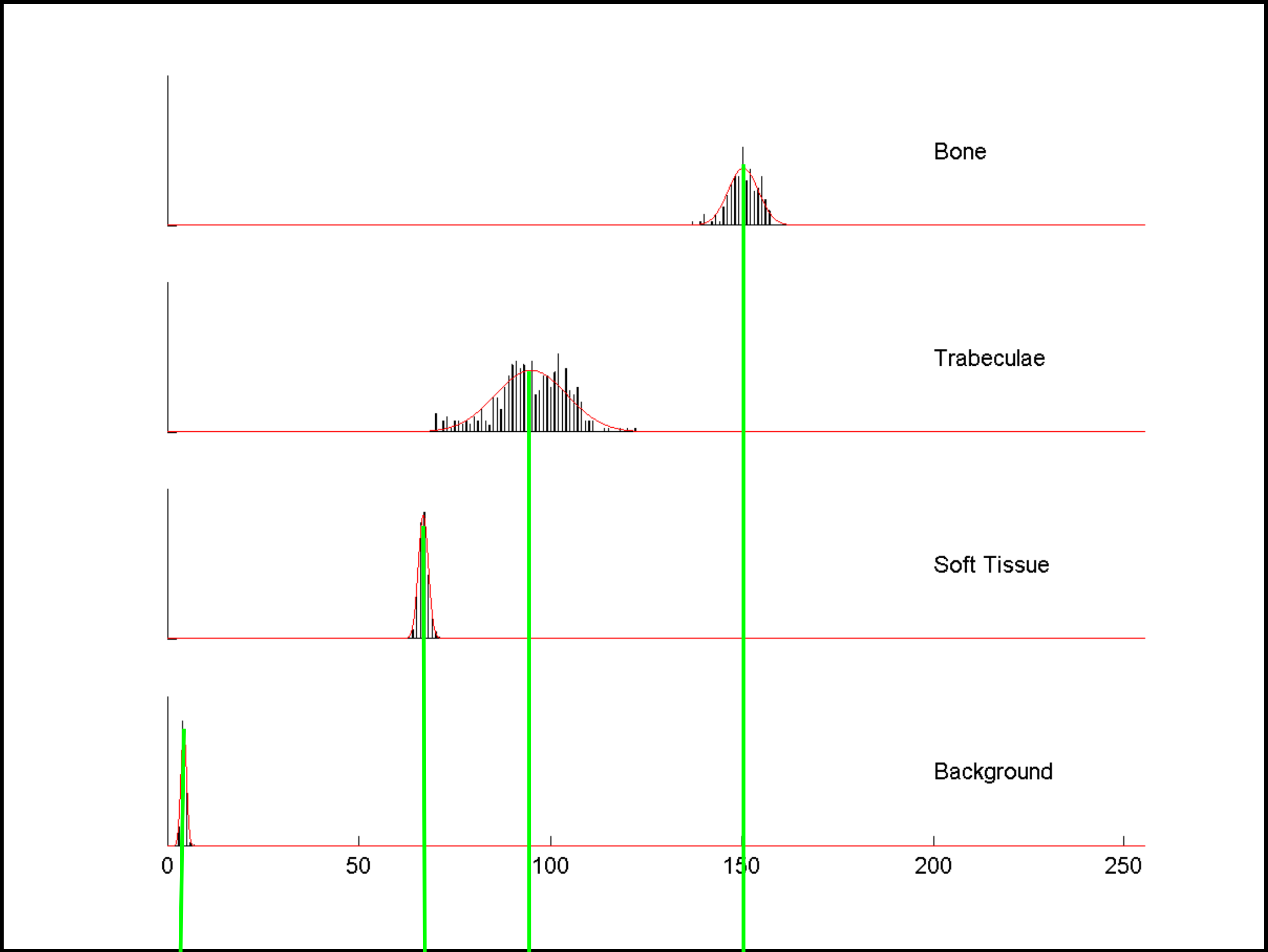
Simple pixel statistics

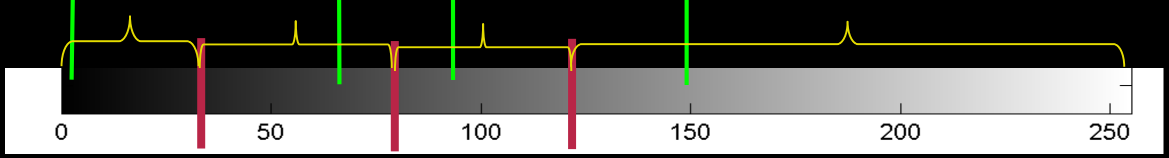
- Calculate the average (mean) and the standard deviation of each class




Bone
Trabeculae
Soft Tissue
Background
0
50
100
150
200
250
0
50
100
150
200
250

Minimum distance classification





Any objections?

The pixel value ranges are not always in good correspondence with the histograms?

Minimum distance classification

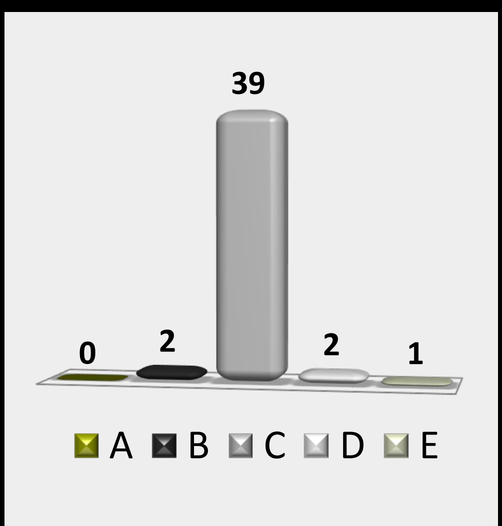
A) Background

B) Soft tissue

C) Fat

D) Bone

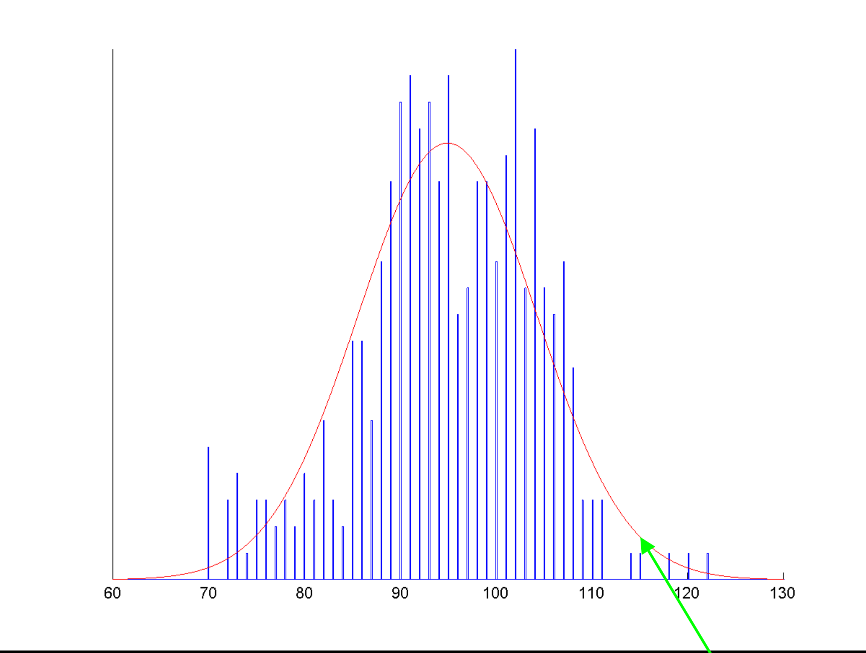
E) None of the above



To make a pixel classification an expert has selected representative regions in the image. They contain background (green), soft tissue (blue), fat (yellow), and bone (purple). The goal is to classify the pixel marked with a light blue circle. Using a minimum distance classifier it is classified as?

5	6	5	81	180	182	222	220
8	9	4	108	181	175	219	221
7	8	132	130	148	182	174	223
58	231	134	133	61	173	178	175
44	250	181	130	117	101	176	174
5	6	7	204	246	94	86	175
156	158	6	7	7	252	173	230
157	161	7	6	6	10	35	227

Parametric classification



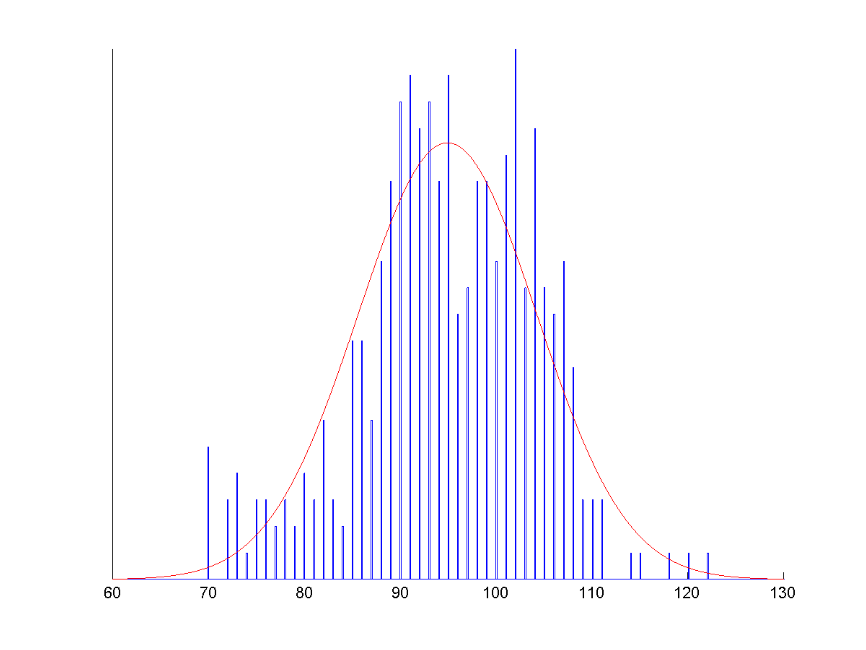
Trabecular bone

- Describe the histogram using a few parameters
- Gaussian/Normal distribution
  - Average μ
  - Standard deviation σ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Only two values needed

Parametric classification



Trabecular bone

Training pixel values $v_1, v_2, \ldots, v_n$,

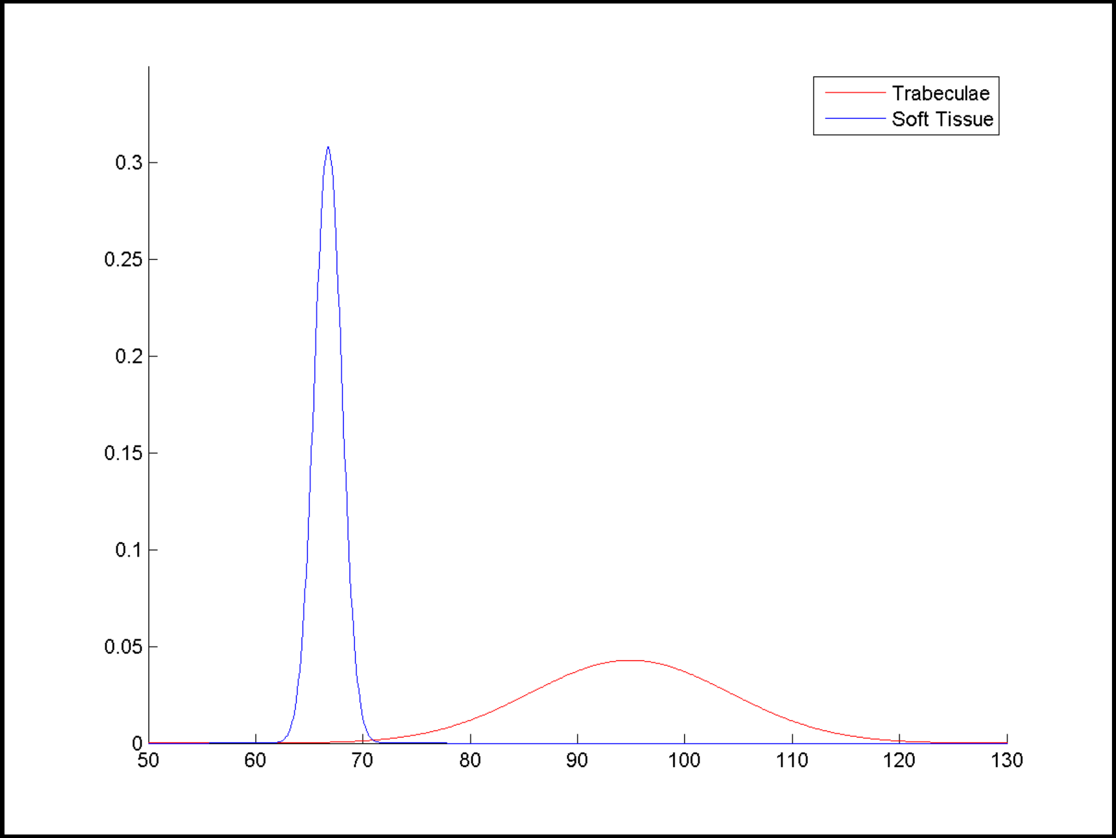
Estimated average $$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} v_i$$

Estimated standard deviation $$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(v_i - \hat{\mu})$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Parametric classification

- Fit a Gaussian to the training pixels for all classes



What do we see here?

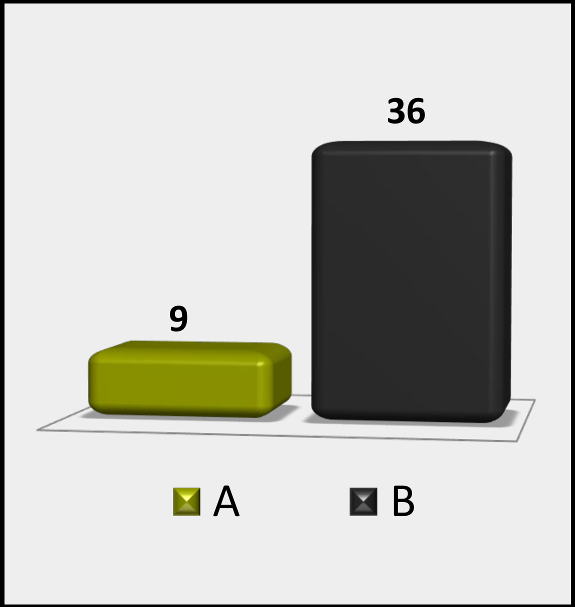
What is the difference between the two classes?

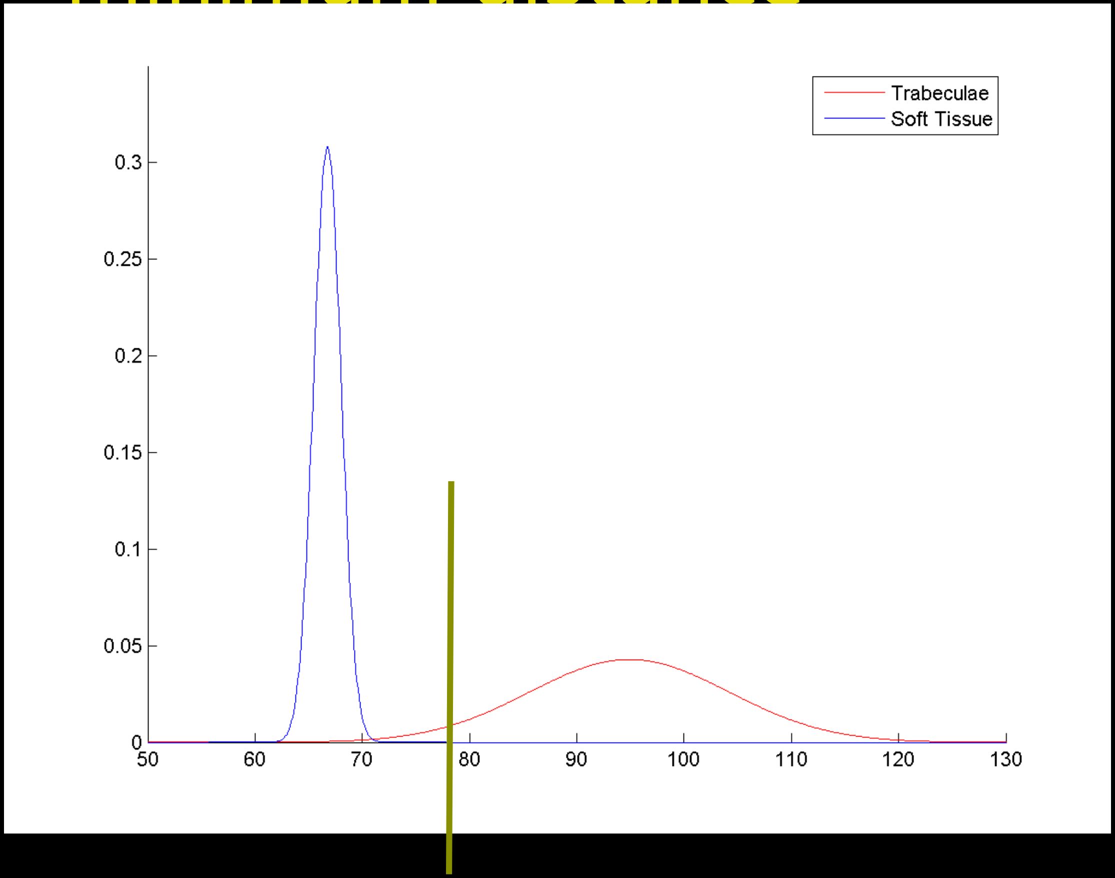
Trabeculae has much higher variation in the pixel values

Two tissue types – minimum distance v = 78

A) Trabeculae

B) Soft-tissue

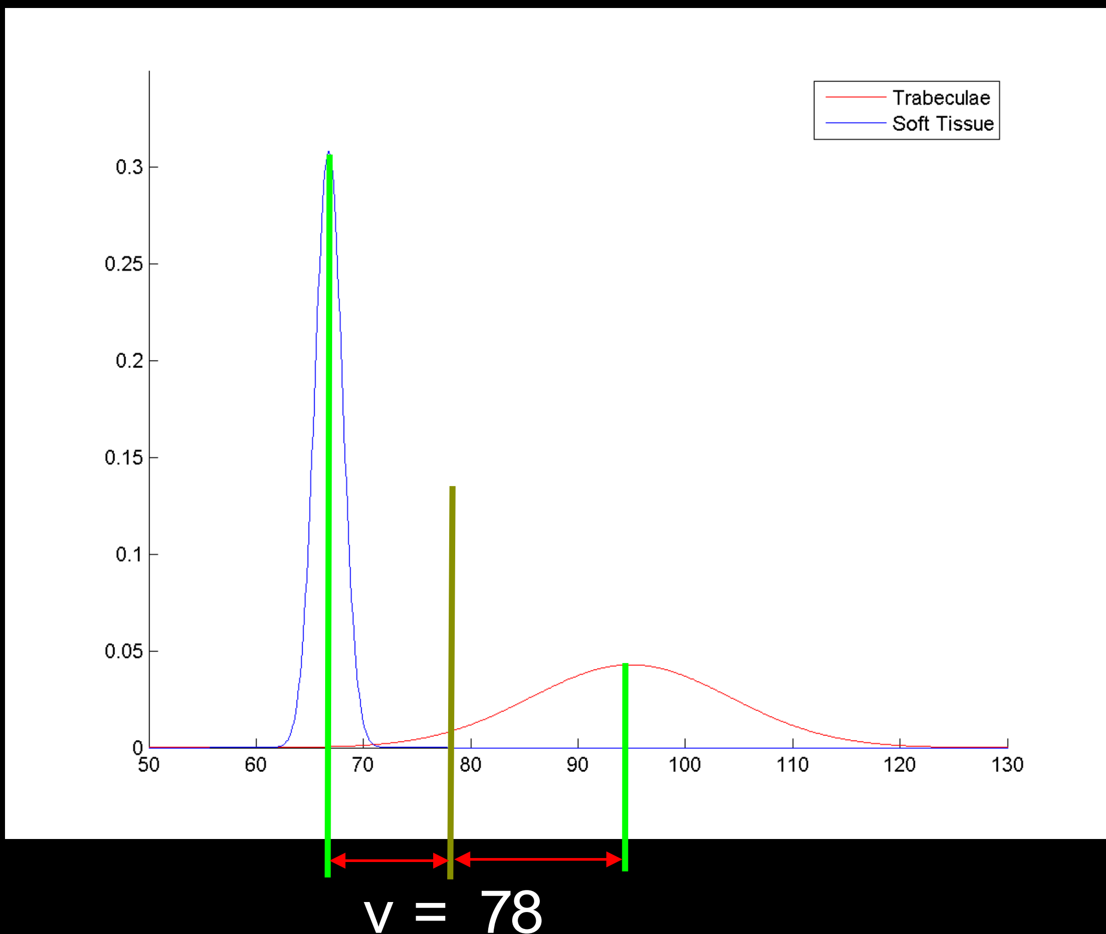




v = 78

Minimum distance classifier

Parametric classification

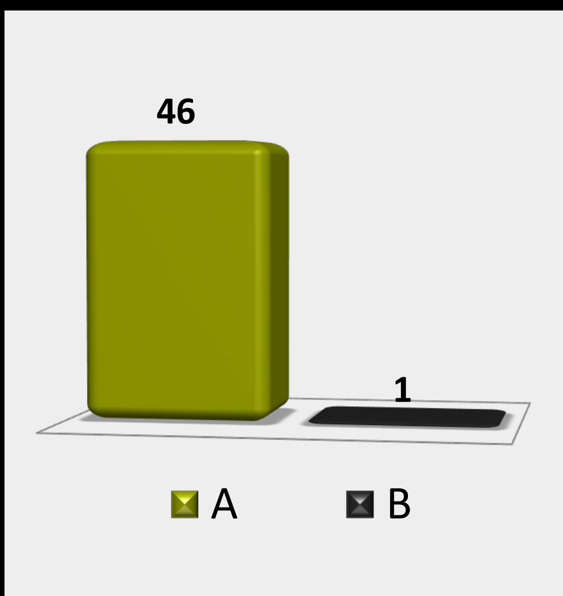


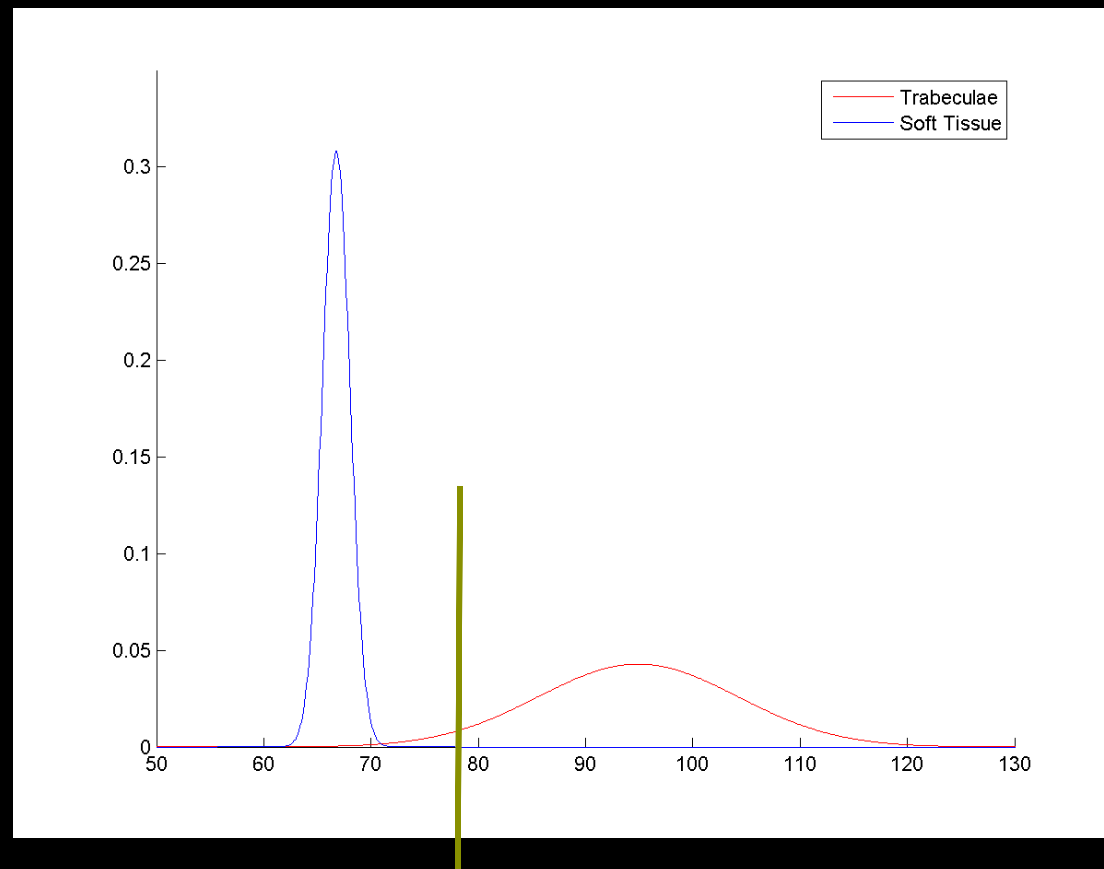
- New pixel with value 78
  - Is it soft-tissue or trabecular bone?
- Minimum distance classifier?
  - Soft-tissue
- Is that fair?
  - Soft-tissue Gaussian says "Extremely low probability that this pixel is soft-tissue"

Two tissue types – parametric classification

A) Trabeculae

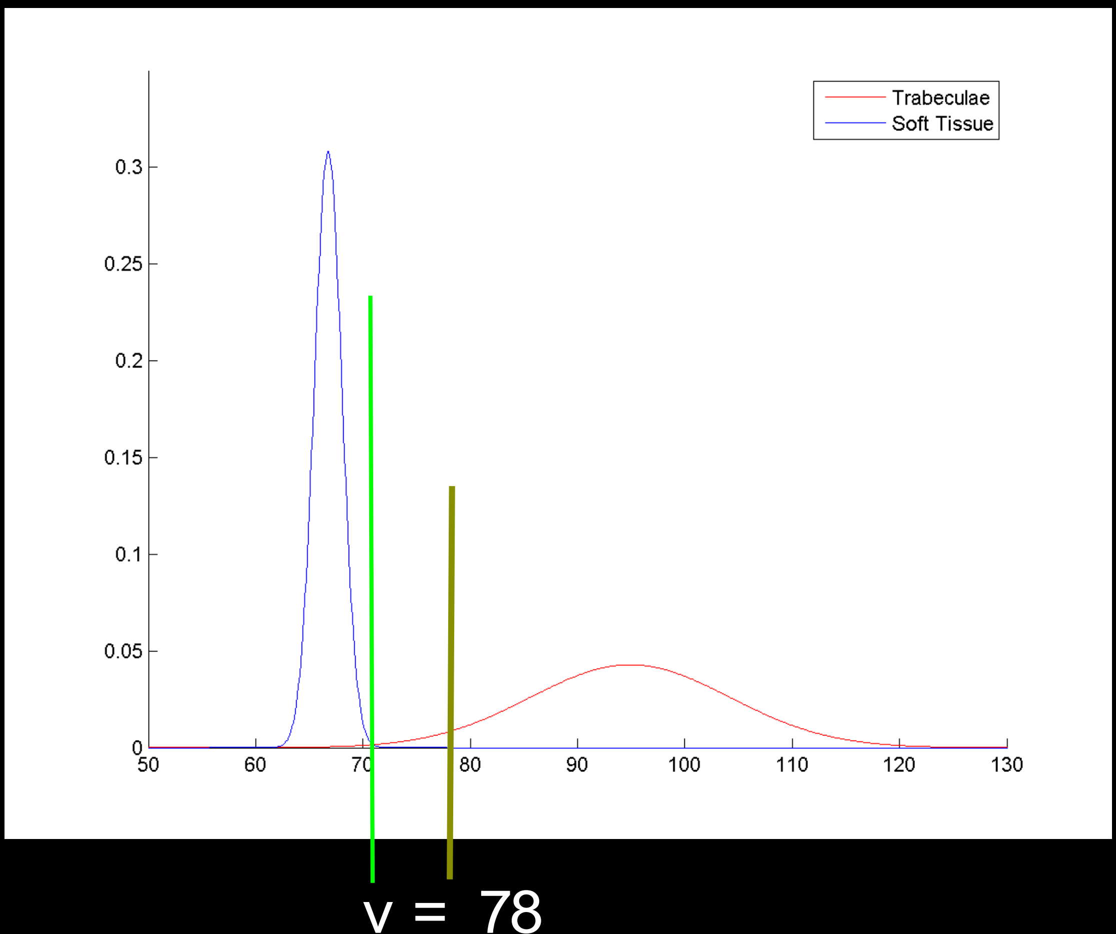
B) Soft-tissue





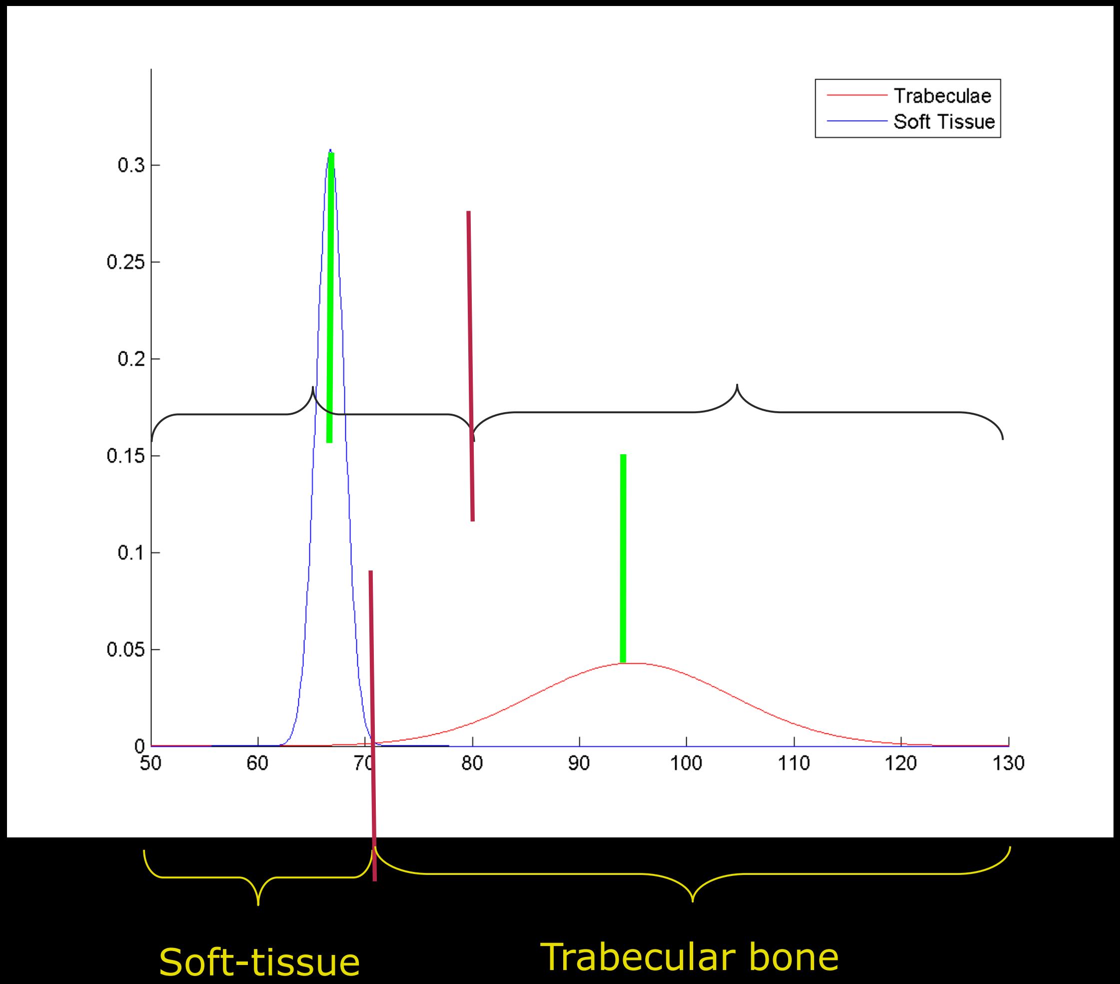
v = 78

Parametric classification – repeat the question



- New pixel with value 78
  - Is it soft-tissue or trabecular bone?
  - Most probably trabecular bone
- Where should we set the limit?
  - Where the two Gaussians cross!

Parametric classification – ranges

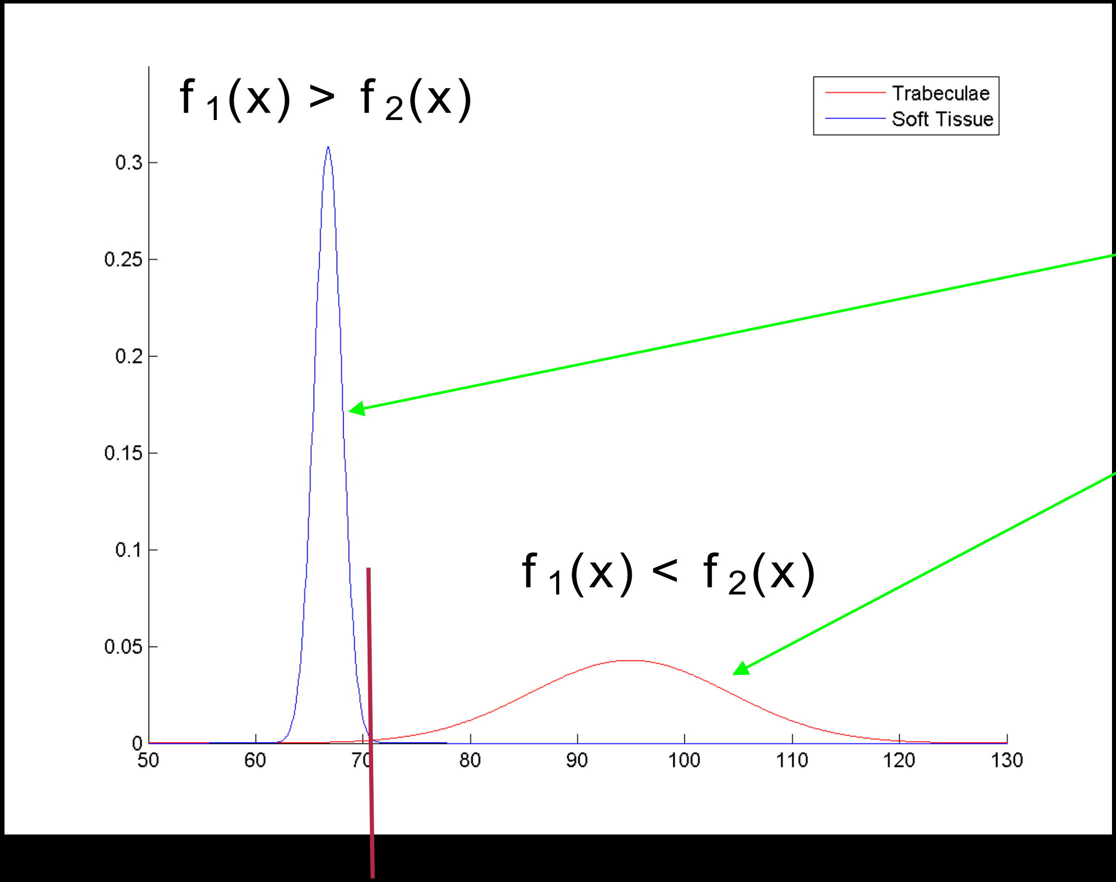


- The pixel value ranges depends on
  - The average
  - The standard deviation
- Compared to the minimum distance classifier
  - Only the average

Parametric classification – how to

- Select training pixels for each class
- Fit Gaussians to each class
- Use Gaussians to determine pixel value ranges
- Little bit difficult with the Gaussians

Parametric classifier - ranges



- We want to compute where they cross

$$f_1(x) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right)$$

$$f_2(x) = \frac{1}{\sigma_2\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right)$$

Create a lookup table:

- Run through all 256 possible pixel values
- Check which Gaussian is the highest
- Store the [value, class] in the table

Alternatively – analytic solution

The two Gaussians

$$\frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{(v-\mu_1)^2}{2\sigma_1^2}\right)=\frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{(v-\mu_2)^2}{2\sigma_2^2}\right)$$

Intercept at

$$v=\frac{{\sigma_1}^2\mu_2-{\sigma_2}^2\mu_1\pm\sqrt{-{\sigma_1}^2{\sigma_2}^2\left(2\,\mu_2\,\mu_1-{\mu_2}^2-2\,{\sigma_2}^2\ln\left(\frac{\sigma_2}{\sigma_1}\right)-{\mu_1}^2+2\,{\sigma_1}^2\ln\left(\frac{\sigma_2}{\sigma_1}\right)\right)}}{-{\sigma_2}^2+{\sigma_1}^2}$$

Class ranges

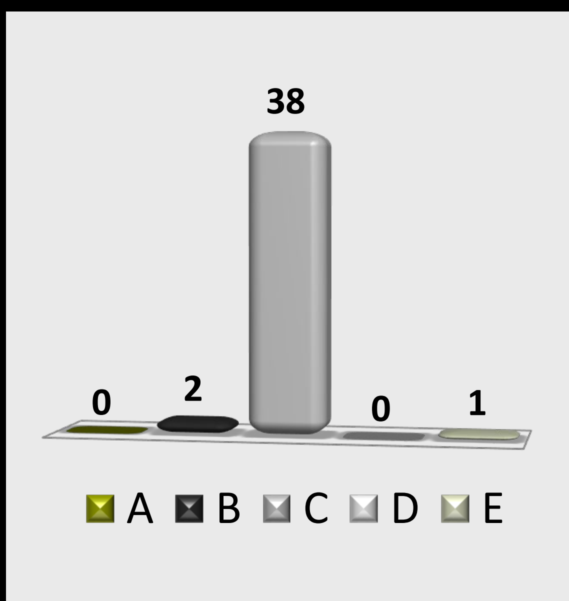
A) [0,45],]45, 75],]75,255]

B) [40,60],]60,100],]100,140]

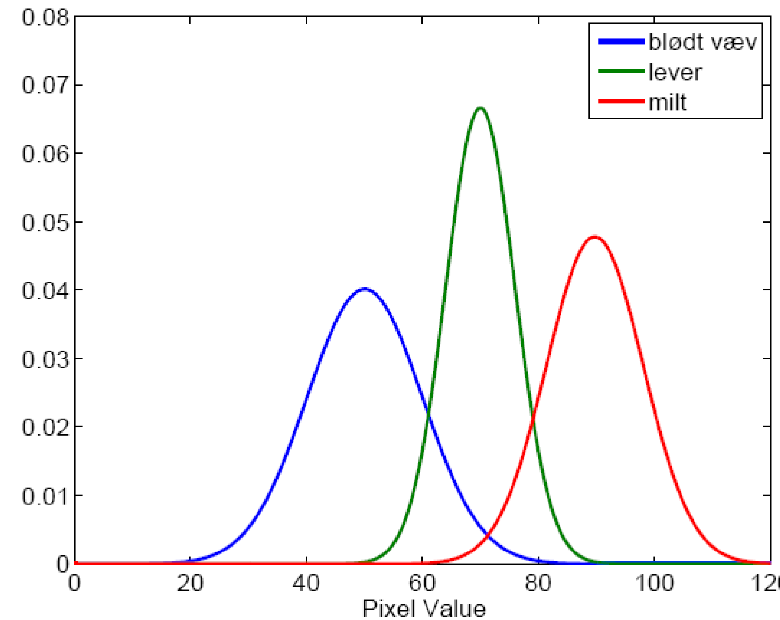
C) [0, 60],]60,80],]80,255]

D) [0,60],]60,100],]100,255]

E) [0,75],[75,100],]100,255]



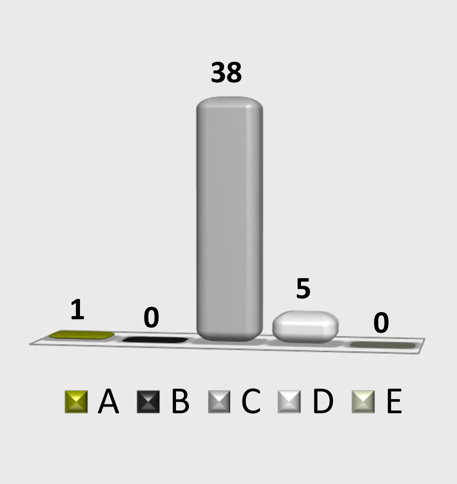
An expert have chosen representative regions in an image that contains soft tissue, liver and spleen. The image pixel minimum and maximum values are 0 and 255. To make a parametric classification, the histograms are parameterized using Gaussian distributions as seen in the image. What are the class ranges?



Parametric classification

In order to perform pixel classification an expert has chosen representitave regions in an image. The regions represent background, soft tissue, pancreas, spleen and bone. The pixel values ranges from 0 to 255. The chosen values can be seen in the table below. A parametric classification is done where the training data is parameterized using Gaussian distributions. A new pixel with value 67 will belong to which class?

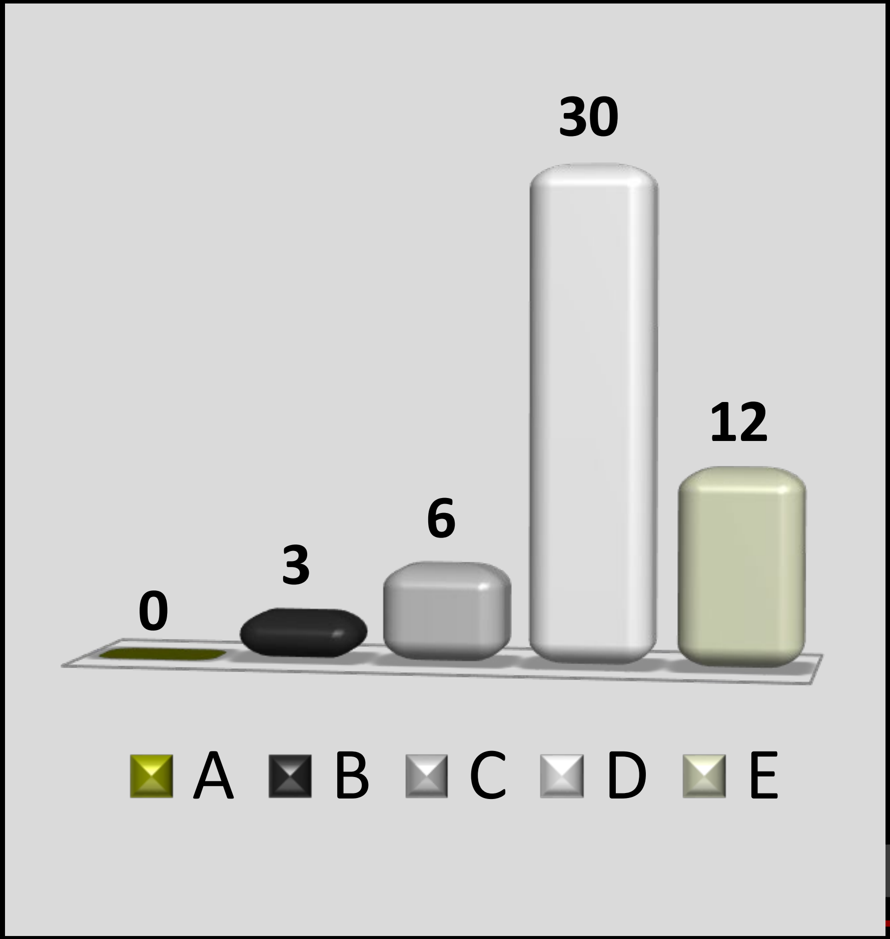
A) Background
B) Soft tissue
C) Pancreas
D) Spleen
E) Bone



Tissue	Pixel values
Background	5, 7, 6
Soft tissue	39, 40, 38
Pancreas	60, 65, 70
Spleen	68, 70, 69
Bone	204, 210, 205

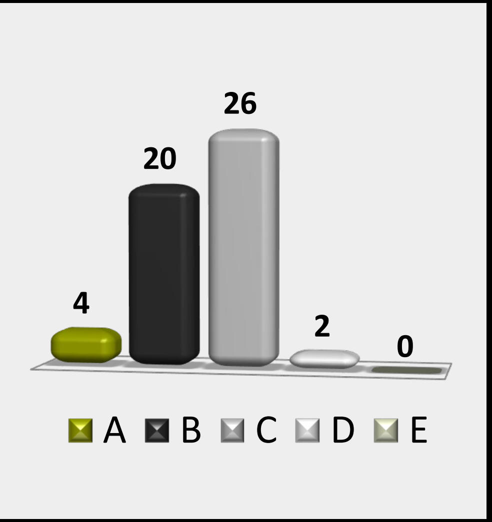
The course until now?

A) I did not learn anything new
B) I did not learn much
C) It is ok
D) I have learnt a good amount
E) I have learnt a lot



Teaching – the speed of the lecture

A) Come oooooon! I am so bored
B) I can easily follow and knit my sweater
C) The speed is fine
D) I need to concentrate a lot to follow
E) Hey! Wait! You are too fast

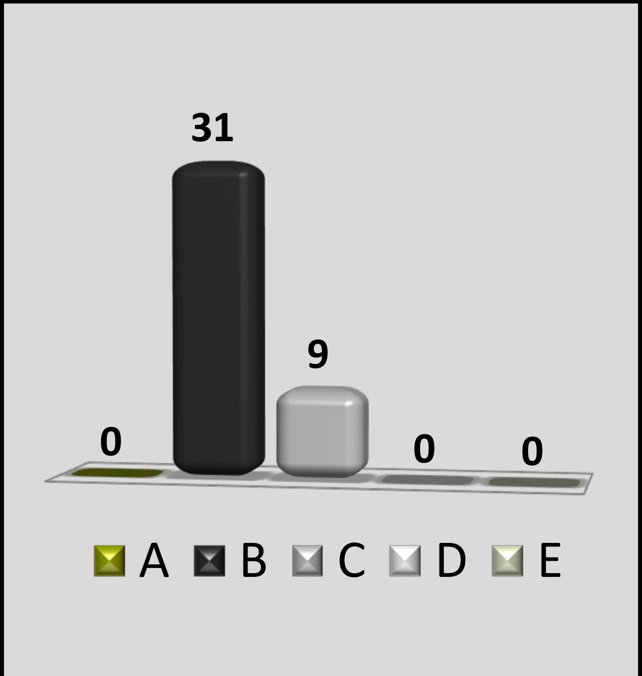


Pixel classification

In order to make a *pixel classification* in images of eyes, an expert has annotated areas in an image containing background, skin, eyebrow, iris and pupil. The original image contains pixel values between 0 and 255. The annotated pixel values are shown in Tabel 1. A *minimum distance classification* is performed on the image. What is the area of the iris in the image in figure 9?

tissue	pixel values
background	176, 178, 183
skin	81, 76, 72
iris	67, 68, 70
pupil	15, 25, 18
eyebrow	25, 42, 32

A) 10
B) 8
C) 5
D) 7
E) 11



181	181	176	80	81	82
180	178	80	74	75	76
177	80	77	74	66	65
80	78	76	68	65	16
80	78	68	67	19	17
79	79	70	69	18	18

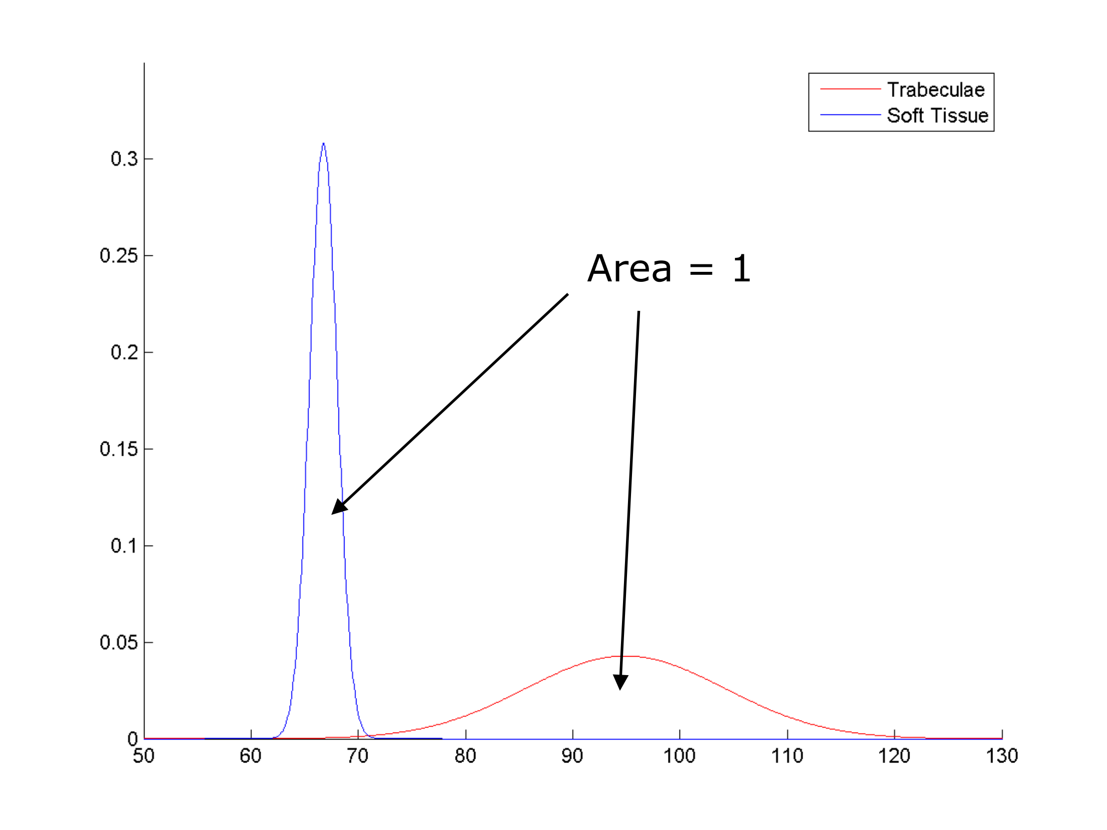
Thomas Bayes

Wikipedia

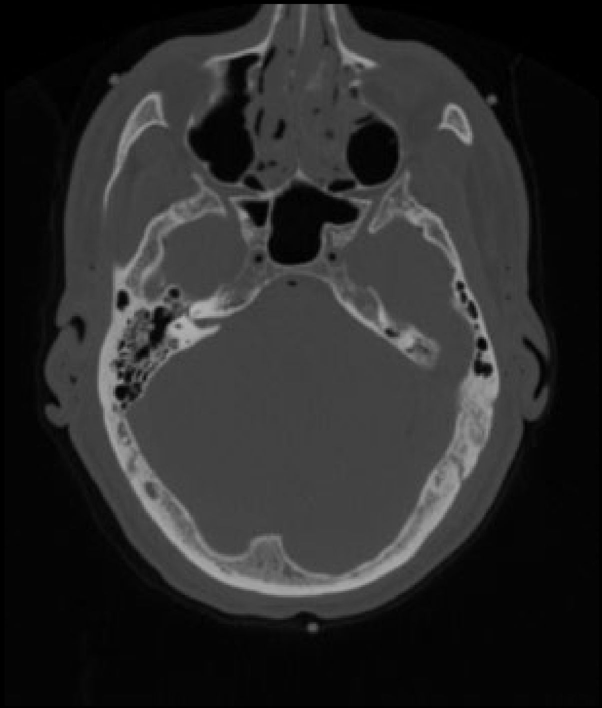
- 1702-1761
- English mathematician and Presbyterian minister
- Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

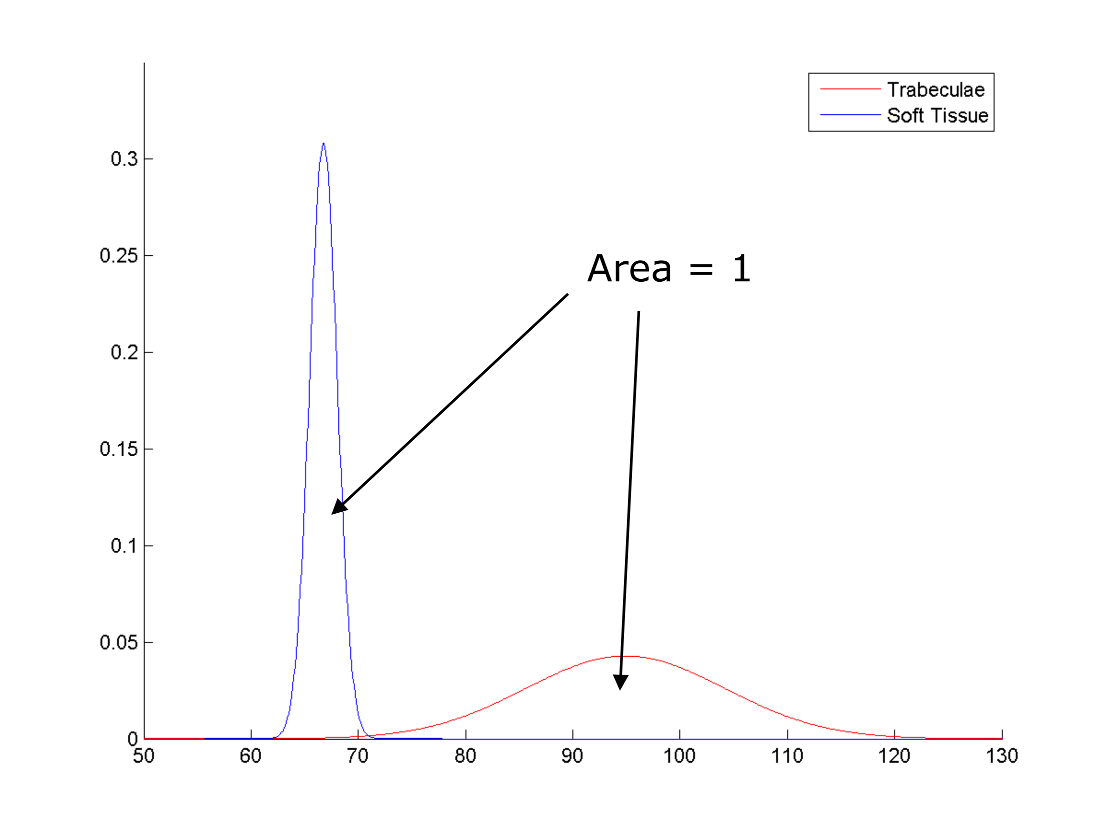
Bayesian Classification



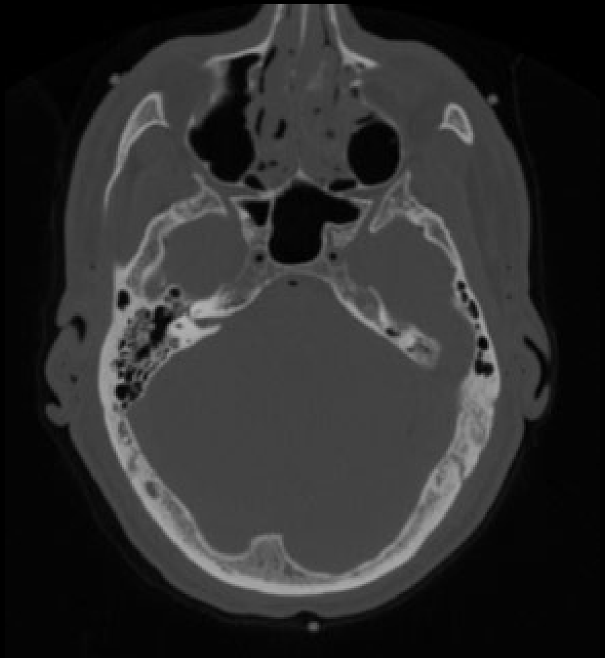
Pure parametric classifier assumes equal amount of different tissue types



Bayesian Classification



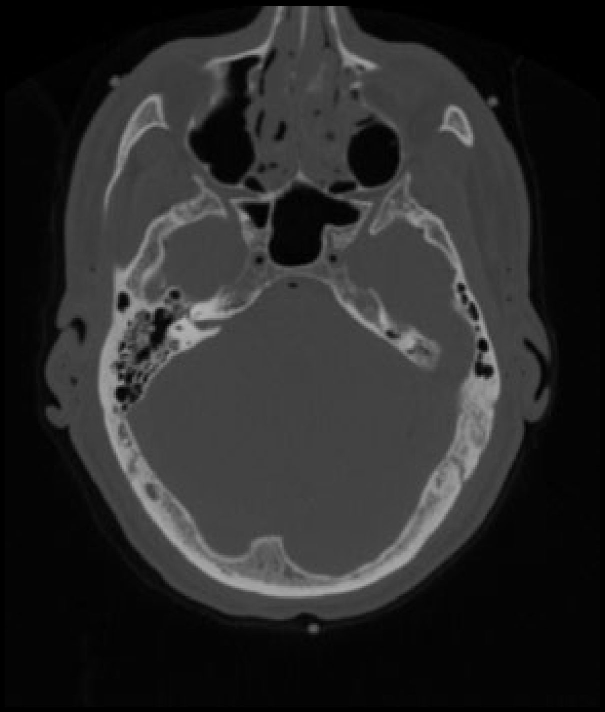
Much more soft-tissue than trabecular bone



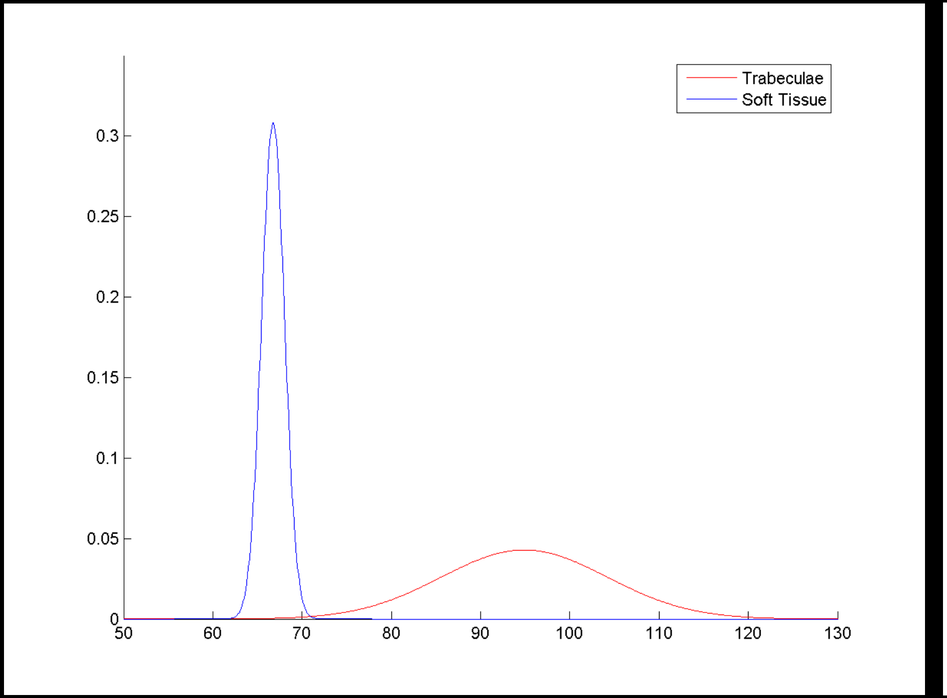
How do we handle that?

Bayesian Classification

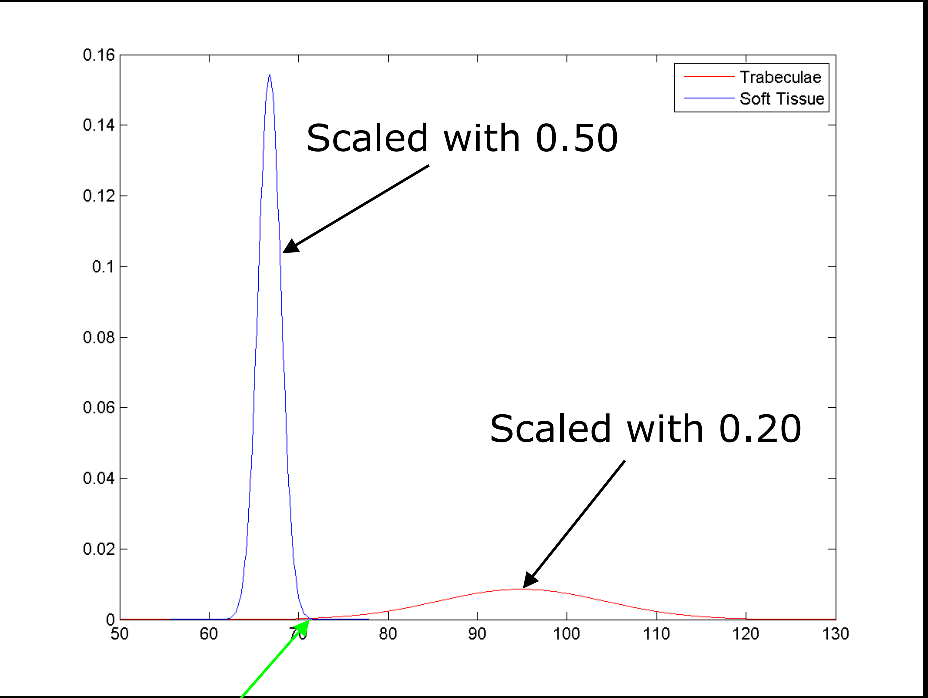
- An expert tells us that a CT scan of a head contains
  - 20% Trabecular bone
  - 50% Soft-tissue
- Picking a random pixel in the image
  - 20% Chance that it is trabecular bone
  - 50% Chance that it is soft-tissue
- How do use that?



Bayesian Classification – histogram scaling



Parametric classifier



Bayesian classifier

Little change in class border

(sometimes significant changes)

Formal definition

- Given a pixel value v
- What is the probability that the pixel belongs to class c_i

Example: If the pixel value is 78, what is the probability that the pixel is bone

$$P(c_i|v) = \frac{P(v|c_i)P(c_i)}{P(v)}$$

Formal definition

Constant – ignored from now on

$$P(c_i|v) = \frac{P(v|c_i)P(c_i)}{P(v)}$$

Formal definition

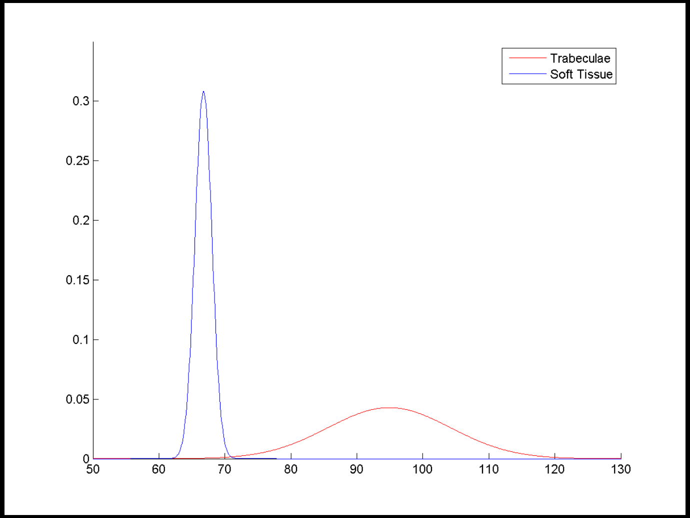
- The *a priori probability* (what is known from before)

> **Example:** From general biology it is known that 20% of a brain CT scan is trabecular bone. Therefore P(trabecular) = 0.20

$$P(c_i|v) = \frac{P(v|c_i)P(c_i)}{P(v)}$$

Formal definition

- The *class conditional probability*
- Given a class, what is the probability of a pixel with value v

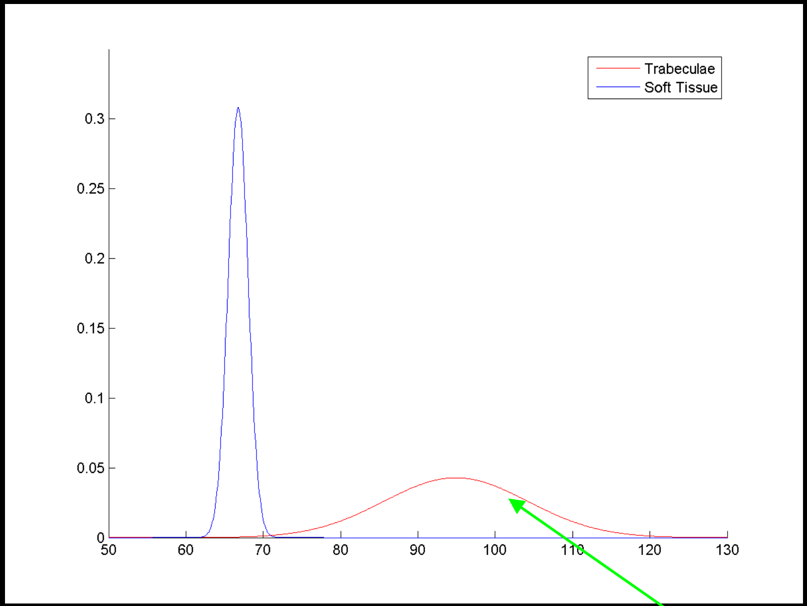


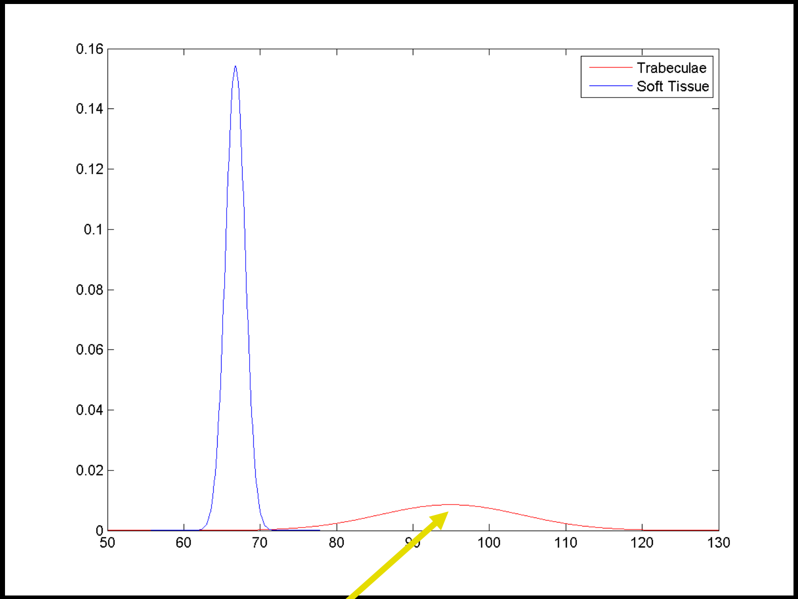
Example: If we consider class = soft-tissue. What is the probability that the pixel value is 78?

Very low

$$P(c_i|v) = \frac{P(v|c_i)P(c_i)}{P(v)}$$

Formal definition – sum up





$$P(c_i|v) = \frac{P(v|c_i)P(c_i)}{P(v)}$$

c_i = trabeculae

Bayesian classification – how to

- Select training pixels for each class
- Fit Gaussians to each class
- Ask an expert for the prior probabilities (how much there normally is in total of each type)
- For each pixel in the image
  - Compute $P(c_i|v)$ for each class (the *a posterior probability*)
  - Select the class with the highest $P(c_i|v)$

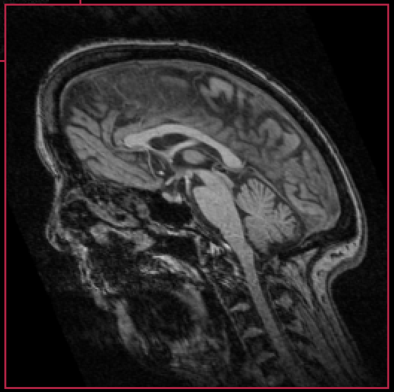
$$P(c_i|v) = \frac{P(v|c_i)P(c_i)}{P(v)}$$

When to use Bayesian classification

- The parametric classifier is good when there are approximately the same amount of all type of tissues
- Use Bayesian classification if there are very little or very much of some types

Next week

- Geometric transformations



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$